Scattering of neutrinos on a polarized electron target as a test for new physics beyond the standard model

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In this paper, we analyze the scattering of the neutrino beam on the polarized electron target, and predict the effects of two theoretically possible scenarios beyond the standard model. In both scenarios, our results are presented in a massless neutrino limit. First, we consider how the existence of CP violation phase between the complex vector V and axial A couplings of the left-handed neutrinos affects the azimuthal dependence of the differential cross section. The azimuthal angle $\phi_{e'}$ of outgoing electron momentum is measured with respect to the transverse component of the initial electron polarization $\boldsymbol{\eta}_e^\perp$. We indicate the possibility of using the polarized electron target to measure the CP violation in the $\nu_\mu e^-$ scattering. The future superbeam and neutrino factory experiments will provide the unique opportunity for the leptonic CP violation studies, if the large magnetized sampling calorimeters with good event reconstruction capabilities are build. Next, we take into account a scenario with the participation of the exotic complex scalar S coupling of the right-handed neutrinos in addition to the standard real vector V and axial A couplings of the left-handed neutrinos. The main goal is to show how the presence of the R-handed neutrinos, in the above process changes the spectrum of recoil electrons in relation to the expected standard model prediction, using the current limits on the nonstandard couplings. The interference terms between the standard and exotic couplings in the differential cross section depend on the angle α between the transverse incoming neutrino polarization and the transverse electron polarization of the target. The detection of the dependence on this angle in the energy spectrum of recoil electrons would be a signature of the presence of the R-handed neutrinos in the neutrino-electron scattering. To make this test feasible, the polarized artificial neutrino source needs to be identified.

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I. INTRODUCTION

Standard model (SM) of electro-weak interactions [1–3] has a vector-axial ($V - A$) Lorentz structure [4], i.e. only left-handed (L-handed chirality states) and massless Dirac neutrinos may take part in the charged and neutral weak interactions. The observed CP violation in the decays of neutral kaons and B-mesons [5] is described by a single phase of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix (CKM) [6].

The vector g_V^L and axial-vector g_A^L neutral-current coupling constants are assumed to be real numbers, which means that $\mathrm{Im}(g_V^L) = \mathrm{Im}(g_A^L) = 0$. The values of these two couplings are derived from neutrino-electron scattering and from $e^+e^- \to l^+l^-$ annihilation studies, but in the fitting procedure the imaginary parts are fixed to their standard model values ([7], page 6).

However, in the general case of complex g_V^L and g_A^L couplings, we have one additional free parameter: the relative phase between these couplings denoted as β_{VA}. The CP-odd interference contribution enters the differential cross section for the scattering of left-handed neutrinos on the polarized electron target (PET), if $|\sin(\beta_{VA})| \neq 0$. The experimental measurement of the azimuthal angle $\phi_{e'}$ of outgoing electron momentum could be used to test the CP symmetry in lepton sector of electro-weak interactions. The observation of asymmetry in the angular distribution of recoil electrons, caused by the interference terms between the standard complex couplings would give additional information about the coupling constants.

The magnetized sampling calorimeters (e.g. MINOS far detector [8]) are composed of many steel layers, which are magnetized using a coil through a hole in the center of the planes to an average field of about 1.5 T. In a piece of magnetized iron, there are lots of unpaired electrons all pointing the same direction. The Moeller polarimeters determine the polarization of the electron beam by measuring the cross section asymmetry in the scattering of polarized electrons by polarized electrons. Polarized electrons are scattered off a polarized ferromagnetic foil, and the foil polarization is determined by measurements of the magnetization of the foil and its thickness. So, there is the well known and commonly used in accelerator physics technique for developing PETs.

Although the SM agrees well with all experimental data up to available energies, the experimental precision of

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present measurements still does not rule out the possible participation of the exotic scalar S, tensor T and pseudoscalar P couplings of the right-handed (R-handed chirality states) Dirac neutrinos beyond the SM [9]. The current upper limits on the all nonstandard couplings, obtained from the normal and inverse muon decay, are presented in the Table I [10]. The coupling constants are denoted as $g^{\gamma}_{\epsilon\mu}$, where $\gamma = S, V, T$ indicates the type of weak interaction, i.e. scalar S, vector V, tensor T; $\epsilon, \mu = L, R$ indicate the chirality of the electron or muon and the neutrino chiralities are uniquely determined for given γ, ϵ, μ. It means that the neutrino chirality is the same as the associated charged lepton for the V interaction, and opposite for the S, T interactions [10,11]. In the SM, only g^{V}_{LL} is nonzero value.

New effects due to the exotic right-handed weak interactions (ERWI) could be detected by the measurement of neutrino observables (NO) which consist only of the interferences between the standard $V - A$ and ERWI, and do not depend on the neutrino mass. The NO include the information on the transverse components of neutrino spin polarization (TCNSP), both T-even and T-odd. These quantities vanish in the SM, so the detection of the nonzero values of the TCNSP would be a direct signature of the R-handed neutrino presence in the weak interactions. The scattering of intense and polarized neutrino beam, coming from the artificial neutrino source, on the polarized electron target could detect the effects from the ERWI. Presently the measurement of such observables is only theoretically possible. We give an example how the polarized neutrino flux can be produced if the exotic scalar coupling C^{R}_{S} is present in the theory of muon capture interaction.

The main goal of the first part of our paper (Sec. II) is to show that the differential cross section for the $(\nu_{\mu} e^{-})$ scattering of left-handed and longitudinally polarized muon neutrinos on the PET may be sensitive to the CP-violating effects, if one assumes the complex standard couplings g^{L}_{V}, g^{L}_{A}. The main goal of the other part (Sec. III) is to show how the presence of the R-handed neutrinos, in the $(\nu_{\mu} e^{-})$ scattering process, changes the energy spectrum of recoil electrons in relation to the expected SM prediction, using the current limits on the nonstandard couplings [10]. We concern the scattering of transversely polarized muon-neutrino beam on the PET to probe ERWI effects and include a theoretical discussion of the possibility of developing such a beam.

TABLE I. Current limits on the nonstandard couplings.

Coupling constants	SM	Current limits
$\lvert g^{V}_{LL}\rvert$	1	>0.960
$\lvert g^{V}_{LR}\rvert$	0	<0.060
$\lvert g^{V}_{RL}\rvert$	0	<0.110
$\lvert g^{V}_{RR}\rvert$	0	<0.039
$\lvert g^{S}_{LL}\rvert$	0	<0.550
$\lvert g^{S}_{LR}\rvert$	0	<0.125
$\lvert g^{S}_{RL}\rvert$	0	<0.424
$\lvert g^{S}_{RR}\rvert$	0	<0.066
$\lvert g^{T}_{LL}\rvert$	0	0
$\lvert g^{T}_{LR}\rvert$	0	<0.036
$\lvert g^{T}_{RL}\rvert$	0	<0.122
$\lvert g^{T}_{RR}\rvert$	0	0

Our analysis is model-independent and all the calculations are made in the limit of vanishing neutrino mass with the Michel-Wightman density matrices [12] for the polarized incoming neutrino beam (Appendix B) and for the polarized electron target, respectively. We use the system of natural units with $\hbar = c = 1$, Dirac-Pauli representation of the γ-matrices and the $(+, -, -, -)$ metric [13].

