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Deformed vortices in (4+1)-dimensional Einstein-Yang-Mills theory

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We study vortex-type solutions in a (4 + 1)-dimensional Einstein-Yang-Mills-SU(2) model. Assuming all fields to be independent on the extra coordinate, these solutions correspond in a four-dimensional picture to axially symmetric multimonopoles, respectively, monopole-antimonopole solutions. By boosting the five-dimensional purely magnetic solutions, we find new configurations which in four dimensions represent rotating regular non-Abelian solutions with an additional electric charge.

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I. INTRODUCTION

Ever since the pioneering work of Kaluza and Klein in the 1920s [1], extra dimensions are intensively discussed in physics. Specifically the result that string theory is only consistent in 10, respectively 26, dimensions for superstring and bosonic string theory has boosted the interest. In string theory [2], the extra dimensions are usually compactified at the Planck length, while in recent years so-called brane world models have emerged which can have noncompact and even infinite extra dimensions [3].

When discussing solutions in higher dimensional theories, two approaches seem possible: either to study solutions with a specific symmetry in the full dimensions or to study solutions with a symmetry in four dimensions which are then trivially extended into the extra dimensions.

Recently, gravitating solutions including non-Abelian gauge fields have been discussed in this context. In [4], (4 + 1)-dimensional generalizations of the Bartnik-McKinnon solutions [5] with an SO(4) symmetry group have been studied in Einstein-Yang-Mills (EYM) theory. However, as was proven in [6] and demonstrated numerically in [4], in this case there are no finite energy solutions unless one considers the inclusion of higher order curvature and/or Born-Infeld–like terms in the action. In contrast to this, if one assumes all fields to be independent on the extra x^5-coordinate, solutions in the "pure" EYM model are possible. These have been constructed in [6] and are spherically symmetric in four dimensions, extending trivially into one extra dimension. Generalizations of this model to n-extra dimensions have been constructed in [7].

In this paper, we extend this model by introducing axial symmetry in four dimensions. Our five-dimensional EYM solutions thus describe deformed vortex-type solutions, which in the (3 + 1)-dimensional effective theory can be interpreted as describing multimonopoles, respectively monopole-antimonopole pairs. Our ansatz admits an interesting Kaluza-Klein picture in the sense that, when boosting the solutions, we obtain new $d = 4$ rotating and electrically charged configurations.

Our paper is organized as follows: In Sec. II, we give the model including the ansatz and the boundary conditions. In Sec. III, we describe our numerical results. In Sec. IV, we comment on the rotating solutions that we obtain by boosting our solutions, and in Sec. V we give our conclusions.

II. THE MODEL

A. Action principle

The five-dimensional EYM-SU(2) system is described by the action

$$I_5 = \int d^5x\sqrt{-g_m}\left(\frac{R}{16\pi G} - \frac{1}{2g^2}\,\mathrm{Tr}\{F_{MN}F^{MN}\}\right), \qquad (1)$$

(throughout this letter, the indices $\{M, N, \ldots\}$ (with M, N running from one to five) will denote the five-dimensional coordinates and $\{\mu, \nu, \ldots\}$ the coordinates of the four-dimensional physical spacetime; the length of the extra dimension x^5 is taken to be one).

Here G is the gravitational constant, R is the Ricci scalar associated with the spacetime metric g_{MN} and $F_{MN} = \frac{1}{2}\tau^a F_{MN}^{(a)}$ is the gauge field strength tensor defined as $F_{MN} = \partial_M A_N - \partial_N A_M + i[A_M, A_M]$, here the gauge field is $A_M = \frac{1}{2}\tau^a A_M^{(a)}$,$\tau^a$ being the Pauli matrices and g the gauge coupling constant.

Variation of the action (1) with respect to g^{MN} and A_M leads to the field equations,

$$R_{MN} - \frac{1}{2}g_{MN}R = 8\pi G T_{MN}, \qquad (2)$$

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$$\nabla_M F^{MN} + i[A_M, F^{MN}] = 0, \tag{3}$$

where the YM stress-energy tensor is

$$T_{MN} = 2\,\mathrm{Tr}\Big(F_{MP}F_{NQ}g^{PQ} - \frac{1}{4}g_{MN}F_{PQ}F^{PQ}\Big). \tag{4}$$

B. The ansatz

In what follows we will consider vortex-type configurations, assuming that both the matter functions and the metric functions are independent on the extra coordinate x^5. Without any loss of generality, we consider a five-dimensional metric parametrization,

$$ds^2 = e^{-a\psi}\gamma_{\mu\nu}dx^\mu dx^\nu + e^{2a\psi}(dx^5 + 2\mathcal{W}_\mu dx^\mu)^2, \tag{5}$$

with $a = 2/\sqrt{3}$. With this assumption, the considered theory admits an interesting Kaluza-Klein (KK) picture. While the KK reduction of the Einstein term in (1) with respect to the Killing vector $\partial/\partial x^5$ is standard, for the reduction of the YM action term, it is convenient to take an SU(2) ansatz,

$$A = \mathcal{A}_\mu dx^\mu + g\Phi(dx^5 + 2\mathcal{W}_\mu dx^\mu), \tag{6}$$

where $\mathcal{W}_\mu$ is a U(1) potential, $\mathcal{A}_\mu$ is a purely four-dimensional gauge field potential, while Φ corresponds after the dimensional reduction to a triplet Higgs field.

