Unified parametrization of quark and lepton mixing matrices

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We present a unified parametrization of quark and lepton mixing matrices. By using some simple relations between the mixing angles of quarks and leptons, i.e., the quark-lepton complementarity, we parametrize the lepton mixing matrix with the Wolfenstein parameters λ and A of the quark mixing matrix. It is shown that the Wolfenstein parameter λ can measure both the deviation of the quark mixing matrix from the unit matrix, and the deviation of the lepton mixing matrix from the exactly bimaximal mixing pattern.

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Introduction—To describe the behaviors of quarks and leptons in a grand unified theory (GUT) is one of main goals of particle physics. Among all the characters of quarks and leptons, the mixing between different generations is one of the fundamental problems. Before more underlying theory of the origin of the mixing is found, to parametrize the quark mixing (CKM) matrix [1] and the lepton mixing (PMNS) matrix [2] phenomenologically is the first step to understand this problem. However, these two mixing matrices were parametrized in isolated ways, with the parameters in these two mixing matrices being uncorrelated with each other. The purpose of this paper is to show that one can parametrize the quark and lepton mixing matrices in a unified way by adopting some simple relations between the mixing angles of quarks and leptons

$$\theta_{23} + \theta'_{23}(\theta_{\rm atm}) = \frac{\pi}{4}, \qquad \theta_{31} \sim \theta'_{31}(\theta_{\rm chz}),$$
$$\theta_{12}(\theta_{\rm C}) + \theta'_{12}(\theta_{\rm sol}) = \frac{\pi}{4}, \tag{1}$$

where θ_{ij} and θ'_{ij} (for $i, j = 1, 2, 3$) are the mixing angles of the i and j generations of the CKM matrix and the PMNS matrix (θ_{12} is the Cabibbo mixing angle $\theta_{\rm C}$). These relations, which have been suggested by Raidal [3] as a support of the grand quark-lepton unification or certain quark-lepton symmetry, are in perfect agreement with experimental data (for example, $\theta_{\rm C} = 12.9^\circ$ and $\theta_{\rm sol} = 32.6^\circ$ at the best fit points, and $\theta_{\rm C} + \theta_{\rm sol} = 45.5^\circ$). The third numerical correlation has been pointed out by Smirnov [4], and is called the quark-lepton complementarity (QLC) [5].

From these relations, we can find that the mixing angles of quarks and leptons are not independent of each other. So we can get the trigonometric functions of the mixing angles of leptons in terms of these of quarks, and link the parameters of the PMNS matrix with these of the CKM matrix. Therefore, we can parametrize the PMNS matrix with the parameters of the CKM matrix, and express the CKM and the PMNS matrices in a same framework.

The quark and lepton mixing matrices—Both quark and lepton mixing matrices can be written as

$$\begin{pmatrix} c_{31}c_{12} & c_{31}s_{12} & s_{31}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{31}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{31}s_{12}e^{i\delta} & s_{23}c_{31} \\ s_{23}s_{12} - c_{23}s_{31}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{31}s_{12}e^{i\delta} & c_{23}c_{31} \end{pmatrix}, \tag{2}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ (for $i, j = 1, 2, 3$), and δ is the CP-violating phase. Altogether there are four parameters in the mixing matrix, describing both the real and the imaginary parts of the mixing matrix.

For the CKM matrix V, the best fit values of the three mixing angles are $\theta_{12}(\theta_{\rm C}) = 12.9^\circ$, $\theta_{23} = 2.4^\circ$, and $\theta_{31} = 0.2^\circ$ [6], and we can find that all the three mixing angles are not large. So the CKM matrix is a small deviation from the unit matrix, and it can be parametrized as [7]

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{3}$$

where λ measures the strength of the deviation of V from the unit matrix ($\lambda = \sin\theta_{\rm C} = 0.2243 \pm 0.0016$), and A, ρ and η are the other three parameters, with the best fit values $A = 0.82$, $\rho = 0.20$ and $\eta = 0.33$ [6].

