# Separate universe and the back reaction of long wavelength fluctuations

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(Received 27 February 2005; published 15 April 2005)

We investigate the back reaction of cosmological long wavelength perturbations on the evolution of the Universe. By applying the renormalization group method to a Friedmann-Robertson-Walker universe with long wavelength fluctuations, we demonstrate that the renormalized solution with the back reaction effect is equivalent to that of the separate universe. Then, using the effective Friedmann equation, we show that only the nonadiabatic mode of long wavelength fluctuations affects the expansion law of the spatially averaged universe.

DOI: 10.1103/PhysRevD.71.084016 PACS numbers: 04.25.Nx, 98.80.Qc

## I. INTRODUCTION

The analysis of large scale cosmological perturbation is an important issue for obtaining information on the initial density fluctuation that was generated during the era of the inflationary expansion of the Universe. However, because of the nonlinear nature of the Einstein equation, linear analysis is not sufficient to investigate the evolution of the early universe. If we consider the expansion law of the early universe, the back reaction effect owing to the long wavelength fluctuations is expected to be important.

Let us consider the Universe with large scale fluctuations of which wavelength is larger than the Hubble horizon. Each spatial region with the Hubble horizon scale in the Universe is causally disconnected and evolves independently in time. Hence the Universe with large scale inhomogeneities can be treated as the collection of quasi-homogeneous and quasi-isotropic Friedmann-Robertson-Walker (FRW) universes. The realization of this idea is the separate universe approach [1] that is equivalent to the lowest order of the gradient expansion of the Einstein equation. This approach is suitable to treat a universe with large scale nonlinear inhomogeneities.

An application of this method is the stochastic approach to inflation [2]. During the inflationary expansion of the Universe, long wavelength stochastic fluctuations are generated and the coarse-grained scalar field in each horizon scale regions behaves as Brownian particles. The random driving force for the coarse-grained scalar field appears as the result of the back reaction of long wavelength quantum fluctuation on the homogeneous background. Another example of the large scale inhomogeneity that is tractable by using the separate universe is the preheating stage after inflation. Long wavelength fluctuations are amplified by the parametric resonance associated with the oscillation of the background inflaton field and the superhorizon scale structure of the Universe evolves to be highly inhomogeneous [3].

On the other hand, our present observable universe is considered to be homogeneous and isotropic, and the evolution of the Universe is determined by the Friedmann equation. Thus, to incorporate the back reaction effect into the Friedmann equation, we have to take the spatial average of an inhomogeneous universe [4–13]. The obtained effective Friedmann equation predicts how the expansion law of the averaged FRW universe is modified by the back reaction effect. The solutions for the cosmological constant problem and the dark energy problem are investigated in this direction [5,6,12].

In this paper, we analyze the back reaction effect by long wave fluctuations using the renormalization group (RG) approach [7–9,14]. First, we consider a homogeneous FRW universe with long wavelength linear fluctuations. We apply the RG method to this system to understand how the long wavelength fluctuations modify the background FRW universe. We found that the effect of the back reaction by long wavelength fluctuations results in spatially dependent constants of integration of a FRW universe and the renormalized variables become solutions of the separate universe approach. Then, by taking the spatial average of the solution of the separate universe, we derive the effective Friedmann equation that involves the back reaction of long wave modes. The obtained equation shows that the back reaction effect on the averaged FRW universe appears only for the nonadiabatic type of fluctuations.

The plan of the paper is as follows. In Sec. II, we review the solution of a FRW universe and long wavelength perturbations about it. Then, the RG method is applied to this system. In Sec. III, we derive the effective Friedmann equation by taking the spatial averaging of the separate universe. Section IV is devoted to summary and conclusion. We use units in which $c = \hbar = 8\pi G = 1$ throughout the paper.

## II. RENORMALIZATION OF LONG WAVELENGTH MODE AND THE SEPARATE UNIVERSE

In this section, we apply the renormalization group method to a FRW universe with long wavelength perturbations and investigate how the long wave modes modify the background FRW universe.

