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Axial-vector tetraquark with $S=+2$

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The possibility of an axial-vector isoscalar tetraquark with $ud\bar{s}\bar{s}$ is discussed. If the pentaquark $\Theta^+(1540)$ has the $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$ configuration, the isoscalar $ud\bar{s}\bar{s}$ (ϑ^+-meson) state with $J^P = 1^+$ is expected to exist in the mass region lower than, or close to, the mass of $\Theta^+(1540)$. Within a flux-tube quark model, a possible resonant state of $ud\bar{s}\bar{s}(J^P = 1^+)$ is suggested to appear around 1.4 GeV with the width $\mathcal{O}(20 \sim 80)$ MeV. We propose that the ϑ^+-meson is a good candidate for the tetraquark search, which would be observed in the $K^+K^+\pi^-$ decay channel.

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I. INTRODUCTION

The possibility of multiquark states has been discussed for a long time [1–11]. In particular, the possible $qq\bar{q}\bar{q}$ states have been suggested in many theoretical efforts to understand light scalar mesons (see, for example, Refs. [1,3,4]). The $4q$ states were proposed in descriptions of $f_0(600)$ and $f_0(980)$, where the strong attraction between $(qq)_{\bar{3}}$ and $(\bar{q}\bar{q})_3$ plays an important role [1,3]. Here, $(qq)_{\bar{3}}$ and $(\bar{q}\bar{q})_3$ denote the color-anti-triplet quark pair and the color-triplet antiquark pair, respectively. On the other hand, the $K\bar{K}$ molecule states were suggested to understand the properties of $f_0(980)$ and $a_0(980)$ [4]. Since the masses of negative-parity mesons are expected to be above 1 GeV in a naive interpretation with P-wave $q\bar{q}$ states, it is considered that these scalar mesons below 1 GeV may be hybrids of P-wave $q\bar{q}$ and compact $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ with meson-meson tails in the outer region, as argued in Ref. [12]. Even if the $4q$ components are dominant in a certain meson whose minimal content is $2q$, it is difficult to find direct evidence of the $4q$ components due to mixing with the conventional $q\bar{q}$ state via the annihilation of $q\bar{q}$ pairs. Our main interest here is the possibility of narrow "tetraquark" states, whose minimal quark content is $4q$.

The recent observation of $D_{sJ}(2317)$ [13] and reports of the pentaquark baryon $\Theta^+(uudd\bar{s})$ [14–22] revived motivation for experimental and theoretical studies on multiquarks in hadron physics, though the existence of Θ^+ is yet to be well established. One of the striking characteristics of the Θ^+ is its narrow width. For a theoretical interpretation of why the Θ^+ is extremely narrow, the possibility of the spin-parity $J^P = 1/2^+$ and $J^P = 3/2^-$ has been discussed by many groups [11,23–29]. Since only the P-wave and D-wave are allowed in NK decays from the $J^P = 1/2^+$ and $J^P = 3/2^-$ states, respectively, the widths should be suppressed due to a high centrifugal barrier. The transition into meson-baryon states should be further suppressed if the pentaquark has an exotic color configuration $(qq)_{\bar{3}} \times (qq)_{\bar{3}}\bar{q}$. The factor $1/3$ in the transition appears from the overlap of the color wave functions of quarks. In addition to suppression due to the color degrees of freedom of 5 quarks, another suppression effect can be considered in flux-tube pictures [6,26,30–32], because the transitions between different flux-tube topologies are suppressed due to a rearrangement of the gluon field. This means that the decays from such exotic flux-tube configurations as $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$ [Fig. 1(e)] into meson-baryon-like $(qqq)_1 \times$

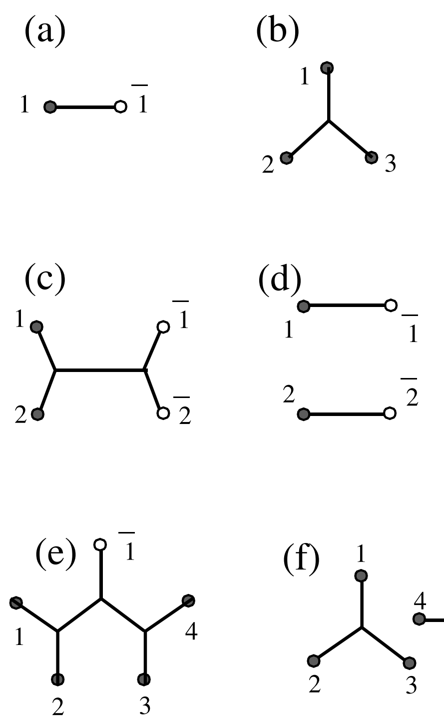

FIG. 1. Flux-tube configurations for $q\bar{q}$ meson (a), q^3 baryon (b), $q^2\bar{q}^2$ states (c),(d), and $q^4\bar{q}$ states (e),(f). For the $q^2\bar{q}^2$ states, the exotic tube configuration (c) correspond, to the $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$, and the disconnected tube (d) represents the meson-meson state, $(q\bar{q})_1(q\bar{q})_1$. The configurations $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$ and $(qqq)_1(q\bar{q})_1$ for the $q^4\bar{q}$ system are illustrated in (e) and (f), respectively.

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$(q\bar{q})_1$ [Fig. 1(f)] are suppressed. In general, the different flux-tube configuration appears in multiquarks that contain more than 3 quarks, as shown in Fig. 1, and the coupling between the disconnected tube and the connected tube topologies should be strongly suppressed.

The predicted spin-particles $J^P = 1/2^+$ and $J^P = 3/2^-$ of the pentaquark $uudd\bar{s}$ are abnormal in a naive quark picture, where $J^P = 1/2^-$ should be the lowest state, while other spin-parity states are expected to be highly excited. Originally, a narrow $J^P = 1/2^+$ was predicted in the Skyrme soliton model by Diakonov *et al.* [33]. For a theoretical explanation of the $J^P = 1/2^+$ state from the point of view of the constituent quarks, the diquark picture and the triquark picture are proposed in Refs. [23,24]. Recently, constituent quark-model calculations [26,27] suggested an abnormal level structure in the compact $uudd\bar{s}$ system with the $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$ configuration, where the masses of the $J^P = 1/2^+$ and/or $J^P = 3/2^-$ states may degenerate with the $J^P = 1/2^-$ state.

