Neutrino 2-3 symmetry and inverted hierarchy

C. S. Lam*

Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, Canada V6T 1Z1
Theory Group, TRIUMF, Vancouver, BC, Canada V6T 2A3

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Neutrino oscillation data indicate the presence of a 2-3 symmetry for the left-handed neutrinos. We show that this symmetry between the second and third generations cannot be extended to include the charged leptons, no matter what basis we use. We point out that if this symmetry is also independently valid for the right-handed neutrinos, then the active neutrino spectrum is inverted, with $m_3 = 0$. This conclusion remains valid even when the left-handed 2-3 symmetry is broken by a nonmaximal atmospheric mixing angle θ_{23}, or a nonzero reactor angle θ_{13}. As previously shown by Mohapatra, Nasri, and Yu, such a symmetry also gives rise to interesting consequences on the leptogenesis asymmetry parameter ϵ_1.

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I. 2-3 SYMMETRY FOR LEFT-HANDED LEPTONS

The leptonic mass Lagrangian can be written symbolically as

$$\mathcal{L} = -\bar{L}_e M_e R_e - L_\nu^T M_\nu L_\nu, \tag{1}$$

where L_e, L_ν are the left-handed fields for the charged leptons and neutrinos, and R_e, R_ν are the corresponding right-handed fields. M_e is the 3×3 charged lepton mass matrix, and M_ν is the symmetric mass matrix for left-handed Majorana neutrinos. In type-I seesaw, this Majorana mass is related to the Dirac mass $-\bar{L}_\nu M_D R_\nu$ and the right-handed Majorana mass $-R_\nu^T M_R R_\nu$ by

$$M_\nu = M_D^T M_R^{-1} M_D. \tag{2}$$

In the basis where $M_e = \mathrm{diag}(m_e, m_\mu, m_\tau)$ is diagonal, M_ν can be diagonalized by the unitary MNS mixing matrix [1] U, $M_\nu = U^T M_\nu^d U$, with $M_\nu^d = \mathrm{diag}(m_1, m_2, m_3)$ being the (generally complex) neutrino mass parameters. Data are consistent [2] with having the atmospheric mixing angle θ_{23} maximal, and the reactor angle θ_{13} zero. In that case, U can be parametrized as

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}c & \sqrt{2}s & 0 \\ -s & c & 1 \\ -s & c & -1 \end{pmatrix}, \tag{3}$$

where $s = \sin\theta_{12}$, $c = \cos\theta_{12}$, and θ_{12} is the solar mixing angle.

The resulting neutrino mass matrix

$$M_\nu = U^T M_\nu^d U = \begin{pmatrix} \nu_{11} & \nu_{12} & \nu_{12} \\ \nu_{12} & \nu_{22} & \nu_{23} \\ \nu_{12} & \nu_{23} & \nu_{22} \end{pmatrix}, \tag{4}$$

with

$$\begin{aligned} \nu_{11} &= c^2 m_1 + s^2 m_2 \\ \nu_{12} &= cs(m_2 - m_1)/\sqrt{2} \\ \nu_{22} &= \frac{1}{2}(s^2 m_1 + c^2 m_2 + m_3) \\ \nu_{23} &= \frac{1}{2}(s^2 m_1 + c^2 m_2 - m_3), \end{aligned} \tag{5}$$

is invariant under the simultaneous interchanges of the second and third columns, together with the second and third rows. We shall refer to this symmetry as the 2-3 symmetry for left-handed neutrinos.

Conversely, the invariance of $\mathcal{L}$ under a permutation of the second and third generations L_ν, via the matrix

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{6}$$

will result in an identity $L_\nu^T P M_\nu P L_\nu = L_\nu^T M_\nu L_\nu$, or equivalently $P M_\nu P = M_\nu$. The most general symmetric matrix satisfying this constraint is of the form given by (4). Moreover, a matrix of this form is known to lead to a maximum θ_{23} and a zero θ_{13} [3,4].

Since left-handed neutrinos have a 2-3 symmetry, one might wonder whether the left-handed charged leptons also have the same symmetry. In the basis where the charged leptons are diagonal, clearly they do not, because $m_\mu \neq m_\tau$. Actually, a 2-3 symmetry cannot be simultaneously valid for the left-handed charged leptons and the left-handed neutrinos, *no matter what basis we choose*. This can be seen as follows.

If both L_ν and L_e are 2-3 symmetric, then $P M_\nu = M_\nu P$, and $P H_e = H_e P$, where $H_e = M_e M_e^\dagger$. Since they commute, there must be a unitary matrix V_ν that can diagonalize P and M_ν simultaneously, and another unitary matrix V_e that can diagonalize P and H_e simultaneously. Now the matrix S below diagonalizes P,

*Electronic address: Lam@physics.ubc.ca

$$S^\dagger PS = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{for} \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}, \tag{7}$$

but since P has two degenerate eigenvalues $+1$, this diagonalization is not unique. The most general unitary matrix to diagonalize P is of the form $V = SW$, with

$$W = \begin{pmatrix} w & 0 \\ 0 & e^{i\phi} \end{pmatrix}, \tag{8}$$

and w any 2×2 unitary matrix. The neutrino mixing matrix $U = V_e^\dagger V_\nu = W_e^\dagger W_\nu$ is therefore block diagonal, implying no mixing of the third-generation neutrino with the other two neutrino generations. This is manifestly false, hence the 2-3 symmetry cannot be simultaneously true for L_e and L_ν in any basis.