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### A. The solution of long wavelength fluctuations in a FRW universe

We consider two scalar fields as the matter fields. The metric and the scalar fields in a flat slice are written as

$$\begin{aligned} ds^2 &= -\left(1 - 2\frac{\delta H}{H_0}\right)\frac{d\alpha^2}{H_0^2} + e^{2\alpha}d\boldsymbol{x}^2, \\ \chi^{(1,2)} &= \chi_0^{(1,2)}(\alpha) + \delta\chi^{(1,2)}(\alpha, \boldsymbol{x}), \end{aligned} \tag{1}$$

where $H_0$ and $\chi_0^{(1,2)}$ are the background quantities, $\delta H$ and $\delta\chi^{(1,2)}$ denote the linear perturbation about a homogeneous FRW universe. In this slice, the logarithm of the scale factor $\alpha$ serves as a time parameter. The background Einstein equation and the scalar field equation are

$$\begin{aligned} -3H_0^2 + \frac{1}{2}\sum_{A=1}^{2}(\Pi_0^{(A)})^2 + V(\chi_0^{(A)}) &= 0, \\ -2H_0H_{0,\alpha} - 3H_0^2 - \frac{1}{2}\sum_{A=1}^{2}(\Pi_0^{(A)})^2 + V(\chi_0^{(A)}) &= 0, \\ H_0\Pi_{0,\alpha}^{(A)} + 3H_0\Pi_0^{(A)} + \frac{\partial V}{\partial\chi_0^{(A)}} &= 0, \\ \Pi_0^{(A)} &\equiv H_0\chi_{0,\alpha}^{(A)}, \\ A &= 1, 2. \end{aligned} \tag{2}$$

where we have introduced the momentum variable $\Pi_0^{(A)}$ and $, \alpha = \partial/\partial\alpha$. By using the Hamilton-Jacobi formalism [15], these equations are combined to the following equations for the Hubble function $H_0(\chi_0^{(A)})$ and the scalar fields $\chi_0^{(A)}$:

$$3H_0^2 = 2\sum_{A=1}^{2}\left(\frac{\partial H_0}{\partial\chi_0^{(A)}}\right)^2 + V(\chi_0^{(A)}), \tag{3}$$

$$H_0\chi_{0,\alpha}^{(A)} = -2\frac{\partial H_0}{\partial\chi_0^{(A)}}. \tag{4}$$

The solution of Eq. (3) is written as

$$H_0 = H_0(\chi_0^{(1)}, \chi_0^{(2)}; d_1, d_2)$$

where $d_1, d_2$ are integration constants. By differentiating Eq. (3) with respect to $d_{1,2}$ and integrating the resulting equation with respect to the time parameter $\alpha$, we obtain the remaining two constants of integration $c_0$ and $f_0$:

$$e^{3\alpha}\frac{\partial H_0}{\partial d_1} \equiv e^{-3c_0}, \qquad e^{3\alpha}\frac{\partial H_0}{\partial d_2} \equiv e^{-3c_0}f_0.$$

These four constants of integration completely specify the background FRW universe. Thus the background solution of the FRW universe can be written as

$$\begin{aligned} \chi_0^{(1,2)} &= \chi_0^{(1,2)}(\alpha + c_0; f_0, d_1, d_2), \\ H_0 &= H_0(\chi_0^{(1)}, \chi_0^{(2)}, d_1, d_2) \end{aligned} \tag{5}$$

Equations for long wavelength linear perturbations $\delta H$ and $\delta\chi^{(A)}$ are

$$-6H_0\delta H + \sum_{A=1}^{2}\Pi_0^{(A)}\delta\Pi^{(A)} + \sum_{A=1}^{2}\frac{\partial V}{\partial\chi_0^{(A)}}\delta\chi^{(A)} = 0, \tag{6}$$

$$\partial_i(\delta H) = -\frac{1}{2}\sum_{A=1}^{2}\Pi_0^{(A)}\partial_i(\delta\chi^{(A)}), \tag{7}$$

$$-2(H_0\delta H)_{,\alpha} - 6H_0\delta H - \sum_{A=1}^{2}\Pi_0^{(A)}\delta\Pi^{(A)} + \sum_{A=1}^{2}\frac{\partial V}{\partial\chi_0^{(A)}}\delta\chi^{(A)} = 0, \tag{8}$$

$$H_0((\delta\Pi^{(A)})_{,\alpha} + 3\delta\Pi^{(A)}) + \delta H(\Pi_{0,\alpha}^{(A)} + 3\Pi_0^{(A)}) + \sum_{B=1}^{2}\frac{\partial^2 V}{\partial\chi_0^{(A)}\partial\chi_0^{(B)}}\delta\chi^{(B)} = 0, \tag{9}$$

$$\delta\Pi^{(A)} = \frac{\Pi_0^{(A)}}{H_0}\delta H + H_0(\delta\chi^{(A)})_{,\alpha}. \tag{10}$$

