

**Asymmetry and minimality of quark mass matrices**

Nobuhiro Uekusa, Atsushi Watanabe, and Koichi Yoshioka  
*Department of Physics, Kyushu University, Fukuoka 812-8581, Japan*  
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We systematically investigate general forms of mass matrices for three-generation up and down quarks, including asymmetrical ones in generation space. Viable zero matrix elements are explored which are compatible with the current observation of masses and mixing angles, and also with the recent measurement of  $CP$  violation in the  $B$ -meson system. The simplest form with the maximal number of vanishing matrix elements is found to be almost consistent with the experimental data, but has an unsatisfactory point that one of the mixing angles is slightly large. At the next-to-minimal level, it is found with help of leptonic generation mixing that only six patterns of mass matrices well describe the experimental data. These sets of mass textures predict all the properties of quarks, including the  $CP$  violation, as well as the large (charged) lepton mixing, which may be appropriate for the atmospheric neutrino in the grand unification scheme.

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**I. INTRODUCTION**

It is certain that one of the most important issues confronting the standard model is the generation structure of quarks and leptons. Various neutrino oscillation experiments have recently been bringing out the generation structure of the leptonic sector. The Super-Kamiokande experiment has established the neutrino oscillation in the atmospheric neutrinos with nearly maximal mixture [1]. As for the solar neutrino problem, the Mikheyev-Smirnov-Wolfenstein solution [2] is strongly suggested by the recent experimental results [3] if there exists the neutrino flavor mixing between the first and second generations [4]. On the other hand, the other mixing (the 1-3 mixing) has been found to be rather small [5] similarly to the quark sector. While these experimental progresses have been giving us a new perspective beyond the standard model, it seems that we are far from a fundamental understanding of the origin of fermion masses and mixing angles.

A promising approach to the issue of the generations is to assume that some of the Yukawa matrix elements are vanishing. An immediate and important consequence of this approach is to reduce the number of free parameters in the theory and to lead to relations among the fermion masses and mixing angles. Moreover, that would provide a clue to find symmetry principles or dynamical mechanisms behind the Yukawa sectors, which are interpreted as remnants of fundamental theory in the high-energy regime. Most of the previous work along this direction, including the systematic analysis by Ramond, Roberts, and Ross [6], assumed that the matrices of Yukawa couplings are symmetric about the generation indices (Hermitian matrices).<sup>1</sup> However, in the context of the standard model and even in grand unified theory, it is not necessarily required that

Yukawa matrices are symmetric. In fact, the present experimental data indicate that the (large) leptonic mixing mentioned above is quite different from the (small) quark mixing. It is interesting that this asymmetrical observation is known to be compatible with quark-lepton grand unification if the fermion mass matrices take asymmetrical forms in generation space [8]. There are also some classes of non-Hermitian ansatz for quark mass matrices [9], which are consistent with the data and cannot be transformed to the solutions obtained in Ref. [6]. It is therefore worthwhile to do systematical examination and to complete the classification of viable asymmetric forms of fermion mass matrices.

In this paper, we investigate phenomenologically viable mass matrices of up and down quarks, assuming that the Yukawa couplings generally take asymmetric forms in generation space. In particular, we look for as simple forms as possible, that is, mass matrices with the maximal number of vanishing elements. Vanishing matrix elements are expected to be deeply connected with underlying physics, such as flavor symmetries, in more fundamental theory to shed some light on constructing realistic models of quarks and leptons. Note that our treatment is general and includes symmetric mass matrices as limited cases.

This paper is organized as follows. In Sec. II, we describe the Yukawa sectors of up and down quarks in the standard model and introduce the parametrization needed in later discussion. Our analysis does not depend on any details of the Higgs field profile, and therefore can be straightforwardly applied to other cases such as grand unified theory and supersymmetric models. Sections III and IV are devoted to analyzing which forms of matrices (vanishing matrix elements) are compatible with the current experimental data. It is found that the minimal (Sec. III) and next-to-minimal (Sec. IV) cases contain only a few types of mass matrices phenomenologically viable. We summarize our results in Sec. V.

<sup>1</sup>Systematic studies and classifications of fermion mass matrices and mixing have also been performed in other approaches [7].

## II. FORMULATION

In this section, we briefly review the formulation of the up- and down-quark Yukawa sectors in the standard model. The  $SU(3) \times SU(2) \times U(1)$  gauge-invariant Yukawa interactions are given by

$$-\mathcal{L}_Y = \bar{Q}_i(Y_u)_{ij}u_{Rj}H^* + \bar{Q}_i(Y_d)_{ij}d_{Rj}H + \text{H.c.}, \quad (2.1)$$

where  $Q_i$  denote the  $SU(2)$  doublets of left-handed quarks, and  $u_R, d_R$  are the right-handed up- and down-type quarks, respectively. The Yukawa couplings  $Y_u$  and  $Y_d$  are  $3 \times 3$  matrices ( $i, j$ , the generation indices), and  $H$  is the  $SU(2)$  doublet Higgs field. After the electroweak symmetry breaking, these Yukawa interactions lead to the following quark mass terms:

$$-\mathcal{L}_m = \bar{u}_{Li}(M_u)_{ij}u_{Rj} + \bar{d}_{Li}(M_d)_{ij}d_{Rj} + \text{H.c.}, \quad (2.2)$$

$$(M_u)_{ij} = (Y_u)_{ij}v, \quad (M_d)_{ij} = (Y_d)_{ij}v,$$

where  $v$  is a vacuum expectation value of the neutral component of the Higgs field  $H$ . If the model is supersymmetrized, the Yukawa terms are described by superpotential in terms of quark and Higgs superfields. The only difference is that in the supersymmetric case two types of Higgses must be introduced to have gauge-invariant Yukawa terms. This procedure does not bring any modification to the structure of Yukawa couplings  $Y_{u,d}$ , and therefore the following analysis is straightforwardly extended to supersymmetric models and also to other scenarios.

The generation mixing is physically described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix which consists of two unitary matrices,

$$V_{\text{CKM}} = V_{uL}^\dagger V_{dL}. \quad (2.3)$$

These unitary matrices diagonalize the mass matrices  $M_u$  and  $M_d$ :

$$M_u = V_{uL}M_u^D V_{uR}^\dagger, \quad M_d = V_{dL}M_d^D V_{dR}^\dagger. \quad (2.4)$$

The diagonal elements of  $M_u^D$  and  $M_d^D$  correspond to the experimentally observed mass eigenvalues.

With phase degrees of freedom of the six quark fields, the number of observable parameters in the CKM matrix is reduced to four (the overall phase rotation is physically irrelevant). However, it is important to distinguish the contributions of  $V_{uL}$  and  $V_{dL}$  from a viewpoint of pursuing clues to find more fundamental theory of quarks and leptons such as grand unification and flavor symmetry. A generic  $3 \times 3$  unitary matrix  $U$  has 9 free parameters and can be parametrized as

$$U = \Phi O_1 \Phi' O_2 O_3 \Phi''. \quad (2.5)$$

The matrices  $O_i$  ( $i = 1, 2, 3$ ) represent the rotations in the index space around the  $i$ th axis,

$$O_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{pmatrix},$$

$$O_2 = \begin{pmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix}, \quad (2.6)$$