We do not understand why the 2-3 symmetry is (at least approximately) true for the left-handed neutrinos but not for the left-handed charged leptons, though there are models that can describe it [4], but data indicate that to be so. From now on we will concentrate solely on the neutrino symmetry.

II. 2-3 SYMMETRY FOR THE RIGHT-HANDED NEUTRINOS

Given a mass matrix M_ν in (4) and (5), the seesaw formula (2) is insufficient to determine both the Dirac mass matrix M_D and the right-handed Majorana mass matrix M_R. M_R, being symmetric, contains 6 complex parameters, but M_D, not having any symmetry, generally contains 9. Since $M_D^T M_R^{-1} M_D$ is symmetric, the seesaw formula contains 6 constraints, thereby leaving 9 complex parameters in M_D and M_R undetermined. If we want to use neutrino oscillation to probe the dynamics beyond the standard model, we need to know those parameters. In any case, some combinations of these parameters are actually required to calculate the asymmetry parameter ϵ_1 in leptogenesis.

One might be able to reduce the number of undetermined parameters if the right-handed neutrinos obey some symmetry. Since the left-handed neutrinos exhibit a 2-3 symmetry, we shall assume the right-handed neutrinos to possess a 2-3 symmetry as well. This includes the possibilities of having the right-handed neutrinos possess a 1-2 or a 1-3 symmetry instead, because a relabeling of generations of the right-handed neutrinos will convert those symmetries into a 2-3 symmetry.

The right-handed 2-3 symmetry might be coupled and linked to the left-handed 2-3 symmetry, or it might be an independent symmetry not linked to the left-handed one. Let us look at these two cases separately.

In the first case, $\bar{L}_\nu M_D R_\nu = \bar{L}_\nu P M_D P R_\nu$ and $R_\nu^T M_R R_\nu = R_\nu^T P M_R P R_\nu$, so the constraints on the mass matrices are $M_D = P M_D P$, and $M_R = P M_R P$. The constraint on M_R is identical to the constraint on M_ν, so M_R contains 6 complex parameters. M_D, not being symmetric, contains 7 complex parameters. The number of undetermined parameters is now reduced from 9 to 7.

As shown by Mohapatra and Nasri [5], and also by Grimus and Lavoura [6], the leptogenesis asymmetry parameter ϵ_1 is then proportional to $\Delta m_\odot^2$, rather than the more generic $\Delta m_{\rm atm}^2$ [7].

If the right-handed 2-3 symmetry is independent of the left-handed 2-3 symmetry, then the constraint on M_R remains the same, but the constraint on M_D becomes stronger. In that case, $\bar{L}_\nu M_D R_\nu = \bar{L}_\nu P M_D R_\nu = \bar{L}_\nu M_D P R_\nu$, hence $M_D = P M_D = M_D P$. In other words, the second and third columns of M_D must be identical, and so must be the second and third rows. Thus M_D is of the form

$$M_D = \begin{pmatrix} a & b & b \\ d & c & c \\ d & c & c \end{pmatrix}, \tag{9}$$

which is specified by 4 complex parameters. In this case the number of unknown parameters is further reduced from 7 to 4.

The conclusion of Ref. [5,6] on ϵ_1 remains valid, but there are now additional consequences which we shall discuss in the rest of this article.

Since M_D has two identical columns, its determinant is zero. Hence M_ν given by (2) also has a zero determinant, and thus a zero eigenvalue. This eigenvalue is m_3, as can be seen as follows. Compute $S^T M_\nu S$, where M_ν is the matrix in (4). It yields a block diagonal matrix, with the 13, 23, 31, and 32 matrix elements zero, and the 33 matrix element equal to $\nu_{22} - \nu_{23}$. Using (5), this difference is equal to m_3. Now compute $S^T M_\nu S$ again, this time using M_ν given by (2). With M_R 2-3 symmetric and M_D given by (9), a straight forward calculation shows that the resulting 33 matrix element is zero. Hence $m_3 = 0$.