II. LEFT-HANDED NEUTRINO SCATTERING ON A POLARIZED ELECTRON TARGET

The standard model (SM) of electro-weak interactions is based on the gauge group SU(2) × U(1). The left-handed fermion fields $\psi_i = \binom{\nu_i}{l_i^-}$ and $\binom{u_i}{d_i'}$ of the i^{th} fermion family transform as doublets under SU(2), where $d_i' \equiv \sum_j V_{ij} d_j$ and V is the Cabibbo-Kobayashi-Maskawa mixing matrix. The vector and axial-vector couplings in SM are

$$g^{L}_{V}(i) \equiv t^{L}_{3}(i) - 2q(i)\sin^2\theta_W, \qquad g^{L}_{A}(i) \equiv t^{L}_{3}(i), \tag{1}$$

where $t^{L}_{3}(i)$ is the weak isospin of fermion i ($+1/2$ for u_i and ν_i; $-1/2$ for d_i and l_i), q_i is the charge of ψ_i in units of e and θ_W is the weak angle. Because of the model-dependent interpretation of the coupling constants values, they are assumed to be real numbers. For example, the total cross section for high energy neutral-current $(\nu_{\mu} e^{-})$ scattering is [14]

$$\sigma_{\rm SM}(\nu_{\mu} + e^{-} \rightarrow \nu_{\mu} + e^{-}) \simeq \frac{2G_F^2 m_e E_{\nu_{\mu}}}{3\pi} \times (g^{L2}_{V} + g^{L2}_{A} + g^{L}_{V} g^{L}_{A}), \tag{2}$$

but in the model-independent (MI) analysis we obtain:

$$\sigma_{\rm MI}(\nu_{\mu} + e^{-} \rightarrow \nu_{\mu} + e^{-}) \simeq \frac{2G_F^2 m_e E_{\nu_{\mu}}}{3\pi} \times (|g^{L}_{V}|^2 + |g^{L}_{A}|^2 + |g^{L}_{V}||g^{L}_{A}|\cos(\beta_{VA})), \tag{3}$$

where $g^{L}_{V} = |g^{L}_{V}|e^{i\beta^{L}_{V}}$, $g^{L}_{A} = |g^{L}_{A}|e^{i\beta^{L}_{A}}$ are the complex coupling constants, $\mathrm{Re}(g^{L}_{V} g^{L*}_{A}) = |g^{L}_{V}||g^{L}_{A}|\cos(\beta_{VA})$ and $\beta_{VA} = \beta^{L}_{V} - \beta^{L}_{A}$ is the relative phase between the g^{L}_{V} and g^{L}_{A} couplings.

The effective vector and axial-vector neutral coupling constants obtained from the absolute neutrino-electron scattering event rate are

$$g_V^L \simeq 0, \qquad g_A^L \simeq \pm 0.5 \qquad \text{or} \qquad g_V^L \simeq \pm 0.5, \qquad g_A^L \simeq 0. \tag{4}$$

However, from our MI expression (3) one can see that the solution (with CP-violating phase):

$$|g_V^L| = |g_A^L| \simeq 0.35 \qquad \text{and} \qquad \beta_{VA} = \pm\frac{\pi}{2} \tag{5}$$

provides to the same total cross section value as the SM fit (4). In the next subsection we present how the existence of nonzero β_{VA} phase is related to CP-odd interference contribution in the differential cross section. The fermion-antifermion pair production cross-sections have only T-even contributions, but their experimental observations are essential to determine a single solution from possible parameters (4). Even if $\beta_{VA} = 0$ the scattering of left-handed neutrinos on the PET provides a new approach to decide which of the two coupling types, (mainly) pure g_A^L or pure g_V^L coupling, is realized in nature. This approach is model independent in contrast to e^+e^- experiments which make the assumption that the neutral current is dominated by the exchange of a single Z^0.

As is well known, CP violation has been observed only in the decays of neutral kaons and B-mesons. The standard model describes the existing data by a single phase of the CKM matrix. However, the baryon asymmetry of the Universe can not be explained by the CKM phase only, and at least one new source of CP violation is required [15]. The first direct confirmation of a time reversal violation has been published by CPLEAR Collaboration in 1998 [16]. Many nonstandard models take into account new CP-violating phases, and can be probed in observables where the SM CP-violation is suppressed, while alternative sources can generate a sizable effect, e.g. the electric