This leads to the four-dimensional action principle,

$$I_4 = \int d^4x\sqrt{-\gamma}\Big[\frac{1}{4\pi G}\Big(\frac{\mathcal{R}}{4} - \frac{1}{2}\nabla_\mu\psi\nabla^\mu\psi - e^{2\sqrt{3}\psi}\frac{1}{4}G_{\mu\nu}G^{\mu\nu}\Big) - e^{2\psi/\sqrt{3}}\frac{1}{2g^2}\mathrm{Tr}\{\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\} - e^{-4\psi/\sqrt{3}}\mathrm{Tr}\{D_\mu\Phi D^\mu\Phi\}$$
$$- 2e^{2\psi/\sqrt{3}}\frac{1}{g}G_{\mu\nu}\mathrm{Tr}\{\Phi\mathcal{F}^{\mu\nu}\} - 2e^{2\psi/\sqrt{3}}G_{\mu\nu}G^{\mu\nu}\mathrm{Tr}\{\Phi^2\}\Big], \tag{7}$$

where $\mathcal{R}$ is the Ricci scalar for the metric $\gamma_{\mu\nu}$, while $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu + i[\mathcal{A}_\mu, \mathcal{A}_\nu]$ and $G_{\mu\nu} = \partial_\mu\mathcal{W}_\nu - \partial_\nu\mathcal{W}_\mu$ are the SU(2) and U(1) field strength tensors defined in $d = 4$.

Here we consider five-dimensional configurations possessing two more Killing vectors apart from $\partial/\partial x^5$, $\xi_1 = \partial/\partial\varphi$, corresponding to an axial symmetry of the four-dimensional metric sector (where the azimuth angle φ rangesfrom 0 to 2π), and $\xi_2 = \partial/\partial t$, with t the time coordinate.

With these assmptions, we consider the following parametrization of the four-dimensional line element:

$$d\sigma^2 = \gamma_{\mu\nu}dx^\mu dx^\nu = \gamma_{tt}dt^2 + d\ell^2$$
$$= -f(r,\theta)dt^2 + \frac{m(r,\theta)}{f(r,\theta)}(dr^2 + r^2d\theta^2) + \frac{l(r,\theta)}{f(r,\theta)}r^2\sin^2\theta d\varphi^2, \tag{8}$$

and the function $\psi(r,\theta)$ depending also on r, θ only.

The YM ansatz in this case is a straightforward generalization of the axial ymmetric $d = 4$ ansatz obtained in the pioneering papers by Manton [8] and Rebbi and Rossi [9], and has been considered to some extent in [10]. For the time and extra-direction translational symmetry, we choose a gauge such that $\partial A/\partial t = \partial A/\partial x^5 = 0$. However, the action of the Killing vector ξ_1 can be compensated by a gauge rotation,

$$\mathcal{L}_\varphi A_N = D_N\Psi, \tag{9}$$

with Ψ being a Lie-algebra valued gauge function. This introduces an winding number n in the ansatz (which is a constant of motion and is restricted to be an integer) and implies the existence of a potential W with

$$F_{N\varphi} = D_N W, \tag{10}$$

where $W = A_\varphi - \Psi$.

Thus, the most general axially symmetric 5D Yang-Mills ansatz contains 15 functions: 12 magnetic and 3 electric potentials and can be easily obtained in cylindrical coordinates $x^i = (\rho, \varphi, z)$ [with $\rho = r\sin\theta$, $z = r\cos\theta$, and r, θ, and φ being the usual spherical coordinates in (3 + 1) dimensions],

$$A_N = \frac{1}{2}A_N^{(\rho)}(\rho,z)\tau_\rho^n + \frac{1}{2}A_N^{(\varphi)}(\rho,z)\tau_\varphi^n + \frac{1}{2}A_N^{(z)}(\rho,z)\tau_z, \tag{11}$$

where the only φ-dependent terms are the SU(2) matrices [composed of the standard (τ_x, τ_y, τ_z) Pauli matrices] $\tau_\rho^n = \cos n\varphi\tau_x + \sin n\varphi\tau_y$, $\tau_\varphi^n = -\sin n\varphi\tau_x + \cos n\varphi\tau_y$. Transforming to spherical coordinates, it is convenient to introduce, without any loss of generality, a new SU(2) basis $(\tau_r^n, \tau_\theta^n, \tau_\varphi^n)$, with $\tau_r^n = \sin\theta\tau_\rho^n + \cos\theta\tau_z$, $\tau_\theta^n = \cos\theta\tau_\rho^n - \sin\theta\tau_z$, which yields

$$A_N = \frac{1}{2}A_N^{(r)}(r,\theta)\tau_r^n + \frac{1}{2}A_N^{(\theta)}(r,\theta)\tau_\theta^n + \frac{1}{2}A_N^{(\varphi)}(r,\theta)\tau_\varphi^n. \tag{12}$$

For this parametrization $2\Psi = n\tau_z = n\cos\theta\tau_r^n - n\sin\theta\tau_\theta^n$. The gauge invariant quantities expressed in terms of these functions will be independent on the angle φ.