With $m_3 = 0$, the magnitudes of the remaining neutrino masses are determined by the atmospheric and solar mass gaps to be

$$\begin{aligned} |m_1| &= \sqrt{\Delta m_{atm}^2} \simeq 52 \text{ meV} \\ |m_2| &= \sqrt{\Delta m_{atm}^2 + \Delta m_\odot^2} \simeq 53 \text{ meV}. \end{aligned} \tag{10}$$

The sum $\sum_{i=1}^2 |m_i|$, just over 0.1 eV, is comfortably below the upper bound of 0.47 eV placed by astrophysical data, including Lyα [8]. The effective neutrino mass measured in tritium β-decay is

$$m_{\nu_e} = \sqrt{\sum_i |U_{ei}^2 m_i^2|} = \sqrt{c^2|m_1|^2 + s^2|m_2|^2}, \qquad (11)$$

which lies between $|m_1|$ and $|m_2|$. This is to be compared with the 2.2 eV upper bound from experiments [9]. Unfortunately, this number is too low to be reached by KATRIN for verification. Finally, the effective mass for the neutrinoless double β-decay is

$$\begin{aligned} m_{ee} &= |\sum_{i=1}^{3} U_{ei}^2 m_i| = |c^2 m_1 + s^2 m_2| \\ &= \sqrt{(c^2|m_1| + s^2|m_2|)^2 - |m_1 m_2|\sin^2 2\theta_\odot \sin^2\frac{\phi_{12}}{2}} \\ &\simeq |m_1|\sqrt{1 - 0.84\sin^2\frac{\phi_{12}}{2}}, \end{aligned} \qquad (12)$$

where ϕ_{12} is the relative Majorana phase angle between m_1 and m_2. The current upper bound from experiment is about 0.3 eV, with a factor of 3 uncertainty from the nuclear matrix elements [10].

III. BREAKING THE LEFT-HANDED 2-3 SYMMETRY

If θ_{23} is not exactly maximal, or $\theta_{13} \neq 0$, then the 2-3 symmetry for the left-handed neutrino is broken. Nevertheless, present experiments indicate that the breaking has to be small.

We can quantify the breaking in the following way. Under a 2-3 permutation of the left-hand neutrino field, $L_\nu \to PL_\nu$, the mass Lagrangian $\mathcal{L}$ can be decomposed into the sum of an even part and an odd part: $\mathcal{L} = \mathcal{L}^e + \mathcal{L}^o$, with $\mathcal{L}^e \to \mathcal{L}^e$ and $\mathcal{L}^o \to -\mathcal{L}^o$. This induces a decomposition of the mass matrix $M_\nu = M_\nu^e + M_\nu^o$, with the constraints $M_\nu^e = PM_\nu^e P$ and $M_\nu^o = -PM_\nu^o P$. Thus M_ν^e is of the form given by (4), but M_ν^o is of the form

$$M_\nu^o = \begin{pmatrix} 0 & \nu'_{12} & -\nu'_{12} \\ \nu'_{12} & \nu'_{22} & 0 \\ -\nu'_{12} & 0 & -\nu'_{22} \end{pmatrix}. \qquad (13)$$

To calculate ν'_{ij}, we assume the atmospheric mixing angle to be maximal, and the reactor angle θ_{13} to be small but not necessarily zero. Then instead of (3), the MNS matrix to $O(\theta)$ is given by

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}c & \sqrt{2}s & \theta \\ -s + c\theta^* & c + s\theta^* & 1 \\ -s + c\theta^* & c + s\theta^* & -1 \end{pmatrix}, \qquad (14)$$

where $\theta \equiv \sin\theta_{13}e^{i\delta}$ and δ is the CP phase. By calculating $M_\nu = U^T M_\nu^d U$ up to $O(\theta)$, we obtain the even part M_ν^e to be given by (4) and (5), and the odd part M_ν^o to be given by (13), with

$$\begin{aligned} \nu'_{12} &= (m_3 - c^2 m_1 - s^2 m_2)\theta/\sqrt{2}, \\ \nu'_{22} &= cs(m_1 - m_2)\theta. \end{aligned} \qquad (15)$$

This is small because of the CHOOZ bound [11].

How does this breaking of the left-handed neutrino affect the 2-3 symmetry of the right-handed neutrino? If its 2-3 symmetry is coupled and linked to the 2-3 symmetry of the left-handed neutrino, we expect a breaking of the right-handed symmetry as well. Since there are no data to guide us how the right-handed neutrinos behave, there are many uncertainties connected with this scenario.

If the 2-3 symmetry of the right-handed neutrino is independent and unlinked to the 2-3 symmetry of the left-handed neutrino, then there is no need to break the right-handed symmetry when the left-handed symmetry is broken. In that case, M_R remains bound by the constraint $M_R = PM_R P$, but M_D is bound only by the constraint $M_D = M_D P$ and *not* the constraint $M_D = PM_D$, as the left-handed 2-3 symmetry is broken. Nevertheless, since the second and third columns of M_D are still identical, its determinant is still zero. The conclusion of having $m_3 = 0$ therefore remain unchanged. However, since $\theta_{13} \neq 0$, the previous estimate of m_{ν_e} should be multiplied by a factor $\cos\theta_{13}$, and that of m_{ee} should be multiplied by a factor $\cos^2\theta_{13}$.

The consequence for the leptogenesis asymmetry parameter ϵ_1 in this scenario is discussed in Ref. [12]. ϵ_1 is now proportional to a linear combination of $\Delta m_\odot^2$ and $|\theta|^2$.

In conclusion, we found that in the presence of a 2-3 symmetry for the right-handed neutrinos, the spectrum for the active neutrinos is inverted, with $m_3 = 0$. This conclusion remains valid whether the left-handed 2-3 symmetry is broken or not.

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