Equation (6) is the Hamiltonian constraint and Eq. (7) is the momentum constraint. The growing mode solution of the long wavelength perturbation is obtained by taking the derivative of the background quantities with respect to the background constants of integration [16]:

$$\delta\chi^{(A)} = C(\boldsymbol{x})\frac{\partial\chi_0^{(A)}}{\partial c_0} + F(\boldsymbol{x})\frac{\partial\chi_0^{(A)}}{\partial f_0}, \tag{11}$$

$$\delta H = -\frac{1}{2}\sum_{A=1}^{2}\Pi_0^{(A)}\delta\chi^{(A)} = \sum_{A=1}^{2}\frac{\partial H_0}{\partial\chi_0^{(A)}}\delta\chi^{(A)}, \tag{12}$$

$$\delta\Pi^{(A)} = (H_0\delta\chi^{(A)})_{,\alpha}, \tag{13}$$

where $C(\boldsymbol{x})$ and $F(\boldsymbol{x})$ are arbitrary functions of spatial coordinates. The gauge invariant variable that corresponds to the spatial curvature perturbation in a comoving slice is

$$\mathcal{R} = -C(\boldsymbol{x}) - \frac{\sum_A \chi_{0,c_0}^{(A)}\chi_{0,f_0}^{(A)}}{\sum_A(\chi_{0,\alpha}^{(A)})^2}F(\boldsymbol{x}). \tag{14}$$

The function $C(\boldsymbol{x})$ corresponds to the adiabatic mode of perturbations and the curvature perturbation owing to this mode is constant in time. The function $F(\boldsymbol{x})$ corresponds to the nonadiabatic mode of perturbation and this mode results in development of the curvature perturbation.

### B. Renormalization of long wavelength fluctuations

We apply the RG method to obtain the back reaction of long wavelength fluctuations on a background FRW universe. Up to the first order of perturbations, the solution of the scalar fields is expressed as

$$\chi^{(A)}(\alpha, \boldsymbol{x}) = \chi_0^{(A)}(\alpha + c_0; f_0) + [C(\boldsymbol{x}) - C(\boldsymbol{x}_0)]\left(\frac{\partial \chi_0^{(A)}}{\partial c_0}\right) + [F(\boldsymbol{x}) - F(\boldsymbol{x}_0)]\left(\frac{\partial \chi_0^{(A)}}{\partial f_0}\right), \quad (15)$$

where we have chosen the functions $C$ and $F$ so that the perturbation vanishes at a spatial point $\boldsymbol{x} = \boldsymbol{x}_0$. We regard the perturbations as the secular terms in the spatial direction and absorb them into the background constants $c_0$ and $f_0$. For this purpose, we prepare a renormalization point $\boldsymbol{x}_\mu = \boldsymbol{x}_0 + \mu(\boldsymbol{x} - \boldsymbol{x}_0)$ and redefine the integration constants as follows:

$$c_0 = c(\mu) + \delta c(\mu; 0), \qquad f_0 = f(\mu) + \delta f(\mu; 0). \quad (16)$$

The counter terms $\delta c$ and $\delta f$ are chosen to cancel the $\boldsymbol{x}_0$ dependence of the perturbation solution:

$$\begin{aligned} &\delta c + [C(\boldsymbol{x}_\mu) - C(\boldsymbol{x}_0)] = 0, \\ &\delta f + [F(\boldsymbol{x}_\mu) - F(\boldsymbol{x}_0)] = 0. \end{aligned} \quad (17)$$

This defines the renormalization transformation: the value of the original constants at the spatial point $\boldsymbol{x}_0$ are mapped to the constants at $\boldsymbol{x}_\mu$. Then the solution of the scalar fields up to the first order becomes

$$\chi^{(A)} = \chi_0^{(A)}(\alpha + c(\mu); f(\mu)) + [C(\boldsymbol{x}) - C(\boldsymbol{x}_\mu)]\chi_{0,\alpha}^{(A)} + [F(\boldsymbol{x}) - F(\boldsymbol{x}_\mu)]\chi_{0,f_0}^{(A)}. \quad (18)$$