We now turn to a discussion on the possibility of tetraquarks. By replacing a ud-diquark in the Θ^+ with an $\bar{s}$ quark, it is natural to expect that a tetraquark with the $ud\bar{s}\bar{s}$ content may exist at nearly the same energy region. In order to search for a narrow tetraquark, we follow the analogy of the theoretical explanation of why the pentaquark Θ^+ can be narrow. First, one should consider those states with unnatural spin and parity, which cannot decay into two light hadrons (pseudoscalar mesons) in the S-wave channel. Second, the exotic flux-tube configurations with connected tubes [Figs. 1(c) and 1(e)] would be essential to stabilize the exotic hadrons. We then propose a $J^P = 1^+$ $ud\bar{s}\bar{s}$ state with the $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ configuration as a candidate of narrow tetraquark states. It should be stressed that two-body KK decays from any $J^P = 1^+$ $ud\bar{s}\bar{s}$ state are forbidden because of conservation of the total spin and parity. The lowest threshold energy of the allowed two-body decays is 1.39 GeV for the $KK^*(895)$ channel. If the mass of the $J^P = 1^+$ $ud\bar{s}\bar{s}$ state lies below the KK^*, two-meson decay channels are closed, and hence its width must be narrow.

The $4q$ states have been proposed by Jaffe in 1977 [1]. The $(qq)_{\bar{3}}$ diquark and the $(\bar{q}\bar{q})_3$ antidiquark correlations play an important role in the stability of the $4q$ state, because there exists attraction between $(qq)_{\bar{3}}$ and $(\bar{q}\bar{q})_3$ due to the confining and the one-gluon-exchange (OGE) potential. Moreover, it is known that the spin-zero flavor-singlet $(qq)_{\bar{3}}$ diquark is favored, because it gains the color-magnetic interaction. Thus, the diquark (antidiquark) correlation leads to confined $4q$ states with the $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ color configuration, which might make the $4q$ system compact and stable. In the $J^P = 1^+$ state of the tetraquark $(ud)_{\bar{3}}(\bar{s}\bar{s})_3$, one can naturally expect that the lowest is the isoscalar $J^P = 1^+$ state with a spin-zero $(ud)_{\bar{3}}$ and a spin-one $(\bar{s}\bar{s})_3$ in a spatially symmetric orbit. In the spatially symmetric $(\bar{s}\bar{s})_3$, the spin-zero (spin-singlet) configuration is forbidden, and hence the $(\bar{s}\bar{s})_3$ must have spin-one. Although the spin-one $(\bar{s}\bar{s})_3$ feels some repulsive color-magnetic interaction, the repulsion is expected to be small because the color-magnetic term in the OGE potential is suppressed by the quark-mass factor m^{-2}. In the flux-tube model, the $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ state has the exotic tube topology shown in Fig. 1(c). Therefore, its coupling with two-hadron states should be small, due to the suppressed transitions between the different tube topologies, (c) and (d).

The tetraquark $ud\bar{s}\bar{s}$ states are discussed in Ref. [1], and noted as $E_{(KK)}$-mesons. In Ref. [1], the theoretical mass for the isoscalar $ud\bar{s}\bar{s}(J^P = 1^+)$ state is predicted to be 1.65 GeV in the MIT bag model [1]. Recently, the isoscalar $ud\bar{s}\bar{s}$ in the flavor $\overline{10}$ group was suggested in analogy with the Θ^+ by Burns *et al.* [34] and by Karliner and Lipkin [35]. In Ref. [35], the possibility of a 0^+ state is discussed. However, we should remark that the $J^P = 0^+$ is not allowed in the isoscalar $ud\bar{s}\bar{s}$ system within the spatially symmetric configuration, and, hence, the $J^P = 0^+$ is expected to be unfavored. The tetraquark $ud\bar{s}\bar{s}$ is called a ϑ^+-meson in Ref. [34], where the $J^P = 1^-$ state with the orbital angular momentum $L = 1$ is predicted in the mass region $\sim$1.6 GeV. Although the $J^P = 1^-$ state may gain color-magnetic attraction, it needs the $L = 1$ excitation energy, and is expected to be higher than the spatially symmetric state. Another claim for the $\vartheta^+(J^P = 1^-)$ state is that it can decay into P-wave KK states. The centrifugal barrier may not be high enough to stabilize the state much above the threshold energy. Therefore, we think that the ϑ^+ $(J^P = 1^+)$ is a better candidate for narrow tetraquarks.

In this paper, we consider the ϑ^+-meson with $J^P = 1^+$ by a constituent quark model. The theoretical method of the calculations is the same as that applied to the pentaquark study in Ref. [26]. Namely, we apply the flux-tube quark model with antisymmetrized molecular dynamics (AMD) [36,37] to the $4q$ systems. Based on the picture of a flux-tube model, we ignore the coupling between configurations shown in Figs. 1(c) and 1(d), and solve the $4q$ dynamics with the variational method in the model space $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ shown in Fig. 1(c). The Coulomb and color-magnetic terms of the OGE potential and the string potential are taken into account. In order to evaluate the $\vartheta^+(J^P = 1^+)$ mass, we adopt the observed Θ^+ mass as an input as well as the normal hadron spectra. We also try to interpret a f_1-meson in the 1.4–1.6 GeV region with the $4q$ state, which would help to check the reliability of the present calculations. The widths of these states are also discussed.

II. HAMILTONIAN

The adopted Hamiltonian is the same as that of previous work [26] as $H = H_0 + H_I + H_f$, where H_0 consists of the mass and kinetic terms, H_I represents the short-range OGE interaction, and H_f is the string potential given by the energy of the flux tubes. The quarks are treated as nonrelativistic spin-$\frac{1}{2}$ fermions. The OGE potential consists of Coulomb and the color-magnetic interactions, as

$$H_I = \alpha_c \sum_{i<j} F_i F_j \left[\frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j} s(r_{ij}) \sigma_i \cdot \sigma_j \right]. \quad (1)$$

Here, α_c is the quark-gluon coupling constant, and $F_i F_j$ is defined by $\sum_{\alpha=1,\ldots,8} F_i^\alpha F_j^\alpha$, where F_i^α is the generator of color SU(3), $\frac{1}{2}\lambda_i^\alpha$ for quarks and $-\frac{1}{2}(\lambda_i^\alpha)^*$ for antiquarks. m_i is the quark mass m_q for u and d quarks, and m_s for a s quark. The usual $\delta(r_{ij})$ function in the spin-spin interaction is replaced by a finite-range Gaussian, $s(r_{ij}) \equiv [\frac{1}{2\sqrt{\pi}\Lambda}]^3 \times \exp[-\frac{r_{ij}^2}{4\Lambda^2}]$.