$$O_3 = \begin{pmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The diagonal phase matrices  $\Phi$ 's are given by  $\Phi = \text{diag.}(e^{i\phi}, e^{i\varphi}, 1)$ ,  $\Phi' = \text{diag.}(1, 1, e^{i\omega})$ , and  $\Phi'' = \text{diag.}(e^{ip}, e^{iq}, e^{ir})$ . There generally exist three rotation angles and six complex phases. When applied to the above quark mixing matrices  $V_{uL}$  and  $V_{dL}$ , the phase factors in  $\Phi''$  are always unphysical degrees of freedom since they can be absorbed by field redefinitions of the quark mass eigenstates. It will also be found that phase matrices  $\Phi$ 's do not appear throughout this work (except for only a few examples discussed at the beginning of Sec. IV) because we will consider  $3 \times 3$  matrices with nonvanishing determinants and at most five independent elements. In this case, a matrix  $M$  is always expressed such that  $M = JM_r J'$  where  $J$  and  $J'$  are the diagonal phase matrices and  $M_r$  contains only real parameters. Thus the matrices  $MM^\dagger$  and  $M^\dagger M$  are diagonalized by real orthogonal matrices, up to overall phase rotations corresponding to  $\Phi$  or  $\Phi''$  in (2.5). On the other hand, the phase factors in  $\Phi$  of the up and down sectors generally contribute to the CKM matrix elements. Thus the CKM mixing matrix is found to be written as

$$V_{\text{CKM}} = O_u^T P O_d, \quad P = \Phi_u^* \Phi_d, \quad O_i = O_{1i} O_{2i} O_{3i}, \quad (2.7)$$

where the subscripts  $i = u, d$  label the up- and down-type quarks, respectively. As mentioned above, two complex phases in  $P$  play an important role for reproducing  $CP$ -violating quantities since their changes generically affect the CKM matrix elements. The numerical results of such phase factors will be discussed in later sections.

The experimentally observable quantities in the quark Yukawa sector are 3 mixing angles with 1 complex phase in the CKM matrix and 6 mass eigenvalues. The measured values of the CKM matrix elements are [10]

$$|V_{\text{CKM}}| = \left| \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \right| = \begin{pmatrix} 0.9739-0.9751 & 0.221-0.227 & 0.0029-0.0045 \\ 0.221-0.227 & 0.9730-0.9744 & 0.039-0.044 \\ 0.0048-0.014 & 0.037-0.043 & 0.9990-0.9992 \end{pmatrix}. \quad (2.8)$$

The various experimental observations of  $CP$ -violating phenomena yield the  $CP$  violation in the standard model, which is translated to [10]

$$J_{CP} = (2.88 \pm 0.33) \times 10^{-5}, \quad (2.9)$$

where  $J_{CP}$  denotes the reparametrization-invariant measure of  $CP$  violation [11]. This value corresponds to the Kobayashi-Maskawa (KM) phase in the standard parametrization as  $\delta_{KM} = 60^\circ \pm 14^\circ$ . Moreover the recent results of studying the decay of the  $B$  mesons to charmoniums indicate [12]

$$\sin 2\phi_1/\beta = 0.726 \pm 0.037, \quad (2.10)$$

where  $\phi_1 \equiv \beta$  is one of the angles of the unitary triangle for the  $B$ -meson system, which is defined as  $\phi_1 = \beta \equiv \arg(V_{cd}^* V_{cb}/V_{td}^* V_{tb})$ . This is the angle that is most precisely known at present and is expected to provide the most stringent constraint. The current-quark masses at the  $Z$ -boson mass scale are evaluated [13] including various effects such as the QCD strong coupling factors and we obtain

$$\begin{aligned} m_u &= 0.000975\text{--}0.00260, & m_d &= 0.00260\text{--}0.00520, \\ m_c &= 0.598\text{--}0.702, & m_s &= 0.0520\text{--}0.0845, \\ m_t &= 170\text{--}180, & m_b &= 2.83\text{--}3.04, \end{aligned} \quad (2.11)$$

in GeV units. The most recent results of the Tevatron CDF and D0 experiments indicate the top-quark mass which is a bit larger than that quoted above [14]. However, if taken into account, the analysis of mass matrix forms presented in this paper is not significantly changed, since the most influential ingredients are practically the masses of lighter generations. In the following analysis, we use these experimental data as input parameters and explore possible forms of quark mass matrices.

### III. THE MINIMAL ASYMMETRIC MATRICES

We would like to systematically search for the mass matrices of up and down quarks which are consistent with the current experimental data. Our analysis is based on possible zero elements in the mass matrices. Namely, the aim of this paper is to investigate how a small number of nonvanishing matrix elements can account for the existing data. Here the number of zeros means independently vanishing elements in a mass matrix. In particular, for a symmetric matrix, “1 zero” implies that a diagonal element or a pair of off-diagonal elements in symmetric positions takes a negligibly small value.

Let us first consider the mass matrix of up-type quarks. In the present work we assume that the up-quark mass

matrix  $M_u$  is symmetric. This assumption is motivated by grand unified theory, where the left- and right-handed up quarks in one generation often belong to the same multiplet of unified gauge symmetry, like  $SU(5)$  and larger. In this case one obtains the identical mixing matrix for left- and right-handed up quarks;  $V_{uL} = V_{uR} \equiv V_u$ .

It is first noticed that three independent, nonvanishing matrix elements are needed to reproduce the observed mass eigenvalues of the three-generation quarks. Moreover a determinant of the mass matrix must be nonzero. The minimal forms of matrices which satisfy these criterions are found to coexist with at most three zeros, and the independent matrices are given by the following three types:

(i) Mu1

$$M_u = \begin{pmatrix} a & & \\ & b & \\ & & c \end{pmatrix}, \quad (3.1)$$

(ii) Mu2

$$M_u = \begin{pmatrix} a & & \\ & b & \\ & & b \end{pmatrix}, \quad (3.2)$$

(iii) Mu3

$$M_u = \begin{pmatrix} & a & \\ a & & b \\ & b & c \end{pmatrix}. \quad (3.3)$$

The matrix elements  $a$ ,  $b$ ,  $c$  are nonzero and the blanks denote vanishing entries. All other forms of matrices consistent with the criterions can be obtained by relabeling the generation indices. For example, by exchanging the first and second generations (both for the left- and right-handed fermions), the matrix Mu2 is converted to the form discussed in [15],

$$M_u = \begin{pmatrix} & & b \\ & a & \\ b & & c \end{pmatrix}.$$

It should be noted that we do not assume any hierarchical orders among the nonvanishing elements. Therefore the analysis of the above three types of matrices (Mu1, Mu2, and Mu3) includes the whole possibility of a symmetric matrix with three zeros. The matrix  $M_u$  is diagonalized as

$$M_u = V_u \begin{pmatrix} m_u & & \\ & -m_c & \\ & & m_t \end{pmatrix} V_u^\dagger. \quad (3.4)$$

The negative sign in front of  $m_c$  is just a convention introduced in order that the mass eigenvalues and the parameter  $c$  are real and positive. With a suitable phase

redefinition of the up-type quarks, we take the nonvanishing matrix elements to be real parameters without loss of generality. These values can be fixed by the three mass eigenvalues from the following three equations:

$$\text{tr } M_u = m_u - m_c + m_t, \quad (3.5)$$

$$\text{tr } M_u^2 = m_u^2 + m_c^2 + m_t^2, \quad (3.6)$$

$$\det M_u = -m_u m_c m_t. \quad (3.7)$$

Thus a unitary matrix  $V_u$  which diagonalizes a three-zero symmetric  $M_u$  is described in terms of the up-quark mass eigenvalues.