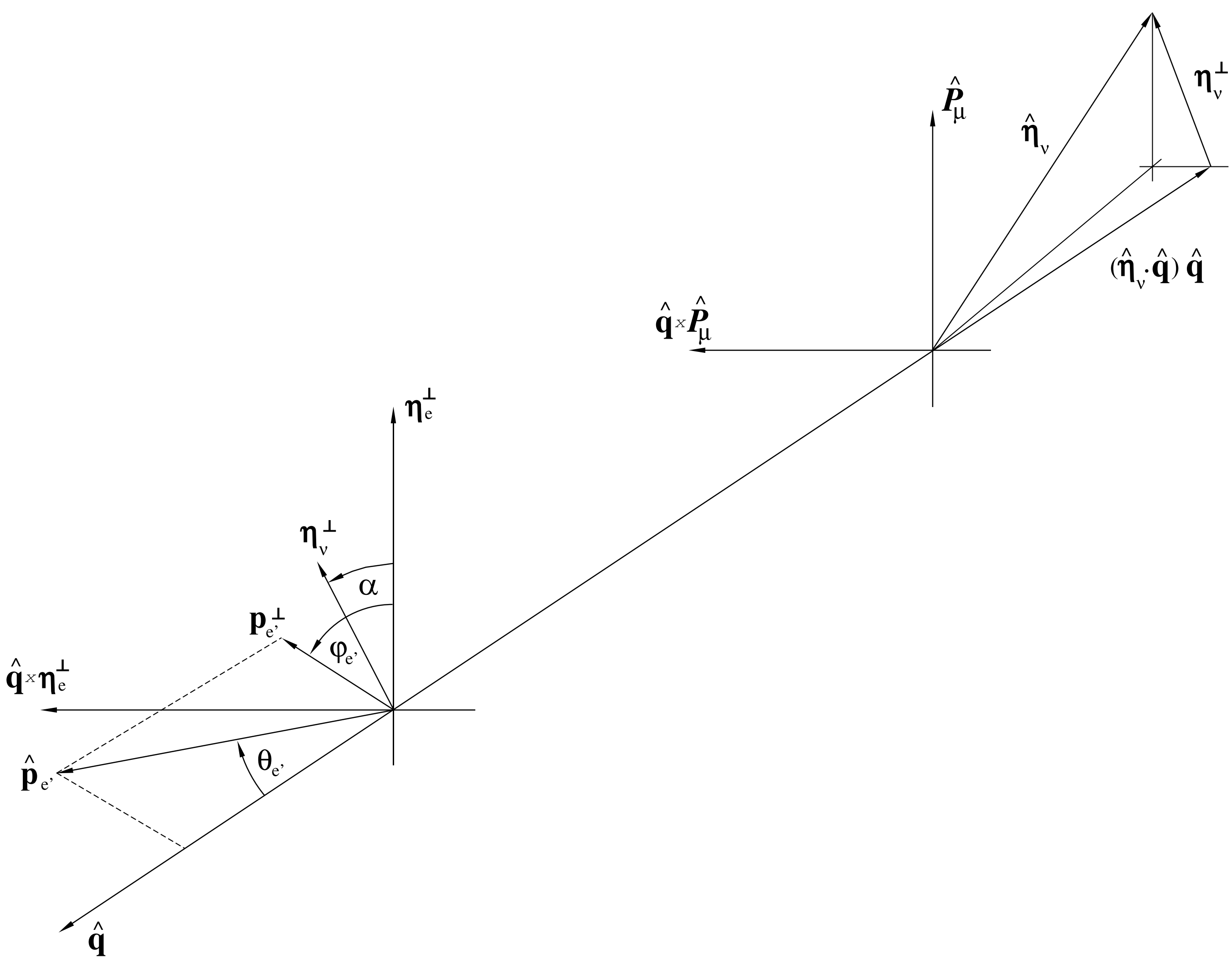

FIG. 1. Figure shows the reaction plane for the $\nu_\mu e^-$ scattering, $\hat{\boldsymbol{\eta}}_\nu$—the unit 3-vector of the initial neutrino polarization in its rest frame, $\boldsymbol{\eta}_e^\perp$—the transverse electron polarization vector of target and the production plane of ν_μ-neutrinos for the reaction of $\mu^- + p \rightarrow n + \nu_\mu$. In Sec. II, the scattering of left-handed neutrinos is considered, which have no transverse polarization $\boldsymbol{\eta}_\nu^\perp = 0$. For the considerations described in Sec. III, the muon capture reaction is used as a source of transversely polarized neutrinos.

dipole moment of the neutron, the transverse lepton polarization in three-body decays of charged kaons K^+ [17,18], transverse polarization of the electrons emitted in the decay of polarized ^{8}Li nuclei [19]. There is no direct evidence of CP violation in the leptonic processes, i.e. a neutrino-electron scattering. However, the future superbeam and neutrino factory experiments [20] will be able to measure the CP violating effects in the lepton sector, where both neutrino and antineutrino oscillation will be observed. We indicate that the scattering of neutrinos on the PET has similar scientific possibilities.

A. CP violation in standard νe scattering

In this subsection, we consider the possibility of the CP violation in the $\nu_\mu e^-$ scattering, when the incoming muon-neutrino beam consists only of the left-handed and longitudinally polarized neutrinos. We assume that these neutrinos are detected in the standard $V-A$ NC weak interactions with the PET and both the recoil electron scattering angle θ'_e and the azimuthal angle of outgoing electron momentum ϕ'_e shown in Fig. 1 are measured with a good angular resolution. Because we allow for the non-conservation of the combined symmetry CP, the amplitude includes the complex coupling constants denoted as g_V^L, g_A^L respectively to the initial neutrino of L-chirality:

$$M_{\nu_\mu e} = \frac{G_F}{\sqrt{2}}\{g_V^L(\overline{u}_{e'}\gamma^\alpha u_e)(\overline{u}_{\nu_{\mu'}}\gamma_\alpha(1-\gamma_5)u_{\nu_\mu}) + g_A^L(\overline{u}_{e'}\gamma^5\gamma^\alpha u_e)(\overline{u}_{\nu_{\mu'}}\gamma_5\gamma_\alpha(1-\gamma_5)u_{\nu_\mu})\}, \quad (6)$$

where u_e and $\overline{u}_{e'}$ (u_{ν_μ} and $\overline{u}_{\nu_{\mu'}}$) are the Dirac bispinors of the initial and final electron (neutrino), respectively. $G_F = 1.16639(1)\times 10^{-5}$ GeV^{-2} [10] is the Fermi constant.

The formula for the differential cross section including the CP-odd contribution $[\hat{\mathbf{q}}\cdot(\hat{\boldsymbol{\eta}}_e\times\hat{\mathbf{p}}_{e'})$ is T-odd and $\mathrm{Im}(g_V^L g_A^{L*}) = |g_V^L||g_A^L|\sin(\beta_{VA})]$, proportional to the magnitude of the transverse electron target spin polarization, with $\hat{\boldsymbol{\eta}}_e \perp \hat{\mathbf{q}}$ is of the form

$$\left(\frac{d^2\sigma}{dyd\phi_{e'}}\right)_{(VA)} = \frac{E_\nu m_e}{4\pi^2}\frac{G_F^2}{2}(1-\hat{\boldsymbol{\eta}}_\nu\cdot\hat{\mathbf{q}})\Bigg\{|g_A^L|^2\left[-\hat{\boldsymbol{\eta}}_e\cdot\hat{\mathbf{p}}_{e'}\sqrt{\frac{2m_e}{E_\nu}+y}\left(\sqrt{y^3}-2\sqrt{y}\right)+\frac{m_e}{E_\nu}y+(y-2)y+2\right] + |g_V^L|^2\left[y^2-\hat{\boldsymbol{\eta}}_e\cdot\hat{\mathbf{p}}_{e'}\sqrt{y^3}\sqrt{\frac{2m_e}{E_\nu}+y}-y\left(\frac{m_e}{E_\nu}+2\right)+2\right] + \mathrm{Im}(g_V^L g_A^{L*})\hat{\mathbf{q}}\cdot(\hat{\boldsymbol{\eta}}_e\times\hat{\mathbf{p}}_{e'})\sqrt{y\left(\frac{2m_e}{E_\nu}+y\right)} + \mathrm{Re}(g_V^L g_A^{L*})\left[\hat{\boldsymbol{\eta}}_e\cdot\hat{\mathbf{p}}_{e'}(y-1)\sqrt{y\left(\frac{2m_e}{E_\nu}+y\right)}+(2-y)y\right]\Bigg\} \quad (7)$$

where $\hat{\boldsymbol{\eta}}_\nu\cdot\hat{\mathbf{q}} = -1$ is the longitudinal polarization of the incoming L-handed neutrino, $\mathbf{q}$—the incoming neutrino momentum, $\mathbf{p}_{e'}$—the outgoing electron momentum, $\hat{\boldsymbol{\eta}}_e$—the unit 3-vector of the initial electron polarization in its rest frame, see Fig. 1. The measurement of the azimuthal angle of outgoing electron momentum $\phi_{e'}$ is only possible when the electron target polarization is known. The polarization vector for electrons is parallel to the magnetic field vector. The variable y is the ratio of the kinetic energy of the recoil electron T_e to the incoming neutrino energy E_ν:

$$y \equiv \frac{T_e}{E_\nu} = \frac{m_e}{E_\nu}\frac{2\cos^2\theta_{e'}}{(1+\frac{m_e}{E_\nu})^2-\cos^2\theta_{e'}}. \quad (8)$$

It varies from 0 to $2/(2+m_e/E_\nu)$. $\theta_{e'}$—the polar angle between the direction of the outgoing electron momentum $\hat{\mathbf{p}}_{e'}$ and the direction of the incoming neutrino momentum $\hat{\mathbf{q}}$ (recoil electron scattering angle), m_e—the electron mass.