However, for the PMNS matrix U, the situation is quite different from the CKM matrix. With the help of the experimental data from KamLAND [8], SNO [9], K2K [10], Super-Kamiokande [11] and CHOOZ [12] experiments, we know that the mixing angles of leptons are not as small as those of quarks [3],

$$\sin^2 2\theta_{\rm atm} = 1.00 \pm 0.05, \qquad \sin^2 2\theta_{\rm chz} = 0 \pm 0.065,$$
$$\tan^2\theta_{\rm sol} = 0.41 \pm 0.05, \tag{4}$$

where $\theta_{\rm atm}$, $\theta_{\rm chz}$, and $\theta_{\rm sol}$ are the mixing angles of atmospheric, CHOOZ and solar neutrino oscillations, and we have $\theta_{\rm atm} = 45.0^\circ \pm 6.5^\circ$, $\theta_{\rm chz} = 0^\circ \pm 7.4^\circ$ and $\theta_{\rm sol} = 32.6^\circ \pm 1.6^\circ$. So the numerical relations in Eq. (1) are satisfied to a good degree of accuracy.

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Therefore, we can get the PMNS matrix and find that almost all the nondiagonal elements of the PMNS matrix are large. According to the results of the global analysis of the neutrino oscillation experimental data, the elements of the modulus of the PMNS matrix are summarized as [13]

$$|U| = \begin{pmatrix} 0.77-0.88 & 0.47-0.61 & <0.20 \\ 0.19-0.52 & 0.42-0.73 & 0.58-0.82 \\ 0.20-0.53 & 0.44-0.74 & 0.56-0.81 \end{pmatrix}. \quad (5)$$

We can see from Eq. (5) that the PMNS matrix deviates from the unit matrix significantly, but it is quite near the bimaximal mixing pattern, which reads

$$\begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -1/2 & 1/2 & \sqrt{2}/2 \\ 1/2 & -1/2 & \sqrt{2}/2 \end{pmatrix}. \quad (6)$$

So, for the parametrization of the PMNS matrix, it is unpractical to imitate the Wolfenstein parametrization of the CKM matrix indiscriminately. The parametrizations of the PMNS matrix on the basis of the bimaximal mixing pattern have been discussed by Rodejohann [14] and us [15]. However, these parametrizations only concern about the experimental data of leptons, without taking into account their relations with quarks. Thus, the CKM and the PMNS matrices are parametrized irrelevantly, and the parameters in them are not correlated with each other. However, with the relations in Eq. (1), we can parametrize the quark and lepton mixing matrices with correlated parameters.

Parametrization of the PMNS matrix—In Wolfenstein parametrization of the CKM matrix, we have (to the order of λ^3)

$$\sin\theta_{12} = \lambda, \quad \cos\theta_{12} = 1 - \frac{1}{2}\lambda^2, \quad \sin\theta_{23} = A\lambda^2, \quad \cos\theta_{23} = 1, \quad \sin\theta_{31}e^{-i\delta} = A\lambda^3(\rho - i\eta), \quad \cos\theta_{31} = 1. \quad (7)$$

For the case of leptons, using Eq. (1), we have (to the order of λ^3)

$$\begin{aligned} \sin\theta'_{12} &= \sin\left(\frac{\pi}{4} - \theta_{12}\right) = \frac{\sqrt{2}}{2}\left(1 - \lambda - \frac{1}{2}\lambda^2\right), \\ \cos\theta'_{12} &= \frac{\sqrt{2}}{2}\left(1 + \lambda - \frac{1}{2}\lambda^2\right), \\ \sin\theta'_{23} &= \frac{\sqrt{2}}{2}(1 - A\lambda^2), \qquad \cos\theta'_{23} = \frac{\sqrt{2}}{2}(1 + A\lambda^2), \\ \sin\theta'_{31}e^{-i\delta'} &= A\lambda^3(\zeta - i\xi), \qquad \cos\theta'_{31} = 1, \end{aligned} \quad (8)$$

where A and λ are just the Wolfenstein parameters of the CKM matrix. So the CKM and the PMNS matrices have only one set of parameters in this unified parametrization. Because there are totally four angles in the mixing matrix (three mixing angles and one *CP*-violating phase angle), and only two precise numerical relations are known (Eq. (1)), we have to introduce another two new parameters ζ and ξ to describe the PMNS matrix fully.

In Eq. (8), we set $\sin\theta'_{31}e^{-i\delta'} = A\lambda^3(\zeta - i\xi)$. Because of the inaccurate experimental data of neutrino oscillations, we have not fixed the value of $|U_{e3}|$, and only known its upper bound [13]. Therefore, we may also set $\sin\theta'_{31}e^{-i\delta'} = A\lambda^2(\zeta' - i\xi')$. Choosing which of them is to be determined by the future experimental data, and we discuss these two cases here, respectively.