For the down sector, the mass matrix is not necessarily symmetric. The minimal criterion for a realistic mass matrix is the same as for symmetric matrices; three independent, nonvanishing matrix elements, and a nonvanishing determinant. To satisfy these requirements, we have at most six zero elements. A matrix with six zeros has only three parameters which correspond to three mass eigenvalues. Such a matrix gives no mixing angle or exchanging generation indices. However since any type of up-quark mass matrices with three zeros cannot be diagonalized with the observed CKM matrix, a matrix with asymmetric six zeros is not suitable for the down sector. We thus find that the most economical candidates for a realistic mass matrix of down-type quarks have five zeros. They can generically describe three eigenvalues and one mixing angle. It is found that there are 36 types of mass matrices with five zeros and nonvanishing determinants. At this stage, since we are not requiring any hierarchy among matrix elements as in the case of up-quark mass matrices, these 36 ( $= 6 \times 6$ ) patterns are related to each other through the permutations of three rows and/or three columns. Namely, one can obtain all the patterns by exchanging the generation labels from a single matrix, e.g.,

$$M_d = \begin{pmatrix} d & & \\ & e & \\ & g & f \end{pmatrix}. \quad (3.8)$$

The matrix elements  $d, \dots, g$  are made real-valued by phase redefinitions of quark fields. In our convention, a permutation of columns corresponds to a rotation of generation indices of the right-handed quarks, and does not change the mass spectrum and the CKM matrix elements. On the other hand, the exchanges of rows, i.e. relabeling three left-handed down quarks, do affect the observable mixing angles. This is because we have already used the label exchange degrees of freedom to reduce the number of matrix patterns for the up-type quarks. Therefore all possible permutations of rows must be taken into account in the down sector to explore the whole combinations of up- and down-quark mass matrices. We thus consider the

following 6 types of mass matrices as the minimal candidates with five zeros:

(i) Md1

$$M_d = \begin{pmatrix} d & & \\ & e & \\ & g & f \end{pmatrix}, \quad (3.9)$$

(ii) Md2

$$M_d = \begin{pmatrix} d & & \\ & e & g \\ & & f \end{pmatrix}, \quad (3.10)$$

(iii) Md3

$$M_d = \begin{pmatrix} d & & \\ g & e & \\ & & f \end{pmatrix}, \quad (3.11)$$

(iv) Md4

$$M_d = \begin{pmatrix} d & g & \\ & e & \\ & & f \end{pmatrix}, \quad (3.12)$$

(v) Md5

$$M_d = \begin{pmatrix} d & & \\ & e & \\ g & & f \end{pmatrix}, \quad (3.13)$$

(vi) Md6

$$M_d = \begin{pmatrix} d & g & \\ & e & \\ & & f \end{pmatrix}. \quad (3.14)$$

As expected, these 6 patterns are transformed into each other by changing the generation indices of the left-handed quarks, up to permutations of the right-handed ones.

Given the possible forms of mass matrices, Mu1–Mu3 and Md1–Md6, we analyze which combinations of mass matrices explain the observed masses and mixing angles. It is easily found that the matrices Mu1 and Mu2 cannot fit the data. This is because they have zero or one finite mixing in the up sector and all the candidates of  $M_d$  can induce only one-generation mixing, that necessarily results in the CKM matrix with more than one vanishing entry. The only remaining possibility is the matrix Mu3 for the up sector. The matrix which diagonalizes Mu3 is approximately written by the mass eigenvalues

$$|O_u| \sim \begin{pmatrix} 1 & \sqrt{\frac{m_u}{m_c}} & \frac{m_c}{m_t} \sqrt{\frac{m_u}{m_t}} \\ \sqrt{\frac{m_u}{m_c}} & 1 & \sqrt{\frac{m_c}{m_t}} \\ \sqrt{\frac{m_u}{m_t}} & \sqrt{\frac{m_c}{m_t}} & 1 \end{pmatrix}. \quad (3.15)$$

This shows that the 1-2 mixing angle from the up sector is roughly given by  $\sqrt{m_u/m_c} = 0.037\text{--}0.066$ . Therefore in order to generate the observed Cabibbo angle, a 1-2 mixing angle from the down sector is required to be of order  $\mathcal{O}(10^{-1})$ , which selects out Md3 or Md4 for an appropriate matrix for the down quarks. The matrices Md3 and Md4 are diagonalized by rotations of the first and second generations and do not affect the third column of  $V_{\text{CKM}}$ . Consequently the combinations (Mu3, Md3) and (Mu3, Md4) predict the CKM angles involving the third generation,

$$|V_{ub}| \simeq \sqrt{\frac{m_u}{m_t}} = 0.00233\text{--}0.00391, \quad (3.16)$$

$$|V_{td}| \simeq \left( |V_{us}| - \sqrt{\frac{m_u}{m_c}} \right) \sqrt{\frac{m_c}{m_t}} = 0.00894\text{--}0.0122, \quad (3.17)$$

$$|V_{cb}| \simeq |V_{ts}| \simeq \sqrt{\frac{m_c}{m_t}} = 0.0576\text{--}0.0643. \quad (3.18)$$

It is found that the first two predictions well agree with the experimental values (2.8) but the mixing between the

second and third generations is slightly larger than the observation.

We have found in this section that the simplest and realistic forms of quark mass matrices can accommodate symmetric three zeros and asymmetric five zeros in the up and down sectors, respectively. Their explicit forms are highly constrained by the experimental values of mass eigenvalues and mixing angles, and there exists only two possibilities which are given by

$$M_u = \begin{pmatrix} a & & \\ a & b & \\ & b & c \end{pmatrix}, \quad M_d = \begin{pmatrix} d & g & \\ & e & \\ & & f \end{pmatrix}, \quad (3.19)$$

and

$$M_u = \begin{pmatrix} a & & \\ a & b & \\ & b & c \end{pmatrix}, \quad M_d = \begin{pmatrix} d & & \\ g & e & \\ & & f \end{pmatrix}. \quad (3.20)$$

It is clear that a simultaneous exchange of the identical generation indices for  $u_L$ ,  $u_R$ , and  $d_L$  completely preserves the physical consequences. Moreover, as noted in the classification, any permutation of the right-handed down quarks (i.e. of the columns of  $M_d$ ) is also allowed phenomenologically. Noticing this fact, one can see, for instance, that the matrix  $M_d$  in (3.20) reconciles the Georgi-Jarlskog ansatz [16], up to unphysical field rotations, and could easily be extended to include the charged-lepton mass matrix. A numerical evaluation for the combination (3.19) presents us an example,

$$M_u = \begin{pmatrix} 0 & 0.000221(\lambda^{5.56}) & 0 \\ 0.000221(\lambda^{5.56}) & 0 & 0.0578(\lambda^{1.88}) \\ 0 & 0.0578(\lambda^{1.88}) & 0.997(\lambda^{0.00}) \end{pmatrix} m_t, \quad (3.21)$$

$$M_d = \begin{pmatrix} 0.00156(\lambda^{4.27}) & 0.00518(\lambda^{3.48}) & 0 \\ 0 & -0.0285(\lambda^{2.35}) & 0 \\ 0 & 0 & 1.00(\lambda^{0.00}) \end{pmatrix} m_b, \quad (3.22)$$

where the couplings are chosen so that we can fit as many observables as possible to the experimental data. A nontrivial phase factor is also required, e.g.  $P = \text{diag.}(e^{-0.7\pi i}, 1, 1)$ , in order to reproduce the observed value of  $CP$  violation (2.9). In this case, however, we find that alternative indication of  $CP$  violation (2.10) cannot be reproduced. The unitary triangle for the  $B$ -meson system is distorted due to a large value of  $|V_{cb}|$ , while the area of the triangle is correct. A similar result is obtained for the combination (3.20) since physical consequences are now determined modulo right-handed mixing of down quarks. In the above example, we have alternatively written down in the parentheses the exponents of a small parameter  $\lambda$  ( $= 0.22$ ) for nonvanishing elements in  $M_u$  and  $M_d$ . Such expressions in terms of an

expansion parameter would be suitable to gain an insight into the fermion masses problem in view of flavor symmetries.