By assuming that the renormalization transformation defined by Eq. (17) forms the Lie group, we can obtain the RG equation by differentiating Eq. (17) with respect to $\mu$:

$$\frac{dc}{d\mu} = (\boldsymbol{x} - \boldsymbol{x}_0) \cdot \nabla C, \qquad \frac{df}{d\mu} = (\boldsymbol{x} - \boldsymbol{x}_0) \cdot \nabla F \quad (19)$$

and the solution of the RG equation is

$$\begin{aligned} c(\mu) &= C[\boldsymbol{x}_0 + \mu(\boldsymbol{x} - \boldsymbol{x}_0)], \\ f(\mu) &= F[\boldsymbol{x}_0 + \mu(\boldsymbol{x} - \boldsymbol{x}_0)]. \end{aligned} \quad (20)$$

The renormalized solution is obtained by setting $\mu = 1$:

$$\chi_{\text{ren}}^{(A)} = \chi^{(A)}|_{\mu=1} = \chi_0^{(A)}(\alpha + C(\boldsymbol{x}); F(\boldsymbol{x})). \quad (21)$$

At the same time, other variables receive the following renormalization:

$$H_0 + \delta H \rightarrow H_{\text{ren}} = H_0(\chi_{\text{ren}}^{(A)}), \quad (22)$$

$$\Pi_0^{(A)} + \delta \Pi^{(A)} \rightarrow \Pi_{\text{ren}}^{(A)} = -2\frac{\partial H_{\text{ren}}}{\partial \chi_{\text{ren}}^{(A)}} = H_{\text{ren}}\chi_{\text{ren},\alpha}^{(A)}, \quad (23)$$

and the renormalized metric becomes

$$ds^2 = -\frac{d\alpha^2}{H_{\text{ren}}^2} + e^{2\alpha}d\boldsymbol{x}^2. \quad (24)$$

By introducing a new time parameter $t = \int d\alpha/(NH_{\text{ren}})$ using an arbitrary lapse function $N(\alpha, \boldsymbol{x})$, the metric becomes

$$ds^2 = -N^2(t, \boldsymbol{x})dt^2 + e^{2\alpha(t,\boldsymbol{x})}d\boldsymbol{x}^2, \quad (25)$$

and the renormalized variables satisfy the following set of equations:

$$3H^2 = \frac{1}{2}\sum_{A=1}^{2}\left(\frac{\dot{\chi}^{(A)}}{N}\right)^2 + V(\chi^{(A)}), \qquad \frac{\dot{\alpha}}{N} = H, \quad (26)$$

$$\partial_i H = -\frac{1}{2}\sum_{A=1}^{2}\dot{\chi}^{(A)}\partial_i\chi^{(A)}, \quad (27)$$

$$\frac{\dot{H}}{N} = -\frac{1}{2}\sum_{A=1}^{2}\left(\frac{\dot{\chi}^{(A)}}{N}\right)^2, \quad (28)$$

$$\frac{1}{N}\left(\frac{\dot{\chi}^{(A)}}{N}\right)^{\cdot} + 3H\left(\frac{\dot{\chi}^{(A)}}{N}\right) + \frac{\partial V}{\partial \chi^{(A)}} = 0. \quad (29)$$

These are the basic equations of the separate universe approach (the lowest order of the gradient expansion).

In the RG approach to the back reaction problem, the effect of the back reaction by long wavelength fluctuations modifies the background constants of integration and the constants acquire spatial dependence associated with long wavelength fluctuations. The renormalized solution with the back reaction effect is equivalent to the solution of the separate universe. Therefore, we can use the separate universe as a starting point to derive the spatially averaged Friedmann equation for an inhomogeneous universe with long wavelength fluctuations.

## III. EFFECTIVE FRIEDMANN EQUATION

In this section, we take the spatial average of the solution of the separate universe and derive the effective Friedmann equation. The purpose is to observe how the expansion law of the spatially averaged FRW universe is modified by the back reaction effect of long wavelength fluctuations.

