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# Semiclassical ultraextremal horizons

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We examine backreaction of quantum massive fields on multiply-degenerate (ultraextremal) horizons. It is shown that, under influence of the quantum backreaction, the horizon of such a kind moves to a new position near which the metric does not change its asymptotics, so the ultraextremal black holes and cosmological spacetimes do exist as self-consistent solutions of the semiclassical field equations.

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There exist two qualitatively distinct classes of black holes—nonextremal ($T_H \neq 0$) and extremal ($T_H = 0$), where $T_H$ is the Hawking temperature. The latter means that the system possesses a degenerate (at least twice) horizon. In turn, in the latter case the subclass of so-called ultraextremal horizons (UEH) is singled out. It represents spacetimes with horizons with triple and higher multiplicity. If one writes the asymptotic form of the metric coefficient $-g_{00} = f$ in the Schwarzschild-like coordinates near the horizon $r_+$ as $f \sim (r - r_+)^n$, then $n \geq 3$ corresponds to UEH which can be, in principle, black hole or cosmological horizons. Ultraextremal horizons appear naturally, for instance, in general relativity and supersymmetrical theories in the spacetimes with the cosmological constants $\Lambda > 0$ and nonzero charge $Q$ (Reissner-Nordström-de Sitter solution (RNdS), provided some special relationships between $\Lambda$ and $Q$ hold [1]. It is worth noting that recently interest to RNdS revived in the context of dS/CFT correspondence [2] and higher-dimensional theories (see. e.g., [3] and references therein).

Up to now, to the best of our knowledge, spacetimes with ultraextremal horizons were considered only classically, with quantum backreaction neglected. Meanwhile, the potential effect of such backreaction on multiple horizons is not evident in advance since it is unclear whether the condition of (ultra)extremality is simply slightly shifted or backreaction pushes classically degenerate horizons away. There was some discussion in literature on the existence of ''ordinary'' ($n = 2$) semiclassical extremal black holes (EBH) and it was established that such solutions do exist [4–6]. On the other hand, recent investigations in two-dimensional dilaton gravity confirmed the existence of semiclassical EBH but showed that semiclassical UEH are forbidden (except some very special exactly solvable models) [7]. It is also worth mentioning another issue (closely related to that of EBH) - quantum-corrected acceleration horizons that arise in the Nariai and Bertotti-Robinson solutions [6,8–15].

In the present article we examine the spherically-symmetric UEH metric with quantum backreaction and show that *self-consistent* solutions possessing triply degenerated horizon do exist. In doing so, we restrict ourselves to the case of massive fields in the large mass limit since only in this case one knows the approximate one-loop stress-energy tensor in terms of the geometry explicitly (see [16–18] and references therein).

The metric under consideration reads

$$ds^2 = -U(r)dt^2 + V^{-1}(r)dr^2 + r^2 d\Omega^2, \qquad (1)$$

and it follows from Einstein equations with the stress-energy tensor $T_\mu^\nu$ that

$$U(r) = e^{2\psi(r)}V(r), \qquad (2)$$

where

$$\psi = 4\pi \int^r drF(r), \quad F(r) = r\frac{T_1^1 - T_0^0}{V}. \qquad (3)$$

As the RNdS-like geometries are not asymptotically flat one can always rescale time that is equivalent to the change of the integration constant in (3) whose particular value is thus unimportant. However, it is unclear in advance whether $\psi$ remains finite when $r$ approaches UEH (see below).

We assume that the right hand side of the Einstein field equations $G_\mu^\nu + \Lambda\delta_\mu^\nu = 8\pi T_\mu^\nu$ is given by $T_\mu^\nu = T_\mu^{\nu(cl)} + T_\mu^{\nu(q)}$, where the first term stems from classical source whereas the second one describes the backreaction of quantum fields on the geometry. As backreaction is considered as a small perturbation, one could try to use the expansion of the metric taking as the main approximation the unperturbed metric. However, such a naive approach suffers from serious shortcomings. It tacitly assumes that parameters of the classical metric such as the charge, mass, etc. are fixed. Then the metric which was extremal classically, in general becomes nonextremal (or vice versa) if

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quantum corrections are taken into account. (To avoid potential confusion, this has nothing to do with the third general law since it is not real transformation of a physical system but, rather, comparison of different configurations in the space of parameters.) Therefore, we prefer to treat the problem in a self-consistent way and use the true (quantum-corrected) horizon value $r_+$ from the very beginning. In what follows we restrict ourselves to the case of the electromagnetic field. Then $T_1^{1(cl)} = T_0^{0(cl)}$ and it follows from (2) and (3) that in the zeroth-order of the approximation (with the backreaction neglected) $U = V$ and we obtain the classical RNdS metric.