Though there is one unsatisfied point for the above two combinations of mass matrices, i.e. a slightly large value of the 2-3 CKM mixing angle ( $|V_{cb}| \simeq \sqrt{m_c/m_t}$ ), one can easily find some remedies. The discrepancy may be removed with radiative corrections, for instance, the renormalization-group effects between the electroweak and high-energy scales. A probable source of such effects is a flexibility of the top-quark Yukawa coupling in the high-energy regime due to its fixed-point behavior in the infrared. To see how the renormalization-group evolution makes the situation better, let us first define the following ratio:

$$\Delta \equiv \frac{\sqrt{m_c/m_t}}{|V_{cb}|}. \quad (3.23)$$

The problem with the mass matrices (3.19) and (3.20) is that their prediction  $\Delta_{\text{pre}} \approx 1$  is not consistent with the experimental results  $\Delta_{\text{exp}} > 1.31$  at the electroweak scale. Within a good approximation that the third-generation Yukawa couplings are dominant, the evolution of the mass ratio  $m_c/m_t$  and the mixing angle  $V_{cb}$  is governed by the equations

$$\frac{d \ln \left( \frac{m_c}{m_t} \right)}{dt} = \frac{-1}{16\pi^2} [\zeta (Y_{u33})^2 + \eta (Y_{d33})^2], \quad (3.24)$$

$$\frac{d \ln |V_{cb}|}{dt} = \frac{-\eta}{16\pi^2} [(Y_{u33})^2 + (Y_{d33})^2], \quad (3.25)$$

where  $t = \ln \mu$  denotes the renormalization scale. The direct contribution from gauge couplings is generally irrelevant to the running of mass ratios and mixing angles. The coefficients  $\zeta$  and  $\eta$  are model-dependent constants, for example,  $\zeta = 3/2$  and  $\eta = -3/2$  in the standard model, and  $\zeta = 3$  and  $\eta = 1$  in supersymmetric standard models (though the  $Y_{d33}$  effect is negligible in the standard model). Thus we obtain

$$\frac{d \ln \Delta}{dt} = \frac{1}{32\pi^2} [(2\eta - \zeta)(Y_{u33})^2 + \eta(Y_{d33})^2]. \quad (3.26)$$

It is noted that the coefficient in front of  $(Y_{u33})^2$  is negative in usual Higgs doublet models. This negative sign suggests that the ratio  $\Delta$  in the high-energy regime is reduced from the value observed at the electroweak scale. Therefore the renormalization-group evolution in fact ameliorates the problem with the mass textures (3.19) and (3.20). It is also noticed that, since the second term on the right-handed side of (3.26) is positive for supersymmetric cases, a smaller value of  $Y_{d33}$  is preferred to cure the mismatch between  $\Delta_{\text{pre}}$  and  $\Delta_{\text{exp}}$ . In the case that only the top Yukawa coupling is dominant, we obtain by integrating (3.26) over the range between  $\mu = 1$  TeV and the unification scale  $\Lambda$ ,

$$\begin{aligned} \frac{\Delta(\Lambda)}{\Delta(\mu)} &= \left( \frac{Y_{u33}(\Lambda)}{Y_{u33}(\mu)} \right)^{(-1)/12} \left( \frac{g_1(\Lambda)}{g_1(\mu)} \right)^{(-13)/1188} \left( \frac{g_2(\Lambda)}{g_2(\mu)} \right)^{(-1)/4} \\ &\times \left( \frac{g_3(\Lambda)}{g_3(\mu)} \right)^{4/27} \approx 0.91 \left( \frac{Y_{u33}(\Lambda)}{Y_{u33}(\mu)} \right)^{(-1)/12} \end{aligned} \quad (3.27)$$

for the minimal supersymmetric standard model. In the second equation, we have roughly assumed that the three gauge couplings of the standard gauge groups  $g_{1,2,3}$  are unified at  $\Lambda$  in the one-loop order estimation. The situation is more improved for the standard model because of a larger negative value of the coefficient  $2\eta - \zeta$  and the negligible positive contribution from the bottom Yukawa coupling, and one typically obtains

$$\frac{\Delta(\Lambda)}{\Delta(\mu)} \sim 0.59 \left( \frac{Y_{u33}(\Lambda)}{Y_{u33}(\mu)} \right)^{(-1)/2} \quad (3.28)$$

at a high-energy scale  $\Lambda \sim 10^{14}$  GeV. It is clearly seen that  $|V_{cb}|$  and  $\sqrt{m_c/m_t}$  at the high-energy regime become closer than around the electroweak scale, and the texture ansatz (3.19) and (3.20) would work better in high-energy theory such as grand unified models.

On the other hand, a more direct resolution to the problem  $\Delta_{\text{pre}} \neq \Delta_{\text{exp}}$  is to incorporate additional nonvanishing elements into the Yukawa matrices. This is the option we will explore in the next section.

#### IV. THE NEXT-TO-MINIMAL ASYMMETRIC MATRICES

The previous analysis has shown that, at classical level, any combination of symmetric three-zeros  $M_u$  and asymmetric five-zeros  $M_d$  are too simplified to be totally consistent with the observed data. In this section we investigate a possibility to relax the constraints on matrix forms and to introduce one more nonvanishing matrix element.

The first case to consider is to work with symmetric two zeros in the mass matrices of up quarks. An interesting observation is that, when the 2-2 element in  $M_u$  is turned on, the five-zeros matrices  $M_d$  in (3.19) and (3.20) may completely explain the data. This is because the mixing angle  $V_{cb}$  could be controlled by a free parameter in  $M_u$ , irrespectively of the charm quark mass, which resolves the difficulty discussed in the previous section. The zero structures of such mass matrices are ruled out at the  $3\sigma$  level by the current experimental data if nonvanishing elements are symmetric and hierarchical valued [17]. Such a special case is obtained from our general form by exchanging the first two indices of right-handed down quarks in (3.20) and identifying the 1-2 and 2-1 matrix elements. Up to relabeling generation indices, there are 4 types of mass textures with symmetric two zeros. Exploring all possible patterns for  $M_u$  (with symmetric two zeros) and  $M_d$  (with asymmetric five zeros), we find that the following mass matrices are successful to explain the experimental data:

| $M_u$   | $M_d$   |
|---|---|
| $\begin{pmatrix} a & & \\ a & d & b \\ & b & c \end{pmatrix}$ | $\begin{pmatrix} e & h & \\ & f & \\ & & g \end{pmatrix}$ |
| $\begin{pmatrix} & a & \\ & b & d \\ a & d & c \end{pmatrix}$ | $\begin{pmatrix} e & & \\ h & f & \\ & & g \end{pmatrix}$ |

All  $4 = 2 \times 2$  combinations of  $M_u$  and  $M_d$  well describe the present experimental data. It should be noted that there are additional solutions with relabeling the generation indices, while physical implications are unchanged. First, any permutation of  $d_R$  (i.e. of the columns of  $M_d$ ) is phenomenologically allowed. In addition, simultaneously exchanges of the

identical generation indices for  $u_L$ ,  $u_R$ , and  $d_L$  completely preserve the physical consequences and also become the solutions. The above sets of mass matrices are consistent with all the properties of quarks, including the recent measurements of  $CP$  violation in the  $B$ -meson system (2.10).

The other case we will pursue in the following is to extend the down-type mass matrix  $M_d$  to contain (asymmetric) four zeros. It is found that there are 81 types of matrices with nonzero determinants, which are related through the permutations of rows and/or columns. All possible forms can be generated from the following four representatives of mass matrices:

$$\begin{pmatrix} d & & h \\ & e & g \\ & & f \end{pmatrix}, \begin{pmatrix} d & & \\ h & e & f \\ & g & \end{pmatrix}, \begin{pmatrix} & e & h \\ d & & f \\ & g & \end{pmatrix}, \begin{pmatrix} d & h \\ & e \\ g & f \end{pmatrix}. \quad (4.1)$$

It is first noticed that the last matrix generates only one mixing angle and does not cure the problem in the previous section, where one of the CKM mixing angles is not entirely consistent with the experimental data. Therefore we can safely drop this matrix (and the other 8 generated by changing the labels) in the analysis below. For the former two mass matrices in (4.1), a permutation of the first two columns produces the same modification as that obtained by permuting the first two rows, since any hierarchical order among the matrix elements is not supposed at this stage. This fact reduces by half the number of independent forms of matrices generated by exchanging the generation labels from the first two representatives in (4.1). The total number of independent mass matrices we will consider is 72 ( $= 18 + 18 + 36$ ).