In a flat slice, the metric and the Hubble function of the separate universe are

$$ds^2 = -\frac{d\alpha^2}{H^2} + e^{2\alpha} d\boldsymbol{x}^2, \tag{30}$$

$$H = H(\chi^{(A)}(\alpha + c(\boldsymbol{x}), f(\boldsymbol{x}))), \tag{31}$$

where $c(\boldsymbol{x})$ and $f(\boldsymbol{x})$ are arbitrary functions of the spatial coordinates. To proceed with the averaging procedure analytically, we adopt the perturbative approach. We expand the solution of the separate universe about a homogeneous FRW background up to the second order of perturbation. By replacing $c(\boldsymbol{x}) \to c + \delta c(\boldsymbol{x})$, $f(\boldsymbol{x}) \to f + \delta f(\boldsymbol{x})$ and expanding the solution with respect to $\delta c$ and $\delta f$, the Hubble function up to the second order of perturbation becomes

$$\begin{aligned} H &= H_0 + H_1 + H_2, \\ H_0[\alpha] &= H(\chi^{(A)}(\alpha + c, f)), \\ H_1[\alpha] &= H_{0,c}\delta c + H_{0,f}\delta f, \\ H_2[\alpha] &= \frac{1}{2}H_{0,cc}(\delta c)^2 + H_{0,cf}\delta c \delta f + \frac{1}{2}H_{0,ff}(\delta f)^2. \end{aligned} \tag{32}$$

The metric up to the second order becomes

$$ds^2 = -\left(1 - \frac{2H_1}{H_0} - \frac{2H_2}{H_0} + 3\left(\frac{H_1}{H_0}\right)^2\right)dt^2 + e^{2\alpha(t)} d\boldsymbol{x}^2, \tag{33}$$

where a time variable $t$ was introduced by

$$t = \int \frac{d\alpha}{H_0(\alpha)}. \tag{34}$$

We can obtain the local scale factor by transforming the metric (33) to a synchronous frame. We define a new time variable $\tau$ by the following coordinate transformation:

$$\begin{aligned} t = \tau + \beta_1(\tau, \boldsymbol{x}) + \beta_2(\tau, \boldsymbol{x}), \qquad \frac{d\beta_1}{d\tau} &= \frac{H_1[\alpha(\tau)]}{H_0[\alpha(\tau)]}, \\ \frac{d\beta_2}{d\tau} &= \beta_1 \frac{d}{d\tau}\left(\frac{H_1}{H_0}\right) + \frac{H_2}{H_0}. \end{aligned} \tag{35}$$

Then we have

$$\begin{aligned} ds^2 &= -d\tau^2 + e^{2\alpha[t(\tau,\boldsymbol{x})]} d\boldsymbol{x}^2, \\ \alpha[t(\tau, \boldsymbol{x})] &= \alpha[\tau + \beta_1 + \beta_2] \equiv \alpha_0 + \alpha_1 + \alpha_2, \\ \alpha_0 &= \alpha(\tau), \qquad \alpha_1 = H_0\beta_1, \\ \alpha_2 &= H_0\beta_2 + \frac{1}{2}\left(\frac{dH_0}{d\tau}\right)(\beta_1)^2. \end{aligned} \tag{36}$$

The metric (36) has the same form as that of a flat FRW universe except its spatial dependence of the scale factor. By assuming that the spatial averaging of the first order variables vanishes $\langle \delta c \rangle = \langle \delta f \rangle = 0$, the spatially averaged Hubble parameter is

$$\tilde{H} \equiv \left\langle \frac{d}{d\tau}\alpha[t(\tau, \boldsymbol{x})] \right\rangle = H_0[\alpha(\tau)] + \left\langle \frac{d\alpha_2}{d\tau} \right\rangle, \tag{37}$$

and the Friedmann equation for the spatially averaged scale factor

$$e^{\tilde{\alpha}} \equiv e^{\langle \alpha \rangle} = e^{\alpha_0}(1 + \langle \alpha_2 \rangle) \tag{38}$$

is

$$3\left(\frac{d\tilde{\alpha}}{d\tau}\right)^2 = 3H_0^2 + \rho_{\rm BR}, \qquad \rho_{\rm BR} \equiv 6H_0\left\langle \frac{d\alpha_2}{d\tau} \right\rangle. \tag{39}$$

The term $\rho_{\rm BR}$ represents the modification of the Friedmann equation due to the long wavelength back reaction effect. The explicit form of $\rho_{\rm BR}$ is given by

$$\begin{aligned} \rho_{\rm BR} &= 3H_0 \frac{d}{d\tau}\left[H_0 \int \frac{d\tau}{H_0}(H_{0,cc}B^2 + 2H_{,cf}B + H_{0,ff})\right] \\ &\quad \times \langle \delta f^2 \rangle, \\ B(\tau) &\equiv H_0 \int d\tau \frac{H_{0,f}}{H_0}. \end{aligned} \tag{40}$$