The stress-energy tensor of the quantized massive scalar (with arbitrary curvature coupling $\xi$), spinor and vector fields [17,18] can be obtained by means of standard methods from the approximate effective action [19,20]

$$W_{ren}^{(1)} = \frac{1}{192\pi^2 m^2}\int d^4x g^{1/2}(\alpha_1^{(s)} R\Box R + \alpha_2^{(s)} R_{\mu\nu}\Box R^{\mu\nu} + \alpha_3^{(s)} R^3 + \alpha_4^{(s)} R R_{\mu\nu}R^{\mu\nu} + \alpha_5^{(s)} R R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \alpha_6^{(s)} R^\mu_\nu R^\nu_\rho R^\rho_\mu + \alpha_7^{(s)} R^{\mu\nu} R_{\rho\sigma} R^{\rho\sigma}_{\mu\nu} + \alpha_8^{(s)} R_{\mu\nu} R^\mu_{\lambda\rho\sigma} R^{\nu\lambda\rho\sigma} + \alpha_9^{(s)} R^{\mu\nu}_{\rho\sigma} R^{\lambda\gamma}_{\mu\nu} R^{\rho\sigma}_{\lambda\gamma} + \alpha_{10}^{(s)} R^{\rho\sigma}_{\mu\nu} R^{\mu\nu}_{\lambda\gamma} R^{\lambda\gamma}_{\rho\sigma}), \quad (4)$$

where the spin-dependent numerical coefficients are tabulated in Table I. As the result of the functional differentiation of $W_R$ with respect to the metric tensor one obtains a rather complicated expression for $T_\mu^{\nu(q)}$ constructed from the curvature tensor and its covariant derivatives. To avoid unnecessary proliferation of long formulas we shall not display it here. An interested reader is referred to [17,18] for results and physical motivation.

In the ultraextremal case which we are interested in, the classical metric functions in (1) read [1]

TABLE I. The coefficients $\alpha_i^{(s)}$ for the massive scalar, spinor, and vector field

| | s = 0 | s = 1/2 | s = 1 |
|---|---|---|---|
| $\alpha_1^{(s)}$ | $\frac{1}{2}\xi^2 - \frac{1}{5}\xi + \frac{1}{56}$ | $-\frac{3}{140}$ | $-\frac{27}{280}$ |
| $\alpha_2^{(s)}$ | $\frac{1}{140}$ | $\frac{1}{14}$ | $\frac{9}{28}$ |
| $\alpha_3^{(s)}$ | $(\frac{1}{6}-\xi)^3$ | $\frac{1}{432}$ | $-\frac{5}{72}$ |
| $\alpha_4^{(s)}$ | $-\frac{1}{30}(\frac{1}{6}-\xi)$ | $-\frac{1}{90}$ | $\frac{31}{60}$ |
| $\alpha_5^{(s)}$ | $\frac{1}{30}(\frac{1}{6}-\xi)$ | $-\frac{7}{720}$ | $-\frac{1}{10}$ |
| $\alpha_6^{(s)}$ | $-\frac{8}{945}$ | $-\frac{25}{376}$ | $-\frac{52}{63}$ |
| $\alpha_7^{(s)}$ | $\frac{2}{315}$ | $\frac{47}{630}$ | $-\frac{19}{105}$ |
| $\alpha_8^{(s)}$ | $\frac{1}{1260}$ | $\frac{19}{630}$ | $\frac{61}{140}$ |
| $\alpha_9^{(s)}$ | $\frac{17}{7560}$ | $\frac{29}{3780}$ | $-\frac{67}{2520}$ |
| $\alpha_{10}^{(s)}$ | $-\frac{1}{270}$ | $-\frac{1}{54}$ | $\frac{1}{18}$ |

$$U(r) = V(r) = -\frac{r^2}{6\rho^2}\left(1-\frac{\rho}{r}\right)^3\left(1+\frac{3\rho}{r}\right), \quad (5)$$

where $\rho$ is the position of the horizon. It is seen from (5) that the static region is confined by $0 < r \le \rho$, $r = 0$ being the singularity. The quantum state of the field we are dealing with is supposed to be the Hartle-Hawking one that physically describes the thermal equilibrium, that, in turn, implies the staticity of the metric in the relevant region. Now, in contrast to the black hole case the horizon under discussion is of cosmological nature in the sense that the metric is static for $r < \rho$ and nonstatic for $r > \rho$. This does not cause obstacles to the existence of the Hartle-Hawking state since cosmological horizons are known to possess thermal properties in a similar way to the black hole case [21]. However, now there is a problem connected with the presence of singularity. In the black hole case (say, for the Schwarzschild metric) the singularity is hidden behind the horizon, the region in which the metric is static is $r > \rho$. Now the situation is opposite. To avoid this difficulty, which is not connected by itself with the issue of UEH, we somewhat modify the system by considering the following model. Relying on the fact that the singularity in the case under discussion is timelike, we smear or simply replace it by some central body with a regular center and the boundary at $r = R$. Then for $R < r \le \rho$ we can use safely the formulas for the stress-energy tensor obtained for the Hartle-Hawking state.