Searching for solutions within the most general ansatz is a difficult task. Therefore we use in this paper a purely magnetic reduced ansatz with six essential non-Abelian potentials and

$$A_r^{(r)} = A_r^{(\theta)} = A_\theta^{(r)} = A_\theta^{(\theta)} = A_\varphi^{(\varphi)} = A_5^{(\varphi)} = A_t^{(a)} = 0.$$

A suitable parametrization of the nonzero components of $A_N^{(a)}$ which factorizes the trivial θ-dependence and admits a straightforward four-dimensional picture is

$$\begin{aligned} A_r^{(\varphi)} &= \frac{1}{r}H_1(r,\theta), \qquad A_\theta^{(\varphi)} = 1 - H_2(r,\theta), \\ A_\varphi^{(r)} &= -n\sin\theta H_3(r,\theta) + 2gJ(r,\theta)\phi_1(r,\theta), \\ A_\varphi^{(\theta)} &= -n\sin\theta(1 - H_4(r,\theta)) + 2gJ(r,\theta)\phi_2(r,\theta), \\ A_5^{(r)} &= \phi_1(r,\theta), \qquad A_5^{(\theta)} = \phi_2(r,\theta), \end{aligned} \tag{13}$$

[note that the SO(3)-symmetric ansatz is recovered for $H_1 = H_3 = \phi_2 = J = 0$ and $H_2 = H_4 = \omega(r)$, $\phi_1 = \phi(r)$].

To fix the residual Abelian gauge invariance we choose the gauge condition,

$$r\partial_r H_1 - \partial_\theta H_2 = 0.$$

We remark that A_φ, A_5 have components along the same directions in isospace. Therefore, the T_φ^5, T_5^φ components of the energy-momentum tensor will be nonzero for axially symmetric YM configurations. This implies the existence, in the five-dimensional metric ansatz (5), of one extradiagonal $g_{5\mu}$ metric function, with

$$\mathcal{W}_\mu = J(r,\theta)\delta_\mu^\varphi. \tag{14}$$

The $d = 5$ EYM configurations extremize also the action principle (7) and can be viewed as solutions of the four-dimensional theory. In this picture, $H_i(r,\theta)$ are the magnetic SU(2) gauge potentials, $\psi(r,\theta)$ is a dilaton, $J(r,\theta)$ is a U(1) magnetic potential, while $\phi_1(r,\theta), \phi_2(r,\theta)$ are the components of a Higgs field. We mention also that, similar to the pure (E)-YMH case, we may define a 't Hooft field strength tensor and an expression for the non-Abelian electric and magnetic charges within the action principle (7).

C. Boundary conditions

1. Metric functions

To obtain asymptotically flat regular solutions with finite energy density, the metric functions have to satisfy the boundary conditions

$$\partial_r\psi|_{r=0} = \partial_r f|_{r=0} = \partial_r m|_{r=0} = \partial_r l|_{r=0} = J|_{r=0} = 0, \tag{15}$$

which result from the requirement of regularity at the origin and

$$f|_{r=\infty} = m|_{r=\infty} = l|_{r=\infty} = 1, \qquad \psi|_{r=\infty} = J|_{r=\infty} = 0, \tag{16}$$

which result from the requirement of asymptotic flatness and finite energy. For solution with parity reflection symmetry (the case considered in this paper), the boundary conditions along the z and ρ axes are (with $z = r\cos\theta$ and $\rho = r\sin\theta$)

$$\begin{aligned} \partial_\theta\psi|_{\theta=0,\pi/2} &= \partial_\theta J|_{\theta=0,\pi/2} = \partial_\theta f|_{\theta=0,\pi/2} = \partial_\theta m|_{\theta=0,\pi/2} \\ &= \partial_\theta l|_{\theta=0,\pi/2} = 0. \end{aligned} \tag{17}$$

Note that the boundary conditions for f, m, l, and ψ are similar to those derived in [11], while the ones for J are newly introduced.

2. Matter functions

A systematic study of the asymptotic behavior of the A_5 component of the gauge field reveals that a general enough set of boundary conditions is given by

$$\lim_{r\to\infty}\phi_1 = \eta\cos m\theta, \qquad \lim_{r\to\infty}\phi_2 = \eta\sin m\theta, \tag{18}$$

with $m = 0, 1, \ldots,$ and η an arbitrary positive constant ($\eta = 0$ implies $A_5 = 0$ which is outside the interest of this paper). This condition fixes the boundary conditions at $r \to \infty$ for the other gauge potentials. In deriving these conditions, we use the asymptotic analysis of the Yang-Mills equations, requiring also the finiteness of the total mass/energy, which implies that $F_{5M}^{(a)}$ vanishes at infinity (see also [12] for a detailed discussion of this issue in a four-dimensional EYMH theory).