In the flux-tube quark model [6,38], the confining string potential is written as $H_f = \sigma L_f - M^0$, where σ is the string tension, L_f is the minimum length of the flux tubes, and M^0 is the zero-point string energy. For the meson and $3q$-baryon systems, the flux-tube configurations are the linear line and the Y-type configuration with a junction, as shown in Figs. 1(a) and 1(b), respectively. For the $q^2\bar{q}^2$ mesons and $q^4\bar{q}$ baryons, the exotic topologies [Figs. 1(c) and 1(e)] appear corresponding to the $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ and $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$, in addition to the normal two-hadron configurations [Figs. 1(d) and 1(f)]. In principle, besides these color configurations, other color configurations are possible in totally color-singlet $q^2\bar{q}^2$ and $q^4\bar{q}$ systems by incorporating a color-symmetric $(qq)_6$ pair. However, since such a string from the $(qq)_6$ is an excited one, and is unfavored in the strong-coupling limit of lattice QCD [39], we should consider only color-3 flux tubes as the elementary tubes. The string potentials given by the tube lengths of the configuration [Fig. 1(b), 1(c), and 1(e)] are supported by lattice QCD calculations [40–42].

In the practical calculation of the expectation values of the string potential $\langle\Phi|H_f|\Phi\rangle$ with respect to a meson state ($\Phi_{q\bar{q}}$), a three-quark state (Φ_{q^3}), a $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ state ($\Phi_{(qq)_{\bar{3}}(\bar{q}\bar{q})_3}$), and a $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$ state ($\Phi_{(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}}$), the minimum length of the flux tubes L_f is approximated by a linear combination of two-body distances r_{ij} as

$$L_f \approx r_{1\bar{1}} \quad \text{in } \langle\Phi_{q\bar{q}}|H_f|\Phi_{q\bar{q}}\rangle, \quad (2)$$

$$L_f \approx \frac{1}{2}(r_{12} + r_{23} + r_{31}) \quad \text{in } \langle\Phi_{q^3}|H_f|\Phi_{q^3}\rangle, \quad (3)$$

$$L_f \approx \frac{1}{2}(r_{12} + r_{\bar{1}\bar{2}}) + \frac{1}{4}(r_{1\bar{1}} + r_{1\bar{2}} + r_{2\bar{1}} + r_{2\bar{2}}) \quad \text{in } \langle\Phi_{(qq)_{\bar{3}}(\bar{q}\bar{q})_3}|H_f|\Phi_{(qq)_{\bar{3}}(\bar{q}\bar{q})_3}\rangle, \quad (4)$$

$$L_f \approx \frac{1}{2}(r_{12} + r_{34}) + \frac{1}{8}(r_{13} + r_{14} + r_{23} + r_{24}) + \frac{1}{4}(r_{\bar{1}1} + r_{\bar{1}2} + r_{\bar{1}3} + r_{\bar{1}4}) \quad \text{in } \langle\Phi_{(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}}|H_f|\Phi_{(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}}\rangle. \quad (5)$$

M^0 depends on the flux-tube topology and is denoted here as $M^0_{q\bar{q}}$, $M^0_{q^3}$, $M^0_{[qq][\bar{q}\bar{q}]}$, and $M^0_{[qq][qq]\bar{q}}$ for the configurations shown in Figs. 1(a)–1(c) and 1(e), respectively. ($[qq]$ and $[\bar{q}\bar{q}]$ indicate $(qq)_{\bar{3}}$ and $(\bar{q}\bar{q})_3$, respectively.)

In the present calculation, we ignore other terms such as tensor and spin-orbit interactions in the OGE potential, and we do not introduce flavor-exchange interactions. As shown later, the major properties of the normal hadron mass spectra are qualitatively reproduced by the present Hamiltonian.

III. MODEL WAVE FUNCTION AND PARAMETERS

We solve the eigenstates of the Hamiltonian with a variational method in the AMD model space proposed in the previous paper. The AMD wave function in a quark model is given as follows:

$$\Phi(\mathbf{Z}) = (1 \pm P)\mathcal{A}[\phi_{Z_1}\phi_{Z_2}\ldots\phi_{Z_{N_q}}\Phi^S\Phi^X], \quad (6)$$

$$\phi_{Z_i} = \left(\frac{1}{\pi b^2}\right)^{3/4} \exp\left[-\frac{1}{2b^2}(r - \sqrt{2}bZ_i)^2 + \frac{1}{2}Z_i^2\right], \quad (7)$$

where $1 \pm P$ is the parity projection operator, $\mathcal{A}$ is the antisymmetrization operator, and the spatial part ϕ_{Z_i} of the ith single-particle wave function given by a Gaussian whose center is located at Z_i in phase space. The spin function, Φ^S, is given as

$$\Phi^S = \sum_{m_1,\ldots,m_{N_q}} c_{m_1\ldots m_{N_q}} |m_1 \ldots m_{N_q}\rangle_S, \quad (8)$$

where $|m\rangle_S (m = \uparrow, \downarrow)$ is the intrinsic-spin function. Φ^X is the flavor-color function. For example, the flavor-color function for the tetraquark $ud\bar{s}\bar{s}$ system with color-configuration $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ is written as

$$\Phi^X = |ud\bar{s}\,\bar{s}\rangle \otimes \epsilon_{abc}\epsilon_{efc}|ab\bar{e}\,\bar{f}\rangle_C. \quad (9)$$

In the present wave function we do not explicitly perform isospin projection, but the wave functions obtained by energy variation are found to be approximately isospin-eigenstates in most cases due to the color-spin symmetry.

As already mentioned, different flux-tube topologies appear in each of the $q^2\bar{q}^2$ and the $q^4\bar{q}$ systems. Since the transitions between the different string configurations are of higher order in the strong-coupling expansion, we ignore the coupling and perform a variational calculation within a single flux-tube topology. In the present calculations, we adopt only the connected flux-tube configurations given in Figs. 1(c) and 1(e), because we are interested in the confined and narrow states. This is regarded as a kind of bound state approximation.

In the numerical calculation, the linear and Coulomb potentials are approximated by seven-range Gaussians. We use the same parameters as those adopted in Ref. [26]:

$$\alpha_c = 1.05, \quad \Lambda = 0.13\ \text{fm}, \quad m_q = 0.313\ \text{GeV}, \quad m_s = 0.513\ \text{GeV}, \quad \sigma = 0.853\ \text{GeV/fm}. \tag{10}$$

Here, the quark-gluon coupling constant (α_c) and the string tension (σ) are chosen so as to fit the mass splitting among N, Δ, and $N^*(1520)$. The width parameter (b) is chosen to be 0.5 fm.