As in the analysis of Sec. III, if one does not add up the rotations of the right-handed down-type quarks (the columns of matrices), only the following 12 types of matrices should be taken into account:

$$\begin{pmatrix} d & & h \\ & e & g \\ & & f \end{pmatrix}, \begin{pmatrix} d & h & \\ & e & f \\ & g & \end{pmatrix}, \begin{pmatrix} d & & \\ h & e & f \\ & g & \end{pmatrix}, \begin{pmatrix} d & h & g \\ & e & \\ & & f \end{pmatrix}, \begin{pmatrix} d & h \\ & e \\ g & f \end{pmatrix}, \begin{pmatrix} d & h \\ & e \\ g & f \end{pmatrix}, \begin{pmatrix} d & h \\ & e \\ g & f \end{pmatrix}, \begin{pmatrix} d & h \\ & e \\ g & f \end{pmatrix}, \begin{pmatrix} d & h \\ & e \\ g & f \end{pmatrix}, \begin{pmatrix} d & h \\ & e \\ g & f \end{pmatrix}, \begin{pmatrix} d & h \\ & e \\ g & f \end{pmatrix}, \begin{pmatrix} d & h \\ & e \\ g & f \end{pmatrix}. \quad (4.2)$$

Since we now drop the degrees of freedom of the right-handed down rotations, physical consequences can be read from the symmetric matrix  $M_d M_d^\dagger$ , which has 6 independent elements. For the above 12 types, a Hermitian matrix  $M_d M_d^\dagger$  contains 5 free parameters and necessarily leads to one vanishing element or one relation among nonvanishing elements. Thus an asymmetric four-zeros matrix gives similar results to that of a symmetric one-zero matrix, as far as mass eigenvalues and left-handed down mixing are concerned. It is noticed that there is a difference between  $M_d M_d^\dagger$  and symmetric mass matrices discussed in [6] that none of the diagonal elements of  $M_d M_d^\dagger$  can be zero for matrices with nonvanishing determinants. It was found in the analysis of Ref. [6] that symmetric two-zeros mass matrices for the down sector are consistent with the experimental data. Given these facts, the above asymmetric four-zeros mass matrices are expected to account for the proper mass eigenvalues and mixing, because they correspond to symmetric one-zero matrices which have one more free parameter. The number of combinations which can explain the data is hence rather large, and the exploration along this line unfortunately does not seem to provide a new perspective for the origin of fermion masses and mixing angles.

Let us proceed by taking into account the roles played by the mixing of right-handed down quarks. In the standard model, the mixing of right-handed [ $SU(2)$ -singlet] fermions is irrelevant to the CKM mixing and unphysical (unobservable) degrees of freedom. This is not necessarily true in various extensions of the standard model. For example, in supersymmetric extensions of the standard model, the right-handed mixing of fermions which diagonalizes a Yukawa matrix is transferred to that of corresponding scalars via supersymmetry-breaking scalar masses. Thus the masses of scalar superpartners generally have generation dependences and cause observable effects, such as flavor-changing decays of heavy fermions. A more interesting situation arises in the frameworks of grand unification. In this case, quarks and leptons are unified into some multiplets of a unified gauge group and the mass matrices of quarks are often closely related to those of leptons. This fact may give rise to an apparent difficulty in simultaneously realizing the small CKM mixing and the observed large lepton mixing, which is described by the Maki-Nakagawa-Sakata (MNS) matrix  $V_{\text{MNS}}$  [18]. The parallelism between quarks and leptons, which is a sign of grand unification, does not seem to work in the Yukawa sector. There is however an interesting observation that the mixing angles of left-handed charged leptons are correlated to those of right-handed down quarks and therefore the CKM mixing does not necessarily connect with the MNS mixing. This idea is easily achieved in  $SU(5)$  grand unification and larger unified theories [8], where a key ingredient is that an anti 5-plet of  $SU(5)$  contains a one-generation right-handed down quark and left-handed lep-

ton doublet, and also there are some multiplicities of anti 5-plets, which are naturally incorporated in  $SO(10)$  or  $E_6$  unified models. This mechanism automatically makes  $M_d$  asymmetric since it changes the property of anti 5-plets only, while keeping that of 10-plets.

In this way the mixing of right-handed fermions is not necessarily an unobservable quantity. In the following analysis, motivated by the grand unification view mentioned above, we investigate possible connections between  $V_{dR}$  and the leptonic mixing matrix which diagonalizes the mass matrix of three-generation charged leptons. In particular, we examine which combinations of mass matrices for the up and down sectors suggest large leptonic mixing, recently observed in various neutrino experiments. Since the flavor rotation of right-handed down quarks is now supposed to change physical consequences, it is not used to reduce the number of candidates for four-zeros matrices as done in the above. We will therefore exhaust all of the most generic 72 candidates for realistic down-quark mass matrices with four vanishing entries.

Assuming that the neutrino oscillations account for the solar and atmospheric neutrino data, the recent experimental results indicate rather large angles for the 1-2 and 2-3 generation mixing in the MNS matrix, but a small one for the 1-3 mixing angle  $(V_{\text{MNS}})_{13} < 0.14\text{--}0.22$  [5]. As for the two large mixing angles, the best fit value for the atmospheric neutrino data is the maximal mixing ( $\theta \simeq \pi/4$ ), and on the other hand, the solar neutrino deficit needs a large but nonmaximal value of the 1-2 mixing angle [4]. In the following analysis, we first consider, just as a first approximation, the maximal angles both for the 1-2 and 2-3 mixing, and then examine possible deviations from these maximal angles. The lepton mixing matrix is defined as

$$V_{\text{MNS}} = V_{eL}^\dagger V_\nu, \quad (4.3)$$

where  $V_{eL}$  rotates the three-generation charged leptons such that the mass matrix of charged leptons is diagonalized, and  $V_\nu$  denotes some mixing matrix in the neutrino sector. Its form crucially depends on the neutrino property and we leave it, together with detailed analysis of neutrino mass texture zeros, to another future task [19]. It should be noted that this does not mean that we take  $V_\nu = 1$  in the following analysis. In fact, our result will show that considerable contribution to lepton mixing needs to come from neutrino mass matrices, which could be realized in a huge variety of neutrino models.

As we mentioned, if embedding the theory into the grand unification scheme, the mixing of charged leptons  $V_{eL}$  may be related to that of right-handed down quarks as  $V_{eL} \simeq V_{dR}$ , up to corrections due to the breaking of unified gauge symmetry. To precisely reproduce the mass eigenvalues of charged leptons, it is in fact needed to take in some breaking effects which split the properties of quarks and leptons. Typical examples of such splitting are provided by the Georgi-Jarlskog factor [16] and higher-dimensional opera-

tors involving Higgs fields that break quark-lepton symmetry. We assume, just for simplicity, that such breaking effects are small enough not to significantly change the analysis below. We are thus interested in the following typical forms of mixing matrices for the down sector, which are associated with large generation mixing of left-handed charged leptons<sup>2</sup>:

$$V_{dR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad (4.4)$$

$$V_{dR} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4.5)$$

$$V_{dR} = \begin{pmatrix} 1/\sqrt{2} & -1/2 & 1/2 \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \quad (4.6)$$

A general procedure for examining viable forms of  $V_{dR}$  is as follows. At first, evaluate the matrix

$$M_d^\dagger M_d = V_{dR} \begin{pmatrix} m_d^2 & & \\ & m_s^2 & \\ & & m_b^2 \end{pmatrix} V_{dR}^\dagger, \quad (4.7)$$

where the matrix  $V_{dR}$  is parametrized as given in Sec. II. We consider in this section the matrices  $M_d$  with asymmetric four zeros. Namely, they contain five free parameters, and a matrix of the form  $M^\dagger M$  has six independent elements. Equation (4.7) therefore imposes one constraint which can be used to eliminate a mixing angle of the right-handed down quarks. As noted above, one or two additional constraints are obtained to reduce the number of independent (mixing) parameters, when one explores the solutions of  $V_{dR}$  with large mixing. For example, we can fix  $(V_{dR})_{32} = 1/\sqrt{2}$  in the case of (4.4). Once the matrix elements in  $M_d$  are solved with respect to the remaining independent parameters, the mixing matrix for left-handed down quarks is expressed as

$$V_{dL} = M_d V_{dR} \begin{pmatrix} m_d^{-1} & & \\ & m_s^{-1} & \\ & & m_b^{-1} \end{pmatrix}. \quad (4.8)$$

Such left-handed down mixing is used to examine which forms of mass matrices produce the observed values of the CKM matrix elements.