For even values of m, the asymptotic boundary conditions of the gauge functions H_i are

$$\begin{aligned} H_1 = 0, \quad H_2 = -m, \quad H_3 = \frac{\cos\theta}{\sin\theta}(\cos m\theta - 1), \\ H_4 = -\frac{\cos\theta}{\sin\theta}\sin m\theta. \end{aligned} \tag{19}$$

while for odd m

$$\begin{aligned} H_1 = 0, \quad H_2 = -m, \quad H_3 = \frac{1}{\sin\theta}(\cos m\theta - \cos\theta), \\ H_4 = -\frac{\sin m\theta}{\sin\theta}. \end{aligned} \tag{20}$$

In this paper we restrict ourselves to the simplest cases, $m = 0$ and $m = 1$, corresponding in a four-dimensional picture to multimonopoles (MM) and monopole-antimonopole (MA) configurations, respectively.

The boundary values at $r = 0$ for $m = 0$ are [13]

$$H_1 = H_3 = 0, \qquad H_2 = H_4 = 1, \qquad \phi_1 = \phi_2 = 0, \tag{21}$$

while for $m = 1$, we impose [14]:

$$\begin{aligned} H_1 = H_3 = 0, \qquad H_2 = H_4 = 1, \\ \cos\theta\partial_r\phi_1 - \sin\theta\partial_r\phi_2 = 0, \qquad \sin\theta\phi_1 + \cos\theta\phi_2 = 0, \end{aligned} \tag{22}$$

which are the known conditions used in the study of four-dimensional MM and MA configurations, respectively. The conditions along the axes are determined by the symmetries and finite energy density requirements. For $m = 0$ solutions we impose [13]

$$\begin{aligned} H_1|_{\theta=0,\pi/2} &= H_3|_{\theta=0,\pi/2} = \phi_2|_{\theta=0,\pi/2} = 0, \\ \partial_\theta H_2|_{\theta=0,\pi/2} &= \partial_\theta H_4|_{\theta=0,\pi/2} = \partial_\theta \phi_1|_{\theta=0,\pi/2} = 0, \end{aligned} \tag{23}$$

while the conditions satisfied by the $m = 1$ configurations are [14]

$$\begin{aligned} H_1|_{\theta=0,\pi/2} &= H_3|_{\theta=0,\pi/2} = \partial_\theta H_2|_{\theta=0,\pi/2} \\ &= \partial_\theta H_4|_{\theta=0,\pi/2} = 0, \\ \partial_\theta \phi_1|_{\theta=0} &= \phi_1|_{\theta=\pi/2} = \phi_2|_{\theta=0} = \partial_\theta \phi_2|_{\theta=\pi/2} = 0. \end{aligned} \tag{24}$$

In addition, regularity on the z-axis requires the conditions $l|_{\theta=0} = m|_{\theta=0}$, $H_2|_{\theta=0} = H_4|_{\theta=0}$ to be satisfied, for any values of the integers (m, n).

D. Other relations

The assumed symmetries together with the YM equations imply the following relations [we use here the relation (10) together with the Yang-Mills equations]:

$$\begin{aligned} K &= \int_V d^3x\sqrt{-g}T^5_\varphi = 2\,\mathrm{Tr}\left\{\int_V d^3x\sqrt{-g}F_{M\varphi}F^{Mt}\right\} \\ &= 2Tr\left\{\oint_\infty dS_\mu\sqrt{-g}WF^{\mu 5}\right\}, \end{aligned} \tag{25}$$

$$\begin{aligned} E_h &= 2\,\mathrm{Tr}\left\{\int_V d^3x\sqrt{-g}F_{M5}F^{M5}\right\} \\ &= 2\,\mathrm{Tr}\left\{\oint_\infty dS_\mu\sqrt{-g}A_5F^{\mu 5}\right\}, \end{aligned} \tag{26}$$

where the volume integral is taken over the three-dimensional physical space.

These relations can easily be evaluated by using the general sets of boundary conditions and the asymptotic expression

$$\phi_1 \sim \eta\left(1 - \frac{Q}{r}\right)\cos m\theta, \qquad \phi_2 \sim \eta\left(1 - \frac{Q}{r}\right)\sin m\theta. \tag{27}$$

Thus we find $E_h = 4\pi\eta^2 Q$, while $K = 4\pi n(1 - (-1)^m)\eta Q$.

This implies that the magnitude of the gauge potentials A_5 should be nonzero at infinity, elsewhere $A_5 \equiv 0$. These relations provide also useful tests to verify the accuracy of the numerical calculation.