Case 1: $\sin\theta'_{31}e^{-i\delta'} = A\lambda^3(\zeta - i\xi)$.

Substituting Eq. (8) into Eq. (2), we can get the PMNS matrix as

$$\begin{aligned} U &= \begin{pmatrix} \frac{\sqrt{2}}{2}\left(1 + \lambda - \frac{1}{2}\lambda^2\right) & \frac{\sqrt{2}}{2}\left(1 - \lambda - \frac{1}{2}\lambda^2\right) & A\lambda^3(\zeta - i\xi) \\ -\frac{1}{2}\left[1 - \lambda + \left(A - \frac{1}{2}\right)\lambda^2 - A\lambda^3(1 - \zeta - i\xi)\right] & \frac{1}{2}\left[1 + \lambda + \left(A - \frac{1}{2}\right)\lambda^2 + A\lambda^3(1 - \zeta - i\xi)\right] & \frac{\sqrt{2}}{2}(1 - A\lambda^2) \\ \frac{1}{2}\left[1 - \lambda - \left(A + \frac{1}{2}\right)\lambda^2 + A\lambda^3(1 - \zeta - i\xi)\right] & -\frac{1}{2}\left[1 + \lambda - \left(A + \frac{1}{2}\right)\lambda^2 - A\lambda^3(1 - \zeta - i\xi)\right] & \frac{\sqrt{2}}{2}(1 + A\lambda^2) \end{pmatrix} \\ &= \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} + \lambda\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} + \lambda^2\begin{pmatrix} -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & 0 \\ -\frac{1}{2}\left(A - \frac{1}{2}\right) & \frac{1}{2}\left(A - \frac{1}{2}\right) & -\frac{\sqrt{2}}{2}A \\ -\frac{1}{2}\left(A + \frac{1}{2}\right) & \frac{1}{2}\left(A + \frac{1}{2}\right) & \frac{\sqrt{2}}{2}A \end{pmatrix} \\ &\quad + \lambda^3\begin{pmatrix} 0 & 0 & A(\zeta - i\xi) \\ \frac{1}{2}A(1 - \zeta - i\xi) & \frac{1}{2}A(1 - \zeta - i\xi) & 0 \\ \frac{1}{2}A(1 - \zeta - i\xi) & \frac{1}{2}A(1 - \zeta - i\xi) & 0 \end{pmatrix} + \mathcal{O}(\lambda^4). \end{aligned} \quad (9)$$

Now we give some discussion about Eq. (9):

(1). The bimaximal mixing pattern is derived naturally as the leading-order approximation. However, it is chosen as the basis for the expansion of the PMNS matrix by hand before [14,15]. So we can even freely choose other bases for the parametrization of the PMNS matrix (for example, to parametrize the PMNS matrix in the tribimaximal mixing pattern [16]). Now we find that the leading-order term of the PMNS matrix must be the bimaximal mixing pattern as long as we accept the numerical relations in Eq. (1).

(2). The Wolfenstein parameter λ can characterize both the deviation of the CKM matrix from the unit matrix (see Eq. (3)), and the deviation of the PMNS matrix from the exactly bimaximal mixing pattern (see the next-to-leading-order term in Eq. (9)). However, in the previous work [14,15], λ in the PMNS matrix is introduced independently, without considering its relation with the Wolfenstein parameter λ in the CKM matrix. Now, we can see that in this unified parametrization these two different deviations of quarks and leptons are correlated essentially, and can be measured by only one single parameter λ, as Raidal pointed out [3].