After the simplification of the vector products and using the complex number identities in the formula (7) for the cross section, we obtain with $\hat{\boldsymbol{\eta}}_e \perp \hat{\mathbf{q}}$ the new form:

$$\left(\frac{d^2\sigma}{dyd\phi_{e'}}\right)_{(VA)} = \frac{E_\nu m_e}{4\pi^2}\frac{G_F^2}{2}(1-\hat{\boldsymbol{\eta}}_\nu\cdot\hat{\mathbf{q}})\Bigg\{|\boldsymbol{\eta}_e^\perp|\sqrt{\frac{m_e}{E_\nu}y\left[2-y\left(2+\frac{m_e}{E_\nu}\right)\right]}\cdot[\cos(\phi_{e'})(2|g_V^L||g_A^L|\cos(\beta_{VA})y + (2-y)|g_A^L|^2-y|g_V^L|^2)-2|g_V^L||g_A^L|\cos(\phi_{e'}+\beta_{VA})] + [(|g_V^L|^2+|g_A^L|^2)(y^2-2y+2) + 2|g_V^L||g_A^L|\cos(\beta_{VA})y(2-y)-\frac{m_e}{E_\nu}y(|g_V^L|^2-|g_A^L|^2)]\Bigg\}. \quad (9)$$

It can be noticed that the interference terms between the standard $g_{V,A}^L$ couplings depend on the value of the β_{VA} phase. However, the angular asymmetry of recoil electrons is not vanishing even if $\beta_{VA} = 0$. The CP-violating phase enters the cross section and changes the angle at which the number of recoil electrons will be maximal ($\phi_{e'}^{\max}$). For $\beta_{VA} = \frac{\pi}{2}$ and $|g_V^L| = |g_A^L| = 0.354$ this angle is quite large $\phi_{e'}^{\max} \simeq \frac{\pi}{3}$, see Fig. 2. In the case of pure vector g_V^L coupling

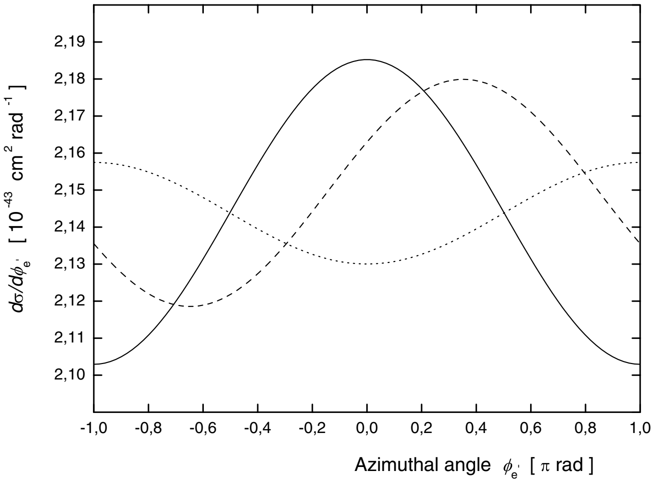

FIG. 2. Plot of the $\frac{d\sigma}{d\phi_e'}{}_{(VA)}$ as a function of the azimuthal angle $\phi_{e'}$ for the $(\nu_\mu e^-)$ scattering, $E_\nu = 1$ GeV, $y = 0.5$, $\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}} = -1$, and $|\boldsymbol{\eta}_e^\perp| = 1$: (a) the case of the pure and real axial-vector coupling, i.e. $g_A^L = 0.5$ and $g_V^L = 0$ (solid line), (b) the case of the pure and real vector coupling, i.e. $g_A^L = 0$ and $g_V^L = 0.5$ (dotted line), (c) CP violation, the case of the complex coupling constants $|g_V^L| = |g_A^L| = 0.354$ with the relative phase $\beta_{VA} = \frac{\pi}{2}$ (dashed line).

we have different azimuthal dependence of the cross section ($\phi_{e'}^{\max} = 0$) than in the case of pure axial vector g_A^L coupling ($\phi_{e'}^{\max} = \pm\pi$).

B. The feasibility of developing the PET

The polarized electron target is a part of all the Moeller polarimeters. The polarized target electrons are produced in ferromagnetic material that is magnetized with using external magnetic field. The target polarization value is determined from measurements of saturation curve and hysteresis loop. At flux density of $\simeq 2T$ the iron becomes magnetically saturated, yielding a target polarization of $\simeq 8\%$ (the magnetic moment per atom $\mu_{Fe} = 2.2\mu_B$ and the number of electrons $Z_{\rm Fe} = 26$) [21]. Thus, we state that the PET with the transverse component of the initial electron polarization $|\boldsymbol{\eta}_e^\perp| \simeq 0.08$ is feasible.

Another issue is event reconstruction, the measurement of the azimuthal angle of outgoing electron momentum ϕ_e' is an challenge. The main advantage of the iron sampling calorimeters with magnetized iron is the ability to measure muon energy by curvature. Magnetized iron allowed one to measure the charged particle momentum by using tracking chambers with an accuracy ordinarily limited by Coulomb scattering. The magnetic field must be known accurately to achieve the required precision on momentum measurements. The MINOS far detector is a magnetized iron calorimeter with 484 planes of scintillator which provide 2D tracking. The MINOS Collaboration experience should be used in R&D studies on the future PETs.

The high resolution detector based on many thin scintillator and magnetized iron plates is rather expensive facility. However, one should have in mind that accelerator neutrino fluxes are expensive, too. It seems reasonable to detect them in various experiments at the same time.

III. SCATTERING OF TRANSVERSELY POLARIZED MUON-NEUTRINO BEAM ON A POLARIZED ELECTRON TARGET

So far the scattering of left-handed and longitudinally polarized neutrino beam on a polarized electron target (SLoPET) was proposed to probe the neutrino magnetic moments [22,23] and the flavor composition of a (anti)-neutrino beam [24].

There were also the ideas of using the scattering of transversely polarized neutrino beam on the unpolarized electron target to probe the nonstandard properties of neutrinos. Barbieri *et al.* proposed to measure the azimuthal asymmetry of recoil electrons caused by the nonvanishing interference between the weak and electro-magnetic interaction amplitudes [25,26]. Ciechanowicz *et al.* [27] indicated that the azimuthal asymmetry of recoil electron event rates could be generated by the interference between the standard $(V, A)_L$ couplings of the L-handed Dirac neutrinos and exotic $(S, T, P)_R$ couplings of the R-handed ones in the laboratory differential cross section. All the terms with interference, proportional to the magnitude of the transverse neutrino polarization, do not vanish in the massless neutrino limit and depend on the azimuthal angle between the transverse neutrino polarization and the outgoing electron momentum. However, in both cases, the neutrino detectors with a good angular resolution would have to measure both the recoil electron scattering angle and the azimuthal angle of outgoing electron momentum.

There exist the nonstandard models, in which the exotic couplings of the right-handed neutrinos can appear. We mean here three classes of such models; left-right symmetric models (LRSM), contact interactions (CI) and leptoquarks (LQ). For example, the CI can be introduced both for the vector coupling of the L-handed neutrinos and scalar, tensor couplings of the R-handed ones, [28]. Such interactions would allow to probe the scale for compositeness of quarks and leptons. As to the LQ models, if the R-handed neutrinos are taken into account, there are possible couplings of these neutrinos to the scalar and vector LQ, [28]. An original discussion concerning the LQ did not allow for the R-handed neutrinos, [29]. Left-right symmetric models were proposed to explain the origin of the parity violation, [30,31]. In such models the R-handed neutrino can couple to R-handed gauge boson with a mass larger than for the observed standard boson ($m_1 = 80$ GeV). Recently TWIST Collaboration [32] has measured the Michel parameter ρ in the normal μ^+ decay and has set new limit on the $W_L - W_R$ mixing angle in the LRSM. Their result $\rho = 0.75080 \pm 0.00044(\text{stat.}) \pm 0.00093(\text{syst.}) \pm 0.00023$ is in good agreement with the

SM prediction $\rho = 3/4$, and sets new upper limit on mixing angle $|\chi| < 0.030(90\%\mathrm{C.L.})$.