<sup>2</sup>The mixture (4.5) would be excluded by the neutrino oscillation experiments as it would generate too large a value of the 1-3 lepton mixing angle, if the atmospheric neutrino angle comes from the neutrino sector. While included in the analysis, as we will show, the mixing pattern (4.5) is already disfavored by the quark data alone.



Numerically exhausting all the possible forms of mass matrices, we find that the down-quark mass matrices with asymmetric four zeros are unfavorable to a sizable value of mixing angle between the first and second generations of right-handed down quarks. This fact is easily understood in the case that there is large mixing only between the first two generations. Such a mixing matrix is given by

$$V_{dR} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4.9)$$

at the leading order of other small mixing angles. The case where  $\theta \sim \pi/4$  corresponds to Eq. (4.5) and could provide a solution to the solar neutrino problem in grand unification schemes. To explain the quark mixing angles for the up-quark matrices Mu1, Mu2, and Mu3, the left-handed mixing in the down sector needs to satisfy

$$V_{dL} \simeq \begin{pmatrix} 1 & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon^3) \\ \mathcal{O}(\epsilon) & 1 & \mathcal{O}(\epsilon^2) \\ \mathcal{O}(\epsilon^3) & \mathcal{O}(\epsilon^2) & 1 \end{pmatrix}, \quad (4.10)$$

where  $\epsilon$  is a small parameter of order  $10^{-1}$ . It is found from the analysis in the previous section that the marginal requirements are sizable contributions from the down sector to  $V_{us}$  and  $V_{cb}$  (and not necessarily to  $V_{ub}$ ). This is translated to lower bounds on the left-handed mixing of down quarks, for example,  $|(V_{dL})_{12}| > 0.16$  and  $|(V_{dL})_{23}| > 0.012$ . When there is a solution for the above-described procedure, the corresponding down-quark mass matrix is given by

$$M_d \simeq \begin{pmatrix} 1 & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon^3) \\ \mathcal{O}(\epsilon) & 1 & \mathcal{O}(\epsilon^2) \\ \mathcal{O}(\epsilon^3) & \mathcal{O}(\epsilon^2) & 1 \end{pmatrix} \begin{pmatrix} m_d \\ m_s \\ m_b \end{pmatrix} \times \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (4.11)$$

It is clearly seen that four zeros in  $M_d$  cannot be realized since the matrix elements in the second and third rows are always nonvanishing for any precise values of  $V_{dL}$  (4.10) satisfying the lower bounds mentioned in the above. The situation might be improved by turning on fluctuations around the exact form of  $V_{dR}$  (4.9). In this case, one is in fact able to take either  $(M_d)_{31}$  or  $(M_d)_{32}$  as zero, if additional  $\mathcal{O}(\epsilon^4)$  mixing in  $V_{dR}$  is introduced. However not all of the elements in the first row become zero; in particular, either  $(M_d)_{11}$  or  $(M_d)_{12}$  can be set to zero. We thus find that the down-quark mass matrices with asymmetric four zeros generically conflict with large 1-2 mixing in the right-handed down sector.

It turns out that solutions with two large mixings like (4.6) are also absent. In the limit of bi-maximal mixing of  $d_R$ , the down-quark mass matrix becomes

$$M_d \simeq \begin{pmatrix} 1 & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon^3) \\ \mathcal{O}(\epsilon) & 1 & \mathcal{O}(\epsilon^2) \\ \mathcal{O}(\epsilon^3) & \mathcal{O}(\epsilon^2) & 1 \end{pmatrix} \begin{pmatrix} m_d \\ m_s \\ m_b \end{pmatrix} \times \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (4.12)$$

The situation is rather different from the case of (4.11); for instance, it is now possible to have vanishing matrix elements in the second row in (4.12) due to the presence of the third-generation large mixing. Note that the third-row elements are necessarily nonvanishing, even if one introduces sizable deviations from the maximal or zero mixing angles in  $V_{dR}$ , i.e.  $\theta_{1,3}^{dR} \neq \pi/4$  and/or  $\theta_2^{dR} \neq 0$ . Accordingly it is enough to consider physical consequences of the matrices  $M_d$  with four zeros placed in the first and second rows (and the other components are nonzero) [20]. If one adopts the up-quark mass matrices Mu1–Mu3, the mixing angles from the down sector have some lower bounds; in particular,  $|(V_{dL})_{12}| > 0.16$  is needed. It is numerically evaluated that the condition  $|(V_{dL})_{12}| > 0.16$  constrains the other mixing angles as  $|(V_{dR})_{21}| < 0.40$  (0.44) for  $|(V_{dR})_{32}| = 0.7$  (0.61). This value is translated, in the limit of negligible 1-3 mixing, to the upper bound of the 1-2 mixing angle  $\theta_3 < 26.1^\circ$ , which is excluded at more than a  $3\sigma$  level by the recent results of neutrino experiments, if generation mixing in the neutrino sector is found to be small. We thus find that any four-zero mass matrix in the down sector is not compatible with bi-large generation mixing of right-handed down quarks.

Finally let us consider the case that the right-handed down mixing between the second and third generations is large and the others are suppressed [Eq. (4.4)]. This implies, if adopting the grand unification, the amount of mixture of the atmospheric neutrinos. Following the general procedure described before, we have exhausted the possible patterns and found that, at classical level, the following mass matrices satisfy the criterion for (charged) lepton mixing, while the quark masses and the CKM matrix elements (including the KM phase) are properly reproduced:

| $M_u$   | $M_d$   |
|---|---|
| $\begin{pmatrix} a & & \\ & b & \\ & b & c \end{pmatrix}$   | $\begin{pmatrix} d & e & \\ & & h \\ & g & f \end{pmatrix}$ |
| $\begin{pmatrix} & a & \\ a & b & \\ & b & c \end{pmatrix}$ | $\begin{pmatrix} & e & \\ d & h & \\ & g & f \end{pmatrix}$ |
|   | $\begin{pmatrix} & e & \\ & & h \\ d & g & f \end{pmatrix}$ |