For the assumed asymptotic behavior, the mass of these solutions is determined by the derivative of the metric function f:

$$M = \frac{1}{2}\lim_{r\to\infty} r^2\partial_r f. \tag{28}$$

When viewed as solutions of the four-dimensional theory, the magnetic charge of the $m = 0$ solutions is n, thus they correspond to a generalization of the gravitating axially symmetric monopoles discussed in [13]. In the same approach, the magnetic charge of the $m = 1$ solutions is zero (although locally the magnetic charge density is nonzero), thus generalizing for a nonzero dilaton and U(1) field the known monopole-antimonopole solutions [14].

III. NUMERICAL SOLUTIONS

By considering the rescalings $r \to r\eta g$ and $\phi \to \phi/\eta$, the field equations depend only on the coupling constants $\alpha = \sqrt{4\pi G}\eta$, yielding the dimensionless mass $\mu = (4\pi G\eta^2)^{-1}M$.

For $\alpha = 0$ (no gravity) and no dependence on the x^5 coordinate, the four-dimensional picture corresponds to the SU(2)-YMH theory in a fixed Minkowski space. Our solutions in this case describe $d = 4$ nongravitating multimonopoles (see e.g. [9]) and monopole-antimonopoles [15], respectively.

A. *m*=0 "multimonopole" solutions

We first constructed a solution for $m = 0$, $n \geq 1$, and varying α. Our numerical analysis strongly suggests that the gravitating solutions exist up to a (n-dependent) maximal value of α, $\alpha_{\max}(n)$. We find $\alpha_{\max}(n = 1) \approx 1.268$ and $\alpha_{\max}(n = 2) \approx 1.275$. In the following, we will refer to this branch of solutions as the "main branch."

In Fig. 1 some data characterizing the monopole solution for $n = 1$ and the multimonopole solutions for $n = 2$ on this branch are shown: the mass per winding number μ/n, the value of the metric function $f(r)$ at the origin, $f(0)$ and the value of the dilaton field $\psi(r)$ at the origin, $\psi(0)$ are given as functions of α. As can be seen from this figure, the

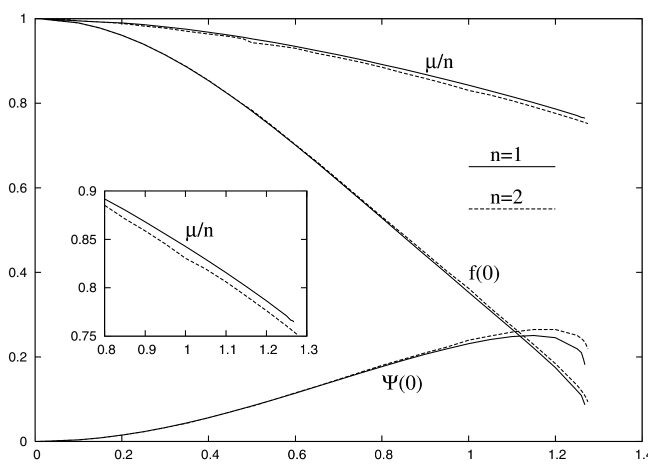

FIG. 1. The dimensionless mass per the winding number μ/n and the values of the metric functions $f(x)$ and $\psi(x)$ at the origin, $f(0)$, $\psi(0)$ are shown as functions of α for $n = 1, 2$ gravitating solutions with $m = 0$.

mass ratio μ/n and the value $f(0)$ decrease for increasing α, while $\psi(0)$ first increases and then decreases starting from $\alpha \sim 1.2$.

Note that $n = 1$ in fact corresponds in this case to SO(3)-symmetric solutions and thus implies $J(r, \theta) = 0$. The solutions coincide with the ones obtained in [6,11]. Here, however, we have used isotropic coordinates as compared to Schwarzschild-like coordinates used previously. The numerical analysis in these latter papers has revealed that several branches of solutions exist. These branches (as illustrated of Figs. 1 and 3 of [11]) have higher mass than the main branch. It is very likely [as suggested e.g. by the parameter $\psi(0)$ at the approach of $\alpha_{\max}$] that other branches of solutions appear also for $n > 1$. The construction of these branches turns out to be numerically difficult and is not attempted in this publication.

In the case $n > 1$, the function J becomes nontrivial although it remains rather small, typically $|J|_{\max} \sim 10^{-2}$ for $\alpha = \alpha_{\max}$ in the case $n = 2$. Another feature of our solutions can be revealed by studying the mass difference $\Delta(n, \alpha) \equiv \mu_{n=1} - \mu_n/n$. This quantity characterizes the binding energy of the monopoles due to gravity. It turns out that it is positive and we find typically, for large α, that $\Delta(2, 1.0) \sim 0.01$. The binding energy values are very close (in fact of the same order of magnitude within the numerical accurancy) to those obtained in a four-dimensional EYMH-dilaton effective theory considered in [11], showing that the supplementary function J has rather little influence on the masses of the solutions.