In this part of our paper we show that the scattering of transversely polarized muon-neutrino beam on a polarized electron target (SToPET) may be sensitive to the interference effects between the L- and R-handed neutrinos in the differential cross section for the $(\nu_\mu e^-)$ scattering process. Our analysis is made for the case, when the outgoing electron direction is not observed. It means that the azimuthal angle of the recoil electron momentum would not be measured and nevertheless the new effects could be observed. We consider the minimal extension of the standard $V - A$ weak interaction to indicate the new tests of the Lorentz structure of the charged- and neutral-current weak interactions. An admittance of all the ERWI does not change qualitatively the conclusions from the investigations.

At present, an experiment with the capture of polarized muon by proton as a strong source of the transversely polarized muon-neutrino beam is still extremely difficult. The main cause is that the initially polarized muons, during atomic cascade down to the muon atom ground state, suffer the depolarization effects to a great extent, and leave over only a small fraction of the initial polarization. This fractional polarization degree in the ground state can be increased due to the external means [33], but not to any substantial degree and less than theoretically predicted. A good solution is a process in which both the initial nucleus and muon are polarized because this allows to end up with a muonic atom ground state polarized to any degree [34]. However, it is worth to have in mind that there are designed a new generation of precision muon lifetime experiments, where the ERWI effects could be measured. We mean here MuCaP [35] and MuLan [36] collaborations at the Paul Scherrer Institute (PSI).

To show how the recoil electron spectrum may depend on the angle between the transverse neutrino polarization and transverse target electron polarization, we use the muon-neutrino beam produced in the reaction where the proton at rest captures the polarized muon $(\mu^- + p \rightarrow n + \nu_\mu)$, see Appendix A. The production plane is spanned by the direction of the initial muon polarization $\hat{\mathbf{P}}_\mu$ (the muon is fully polarized, i.e. $|\mathbf{P}_\mu| = \mathbf{1}$) and of the outgoing neutrino momentum $\hat{\mathbf{q}}$, Fig. 1. $\hat{\mathbf{P}}_\mu$, and $\hat{\mathbf{q}}$ are assumed to be perpendicular to each other because this leads to the unique conclusions as to the possible presence of the R-handed neutrinos. When admitting additional exotic scalar coupling C_S^R in muon capture interaction, Eq. (A1), the outgoing muon-neutrino flux is a mixture of the L-handed neutrinos produced in the standard $V - A$ charged weak interaction and the R-handed ones produced in the exotic scalar S charged weak interaction (the transition amplitude and the neutrino observables are presented in Appendix A). This mixture is detected in the neutral-current (NC) weak interaction. We mean that the incoming L-handed neutrinos are detected in the standard $V - A$ neutral weak interaction, while the initial R-handed ones are detected in the exotic scalar S one. Then in the final state all the neutrinos are L-handed. Below we give the transition amplitude for this type of neutral current:

$$M_{\nu_\mu e} = \frac{G_F}{\sqrt{2}}\Big\{(\overline{u}_{e'}\gamma^\alpha(g_V^L - g_A^L\gamma_5)u_e)(\overline{u}_{\nu_{\mu'}}\gamma_\alpha(1-\gamma_5)u_{\nu_\mu}) + \frac{1}{2}g_S^R(\overline{u}_{e'}u_e)(\overline{u}_{\nu_{\mu'}}(1+\gamma_5)u_{\nu_\mu})\Big\}, \tag{10}$$

The coupling constants are denoted with the superscripts L and R as g_V^L, g_A^L, and g_S^R respectively to the incoming neutrino of left- and right-chirality. Standard couplings g_V^L, g_A^L are assumed to be real, i.e. $\beta_V^L = 0$, $\beta_A^L = 0$.

A. Laboratory differential cross section

Because we consider the case when the outgoing electron direction is not observed, the formula for the laboratory differential cross section is presented after integration over the azimuthal angle $\phi_{e'}$ of the recoil electron momentum. The result of the calculation performed with the amplitude $M_{\nu_\mu e}$, in Eq. (10), is divided into three parts, standard (V, A), exotic (S) and interference $(VS + AS)$:

$$\frac{d\sigma}{dy} = \left(\frac{d\sigma}{dy}\right)_{(V,A)} + \left(\frac{d\sigma}{dy}\right)_{(S)} + \left(\frac{d\sigma}{dy}\right)_{(VS+AS)}, \tag{11}$$

with

$$\left(\frac{d\sigma}{dy}\right)_{(V,A)} = \frac{E_\nu m_e}{2\pi}\frac{G_F^2}{2}(1 - \hat{\boldsymbol{\eta}}_\nu\cdot\hat{\mathbf{q}})\Big\{(g_V^L + g_A^L)^2(1 + \hat{\boldsymbol{\eta}}_\mathrm{e}\cdot\hat{\mathbf{q}}) + (g_V^L - g_A^L)^2\Big[1 - (\hat{\boldsymbol{\eta}}_\mathrm{e}\cdot\hat{\mathbf{q}})\Big(1 - \frac{m_e}{E_\nu}\frac{y}{(1-y)}\Big)\Big] \times (1-y)^2 - [(g_V^L)^2 - (g_A^L)^2] \times (1 + \hat{\boldsymbol{\eta}}_\mathrm{e}\cdot\hat{\mathbf{q}})\frac{m_e}{E_\nu}y\Big\}, \tag{12}$$

where, $\boldsymbol{\eta}_\nu\cdot\hat{\mathbf{q}}$ is the longitudinal polarization of the incoming L-handed neutrino. We shall point out that the standard (SM) part has been already published in Ref. [22]. The exotic part of the cross section may contribute merely for the right-handed neutrino scattering $(\hat{\boldsymbol{\eta}}_\nu\cdot\hat{\mathbf{q}} \simeq +1)$:

$$\left(\frac{d\sigma}{dy}\right)_{(S)} = \frac{E_\nu m_e}{2\pi}\frac{G_F^2}{2}(1 + \hat{\boldsymbol{\eta}}_\nu\cdot\hat{\mathbf{q}})|g_S^R|^2\frac{1}{8}\Big(2\frac{m_e}{E_\nu} + y\Big)y, \tag{13}$$

In the interference part we have angular correlations with the transverse component of the neutrino polarization $\boldsymbol{\eta}_\nu^\perp$, both T-odd and T-even. The correlation coefficients depend linearly on the exotic coupling constant g_S^R. Hence, this contribution could be a tool suitable to investigate the effects due to scalar interactions of the R-handed neutrinos:

$$\left(\frac{d\sigma}{dy}\right)_{(VS+AS)} = -\frac{E_\nu m_e}{4\pi}\frac{G_F^2}{2}y\Big\{\hat{\mathbf{q}}\cdot(\boldsymbol{\eta}_e^\perp\times\boldsymbol{\eta}_\nu^\perp)\Big[\mathrm{Im}(g_V^L g_S^{R*})\times\Big(1+\frac{m_e}{2E_\nu}y\Big)+\mathrm{Im}(g_A^L g_S^{R*})\times\Big(1-\frac{m_e}{2E_\nu}(y-4)\Big)\Big]+(\boldsymbol{\eta}_e^\perp\cdot\boldsymbol{\eta}_\nu^\perp)\times\Big[\mathrm{Re}(g_V^L g_S^{R*})\Big(1+\frac{m_e}{2E_\nu}y\Big)+\mathrm{Re}(g_A^L g_S^{R*})\Big(1-\frac{m_e}{2E_\nu}(y-4)\Big)\Big]\Big\}. \quad (14)$$

It can be noticed that the occurrence of the interference terms between the standard $g_{V,A}^L$ and exotic g_S^R couplings does not depend on the neutrino mass and they pertain in the massless neutrino limit. The independence on the m_ν makes the measurement of the relative phases between these couplings possible. The terms with the interference between the standard $g_{V,A}^L$ and exotic g_S^R couplings, Eqs. (14), include only the contributions from the transverse component of the initial neutrino polarization $\boldsymbol{\eta}_\nu^\perp$ and the transverse component of the polarized electron target $\boldsymbol{\eta}_e^\perp$. Both transverse components are perpendicular with respect to the $\hat{\mathbf{q}}$.