All  $6 = 2 \times 3$  combinations of  $M_u$  and  $M_d$  are consistent with the present experimental data. Note that one type of

mass texture for  $M_d$  [the second one in (4.2)] almost describes the data we have listed in Sec. II. It is however found that the whole parameter space of that texture is excluded by the measured value of  $CP$  violation in the  $K$ -meson system (the  $\epsilon_K$  constraint). All the other combinations of mass textures are not compatible with the experimentally allowed parameter region for the atmospheric neutrino problem at more than a  $6\sigma$  level, unless there is sizable contribution to mixing angles from the neutrino sector. The examples of numerical fits for these matrix elements are shown in the Appendix. It should be noted that there are additional solutions with relabeling the generation indices, while physical implications are unchanged. The combinations obtained by exchanging the second and third columns of  $M_d$  are viable. This is simply because we now consider the situation that the second and third generations of  $d_R$  are largely mixed. In addition, simultaneous exchanges of the identical generation indices for  $u_L$  (and  $u_R$  for symmetric textures) and  $d_L$  completely preserve the physical consequences and also become the solutions. No other exchanging symmetry exists. The above sets of mass matrices are consistent with all the properties of quarks, including the recent measurements of  $CP$  violation in the  $B$ -meson system, as well as large lepton mixing for the atmospheric neutrino problem. More conservatively, they provide sizable contribution to leptonic 2-3 mixing, while satisfying the experimental results of quark masses and the CKM matrix elements. It is interesting to see that, in the down sector, the numerical exploration of parameter space shows that a correlation  $f \simeq g$  should hold for all the above solutions. The above list of textures contains the Fritzsch ansatz [21], but  $M_u$  is symmetric while  $M_d$  is not [22], leading to large generation mixing in  $V_{dR}$ . As mentioned in the beginning of this section, such an asymmetric form of mass matrix often plays a key role for neutrino physics in grand unified theory [8]. We have shown that the above three forms of  $M_d$  are the minimal extensions of the ansatz  $f \sim g$  to include the first generation. The obtained matrices are successful to explain the observed quark masses and mixing angles and have the maximal number of vanishing elements.

## V. SUMMARY AND DISCUSSIONS

The study of the origin of fermion masses and mixing angles is one of the most important unresolved issues in particle physics. As a plausible approach to this issue, possible zero elements in mass matrices have been extensively examined and the obtained results have suggested useful guides for realistic model construction. In this paper we have systematically investigated what types of quark mass matrices with nonsymmetrical forms can be consistent with the experimentally obtained CKM matrix and mass eigenvalues. Our first principle is that a mass matrix has as simple form as possible, namely, to search for the minimal number of free parameters in the mass matrices. This leads us to consider some of the mass matrix elements to be vanishing. The existence and structures of zero matrix

elements are expected to be deeply connected with underlying physics, such as flavor symmetries, in more fundamental theory of quarks and leptons. We have first examined experimentally viable mass matrices in the case where the up-quark sector has symmetric three zeros and the down-quark sector asymmetric five zeros. This is the simplest possibility that apparently does not conflict with the experimental data, and can almost explain the observed quark masses and mixing angles. The situation is rather different from the case where the down-quark mass matrix contains at most four vanishing elements. We then find that there exist various forms of mass matrices consistent with the existing experimental data, and it seems difficult to find some clues to understanding the generation structure. Additional information comes from the recent observation of neutrino generation mixing. If working with the grand unification hypothesis, the mixing of  $SU(2)$ -doublet leptons is correlated to that of  $SU(2)$ -singlet down-type quarks. To investigate the implications of large mixing angles in the lepton sector, we have searched viable solutions which induce large right-handed mixing in the down sector, and found that there only exist six patterns of mass matrices with a large mixture between the second and third generation of right-handed down quarks. Furthermore it turns out in our framework that the large angle solution for the solar neutrino problem cannot be realized from the charged-lepton sector with asymmetric four-zeros  $M_d$  ( $M_e$ ).

The observed large amount of mixing angle of solar neutrinos then should come from the neutrino sector. If the minimality principle is applied to neutrino mass matrices, the simplest matrix forms (i.e. with the maximal number of zero matrix elements) could be found out. However the neutrinos have rich phenomenology and their property has not been fixed experimentally. In particular, there still exist wide possibilities for the neutrino mass spectrum, a fact which generally makes the thorough analysis of neutrino mass textures laborious. We here briefly discuss several results for possible forms of neutrino mass matrices which lead to the large lepton mixing between the first and second generations and have the maximal number of allowed zero matrix elements. First, consider the effective neutrino mass operator  $\kappa_{ij}\bar{L}_i L_j H^* H$  where  $L_i$  denote the three-family lepton doublets. The coefficient matrix  $\kappa_{ij}$  is symmetric in the generation space. This higher-dimensional operator induces the Majorana neutrino mass matrix  $M_L = \kappa\langle H^* H \rangle$  after the electroweak gauge symmetry breaking. We find two types of the simplest forms of  $M_L$  which have symmetric four zeros and are given by

$$M_L = \begin{pmatrix} & l \\ l & n \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} n & l \\ l & \end{pmatrix}. \quad (5.1)$$

It is interesting to note that this form of neutrino mass

matrix predicts the spectrum with the inverted mass hierarchy and an exactly massless neutrino for the third generation. Moreover, taking into account the observed neutrino mass differences, such  $M_L$  leads to an almost maximal mixing angle between the first two generations. As a result, it can be matched with only four of six combinations of quark mass matrices found in the previous section. If the minimality analysis is extended to the next-to-minimal level, i.e.  $M_L$  with symmetric three zeros, we find 13 patterns are allowed, each of which predicts a characteristic mass spectrum of light Majorana neutrinos. Another well-known scheme for light neutrinos is to consider a  $(3 \times 3)$  Dirac neutrino mass matrix  $M_\nu$  and a right-handed Majorana one,  $M_R$ . Also in this case, the analysis is quite different from the quark sector, mainly because one neutrino can be massless. We find from the exhaustive exploration that the maximal number of vanishing matrix elements is ten which consists of asymmetric seven (six) zeros in  $M_\nu$  and symmetric three (four) zeros in  $M_R$ . There are four patterns of the seven-zeros  $M_\nu$  cases, which contain as an example

$$M_\nu = \begin{pmatrix} s & & \\ & t & \\ & & \end{pmatrix}, \quad M_R = \begin{pmatrix} & u & \\ u & & v \\ & v & w \end{pmatrix}, \quad (5.2)$$

and six patterns for the six-zeros  $M_\nu$  cases, for example,

$$M_\nu = \begin{pmatrix} & s & \\ t & & u \\ & & \end{pmatrix}, \quad M_R = \begin{pmatrix} & & v \\ v & & \\ & & w \end{pmatrix}. \quad (5.3)$$

All ten patterns of  $M_\nu$  and  $M_R$  generate light neutrino mass matrix  $M_L$  in the form of (5.1) after the seesaw operation. Therefore the resultant mass spectrum and possible partners for quark mass matrices are the same as the cases (5.1). Some different phenomenology may appear through lepton flavor-violating processes induced by lepton Yukawa couplings [23]. The detailed analysis of minimal lepton mass matrices and their phenomenological implications will be presented in a separate paper [19].

In the analysis of this paper, except for the discussion at the end of Sec. III, we have not taken into account the dependence of matrix forms on the renormalization scale, but we have considered generic features of  $3 \times 3$  quark Yukawa couplings including asymmetrical matrices. For more precise treatment, the renormalization-group evolution of Yukawa couplings needs to be evaluated, because zeros of matrix elements should be implemented at some high-energy scale such as a grand unification scale. The observable quantities at the electroweak scale deviate to some extent from the values estimated in a high-energy regime. However one of the most important points is that the fermion mass ratios of the first to second generations is almost insensitive to radiative corrections due to the fact that the dominant contribution to flavor-changing evolution comes from the Yukawa couplings of the third gen-

eration. In our analysis, the selection of viable forms of mass matrices has mainly depended on whether the down-quark matrices satisfy the experimental value of the 1-2 CKM mixing in conspiracy with the up sector. It is therefore expected that the renormalization-group analysis does not destabilize the results of our analysis of possible zero elements, while there certainly exist some scale dependences of nonvanishing matrix elements in the presence of significant contributions from the gauge and top-quark Yukawa couplings. This latter fact is supposed to only change ‘‘initial’’ values of nonvanishing Yukawa couplings at a high-energy scale. The results presented in this paper are also useful for explaining the flavor structures of quarks and leptons in grand unification schemes.