IV. RESULTS AND DISCUSSIONS

A. Mesons and baryons

In Fig. 2, we display the calculated masses of the conventional mesons and baryons compared with the experimental data. The zero-point energy of the string potential for the $3q$ system is chosen to be $M^0_{q^3} = 972$ MeV to fit the nucleon mass, while $M^0_{q\bar{q}}$ for the $q\bar{q}$ is adjusted to be 584 MeV to reproduce the ρ-meson mass. It is shown that the systematics of the mass spectra are reasonably reproduced by the present calculations, except for the pseudoscalar mesons.

B. ϑ^+-meson ($I=0$, $J^P=1^+$)

As mentioned before, the zero-point energy (M^0) of the string potential depends on the flux-tube topology. We, here, phenomenologically deduce the unknown $M^0_{[qq][\bar{q}\bar{q}]}$ for the $4q$ system with the help of the systematics of $M^0_{q\bar{q}}$, $M^0_{q^3}$, and $M^0_{[qq][qq]\bar{q}}$ for the normal meson, baryon, and pentaquark systems. In a previous paper, we applied the present method to the $uudd\bar{s}$ system and studied the properties of Θ^+. In the results, it was predicted that the three narrow states, $I = 0$, $J^P = 1/2^+, 3/2^+$ states, and the $I = 1$, $J^P = 3/2^-$ state may degenerate in almost the same mass region. $M^0_{[qq][qq]\bar{q}} = 2375$ MeV was chosen to fit the theoretical mass to the observed Θ^+ mass.

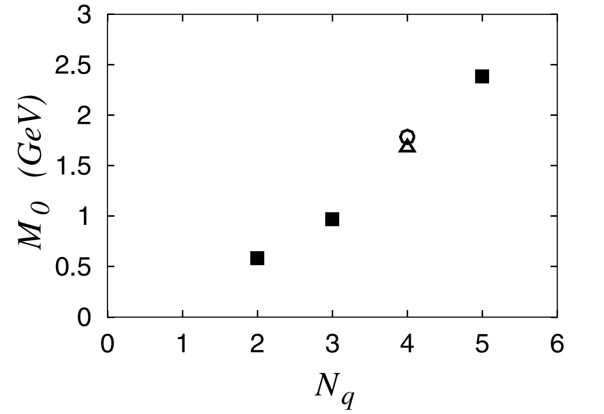

FIG. 3. Adopted zero-point energy (M^0) of the string potential as a function of the quark number N_q. $M^0_{q\bar{q}}$, $M^0_{q^3}$, and $M^0_{[qq][qq]\bar{q}}$ are adjusted to reproduce the ρ-meson, nucleon, and Θ^+ masses, respectively. The $M^0_{[qq][\bar{q}\bar{q}]}$ for the $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ is deduced by assuming the linear function $M^0(N_q) = a_0 N_q + b_0$, where the parameters a_0 and b_0 are determined by fitting (i) $M^0(N_q = 2) = M^0_{q\bar{q}} = 584$ MeV and $M^0(N_q = 5) = M^0_{[qq][qq]\bar{q}} = 2375$ MeV, or (ii) $M^0(N_q = 3) = M^0_{q^3} = 972$ MeV and $M^0(N_q = 5) = M^0_{[qq][qq]\bar{q}} = 2375$ MeV. The circle and triangle indicate the $M^0_{[qq][\bar{q}\bar{q}]}$ values obtained by the former fitting (i) and the latter one (ii), respectively.

In Fig. 3, the adopted $M^0_{q\bar{q}}$, $M^0_{q^3}$, and $M^0_{[qq][qq]\bar{q}}$ are shown as a function of the quark number (N_q). If the string potential is assumed to be a two-body linear potential, $-a_s \sum_{ij} F_i F_j (r - r_0)$, the potential is equivalent to the approximated flux-tube potential in Eqs. (2)–(5) with the relations $\sigma = \frac{4}{3} a_s$ and $M^0 = \frac{\sigma r_0}{2} N_q$. As a result, M^0 should be proportional to N_q in the pairwise confining potential,

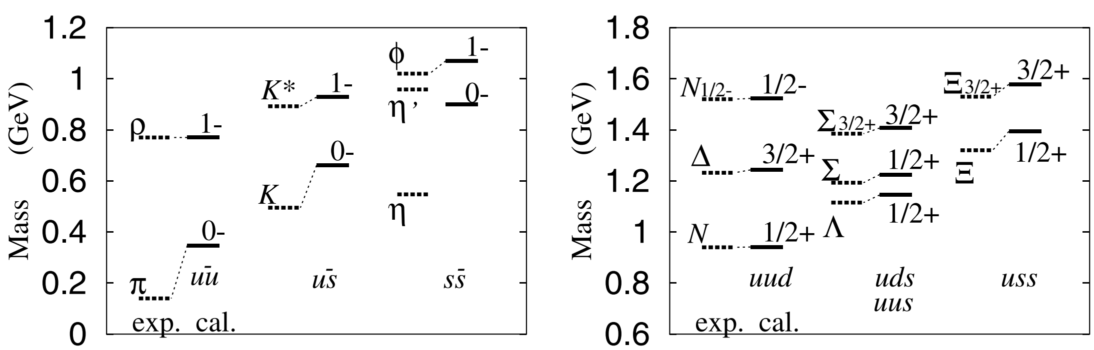

FIG. 2. Mass spectra for $q\bar{q}$ mesons and $3q$ baryons. The experimental and calculated masses are shown by dashed and solid lines, respectively. We adjust the zero-point energy of the string potential for the $q\bar{q}$ system as $M^0_{q\bar{q}} = 584$ MeV to fit the experimental ρ-meson mass. For the $3q$ system, $M^0_{q^3}$ is chosen to be $M^0_{q^3} = 972$ MeV to reproduce the nucleon masses.

which leads to the relation $M^0_{[qq][qq]\bar{q}}/M^0_{q^3} = 5/3$. However, as shown in Fig. 3, the ratio $M^0_{[qq][qq]\bar{q}}/M^0_{q^3}$ is larger than 5/3 in the present model, and also in the pentaquark study with a constituent quark-model calculation in Ref. [27]. This means that we need an extra attraction in the $[qq][qq]\bar{q}$ system in addition to the pairwise confining potential to understand the absolute mass of the $\Theta^+(1.54)$ within constituent quark models. We consider the dependence of M^0 on the tube topology as the many-body potential, and we here accept the value $M^0_{[qq][qq]\bar{q}} = 2375$ MeV [26] adjusted to the experimental Θ^+ mass. Then we phenomenologically determine the $M^0_{[qq][\bar{q}\bar{q}]}$ for $4q$ states by using the $M^0_{[qq][qq]\bar{q}}$ as an input as follows.