If one assumes the production of only left-handed neutrinos in the standard $(V-A)$ and nonstandard S weak interactions, there is no interference between the $g_{V,A}^L$ and g_S^L couplings in the differential cross section, when $m_\nu \to 0$. We do not consider this scenario.

B. *CP* conservation in νe scattering at low-energy

In this subsection, we will consider the CP-symmetric scenario with the standard $(V-A)_L$ and S_R weak interactions. From the general formula for the cross section, we get with $\hat{\boldsymbol{\eta}}_e \perp \hat{\mathbf{q}}$ for $|\boldsymbol{\eta}_e^\perp| = 1$:

$$\frac{d\sigma}{dy} = \left(\frac{d\sigma}{dy}\right)_{(V,A)} + \left(\frac{d\sigma}{dy}\right)_{(S)} + \left(\frac{d\sigma}{dy}\right)_{(VS+AS)}, \quad (15)$$

$$\left(\frac{d\sigma}{dy}\right)_{(VS+AS)} = -\frac{E_\nu m_e}{4\pi}\frac{G_F^2}{2}y|\boldsymbol{\eta}_\nu^\perp|\cos(\alpha)|g_S^R|\Big\{\Big(1+\frac{m_e}{2E_\nu}y\Big)\times|g_V^L|+\Big(1-\frac{m_e}{2E_\nu}(y-4)\Big)|g_A^L|\Big\}, \quad (16)$$

where $(d\sigma/dy)_{(V,A)}$, $(d\sigma/dy)_{(S)}$ are given by Eqs. (12) and (13) and α is the angle between the $\boldsymbol{\eta}_\nu^\perp$ and $\boldsymbol{\eta}_e^\perp$, Fig. 1. We see that the CP-even interference terms enter the cross section and will be large at the $\alpha = 0, \pi$, and they vanish for $\alpha = \pi/2, 3\pi/2$.

Analyzing the assumption that the R-handed neutrinos are created and detected in the exotic S weak interaction, we have used the same upper limit on the NC coupling g_S^R as for the μ-capture coupling C_S^R, i.e. $|g_S^R| < 0.974$, if allowing for the qualitative argument of weak interactions universality. Moreover, with the presence of the exotic scalar coupling $|g_S^R| = 0.974$, we have shifted the numerical values of the V and A couplings to the new ones: $g_V^{\prime L} = -0.041$, $g_A^{\prime L} = -0.517$, which still lie within the experimental bars on the SM results: $g_V^L = -0.040 \pm 0.015$, $g_A^L = -0.507 \pm 0.014$ [10], when $\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}} = -1$. Independently, in Appendix A, we have estimated the transverse and longitudinal components for the neutrino in μ-capture: $|\boldsymbol{\eta}_\nu^\perp| = 0.318$, $\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}} = -0.948$. Finally, with these estimates and the couplings $g_V^{\prime L}$, $g_A^{\prime L}$ and $|g_S^R|$, the correlation coefficients, Eq. (16), and the cross section, Eq. (11), have been calculated in order to show the effect coming from the R-handed muon-neutrinos.

C. *CP* violation in νe scattering at low-energy

In this subsection, we analyze the case of the violation of combined symmetry CP. The formula for the differential cross section including the interference contribution between the standard $(V-A)_L$ and exotic S_R weak interactions with $\hat{\boldsymbol{\eta}}_e \perp \hat{\mathbf{q}}$ for $|\boldsymbol{\eta}_e^\perp| = 1$ is of the form

$$\frac{d\sigma}{dy} = \left(\frac{d\sigma}{dy}\right)_{(V,A)} + \left(\frac{d\sigma}{dy}\right)_{(S)} + \left(\frac{d\sigma}{dy}\right)_{(VS+AS)}, \quad (17)$$

$$\left(\frac{d\sigma}{dy}\right)_{(VS+AS)} = -\frac{E_\nu m_e}{4\pi}\frac{G_F^2}{2}y|\boldsymbol{\eta}_\nu^\perp||g_S^R|\Big\{\Big(1+\frac{m_e}{2E_\nu}y\Big)|g_V^L|\times\cos(\alpha+\beta_{VS})+\Big(1-\frac{m_e}{2E_\nu}(y-4)\Big)|g_A^L|\times\cos(\alpha+\beta_{AS})\Big\}, \quad (18)$$

where $\beta_{VS} \equiv \beta_V^L - \beta_S^R$, $\beta_{AS} \equiv \beta_A^L - \beta_S^R$ are the relative phases between the g_V^L, g_S^R and g_A^L, g_S^R couplings, respectively.

It can be seen that the CP-odd interference contribution enters the cross section and will be substantial at the $\alpha + \beta_{VS} = 0, \pi$ and $\alpha + \beta_{AS} = 0, \pi$, and it vanishes for the $\alpha + \beta_{VS} = \pi/2$ and $\alpha + \beta_{AS} = \pi/2$, respectively. The situation is illustrated in the Fig. 3 for the same limits as for the CP-symmetric case with $E_\nu = 100$ MeV (long-dashed and short-dashed lines, respectively). The phases β_{VS} and β_{AS} in Eq. (18), when different from 0 or π, may result from CP-violation in NC weak interaction ($\nu_\mu e^-$). The angle α is defined in accordance with Fig. 1 and relates the direction of $\boldsymbol{\eta}_\nu^\perp$ to the direction of $\boldsymbol{\eta}_e^\perp$. So with the proper choices of α, the phases β_{VS} and β_{AS} could be detected by measuring the maximal asymmetry of the cross section $d\sigma/dy$.