Finally, we would like to comment on some phenomenology related to the solutions obtained in Sec. IV. These solutions predict similar sizes of off-diagonal elements to the 3-3 elements, and radiative corrections from Yukawa couplings are important for flavor physics. For example, if the theory is supersymmetrized, flavor violation in the Yukawa sectors is translated to off-diagonal components of supersymmetry-breaking scalar masses through the radiative corrections. That could induce sizable rates of flavor-changing neutral currents for quarks and charged leptons [23] in supergravity models. Further searches of flavor-violating processes will provide us a new perspective of flavor structures in a high-energy regime.

In the viewpoint of distinguishing possible solutions, it is important to examine observable signals of underlying theory. In addition to signals of underlying symmetries or dynamics, the improved measurements of low-energy observable quantities allow us to discriminate discrete ambiguities of possible matrices. As for the solutions 1–6 presented in the Appendix, it can be seen from the numerical analysis that the solutions 3 and 4 have sizable contributions to  $(V_{dR})_{31}$  components. This means that they predict  $(V_{MNS})_{13} \sim \mathcal{O}(10^{-1})$  if there appears no fine tuning of parameters in  $V_{dR}$  and  $V_\nu$ . Since the planned improvements in the sensitivity to  $(V_{MNS})_{13}$  are expected to reach 0.05 [24], these solutions would be supported or disfavored when a precise value of  $(V_{MNS})_{13}$  is measured. For other generation mixing, the solutions 3–6 are found to have relatively larger values of  $(V_{dR})_{21}$  (of the order of the Cabibbo angle) than the other solutions. This fact also distinguishes possible textures, for example, if the theory is extended to incorporate supersymmetry (breaking) or grand unification. Together with these issues stated above and others, it is hoped to find what underlying theory governs the masses and mixing angles of quarks and leptons.

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TABLE I. The typical orders of matrix elements for six possible mass textures. We have not explicitly included  $O(1)$  coefficients, which would be needed to precisely reproduce the experimental data. Note that there are also additional solutions obtained (i) by exchanging the second and third generations of  $d_R$  (columns in  $M_d$ ) and/or (ii) by identically relabeling generation indices for  $u_L$ ,  $u_R$ , and  $d_L$ .

|   | $M_u/m_t$   | $M_d/m_b$   |
|---|---|---|
| 1 | $\begin{pmatrix} \lambda^8 & & \\ & \lambda^2 & \lambda^2 \\ & \lambda^2 & \lambda^0 \end{pmatrix}$                     | $\begin{pmatrix} \lambda^4 & \lambda^3 & \\ & & \lambda^2 \\ & \lambda^0 & \lambda^0 \end{pmatrix}$           |
| 2 | $\begin{pmatrix} & \lambda^6 & \\ \lambda^6 & & \lambda^2 \\ \lambda^2 & \lambda^2 & \lambda^0 \end{pmatrix}$           | $\begin{pmatrix} \lambda^4 & \lambda^3 & \\ & & \lambda^2 \\ & \lambda^0 & \lambda^0 \end{pmatrix}$           |
| 3 | $\begin{pmatrix} \lambda^8 & & \\ & \lambda^2 & \lambda^2 \\ & \lambda^2 & \lambda^0 \end{pmatrix}$                     | $\begin{pmatrix} & \lambda^3 & \\ \lambda^3 & & \lambda^2 \\ & \lambda^0 & \lambda^0 \end{pmatrix}$           |
| 4 | $\begin{pmatrix} & \lambda^6 & \lambda^2 \\ \lambda^6 & & \lambda^2 \\ \lambda^2 & \lambda^2 & \lambda^0 \end{pmatrix}$ | $\begin{pmatrix} & \lambda^3 & \lambda^2 \\ \lambda^3 & & \lambda^2 \\ & \lambda^0 & \lambda^0 \end{pmatrix}$ |
| 5 | $\begin{pmatrix} \lambda^8 & & \\ & \lambda^2 & \lambda^2 \\ & \lambda^2 & \lambda^0 \end{pmatrix}$                     | $\begin{pmatrix} & \lambda^3 & \lambda^2 \\ \lambda^1 & \lambda^0 & \lambda^0 \end{pmatrix}$                  |
| 6 | $\begin{pmatrix} & \lambda^6 & \lambda^2 \\ \lambda^6 & & \lambda^2 \\ \lambda^2 & \lambda^2 & \lambda^0 \end{pmatrix}$ | $\begin{pmatrix} & \lambda^3 & \lambda^2 \\ \lambda^1 & \lambda^0 & \lambda^0 \end{pmatrix}$                  |

discoveries of Higgs boson and supersymmetry” (No. 16081209).

### APPENDIX: THE ORDER ESTIMATION AND NUMERICAL EVALUATIONS OF THE QUARK MASS MATRICES

In this appendix, we would like to present the order estimation of quark mass matrix elements and typical examples of numerical fitting for the solutions obtained

TABLE II. Numerical examples for the predictions of the texture combinations given in Table I. The mass eigenvalues are denoted in GeV units.

|                      | 1       | 2       | 3       | 4       | 5       | 6        |
|----------------------|---------|---------|---------|---------|---------|----------|
| $m_u$                | 0.00179 | 0.00104 | 0.00179 | 0.00223 | 0.00179 | 0.000983 |
| $m_d$                | 0.00387 | 0.00470 | 0.00283 | 0.00321 | 0.00511 | 0.00271  |
| $m_s$                | 0.0562  | 0.0683  | 0.0637  | 0.0805  | 0.0566  | 0.0542   |
| $m_c$                | 0.613   | 0.611   | 0.652   | 0.633   | 0.601   | 0.621    |
| $m_b$                | 2.90    | 3.01    | 2.94    | 2.99    | 2.91    | 2.87     |
| $m_t$                | 178     | 177     | 175     | 176     | 179     | 171      |
| $ V_{us} $           | 0.225   | 0.225   | 0.223   | 0.223   | 0.224   | 0.223    |
| $ V_{cb} $           | 0.0433  | 0.0430  | 0.0433  | 0.0433  | 0.0428  | 0.0434   |
| $ V_{ub} $           | 0.00403 | 0.00414 | 0.00426 | 0.00429 | 0.00377 | 0.00437  |
| $J_{CP}/10^{-5}$     | 2.90    | 2.60    | 3.16    | 2.79    | 2.91    | 2.83     |
| $\sin 2\phi_1/\beta$ | 0.709   | 0.692   | 0.762   | 0.735   | 0.694   | 0.742    |
| $ (V_{dR})_{21} $    | 0.0116  | 0.00990 | 0.144   | 0.167   | 0.285   | 0.135    |
| $ (V_{dR})_{32} $    | 0.681   | 0.682   | 0.643   | 0.642   | 0.656   | 0.707    |
| $ (V_{dR})_{31} $    | 0.0108  | 0.00922 | 0.125   | 0.147   | 0.0136  | 0.00918  |

in Sec. IV, where the up-quark matrices have symmetric three zeros and the down-quark ones asymmetric four zeros. Since the observed values of masses and mixing angles are hierarchical, one could parametrize matrix elements by integer exponents of a small parameter  $\lambda$  ( $= 0.22$ ) times  $O(1)$  coefficients, which originate from the ambiguities of Yukawa coupling constants. Such expressions with integer exponents might be useful for getting ideas of constructing fermion mass matrix models with flavor symmetries. We have found in Sec. IV that there are  $6 = 2 \times 3$  combinations of up- and down-quark mass textures that well describe the current experimental data. The order estimation of these mass matrix elements are presented in Table I, where we have not explicitly written down  $O(1)$  coefficients mentioned above.

Suitably choosing the  $O(1)$  coefficients (i.e. Yukawa couplings) in the textures listed in Table I, we obtain numerical examples for these six solutions (Table II).

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