(3). Comparing with the parametrizations in bimaxiaml mixing pattern [14,15], we can see that this unified parametrization is equivalent to them to the leading and next-to-leading orders. In [14], the elements of the PMNS matrix are set to be $U_{e2} = \frac{\sqrt{2}}{2}(1-\lambda)$, $U_{e3} = A\lambda^n e^{-i\delta}$, and $U_{\mu 3} = \frac{\sqrt{2}}{2}(1 - B\lambda^m)$. If we let $B \to A$ and fix m to be 2, and n to be 3, we can find that the parametrization in [14] is just the unified parametrization here. Similarly, in [15], $U_{e1} = \frac{\sqrt{2}}{2} + \lambda$, $U_{e3} = b\lambda^2$, and $U_{\mu 3} = \frac{\sqrt{2}}{2} + a\lambda^2$. If we rescale $\lambda \to \frac{\sqrt{2}}{2}\lambda$ and $a \to -\sqrt{2}A$, we can find that the first two terms of the expansion in [15] are just the same as Eq. (9) (in [15], U_{e3} is set to be $b\lambda^2$, not $b\lambda^3$, but this only affects the terms of higher orders). So the parametrizations in [14,15] have been rederived as the natural results in this unified parametrization.

(4). The range of λ in [15] is calculated in detail, $0.08 < \lambda < 0.17$. Now, in this unified parametrization, λ here is just the Wolfenstein parameter of the CKM matrix, $\lambda = \sin\theta_C = 0.2243$. As discussed in (3), if we rescale λ, and divide it by $\sqrt{2}$, we get $\lambda = 0.1586$. We can see that the value of the rescaled λ is just in the range calculated in [15]. So this unified parametrization is reasonable compared with the experimental data.

(5). The values of ρ and η in the CKM matrix have been measured by many experiments [6], and the typical values are $\rho = 0.20$ and $\eta = 0.33$. On the contrary, the inaccuracy of the current experimental data of neutrinos makes it difficult to fix the values of the elements of the PMNS matrix to a very good degree of accuracy. So the values of ζ and ξ have not been determined by now. At present, the best fit point of $\sin^2\theta'_{31}$ is 0.006 [17], so we have $A\lambda^3\sqrt{\zeta^2+\xi^2} \sim 0.077$, and $\sqrt{\zeta^2+\xi^2} \sim 8.2$. Therefore, both ζ and ξ are of $\mathcal{O}(1)$.

Furthermore, ζ and ξ are related with the CP-violating process [18], and the rephasing-invariant measurement of the lepton CP-violation is described by the Jarlskog parameter J [19], $J = \mathrm{Im}(U_{e2}U_{\mu 3}U^*_{e3}U^*_{\mu 2})$. In this unified parametrization, from Eq. (9), J can be expressed in a simple form (to the order of λ^5),

$$J = \frac{1}{4}A\lambda^3\xi(1-2\lambda^2) = 0.0022\xi. \tag{10}$$

We can see from Eq. (10) that J is only related with the parameter ξ. So if we can observe the lepton CP-violating process in the future neutrinoless $\beta\beta$ decay reaction, and can determine the value of J, then the value of ξ can be fixed. And with the more precise experimental data of $|U_{e3}|$, we can determine the value of ζ ultimately. Thus we can get a full understanding of the structure of the PMNS matrix.

Case 2: $\sin\theta'_{31}e^{-i\delta'} = A\lambda^2(\zeta' - i\xi')$.

Repeating the former process, we get

$$\begin{aligned}
U &= \begin{pmatrix}
\frac{\sqrt{2}}{2}\left(1+\lambda-\frac{1}{2}\lambda^2\right) & \frac{\sqrt{2}}{2}\left(1-\lambda-\frac{1}{2}\lambda^2\right) & A\lambda^2(\zeta'-i\xi') \\
-\frac{1}{2}\left\{1-\lambda-\left[\frac{1}{2}-A(1+\zeta'+i\xi')\right]\lambda^2\right\} & \frac{1}{2}\left\{1+\lambda-\left[\frac{1}{2}-A(1-\zeta'-i\xi')\right]\lambda^2\right\} & \frac{\sqrt{2}}{2}(1-A\lambda^2) \\
\frac{1}{2}\left\{1-\lambda-\left[\frac{1}{2}+A(1+\zeta'+i\xi')\right]\lambda^2\right\} & -\frac{1}{2}\left\{1+\lambda-\left[\frac{1}{2}+A(1-\zeta'-i\xi')\right]\lambda^2\right\} & \frac{\sqrt{2}}{2}(1+A\lambda^2)
\end{pmatrix} \\
&= \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} + \lambda \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} + \lambda^2 \begin{pmatrix}
-\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & A(\zeta'-i\xi') \\
\frac{1}{2}\left[\frac{1}{2}-A(1+\zeta'+i\xi')\right] & -\frac{1}{2}\left[\frac{1}{2}-A(1-\zeta'-i\xi')\right] & -\frac{\sqrt{2}}{2}A \\
-\frac{1}{2}\left[\frac{1}{2}+A(1+\zeta'+i\xi')\right] & \frac{1}{2}\left[\frac{1}{2}+A(1-\zeta'-i\xi')\right] & \frac{\sqrt{2}}{2}A
\end{pmatrix} + \mathcal{O}(\lambda^3).
\end{aligned} \tag{11}$$

Similar to *Case 1*, we can see that:

(1). The bimaximal mixing pattern is derived as the leading-order term naturally.