On the other hand, if knowing these phases prior to ($\nu_\mu e^-$) scattering, it would be possible to test CP-symmetry in muon capture. In case of CP-violation, the neutrino transverse polarization vector $\boldsymbol{\eta}_\nu^\perp$ would be turned aside from the production plane $(\hat{\mathbf{q}}, \hat{\mathbf{P}}_\mu)$, having the

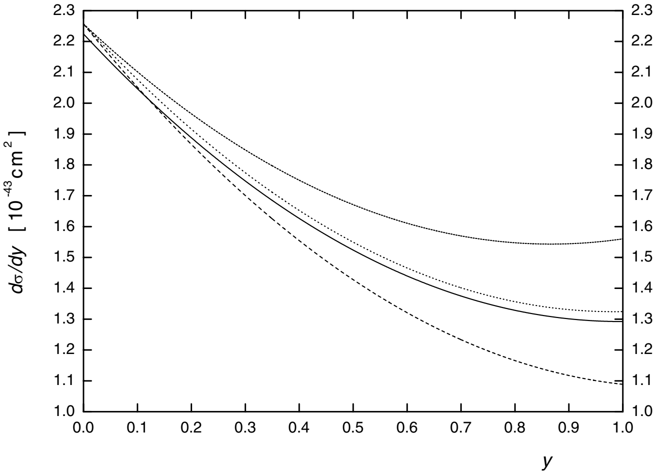

FIG. 3. Plot of the $\frac{d\sigma}{dy}$ as a function of y for the $(\nu_\mu e^-)$ scattering after integration over $\phi_{e'}$, $E_\nu = 100$ MeV: (a) SM with the L-handed neutrino (solid line), (b) CP conservation, the case of the exotic S coupling of the R-handed neutrinos for $\alpha = 0$ (long-dashed line), $\alpha = \pi$ (short-dashed line) and $\alpha = \pi/2$, $3\pi/2$ (dotted line), respectively, (c) CP violation, the case of the exotic S coupling of the R-handed neutrinos for $\alpha + \beta_{VS} = 0$ and $\alpha + \beta_{AS} = 0$ (long-dashed line), $\alpha + \beta_{VS} = \pi$ and $\alpha + \beta_{AS} = \pi$ (short-dashed line), respectively.

CP-breaking component $\langle \mathbf{S}_\nu \cdot (\hat{\mathbf{P}}_\mu \times \hat{\mathbf{q}}) \rangle_f$, see Fig. 1 and Eq. (A3). For illustration purposes let us take $\beta_{VS} = \beta_{AS} = 0$ (i.e. CP-symmetry in $\nu_\mu e^-$). Next, we measure the angular correlation $\boldsymbol{\eta}_e^\perp \cdot \boldsymbol{\eta}_\nu^\perp \sim \cos(\alpha)$, in order to see the direction of $\boldsymbol{\eta}_e^\perp$ along which the maximal asymmetry is oriented. If this direction were turned aside from $\hat{\mathbf{P}}_\mu$, it would be evidence for CP-breaking.

IV. CONCLUSIONS

In the first part of the paper, we have shown that the SLoPET can be used to measure the CP violation in the pure leptonic process, Fig. 2. The azimuthal asymmetry of the recoil electrons does not depend on the neutrino mass and is not vanishing even if $\beta_{VA} = 0$. The CP-breaking phase β_{VA} could be detected by measuring the maximal asymmetry of the cross section. The future superbeam and neutrino factory experiments will provide the unique opportunity for the leptonic CP violation studies, if the large magnetized sampling calorimeters with good event reconstruction capabilities are build.

In the second part, we have shown that the SToPET may be used to detect the effects caused by the interfering L- and R-handed neutrinos. In spite of the integration over the azimuthal angle $\phi_{e'}$ of the recoil electron momentum, the terms with the interference between the standard $(V, A)_L$ and exotic S_R couplings in the laboratory differential cross section depend on the angle α, Fig. 1, between the transverse incoming neutrino polarization and the transverse electron polarization of the target and are present even in the massless neutrino limit. The observation of the dependence on this angle α in the recoil electron energy spectrum would be a clear signal of the R-handed neutrinos in the νe scattering.

It can be noticed that the disagreement with the SM would be substantial for the small polar angle $\theta_{e'}$, both for the CP-even and CP-odd cases, Fig. 3.

To search for the effects connected with the ERWI, the strong polarized neutrino beam and the polarized electron target is required. The electron target should be polarized perpendicular to the direction of the incoming neutrino beam, $\hat{\boldsymbol{\eta}}_e \cdot \hat{\mathbf{q}} = 0$, because it leads to the unique conclusions as to the R-handed neutrinos. If one has the polarized artificial neutrino source, the direction of the transverse neutrino polarization with respect to the production plane will be fixed. So having the assigned direction of the polarization axis of electron target and turning the polarization axis of neutrino source, the dependence of the event number on the angle α could be tested.

It seems worthy of exploring high energy region in the L- and R- handed neutrinos interference. Because of the angular correlation between the transverse spin polarizations of the neutrino and electron and at the large kinetic energy transfer to the recoil electron, we see strong angular asymmetry in the cross section $d\sigma/dy$ for the small values of the polar recoil angle, see Eq. (16) and (18). We expect for this fact some interest in the accelerator laboratories working with neutrino beams, accompanied by the progress in the spin polarization engineering. For the future outline, we shall inspect the other examples, which could be interesting from the point of observable effects caused by the exotic neutrino states. We plan to work mainly on the weak interaction processes that are known from experiment or have been already under consideration in the literature.

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APPENDIX A: MUON CAPTURE BY PROTON

The amplitude for the muon capture by proton $(\mu^- + p \rightarrow n + \nu_\mu)$, as a production process of massive muon-neutrinos, is commonly of the form

$$\begin{aligned} M_{\mu^-} = {} & (C_V^L + 2Mg_M)(\overline{u}_\nu \gamma_\lambda (1 - \gamma_5) u_\mu)(\overline{u}_n \gamma^\lambda u_p) \\ & + \left(C_A^L + m_\mu \frac{q}{2M} g_P \right)(\overline{u}_\nu i\gamma_5 \gamma_\lambda (1 - \gamma_5) u_\mu) \\ & \times (\overline{u}_n i\gamma^5 \gamma^\lambda u_p) + C_S^R (\overline{u}_\nu (1 - \gamma_5) u_\mu)(\overline{u}_n u_p), \quad \text{(A1)} \end{aligned}$$

where the fundamental coupling constants are denoted as C_V^L, C_A^L, and C_S^R respectively to the outgoing neutrino of L- and R-chirality. Since we do not preclude the

CP-asymmetry between SM and exotic sectors, we allow for these coupling constants to be the complex numbers. g_M, g_P—the induced weak couplings of the left-handed neutrinos, i.e. the weak magnetism and induced pseudoscalar, respectively; m_μ, q, E_ν, m_ν, M—the muon mass, the absolute value of the neutrino momentum, its energy, its mass and the nucleon mass; u_p, $\overline{u}_n$—the Dirac bispinors of initial proton and final neutron; u_μ, $\overline{u}_\nu$—the Dirac bispinors of initial muon and final neutrino.

Following the results of Ref. [37], in the case of nonvanishing neutrino mass ($m_\nu \neq 0$), we take the transverse components of the neutrino spin polarization, T-even:

$$\langle \mathbf{S}_\nu \cdot \hat{\mathbf{P}}_\mu \rangle_f = |\mathbf{P}_\mu| \bigg\{ \bigg(1 + \frac{q}{E_\nu}\frac{q}{2M}\bigg) \mathrm{Re}((C_V^L + 2Mg_M)C_S^{R*}) + \frac{1}{2}\frac{m_\nu}{E_\nu}\bigg(|C_V^L + 2Mg_M|^2 - \bigg| C_A^L + m_\mu \frac{q}{2M} g_P \bigg|^2 + |C_S^R|^2 \bigg) \bigg\}, \tag{A2}$$

and T-odd:

$$\langle \mathbf{S}_\nu \cdot (\hat{\mathbf{P}}_\mu \times \hat{\mathbf{q}}) \rangle_f = -|\mathbf{P}_\mu| \bigg(\frac{q}{E_\nu} + \frac{q}{2M} \bigg) \mathrm{Im}((C_V^L + 2Mg_M)C_S^{R*}). \tag{A3}$$

Here, $\mathbf{S}_\nu$ is the neutrino spin operator, $s = 1/2$ is the neutrino spin; $\hat{\mathbf{P}}_\mu$ is the unit vector of the muon polarization in the muonic atom $1s$ state, and $\hat{\mathbf{q}}$ is the unit vector of the neutrino momentum. $\hat{\mathbf{P}}_\mu$ and $\hat{\mathbf{q}}$ are perpendicular to each other, $\hat{\mathbf{P}}_\mu \cdot \hat{\mathbf{q}} = 0$.