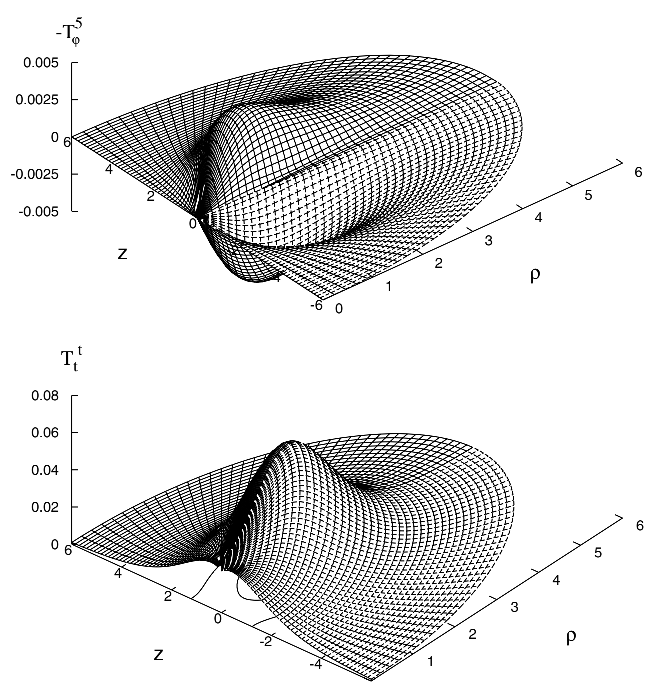

FIG. 2. The components T^5_φ and T^t_t of the energy-momentum tensor are shown for a typical $m = 0, n = 2$ solution, with $\alpha = 0.95$.

In Fig. 2 we show the energy density $\epsilon = -T^t_t$ and the extradiagonal component T^5_φ (which can be interpreted as the momentum flux of the extra dimension across a surface with $\varphi = \text{const}$) of a typical $m = 0$, $n = 2$ solution as a function of the coordinates $z = r\cos\theta$ and $\rho = r\sin\theta$ for a typical $n = 2$ solution with $\alpha = 0.95$. As seen from this figure, the distributions of the mass-energy density $-T^t_t$ can be different from those of spherical configurations, showing a pronounced peak along the ρ-axis and decreasing monotonically along the z-axis. Equal density contours reveal a toruslike shape of the solutions. The picture is different for the T^5_φ-component which vanishes on the ρ-axis and changes the sign as $z \to -z$.

B. m=1 "monopole-antimonopole" solutions

A very different picture is found by taking $m = 1$ in the asymptotic boundary conditions (18) and (20) (here we consider the case $n = 1$ only). When α is increased from zero, a branch of $m = 1$ solutions emerges from the uplifted version of the $d = 4$ flat spacetime MA configurations. This branch ends at a critical value $\alpha_{cr} \approx 0.65$. As $\alpha \to \alpha_{\text{cr}}$, the geometry remains regular with no event horizon appearing, and the mass approaches a finite value (see Fig. 3).

Apart from this fundamental branch, the $m = 1$ solutions admit also excited configurations, emerging in the $\alpha \to 0$ limit (after a rescaling) from the spherically symmetric solutions with $A_5 = 0$ (corresponding after dimensional reduction to solutions of a $d = 4$ EYM-dilaton theory). The lowest excited branch, originating from the one-node spherically symmetric solution, evolves smoothly from $\alpha = 0$ to α_{cr} where it bifurcates with the fundamental branch.

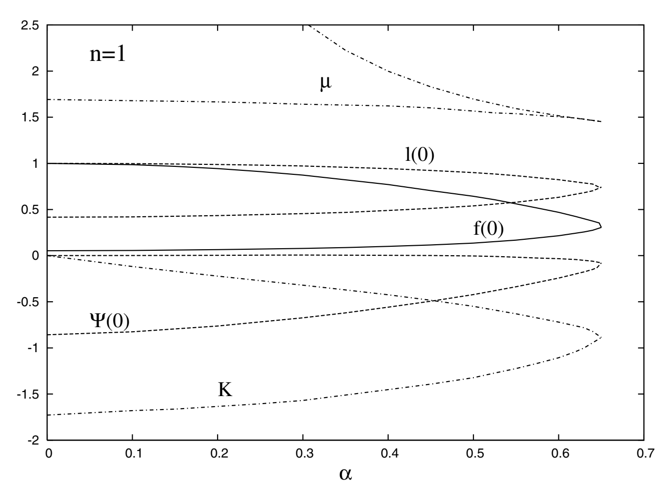

FIG. 3. The dimensionless mass μ, the value at the origin of the metric functions f, l and ψ as well as the integral of the T^5_φ component, K, are shown as functions of α for $n = 1$ gravitating solutions with $m = 1$.