We assume the linear function $M^0(N_q) = a_0 N_q + b_0$ and determine the parameters (a_0, b_0) by fitting the M^0 values for the $q\bar{q}$ and $[qq][qq]\bar{q}$ systems as (i) $M^0(N_q = 2) = M^0_{q\bar{q}} = 584$ MeV and $M^0(N_q = 5) = M^0_{[qq][qq]\bar{q}} = 2375$ MeV. We also use another parameter set for (a_0, b_0) by fitting the M^0 values for the q^3 and $[qq][qq]\bar{q}$ systems as (ii) $M^0(N_q = 3) = M^0_{q^3} = 972$ MeV and $M^0(N_q = 5) = M^0_{[qq][qq]\bar{q}} = 2375$ MeV. With the obtained parameter sets a_0 and b_0, we obtain $M^0_{[qq][\bar{q}\bar{q}]} = 1785$ MeV and $M^0_{[qq][\bar{q}\bar{q}]} = 1679$ MeV from $M^0(N_q) = a_0 N_q + b_0$, $(N_q = 4)$ for the former fitting (i) and the latter one (ii), respectively.

Now, we apply the flux-tube quark model with AMD to the $(ud)_{\bar{3}}(\bar{s}\bar{s})_3$ system and calculate the $\vartheta^+(J^P = 1^+)$ mass. We use the above-determined zero-point energies, (i) $M^0_{[qq][\bar{q}\bar{q}]} = 1785$ MeV and (ii) $M^0_{[qq][\bar{q}\bar{q}]} = 1679$ MeV. The calculated ϑ^+ $(I = 0, J^P = 1^+)$ mass is 1.37 GeV in case (i) and 1.46 GeV in case (ii). The results indicate that the $\vartheta^+(J^P = 1^+)$-meson may exist around 1.4 GeV, near the KK^* threshold (Fig. 4).

Although we do not put *a priori* assumptions for spin and spatial configurations of 4 particles, the $\vartheta^+(J^P = 1^+)$ wave function obtained by the energy variation is dominated by the component with the spin-zero $(ud)_{\bar{3}}$ and the spin-one $(\bar{s}\bar{s})_3$ in the spatially symmetric orbit, $(0s)^4$. The spin-zero $(ud)_{\bar{3}}$ gain the color-magnetic interaction, while only the spin-one configuration is allowed in the spatially symmetric $(\bar{s}\bar{s})_3$ pair. Therefore our result is consistent with the naive expectation in the diquark picture.

We comment on the accuracy of approximations Eqs. (3)–(5) for the tube length in the Hamiltonian. As discussed in a previous paper [26], the tube length (L_f) is reasonably simulated by the approximated tube length L_{app} given by Eqs. (3)–(5), while L_{app} is exactly equal to L_f in the $q\bar{q}$ system. If we assume the harmonic oscillator $(0s)^{N_q}$ configurations and ignore the antisymmetrization of quarks, we can calculate the ratio of the L_{app} to the exact tube length (L_f), which is denoted by $L^{(0s)}_{app}/L^{(0s)}_f$. The ratio $L^{(0s)}_{app}/L^{(0s)}_f$ is 0.91, 0.86, and 0.84 for the q^3, $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$, and $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$ systems, respectively. In order to examine the effect of this factor on the tetraquark mass, we scale the tube length as $L_f \approx L^{(0s)}_f/L^{(0s)}_{app} \times L_{app}$ and estimate the expectation value of the Hamiltonian for the present wave functions. With the obtained energies, we retune the M_0 by fitting the ρ-meson, nucleon, and pentaquark masses, and reexamine the ϑ mass. We then find that the modification of the tetraquark mass by the scaled L_f is slight; the $\vartheta^+(J^P = 1^+)$ mass can decrease by 10 MeV for case (i) and 4 MeV for case (ii).

C. f_1-meson

Here, we discuss the possibility of a $f_1(J^PC = 1^{++})$-meson with the $4q$ component. Since its dominant decay mode should be $K\bar{K}^*$, it may have some analogy with the tetraquark $\vartheta(J^PC = 1^{++})$. If we ignore the $q\bar{q}$ annihilation, we can calculate the mass of the $J^{PC} = 1^{++}$ $(us)_{\bar{3}} \times (\bar{u}\bar{s})_3$ state within the present framework in the same way as for the tetraquark ϑ-meson. The $(us)_{\bar{3}}(\bar{u}\bar{s})_3$ $(J^{PC} = 1^{++})$ state corresponds to an isoscalar f_1-meson and an isovector a_1-meson, which degenerate in the present Hamiltonian. We concentrate on the f_1-meson in the present paper. By using the same zero-point energies (i) $M^0_{[qq][\bar{q}\bar{q}]} =$

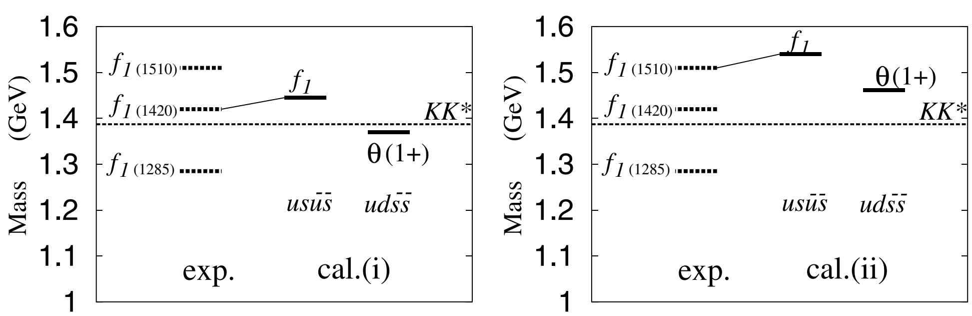

FIG. 4. Masses of the $\vartheta^+(J^{PC} = 1^+)$-meson and f_1-mesons. The theoretical values (solid lines) shown in the left and right panels were obtained by using (i) $M^0_{[qq][\bar{q}\bar{q}]} = 1785$ MeV and (ii) $M^0_{[qq][\bar{q}\bar{q}]} = 1679$ MeV, respectively. The mass of the f_1 with the $4q$ component was obtained by calculations of the $us\bar{u}\bar{s}$ with $J^{PC} = 1^{++}$. The dashed lines are experimental masses of the f_1-mesons. The experimental KK^* threshold energy is also displayed by dotted lines.

1785 MeV and (ii) $M^0_{[qq][\bar{q}\bar{q}]} = 1679$ MeV, the $(us)_{\bar{3}}(\bar{u}\bar{s})_3$ state with $J^{PC} = 1^{++}$ for the f_1-meson is calculated to be 1.45 GeV in case (i) and 1.54 GeV in case (ii) (Fig. 4).