We notice that the expression (40) does not contain $\delta c$ which is the source of the adiabatic mode of perturbations. Therefore, for the pure adiabatic type of fluctuation $\delta c \neq 0, \delta f = 0$, we have $\rho_{\rm BR} = 0$. Hence, the effective Friedmann equation does not contain the back reaction terms and long wavelength fluctuations owing to the adiabatic mode do not alter the expansion of the FRW universe. This result is consistent with the previous analysis of the back reaction problem [7–9]; the back reaction effect appears from $O(k^2)$ in the long wavelength expansion and there is no back reaction in the long wavelength limit. For the nonadiabatic type of fluctuation $\delta f \neq 0$, we have a nonzero value of $\rho_{\rm BR}$ and the back reaction of long wavelength fluctuations modifies the expansion law of the FRW universe. Although these results were previously obtained for the Universe with inflationary expansion [10,11], our analysis shows that we need a generally nonadiabatic type of fluctuation to obtain the long wavelength back reaction effect on the spatially averaged FRW universe. This result is independent of the expansion law of the background FRW universe.

## IV. CONCLUSION

In this paper, we discussed the back reaction effect owing to long wavelength fluctuations from two different perspectives: One is renormalization of constants by long wavelength fluctuations. The back reaction of long wavelength modes leads to the renormalization of constants contained in the solution of the background FRW universe. The other is the averaging of the inhomogeneous universe.

From the first perspective, the long wavelength mode generates spatial dependence of constants of a FRW universe and a homogeneous universe becomes an inhomogeneous one owing to the back reaction effect of long wavelength modes. From the second perspective, the effective Friedmann equation gets the additional contribution of the energy density from long wavelength fluctuations and the expansion law of the averaged universe becomes different from that of the original background FRW universe.

In this paper, we derived a general formula for $\rho_{\mathrm{BR}}$ but have not examined what type of expansion law can be obtained by the back reaction of long wavelength modes. This subject will be reported in a separate publication.

## ACKNOWLEDGMENTS

This work was supported in part by a Grant-In-Aid for Scientific Research of the Ministry of Education, Science, Sports, and Culture of Japan (15540265).

*Note added in proof.*—After completion of this work, we noticed the work by Kolb *et al.* [17]. They consider the second order long wavelength perturbation and derive the conclusion that only the nonadiabatic mode of the first order perturbation affects the local Hubble parameter.


[1] D. Wands, K. A. Maik, D. H. Lyth, and A. R. Liddle, Phys. Rev. D **62**, 043527 (2000).

[2] A. Starobinski, in *Current Topics in Field Theory, Quantum Gravity and Strings*, edited by H. J. de Vega and N. Sanchez, Lecture Notes in Physics (Springer, Heidelberg, 1986), Vol. 206, p. 107.

[3] T. Tanaka and B. Basset, astro-ph/0302544.

[4] V. M. Mukhanov, L. R. Abramo, and R. H. Brandenberger, Phys. Rev. Lett. **78**, 1624 (1997).

[5] L. R. Abramo, R. H. Brandenberger, and V. M. Mukhanov, Phys. Rev. D **56**, 3248 (1997).

[6] L. R. Abramo, Phys. Rev. D **60**, 064004 (1999).

[7] Y. Nambu, Phys. Rev. D **62**, 104010:1 (2000).

[8] Y. Nambu, Phys. Rev. D **63**, 044013:1 (2001).

[9] Y. Nambu, Phys. Rev. D **65**, 104013:1 (2002).

[10] G. Geshnizjani and R. H. Brandenberger, Phys. Rev. D **66**, 123507 (2002).

[11] G. Geshnizjani and R. H. Brandenberger, hep-th/0310265.

[12] R. H. Brandenberger and C. S. Lam, hep-th/0407048.

[13] L. R. Abramo, Ph.D. thesis, Brown University, 1997, gr-qc/9709049.

[14] Y. Nambu and Y. Y. Yamaguchi, Phys. Rev. D **60**, 104011 (1999).

[15] D. S. Salopek and J. M. Stewart, Classical Quantum Gravity **9**, 1943 (1992).

[16] Y. Nambu and A. Taruya, Classical Quantum Gravity **15**, 2761 (1998).

[17] E. W. Kolb, S. Matarrese, A. Notari, and A. Riotto, astro-ph/0410541.