To evaluate the role of backreaction, we proceed along the same line as in [6]. We perform two steps: (i) we show that the triple root of $V(r)$ does exist and find corresponding quantum-corrected relationships between parameters $M$, $Q$, $\Lambda$; (ii) check that the function $F(r)$ is finite, so that $U(r)$ has the same triple root as $V(r)$. It is convenient to write

$$V = 1 - \frac{2m(r)}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}, \quad (6)$$

where

$$m(r) = M + m_q(r), \quad (7)$$

$$M = \frac{r_+}{2} + \frac{Q^2}{r_+} - \frac{\Lambda r_+^3}{6}. \quad (8)$$

$m_q = -4\pi\int_{r_+}^r dr' r'^2 T_0^{0(q)}$ is the contribution of quantum fields, $m(r_+) = 0$. To get rid off the denominators in (6), it is convenient to introduce $g(r) \equiv r^2 V(r)$.

In the vicinity of the horizon

$$m(r) = M + A(r - r_+) + \ldots, \quad (9)$$

where $A = -4\pi T_0^{0(q)} r_+^2$ is a small parameter responsible for backreaction. We should check that equations

$$g(r_+) = 0 = g'(r_+) = g''(r_+) \quad (10)$$

are self-consistent. The form (9) is inappropriate for analysis of the aforementioned equations since the equations would contain as the parameter their own root that, as is shown in [6] leads to difficulties connected with the appearance of spurious roots [4]. To avoid this difficulty, we redefine $M_0 = M - Ar_+$ and substitute in (6) the expression $m(r) = M_0 + Ar$. Then it is straightforward to show that all three equations (10) are mutually consistent, with

$$r_+^2 = \frac{1}{2\Lambda}(1 - 2A), \qquad (11)$$

$$M_0 = \frac{1}{\sqrt{2\Lambda}}(1 - 3A), \qquad (12)$$

$$Q^2 = \frac{1}{4\Lambda}(1 - 4A). \qquad (13)$$

For the massive scalar, spinor and vector fields considered in this paper one has respectively

$$A^{(0)} = \frac{\Lambda^2(3780\eta^3 + 63\eta - 8)}{5670\pi m^2}, \quad A^{(1/2)} = -\frac{\Lambda^2}{252\pi m^2}, \qquad A^{(1)} = -\frac{2\Lambda^2}{105\pi m^2}, \qquad (14)$$

where superscripts denote the value of spin and $\eta = \xi - 1/6$ and the form (14) implies that the parameters $M$, $Q$, $\Lambda$ are not arbitrary but connected by the relationship inherent to the ultraextremal case.

Now we pass to the next step and substitute into the expression for $T_\mu^{\nu(q)}$ the metric (1) in which we put, in the main approximation, $U = V = V_0$. The function $\psi(r)$ is given by Eq. (3) with

$$F(r) = r\frac{T_1^{1(q)} - T_0^{0(q)}}{V}. \qquad (15)$$

If $\psi(r_+)$ is bounded, the backreaction does not change qualitatively the character of the metric. It is worth noting that the finiteness of $\psi(r_+)$ is equivalent of the finiteness of the energy measured by an observer who moves along the radial geodesics [22]. Specifically, for the line element (5) one obtains after calculations

$$\psi(r) = \frac{1}{\pi m^2}\sum_{i=4}^{8}\Lambda^{-(i-4)/2}B_i^{(s)}r^{-i} + C_1^{(s)}, \qquad (16)$$

where $B_i^{(s)}$ with $B_5^{(s)} = 0$ are coefficients depending on the parameters of the theory and $C_i^{(s)}$ are integration constants. Specifically, for the massive $s = 0, 1/2, 1$ fields one has

$$B_4^{(s)} = -h(0)\left(\frac{\eta}{120} - \frac{1}{4320}\right) - \frac{h(1/2)}{2880} - h(1)\frac{23}{4320}, \qquad (17)$$

$$B_6^{(s)} = h(0)\left(\frac{14}{135}\eta - \frac{7}{3240}\right) + h(1/2)\frac{13}{1080} + h(1)\frac{211}{1080}, \qquad (18)$$

$$B_7^{(s)} = -2^{1/2}\left[h(0)\left(\frac{4}{45}\eta - \frac{17}{6615}\right) + h(1/2)\frac{13}{1470} + h(1)\frac{1223}{6615}\right], \qquad (19)$$

$$B_8^{(s)} = h(0)\left(\frac{13}{320}\eta - \frac{47}{26880}\right) + h(1/2)\frac{37}{17920} + h(1)\frac{2141}{26880}, \qquad (20)$$

where $h(s)$ is the number of fields with spin $s$. We see that $\psi(r_+)$ is bounded and this key feature entails immediately the conclusion that semiclassical UEH of the type under discussion do exist.

To summarize, we showed that there exist self-consistent UEH solutions of semiclassical field equations, if the corresponding classical system admits them. Although we restricted ourselves by one concrete physically relevant example (RNdS metric) the general approach applies to a more general situation when the function $f$ has near the asymptotics $(r - r_+)^n$, $n \geq 3$. In this case it also turns out that the functions $F(r)$ and $\psi(r)$ are finite near the horizon, so that $U(r)$ has the root of same multiplicity $n$. In this sense, ultraextremal horizons are stable against backreaction of massive fields. In particular, this confirms the significance of ultraextremal black holes in addition to ordinary EBH as potential candidates to the role of stable remnants after black hole evaporation.


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