There are various theoretical interpretations of scalar and axial-vector mesons as P-wave $q\bar{q}$ states, $4q$ states, and hybrid $q\bar{q}g$ states. In the mass region 1–1.6 GeV, three f_1-mesons, $f_1(1285)$, $f_1(1420)$, and $f_1(1510)$, are known, though the $f_1(1510)$ is not well established [43]. In the P-wave $q\bar{q}$ state, two f_1-mesons are expected to appear in this energy region as partners in the $q\bar{q}$ nonet. It is considered that the lower one is dominated by the light-quark component ($n\bar{n} \equiv u\bar{u} + d\bar{d}$), and the major component of the higher one is the $s\bar{s}$ state. In the standard interpretation, the lowest $f_1(1285)$ is regarded as the $n\bar{n}$ state. On the other hand, $f_1(1420)$ and $f_1(1510)$ are candidates for the partner of the $f_1(1285)$ in the $q\bar{q}$ nonet, but the assignment is not yet confirmed.

In the constituent quark-model calculation of $q\bar{q}$ systems [44], the masses of two 1^{++} states in the P-wave $q\bar{q}$ nonet are 1.24 and 1.48 GeV. The theoretical mass spectra of the 1^{++} $q\bar{q}$ states seems to be consistent with the experimental ones if $f_1(1510)$ is assigned to be a partner of the $f_1(1285)$ in the flavor nonet. This is consistent with the assignment in Ref. [45]. On the other hand, an alternative interpretation that the $f_1(1285)$ and $f_1(1420)$ are $q\bar{q}$ partners is claimed in Refs. [12,43,46].

These interpretations lead to an indication that one of $f_1(1420)$ and $f_1(1510)$ may be a non-$q\bar{q}$ meson, while the other can be understood as being partners of the $f_1(1285)$ in the conventional P-wave $q\bar{q}$ states. In the present calculation of the $(us)_{\bar{3}}(\bar{u}\bar{s})_3$ state with $J^{PC} = 1^{++}$, the theoretical mass in case (i) 1.45 GeV seems to be consistent with the $f_1(1420)$, while the mass in case (ii) 1.54 GeV energetically corresponds to the $f_1(1510)$, as shown in Fig. 4. The present results suggest the assignment of a f_1-meson in the ~1.4–1.6 GeV mass region as the $4q$ state.

D. Width of ϑ^+-meson

As mentioned above, we suggest that the $\vartheta^+(J^P = 1^+)$-meson may appear in the energy region ~1.4 GeV near the KK^* threshold. The expected decay modes are KK^* and $KK\pi$. The width for the $KK\pi$ decay can be enhanced by the broad resonance, $\kappa(800)$ state, in the scalar $K\pi$ channel. On the other hand, the phase space for the direct three-body decay is generally suppressed. In fact, the typical width of three-body decays is of the order of a several MeV in the ω-meson width, for example. Therefore, if the branchings into two-hadron decays, KK^* and $K\kappa(800)$, are small enough, the width should be narrow. In order to discuss the stability of the ϑ^+-meson, we here consider here only the two-hadron decay modes. We should note that the decay mechanism of the $\vartheta^+(J^P = 1^+)$ may be analogous with that of the f_1-meson with the $us\bar{u}\bar{s}(J^{PC} = 1^{++})$ state, where the $K\bar{K}^*$ and $K\bar{K}\pi$ $(K\bar{\kappa})$ are expected to be dominant decay modes. In fact, in the decay of $f_1(1420)$ and $f_1(1510)$, which are the candidates of the $us\bar{u}\bar{s}(J^{PC} = 1^{++})$, as mentioned before, the $K\bar{K}^*$ and/or $K\bar{K}\pi$ modes are experimentally seen, and the former is dominant in the $f_1(1420)$ decay modes [43]. For decay into the $K\bar{K}^*$, the S-wave channel is allowed, while the $K\bar{\kappa}$ decays should be a P-wave.

First, we give a rough estimation of the ϑ^+ width for the KK^* decay by assuming that the coupling for $f_1 \to K\bar{K}^*$ ($g_{f_1K\bar{K}^*}$) is the same as that for $\vartheta^+ \to KK^*$ ($g_{\vartheta KK^*}$). The width is approximated by the product of the coupling and the phase space. We take into account only the S-wave decay, and estimate the phase space for the KK^* decay by the imaginary part of the one-loop self-energy integral $I(p)$ for the scalar mesons,

$$\mathrm{Im}[I(p)] = \frac{1}{16\pi^2}\,\mathrm{Im}\Big[-a_1\ln(m_1^2) - a_2\ln(m_2^2) - q\ln\frac{(q+a_1)(q+a_2)}{(q-a_1)(q-a_2)}\Big], \quad (11)$$

where

$$q \equiv \frac{1}{2}\Big[\Big(1-\frac{(m_1+m_2)^2}{p^2}\Big)\Big(1-\frac{(m_1-m_2)^2}{p^2}\Big)\Big]^{1/2}$$
$$a_1 \equiv \frac{1}{2}\Big(1-\frac{m_1^2-m_2^2}{p^2}\Big) \quad a_2 \equiv \frac{1}{2}\Big(1-\frac{m_2^2-m_1^2}{p^2}\Big), \quad (12)$$

and m_1 and m_2 are the masses of the daughter particles. We take the rest frame $p = (p_0, \mathbf{0})$. As is well known, if the m_1 and m_2 are real values, $\mathrm{Im}[I(p)]$ is equal to the usual phase space q above the threshold ($p_0 > m_1 + m_2$), and is zero below the threshold ($|m_2 - m_1| < p_0 < m_1 + m_2$). However, since the daughter particle K^* has a width of $\Gamma_{K^*} \sim 50$ MeV for $K\pi$ decays, the phase space should be finite, even below the threshold. In order to estimate the effect of Γ_{K^*} on the phase space, we take the masses of the daughter particles to be $m_1 = m_{K^*} - i\Gamma_{K^*}/2$ and $m_2 = m_K$ in Eq. (12), as is done in Ref. [47]. We use the masses of K and K^* as $m_K = 0.495$ GeV and $m_{K^*} = 0.892$ GeV, and

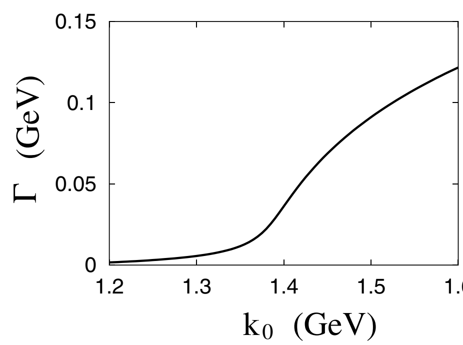

FIG. 5. Energy dependence of the two-meson $KK^*(892)$ decay width Γ. The coupling is chosen to fit the full-width $\Gamma = 55$ MeV of the $f_1(1420)$ at $m_0 = 1.42$ GeV, and is assumed to be energy independent.

evaluate the width of the parent particle as $\Gamma = 2g^2 \,\mathrm{Im}[I(m_0)]$, where m_0 is the parent particle mass and g is the coupling.