Not surprisingly, the $m = 1$ solutions share a number of common properties with the $d = 4$ MA configurations in EYMH theory discussed in [14]. The functions H_i, ϕ_i and f, l, m present a shape similar to the case considered in [14]. The energy density $\epsilon = -T_t^t$ possesses maxima at $z = \pm d/2$ and a saddle point at the origin, and presents the typical form exhibited in the literature on MA solutions [12,14,15]. The modulus of the fifth component of the gauge potential possesses always two zeros at $\pm d/2$ on the z-symmetry axis. The excited solutions become infinitely heavy as $\alpha \to 0$ while the distance d tends to zero. The metric function $J(r, \theta)$ presents a nontrivial angular dependence, behaving asymptotically as $J \sim J_0 \sin^2\theta/r$ (for $m = 0$, J decays as $1/r^2$ in the same limit).

The solutions mass, the integral (25) of the T_φ^5 component of the energy-momentum tensor, and the values of the metric functions f, l, ψ at the origin are plotted in Fig. 3 as functions of α. In Fig. 4 we plot the energy density $\epsilon = -T_t^t$ and the extradiagonal component T_φ^5 of a typical $m = 1$ solution as a function of the coordinates ρ, z, for $\alpha = 0.45$. Note the different shape of T_φ^5 as compared to the $m = 0$ case, which implies in this case a nonzero value of the volume integral (25).

Although we have restricted the analysis here to the simplest sets of solutions, other excited $m = 1$ branches should exist (these solutions have been found in the $d = 4$ EYMH theory [14]). These solutions do not possess counterparts in flat spacetime and their $\alpha \to 0$ limit corresponds always to (higher nodes) vortex solutions of the EYM theory with $A_5 = 0$.

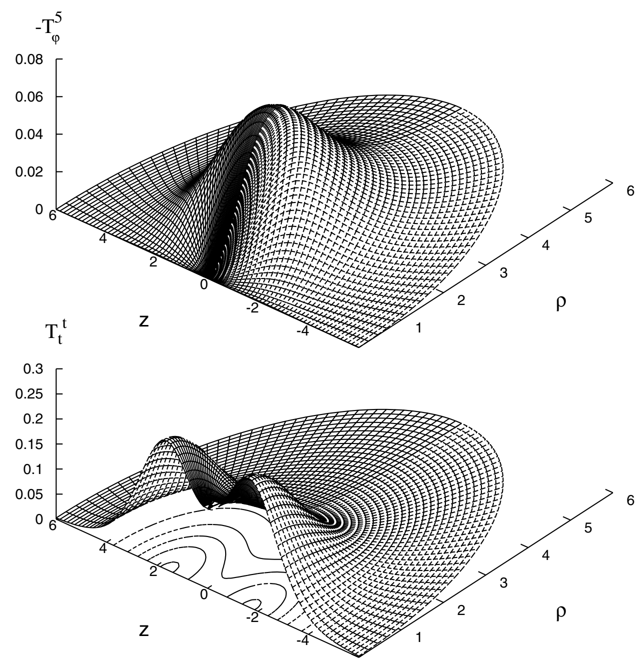

FIG. 4. The components T_φ^5 and T_t^t of the energy-momentum tensor are shown for a typical ($m = 1$, $n = 1$) solution, with $\alpha = 0.45$.

IV. REMARKS ON ROTATING SOLUTIONS

As remarked in [16], there is a simple way to generate electrically charged solutions in a $d = 4$ YMH theory without a Higgs potential, by starting with a pure magnetic configuration $(\vec{A}, \Phi_0)$ and using the transformation $\vec{A} \to \vec{A}$, $\Phi \to \Phi_0 \cosh\beta$, $A_t \to \Phi_0 \sinh\beta$, with β an arbitrary real constant. In a five-dimensional picture, this corresponds to boosting in the (x^5, t) plane a purely magnetic YM vortex, according to

$$x^5 = \cosh\beta U + \sinh\beta T, \qquad t = \sinh\beta U + \cosh\beta T. \tag{29}$$

Also, for vacuum solutions extremizing (1), it has been known for some time that, by taking the product of the $d = 4$ Schwarzschild solution with a circle and boosting it in the fifth direction, the entire family of electrically charged (magnetically neutral) KK black holes is generated.

A more complicated picture is found in the presence of non-Abelian matter fields. However, it can be proven that, given a $d = 5$ initial configuration $(\psi, \mathcal{W}_\varphi, \gamma_{\mu\nu}, A_N)$, with two Killing vectors $\partial/\partial t$, $\partial/\partial x^5$, by applying the coordinate transformation (29), the new form of the EYM solution is given by the line element,

$$ds^2 = e^{-a\bar{\psi}}\bar{\gamma}_{\mu\nu}dx^\mu dx^\nu + e^{2a\bar{\psi}}(dU + 2\bar{\mathcal{W}}_\varphi d\varphi + 2\bar{\mathcal{W}}_T dT)^2, \tag{30}$$

the same SU(2) potentials A_r, A_θ, A_φ and

$$A_T = \sinh\beta\left(\phi_1 \frac{\tau_r^n}{2} + \phi_2 \frac{\tau_r^\theta}{2}\right), \qquad A_U = \cosh\beta\left(\phi_1 \frac{\tau_r^n}{2} + \phi_2 \frac{\tau_r^\theta}{2}\right). \tag{31}$$