(2). The deviation of the CKM matrix from the unit matrix, and the deviation of the PMNS matrix from the exactly bimaximal mixing pattern can be characterized by only one parameter λ.

(3). Parametrizations in [14,15] can be transformed into this unified parametrization, if we let $B \to A$ and fix m and n to be 2 in [14], and rescale $\lambda \to \frac{\sqrt{2}}{2}\lambda$ and $a \to -\sqrt{2}A$ in [15].

(4). The expressions of the leading-order and next-to-leading-order terms in *Case 2* are the same as those in *Case 1*, because the difference between them is caused by the introductions of U_{e3} at the second and the third orders. So the expressions of the first two orders must be the same in these two cases. Also, λ in *Case 2* is still consistent with the range $0.08 < \lambda < 0.17$ after rescaling.

(5). The Jarlskog parameter J can be expressed now in the form

$$J = \frac{1}{4} A\lambda^2 \xi'(1 - 2\lambda^2) = 0.0099\xi'. \tag{12}$$

Similarly, we can fix the value of ξ' by observing the lepton CP-violating process, and then can determine the value of ζ'. Now $\sqrt{\zeta'^2 + \xi'^2} \sim 1.8$, and ζ' and ξ' are still of $\mathcal{O}(1)$. Of course, ζ' and ξ' in *Case 2* are not the ζ and ξ in *Case 1*, and they are equivalent to the ζ and ξ in *Case 1* by rescalings $\zeta' \to \lambda\zeta$ and $\xi' \to \lambda\xi$.

Then we can see the merits of these two cases. If $\sin^2\theta'_{31} \sim 0.006$ as a preliminary estimate shows, then *Case 2* is preferable, because ζ' and ξ' in *Case 2* are more close to 1 in magnitude. However, if $\sin^2\theta'_{31} \sim 0.0001$ or less, then *Case 1* is to be preferred.

Conclusions—We present a unified parametrization of the quark and lepton mixing matrices, which is based on the simple relations between the mixing angles of quarks and leptons. Although the physical explanation of these relations remains to be explored, we believe that there must be some deeper principle behind these elegant correlations, which are in perfect agreement with the current experimental data.

If the numerical relations in Eq. (1) violate a little, we can maintain the expressions in Eq. (8), and only need to redefine the parameters λ and A. For example, we can still set $\sin\theta'_{23} = \frac{\sqrt{2}}{2}(1 - A'\lambda'^2)$. Thus, the parameter λ' and A' are not the same as the Wolfenstein parameters λ and A, and the symmetry between the quark and lepton mixing matrices will break slightly. This is a more general parametrization, and can work whether Raidal's numerical relations keep or not. However, A' and λ' in this more general parametrization are still the Wolfenstein-like parameters. The forms of s'_{ij}, c'_{ij} and expansion of the PMNS matrix will still keep invariant, and the leading-order term is still the bimaximal mixing pattern, only with the transition $\lambda \to \lambda'$, and $A \to A'$.

In conclusion, although all sorts of parametrization of the quark and lepton mixing matrices are not based on deep theoretical foundation, and applying any of them may not have specific physical significance, however, it is quite likely that this unified parametrization does have its advantages. For instance, the number of the free parameters in this unified parametrization is fewer than the parametrizations in [14,15], the bimaximal mixing pattern as the leading-order term is derived naturally, and the next-to-leading-order is the same as [14,15] after rescalings. Also, the Wolfenstein parameter λ can measure both the deviation of the CKM matrix from the unit matrix, and the deviation of the PMNS matrix from the bimaximal mixing pattern. So if this unified parametrization is tested to be consistent with more precise experimental data in the future, we can get a comprehensive understanding of the mixings of quarks and leptons, and push forward the exploration of grand unification.

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