We show the mass m_0 dependence of the width $\Gamma(m_0)$ for the KK^* decays in Fig. 5, where the coupling g is adjusted to reproduce the experimental full width $\Gamma_{f_1(1420)} = 56$ MeV at $m_0 = 1.42$ GeV, and is assumed to be energy independent. If we adopt the case (i) calculation and the assignment of the $f_1(1420)$ as the $4q$ state, the width of the $\vartheta^+(1^+)$ is estimated to be $\Gamma_\vartheta \sim 20$ MeV. When we assign the $f_1(1510)$ as the $4q$ state and choose the coupling constant g to fit the width 100 MeV of the $f_1(1510)$ at $m_0 = 1.51$ GeV, we obtain $\Gamma_\vartheta \sim 80$ MeV based on the case (ii) calculation. Here, we adopt the upper limit of the $f_1(1510)$ width in Ref. [43], though there still remains an ambiguity in the experimental data.

Next, we discuss the effect of $K\kappa$ decay on the width. In the above estimation of Γ_ϑ, the effect of this mode is already included in the total width of the f_1-meson. In contrast to the S-wave decay in the KK^* channel, the 1^+ states decay into the P-wave $K\kappa$ state. Even though the P-wave decay is unfavored compared to the S-wave decay, we should consider its effect, because the $K\kappa$ threshold ($\sim$ 1.3 GeV) is lower than the KK^* threshold (1.39 GeV). We again assume that the coupling for $f_1 \to K\bar{\kappa}$ is the same as that for $\vartheta^+ \to K\kappa$, and estimate the width by using the ratio of the phase space for the ϑ ($m_0 = 1.37$ GeV), q_ϑ, to that for the $f_1(1420)$ ($m_0 = 1.42$ GeV), q_{f_1}, as $\Gamma_\vartheta = \Gamma_{f_1} \times q_\vartheta / q_{f_1}$. Since $\kappa(800)$ is a broad resonance with $\Gamma = 300 \sim 800$ MeV, we may not apply the previous method by the single-pole approximation of the propagator in the one-loop diagram to estimate the phase space. Instead, we consider the phase space for the P-wave decay, $|m_0 - m_\kappa - m_K|^{3/2}$, and take into account the broad mass range of m_κ, where m_κ is the κ mass and is a real value. When we adopt the case (i) calculation, we obtain the ratio (q_ϑ / q_{f_1}) of the phase space for the ϑ ($m_0 = 1.37$ GeV) to that for the $f_1(1420)$ ($m_0 = 1.42$ GeV) as 0.5 for $m_\kappa = 800$ MeV. The ratio q_ϑ / q_{f_1} is 0.7 for the lower limit $m_\kappa = 640$ ($K\pi$ threshold) MeV, while $q_\vartheta / q_{f_1} = 0$ for $m_\kappa \geq 900$ MeV. On the average, the ratio q_ϑ / q_{f_1} in the $K\kappa$ decay is almost the same as that in the KK^* decay. Even if the branching ratio of the $K\kappa$ ($K\bar{\kappa}$) mode is 100% in $\vartheta^+(1^+)$ (f_1), an upper limit of $\Gamma_\vartheta = 40$ MeV is obtained from the $\Gamma_{f1} = 56$ MeV and the lower limit of m_κ. Also, in calculation (ii), the phase space ratio (q_ϑ / q_{f_1}) for $f_1(1510) \to K\bar{\kappa}$ to that for the $\vartheta \to K\bar{\kappa}$ in the mass range $m_\kappa \geq 640$ MeV is calculated as $q_\vartheta / q_{f_1} < 0.8$, which is consistent with that in the KK^* decay. As a result, it is concluded that the ϑ-meson width is estimated to be $\mathcal{O}$(20–80 MeV), no matter how great the branching ratio in the KK^* channel and that in the $K\kappa$ channel.

In the above discussion, we roughly evaluate the width of the $\vartheta^+(1^+)$-meson by assuming the same coupling for $\vartheta^+(1^+) \to KK^*$ and $\vartheta^+(1^+) \to K\kappa$ as those for $f_1 \to K\bar{K}^*$ and $f_1 \to K\bar{\kappa}$, respectively. In the present estimation we choose the couplings to fit the full width of f_1. In the case that the couplings are enhanced via the annihilation and creation of a $q\bar{q}$ pair in the f_1-meson, the width of the $\vartheta^+(1^+)$-meson might be smaller than the present estimation.

V. SUMMARY AND DISCUSSION

We discussed the possibility of the $J^P = 1^+$ state of the isoscalar tetraquark ($S = +2$), ϑ^+-meson, with the $ud\bar{s}\bar{s}$ content. If the pentaquark $\Theta^+(1540)$ has the $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$ configuration, the $\vartheta^+(J^P = 1^+)$ is expected to exist at an energy lower than, or close to, the pentaquark $\Theta^+(1540)$ mass. This leads to a possible appearance of the ϑ^+-meson as a resonant state, which may be observed in the $K^+K^+\pi^-$ channel.

We investigated the $\vartheta^+(J^P = 1^+)$ with a constituent quark model. The flux-tube quark model with antisymmetrized molecular dynamics (AMD) was applied to $4q$ systems in the same way as in the pentaquark study in Ref. [26]. Based on the picture of a flux-tube model, we solved the $4q$ dynamics in the $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ model space by the variational method. The results suggest that the $\vartheta^+(J^P = 1^+)$ may exist at around 1.4 GeV. Since the predicted mass of the $\vartheta^+(J^P = 1^+)$ is close to the lowest (KK^*) threshold in the allowed two-hadron decays, the present results imply that the $\vartheta^+(J^P = 1^+)$ may exist as a resonance, and its width may be not very broad.