The new quantities in (30) are defined by

$$\begin{aligned} e^{2a\bar{\psi}} &= e^{2a\psi}\cosh^2\beta + e^{-a\psi}\gamma_{tt}\sinh^2\beta, \\ \bar{\mathcal{W}}_\varphi &= \frac{e^{2a\psi}\cosh\beta\,\mathcal{W}_\varphi}{e^{2a\psi}\cosh^2\beta + e^{-a\psi}\gamma_{tt}\sinh^2\beta}, \\ \bar{\mathcal{W}}_T &= \frac{1}{2}\frac{(e^{2a\psi} + e^{-a\psi}\gamma_{tt})\sinh\beta\cosh\beta}{e^{2a\psi}\cosh^2\beta + e^{-a\psi}\gamma_{tt}\sinh^2\beta}, \\ \bar{\gamma}_{\mu\nu}dx^\mu dx^\nu &= e^{a(\bar{\psi}-\psi)}(\gamma_{tt}(dT - 2\sinh\beta\,\mathcal{W}_\varphi d\varphi)^2 + d\ell^2). \end{aligned} \tag{32}$$

For the metric ansatz (8), we find

$$\bar{f} = \frac{f e^{a\psi}}{\sqrt{e^{2a\psi}\cosh^2\beta - e^{-a\psi} f \sinh^2\beta}}, \quad \bar{m} = m, \quad \bar{l} = l. \tag{33}$$

The following "reality condition" follows straightforward

$$e^{2a\psi}\cosh^2\beta - e^{-a\psi} f \sinh^2\beta > 0, \qquad (34)$$

which turn out to be satisfied by all considered configurations (although we could not find an analytical argument).

The dimensional reduction of these configurations along the U-direction provides new solutions in the $d = 4$ EYMH-U(1)-dilaton theory. As different from the original configurations, the boosted configurations present a nonzero $\gamma_{\varphi T}$ term, thus corresponding to rotating electrically charged solutions. The angular momentum density of these $d = 4$ configurations is given by $\cosh\beta T_\varphi^5$ and has the typical shape presented in Figs. 2 and 4. However, although they will rotate locally, the total angular momentum of the MM solutions is zero, and the spacetime consists in two regions rotating in opposite directions.[1] The situation is different for MA configurations, whose ADM angular momentum becomes proportional with the constant K of the static solutions as defined by (25).

V. CONCLUSIONS

In this paper we consider axially symmetric vortex-type solutions in the $d = 5$ EYM-SU(2) theory. Our motivation is twofold: first, such solutions are interesting in their own right. Second, it has been shown that, after dimensional reduction, the system corresponds to a particular EYMH-U(1)-dilaton model. Thus, we may hope to learn more about physics of the gravitating YMH model in four dimensions. This is interesting especially in connection with the question of gravitating rotating solutions. In this context, we have presented a simple procedure to generate $d = 4$ rotating solutions with non-Abelian matter fields starting with static $d = 5$ EYM vortex-type solutions.

[1] In Einstein-Maxwell theory, a zero total angular momentum implies a static configuration. The situation may be different for a more general matter content. The existence of a rotating solution of the Einstein equation with a vanishing ADM angular momentum has been noticed in [17].

The $d = 4$ rotating solutions we find by taking $m = 1$ in the general set of boundary conditions generalize the MA configurations discussed recently in [18] and may help to clarify the issue of the limiting solutions, left unsolved in that paper. Even more interesting is the $m = 0$ case. As yet to the best of our knowledge, there is no example of globally regular rotating non-Abelian solutions with a nonvanishing magnetic charge presented in the literature. However, the boosted $m = 0$ solutions correspond in $d = 4$ to globally regular dyons solutions with a nonzero extra-diagonal metric component $\gamma_{\varphi T}$ associated with rotation. Although these configurations rotate locally, their global angular momentum vanishes as predicted in [10,19]. This suggests that similar configurations should exist also in EYMH theory.

We expect also that $d = 5$ EYM theory possesses a whole sequence of solutions, obtained within the ansatz (13) for an arbitrary $m > 1$. By boosting these solutions, new $d = 4$ configurations describing rotating chains and vortex rings [20] can be generated.

We close by pointig out another possible interpretation of the solutions discussed in this paper. Since $A_t = 0$ in the matter ansatz (12) and there is also no time dependence, our configurations will solve also the $d = 5$ EYM equations on the Euclidean section, obtained by analytically continuing $t \rightarrow i\tau$ (with an arbitrary periodicity of τ for these regular solutions). Now, the KK reduction with respect to the Killing vector $\partial/\partial\tau$ corresponds in a four-dimensional picture to rotating regular instantons in an EYM-dilaton theory. The U(1) and Higgs field are zero in this case while ϕ_i corresponds to electric SU(2) potentials.

More details on these globally regular solutions will be given elsewhere.

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