It can be noticed that in the limit of vanishing neutrino mass, these observables consist only of the interference term between the standard C_V^L coupling and exotic C_S^R one. There is no contribution to these observables from the SM in which neutrinos are only L-handed and massless. The neutrino mass terms, m_ν/E_ν, in the above observables give a very small contribution in relation to the main one coming from the interference terms and they are neglected in the considerations. As the induced weak couplings enter additively the fundamental $C_{V,A}^L$ couplings, they are omitted in the considerations, for their presence does not change qualitatively the conclusions concerning the transverse neutrino polarization.

Using the current data [10], we calculate the lower limits on the SM couplings: $|C_V^L| > 0.850(4G_F/\sqrt{2})\cos\theta_c$ and $|C_A^L| > 1.070(4G_F/\sqrt{2})\cos\theta_c$, and upper limit on the exotic scalar: $|C_S^R| < 0.974(4G_F/\sqrt{2})\cos\theta_c$. Now, we may give the upper bound on the magnitude of the transverse neutrino polarization in the massless neutrino limit:

$$|\boldsymbol{\eta}_{\boldsymbol{\nu}}^{\perp}| = \frac{1}{s}\sqrt{\langle \mathbf{S}_\nu \cdot (\hat{\mathbf{P}}_\mu \times \hat{\mathbf{q}}) \rangle^2 + \langle \mathbf{S}_\nu \cdot \hat{\mathbf{P}}_\mu \rangle^2}. \tag{A4}$$

The transverse components, Eqs. (A2) and (A3), are calculated with the amplitude M_{μ^-} and normalized with the μ-capture probability $\langle \mathbf{1} \rangle_{\mathbf{f}}$:

$$|\boldsymbol{\eta}_{\boldsymbol{\nu}}^{\perp}| = \frac{1}{s}\bigg|\frac{C_S^R}{C_V^L}\bigg| \bigg(1 + \frac{q}{2M}\bigg) \bigg[1 + \frac{q}{M} + \bigg(3 + \frac{q}{M}\bigg) \bigg| \frac{C_A^L}{C_V^L} \bigg|^2 + \bigg| \frac{C_S^R}{C_V^L} \bigg|^2 - 2\frac{q}{M} \bigg| \frac{C_A^L}{C_V^L} \bigg| \cos(\alpha_{AV}^L) \bigg]^{-1}. \tag{A5}$$

After inserting from above the limits on coupling constants and with the relative phase between the standard C_A^L and C_V^L couplings $\alpha_{AV}^L \equiv \alpha_A^L - \alpha_V^L = \pi$, under the condition that $|\hat{\boldsymbol{\eta}}_\nu| = 1$, one obtains; $|\boldsymbol{\eta}_{\boldsymbol{\nu}}^{\perp}| \leq 0.318$, which means that the value of the longitudinal neutrino polarization is equal to $\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}} = -0.948$.

APPENDIX B: FOUR-VECTOR NEUTRINO POLARIZATION AND MICHEL-WIGHTMAN DENSITY MATRIX

The formulas for the 4-vector of the massive neutrino polarization S in its rest frame and for the initial neutrino moving with the momentum $\mathbf{q}$, respectively, are as follows:

$$S = (0, \hat{\boldsymbol{\eta}}_\nu), \tag{B1}$$

$$S' = \frac{\hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q}}{E_\nu} \cdot \frac{1}{m_\nu} \begin{pmatrix} E_\nu \\ \mathbf{q} \end{pmatrix} + \begin{pmatrix} 0 \\ \hat{\boldsymbol{\eta}}_\nu \end{pmatrix} - \frac{\hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q}}{E_\nu(E_\nu + m_\nu)} \begin{pmatrix} 0 \\ \mathbf{q} \end{pmatrix}, \tag{B2}$$

$$S^{0'} = \frac{|\mathbf{q}|}{m_\nu}(\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}}), \tag{B3}$$

$$\mathbf{S}' = \frac{E_\nu}{m_\nu}(\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}})\hat{\mathbf{q}} + \hat{\boldsymbol{\eta}}_\nu - (\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}})\hat{\mathbf{q}}, \tag{B4}$$

where $\hat{\boldsymbol{\eta}}_\nu$—the unit vector of the initial neutrino polarization in its rest frame. The formula for the Michel-Wightman density matrix [12] is given by

$$\Lambda_\nu^{(s)} = \sum_{r=1,2} u_r \overline{u}_r \sim [(q^\mu \gamma_\mu) + m_\nu + \gamma_5 (S'^\mu \gamma_\mu)(q^\mu \gamma_\mu) + \gamma_5 (S'^\mu \gamma_\mu) m_\nu], \tag{B5}$$

$$(S'^\mu \gamma_\mu) = \frac{\hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q}}{E_\nu m_\nu}(q^\mu \gamma_\mu) - \bigg(\hat{\boldsymbol{\eta}}_\nu - \frac{(\hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q})\mathbf{q}}{E_\nu(E_\nu + m_\nu)} \bigg) \cdot \boldsymbol{\gamma}, \tag{B6}$$

$$(S'^\mu \gamma_\mu)(q^\mu \gamma_\mu) = \frac{m_\nu}{E_\nu} \hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q} - \bigg(\hat{\boldsymbol{\eta}}_\nu - \frac{(\hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q})\mathbf{q}}{E_\nu(E_\nu + m_\nu)} \bigg) \cdot \boldsymbol{\gamma}(q^\mu \gamma_\mu), \tag{B7}$$

$$(S'^\mu \gamma_\mu) m_\nu = \frac{\hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q}}{E_\nu}(q^\mu \gamma_\mu) - m_\nu \bigg(\hat{\boldsymbol{\eta}}_\nu - \frac{(\hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q})\mathbf{q}}{E_\nu(E_\nu + m_\nu)} \bigg) \cdot \boldsymbol{\gamma}, \tag{B8}$$

and in the limit of vanishing neutrino mass m_ν, we have

$$\lim_{m_\nu \to 0} \Lambda_\nu^{(s)} = \left[1 + \gamma_5 \left(\frac{\hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q}}{|\mathbf{q}|} - \left(\hat{\boldsymbol{\eta}}_\nu - \frac{(\hat{\boldsymbol{\eta}}_\nu \cdot \mathbf{q})\mathbf{q}}{|\mathbf{q}|^2} \right) \cdot \boldsymbol{\gamma} \right) \right] \times (q^\mu \gamma_\mu). \quad \text{(B9)}$$

We see that in spite of the singularities m_ν^{-1} in the polarization four-vector S', the density matrix $\Lambda_\nu^{(s)}$ remains finite including the transverse component of the neutrino spin polarization.

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