We also calculated the $J^{PC} = 1^{++}$ $(us)_{\bar{3}}(\bar{u}\bar{s})_3$ state, which is associated with a non-$q\bar{q}$ f_1-meson. The calculated mass of the f_1 with the $4q$ configuration suggests an interpretation that the $4q$ state may correspond to one of the f_1-mesons in the ~1.4–1.6 mass region. $f_1(1420)$ and $f_1(1510)$ are candidates of the $4q$ state. If we assume that the couplings for $f_1 \to K\bar{K}^*$ and $f_1 \to K\bar{\kappa}^*$ are the same as those for $\vartheta^+(J^P = 1^+) \to KK^*$ and $\vartheta^+(J^P = 1^+) \to K\kappa$, respectively, we can evaluate the width of the ϑ^+ to be 20–80 MeV from the phase space for these two-hadron decay modes. Provided that the coupling for the two-hadron decays in the $\vartheta^+(J^P = 1^+)$ is small enough, the dominant decay should be a direct three-hadron decay, $\vartheta^+ \to KK\pi$, with a small phase space, and hence the $\vartheta^+(J^P = 1^+)$ should have a small width.

Recently, the ϑ^+-meson was discussed by Burns *et al.* [34] and by Karliner and Lipkin [35]. In Ref. [35], it was mentioned that the $\vartheta(J^P = 0^+)$ can be narrow, because the lowest allowed decay mode is a four-body $KK\pi\pi$ channel with a small phase space. However, the $J^P = 0^+$ state is forbidden in the isoscalar $ud\bar{s}\bar{s}$ system within spatially symmetric configurations, and, hence, the $\vartheta(J^P = 0^+)$ is expected to be energetically unfavored. Burns *et al.* predicted the $\vartheta^+(J^P = 1^-)$ with $L = 1$ at ~1.6 GeV with a width of O(10–100) MeV, which can decay into K^+K^0. Although the color-magnetic attraction may be larger in the $\vartheta(J^P = 1^-)$ state than in the $\vartheta(J^P = 1^+)$ state, the $\vartheta(J^P = 1^-)$ must have the $L = 1$ excitation energy. Another claim for the $\vartheta^+(J^P = 1^-)$ state is that it can decay into P-wave KK states. The centrifugal barrier may not be high enough to stabilize the state much above the threshold energy. Therefore, we consider that the $\vartheta^+(J^P = 1^+)$ is a better candidate of narrow tetraquarks.

Our calculations of the tetraquark $\vartheta^+(J^P = 1^+)$ and the pentaquark Θ^+ are based on the color-configurations $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ and $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$. We ignore $(qq)_6$ configurations because the $(qq)_6$ is an excited configuration, and is unfavored in the strong-coupling picture. We should remark that the preference of the spin-parity $J^P = 1^+$ in the ϑ^+ system does not depend on the color configuration. This is because, in the isoscalar $ud\bar{s}\bar{s}$ with a spatially symmetric configuration, other spin parities are forbidden, and therefore the spin parity is uniquely determined to be $J^P = 1^+$. This means that, even in the color-configuration $(ud)_6(\bar{s}.05pt\bar{s})_{\bar{6}}$, as suggested in Refs. [34,35], only $J^P = 1^+$ is allowed in the spatially symmetric ϑ state. If the Θ^+ has a triquark-diquark structure $((ud)_6\bar{s})_3(ud)_{\bar{3}}$ with the relative P-wave motion as proposed by Karliner and Lipkin [24], it is expected that the $\vartheta(J^P = 1^+)$ mass with a spatially symmetric $(ud)_6(\bar{s}\bar{s})_{\bar{6}}$ configuration would be close to, or could be smaller than, the Θ^+ mass due to the kinematic energy gain. Another important channel is the meson-meson $(q\bar{q})_1(q\bar{q})_1$ configuration, though it can be expressed by a linear combination of the $(ud)_{\bar{3}}(\bar{s}\bar{s})_3$ and $(ud)_6(\bar{s}\bar{s})_{\bar{6}}$ configurations. If the $\vartheta^+(J^P = 1^+)$ mass is smaller than the lowest meson-meson threshold KK^*, mixing of the $(q\bar{q})_1(q\bar{q})_1$ state may not have a significant effect on the ϑ width, because the $\vartheta^+(J^P = 1^+)$ is bound in the KK^* channel. In the case that the $\vartheta^+(J^P = 1^+)$ is heavier than the KK^* threshold, channel mixing should be taken into consideration to estimate its width. In IV D, we gave an analysis of the widths by assuming that the decay mechanism and the coupling in the $\vartheta^+(J^P = 1^+)$ are the same as those in the f_1-meson. Within this assumption, the channel-coupling effect on the widths is effectively included in the adopted total width of the f_1-meson. For a further detailed discussion of the stability of the $\vartheta(J^P = 1^+)$, coupled calculations of different color configurations are required.

We should point out that the allowed decay channels are different among these three predictions ($J^P = 1^+$, 1^-, and 0^+ states) of the ϑ^+-mesons. For the $\vartheta^+(J^P = 1^+)$, $\vartheta^+(J^P = 1^-)$, and $\vartheta^+(J^P = 0^+)$, the decay modes are $KK\pi$, KK, and $KK\pi\pi$, respectively. In order to establish the $ud\bar{s}\bar{s}$ content in the tetraquark ϑ^+-meson, it is necessary to observe not K^0, but at least two K^+s, because the K^0 contains the $s\bar{d}$ component as well as the $d\bar{s}$. In that sense, the $K^+K^+\pi^-$ decay from the $\vartheta(J^P = 1^+)$ predicted in the present work is suitable for an experimental tetraquark search.

In the constituent quark-model calculations of Refs. [4,6], there is no indication for the existence of multiquark hadrons, except for the $f_0(980)$ and $a_0(980)$ as $K\bar{K}$ molecules [4], while interpretations of scalar mesons, like $f_0(600)$, $\kappa(800)$, and $D_{sJ}(2317)$, by four-quark states have been suggested in several quark-model calculations [1–3,48–51]. In Ref. [6], the absence of the pentaquark was claimed, which is inconsistent with the observation of the pentaquark Θ^+. In order to explain the absolute mass of the $\Theta^+(1540)$ within constituent quark models, we need an extra attraction for the multiquark system in addition to a pairwise interaction. In the present calculation, we did not obtain a quantitative value of this extra attraction from fundamental theory; instead, the many-body potential was taken into account in terms of the flux-tube potential, and we phenomenologically evaluated it by using the observed Θ^+ mass as a input.

In the present work, we used a simple Hamiltonian with the confining force, and the Coulomb and color-magnetic terms in the OGE potential. For a systematic description of the hadron spectra, there still remain such problems as fine-tuning of the interaction parameters and inclusion of the tensor and spin-orbit interactions in the Hamiltonian. To obtain theoretical insights into multiquark hadron physics, an experimental search for the tetraquark is necessary as well as further experimental studies on the pentaquark. We conclude that the $\vartheta^+(J^P = 1^+)$-meson is proposed as a good candidate of the tetraquark, which would be observed in the $K^+K^+\pi^-$ decay channel.

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