

**Scalar field perturbations of the Schwarzschild black hole in the Gödel universe**

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We investigate the scalar field perturbations of the  $4 + 1$ -dimensional Schwarzschild black hole immersed in a Gödel universe, described by the Gimon-Hashimoto solution. This may model the influence of the possible rotation of the universe upon the radiative processes near a black hole. In the regime when the scale parameter  $j$  of the Gödel background is small, the oscillation frequency is linearly decreasing with  $j$ , while the damping time is increasing. The quasinormal modes are damping, implying stability of the Schwarzschild-Gödel space-time against scalar field perturbations. The approximate analytical formula for large multipole numbers is found.

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**I. INTRODUCTION**

A black hole's behavior is often crucially dependent upon the cosmological background in which the black hole is immersed. The simple case of a black hole immersed in an asymptotically flat space-time is described by the Schwarzschild solution. A natural extension is to consider a cosmological term which is described in terms of the Schwarzschild-de Sitter or Schwarzschild-anti-de Sitter solutions. Recent investigations of radiative processes around such black holes show that the radiative features are dependent upon the asymptotic conditions at infinity. Thus, for example, the quasinormal spectrum and late time tails are totally different for Schwarzschild [1], Schwarzschild-de Sitter [2], and Schwarzschild-anti-de Sitter black holes [3].

The cosmological background we are interested in here is the rotating universe. The rotation seems to be a universal phenomenon: all compact objects in the universe rotate. Yet the standard Friedman-Robertson-Walker metric represents a rather idealized model of an isotropic homogeneous world filled with a perfect fluid. It looks improbable that such a finely tuned universe can exist starting from the big bang to the present stage. In the beginning of the investigations of rotating cosmological models it was suggested that one should observe the anisotropy of the microwave background radiation (MBR), yet, as shown later, that the rotating models with no anisotropy of MBR or broken causality can exist [4]. In addition, apparent anisotropy in distribution of the observed angles between the polarization vectors and position of the major axis of radio sources can be related to a possible rotation of the universe [5]. For a further advance in the possibility of observation of global rotation, see the review [6].

An exact solution for the rotating universe was found by Gödel [7]. His solution was originally proposed for a four dimensional space-time. It possesses, among others, the following properties: it is homogeneous, has rotational symmetry, and allows the definition of the direction of positive time consistently in the whole solution, and,

what was in the focus of further research, it allows closed timelike curves, i.e. the time machine.

Recently, the Gödel universes have been of considerable interest [8], because in five-dimensional minimal supergravity the maximally supersymmetric Gödel-type universes are U-dual to pp-waves. Thereby, the Gödel-type universes are important as an opportunity of quantizing strings in this background as well as due to its relation to the corresponding limit of super-Yang Mills theories. On the gravitational side, the pp-waves dual to the Gödel universe correspond to the Penrose limit of near-horizon geometries.

As far as we are aware, until recently, an exact solution for a stationary black hole immersed in a rotating universe was not known. Nevertheless, such a solution has been obtained by Gimon and Hashimoto within the above mentioned five-dimensional minimal supergravity [9]. This solution represents the  $4 + 1$ -dimensional Schwarzschild space-time when the scale parameter of the Gödel background  $j$  goes to zero, and to the five-dimensional Gödel universe when the black hole mass vanishes, thereby giving us the model for the Schwarzschild black hole immersed in the rotating universe. The different features of this solution have been investigated recently in a series of papers [10]. The generalizations of the Gimon-Hashimoto solution were obtained in [11].

Yet, here we are interested in this solution from a rather different point of view, namely, we would like to find out what will happen with black hole (classical) radiation in the rotating universe. The straightforward way to know it is to investigate the quasinormal modes which govern the black hole response to external perturbations at late times. The quasinormal spectrum is sensitive to boundary conditions both at the event horizon and at spatial infinity, so the spectrum must be considerably affected by the rotating cosmological background.

In the present paper we had to be limited by the case of "slow rotation," i.e. the case when the influence of the cosmological background is weak. This happens when the above mentioned parameter  $j$  is small. We found that, at

least in the regime of small  $j$ , the black hole is stable against scalar field perturbations and as a result all found modes are damping. Because of cosmological rotation the real oscillation frequencies are decreasing and are roughly proportional to  $j$ , while the damping rates are decreasing nonlinearly with  $j$ . In addition we derive an approximate analytical formula for quasinormal (QN) modes with large multipole number  $L$ . Fortunately, the regime of small  $j$  seems to be the most reasonable phenomenologically.

The paper is organized as follows: in Sec. II we give some introductory material on the Schwarzschild-Gödel metric. In Sec. III, the scalar field equations is obtained in the limit of small scale parameter  $j$  of the cosmological background. For any  $j$ , the Klein-Gordon equation is not separable at least in the coordinates we have considered. In Sec. IV we find the quasinormal frequencies for the scalar field perturbations. In the end we discuss the obtained results and future perspectives.

## II. PRELIMINARIES OF THE SCHWARZSCHILD-GÖDEL SPACE-TIME

The bosonic fields of the minimal (4 + 1)- supergravity theory consist of the metric and the one-form gauge field, which are governed by the equations of motion:

$$R_{\mu\nu} = 2\left(F_{\mu\alpha}F_{\nu}^{\alpha} - \frac{1}{6}g_{\mu\nu}F^2\right), \quad (1)$$

$$D_{\mu}F^{\mu\nu} = \frac{1}{2\sqrt{3}}\varepsilon^{\alpha\lambda\gamma\mu\nu}F_{\alpha\lambda}F_{\gamma\mu}. \quad (2)$$

Here,  $\varepsilon_{\alpha\lambda\gamma\mu\nu} = \sqrt{-g}\epsilon_{\alpha\lambda\gamma\mu\nu}$ .

In the Euler coordinates  $(t, \theta', \psi', \phi')$ , the solution of the equations of motion (1) and (2), describing the Gödel universe, has the form [9]

$$ds^2 = -(dt + j(r^2)\sigma_L^3)^2 + dr^2 + \frac{r^2}{4}(d\theta'^2 + d\psi'^2 + d\phi'^2 + 2\cos\theta'd\psi'd\phi'), \quad (3)$$

where  $\sigma_L^3 = d\phi' + \cos\theta'd\psi'$ . The parameter  $j$  defines the scale of the Gödel background. At  $j = 0$  we have the Minkowski space-time. The solution for the Schwarzschild black hole in the Gödel universe is given by [9]

$$ds^2 = -f(r)dt^2 - g(r)r\sigma_L^3 dt + h(r)r^2(\sigma_L^3)^2 + k(r)dr^2 + \frac{r^2}{4}(d\theta'^2 + d\psi'^2 + d\phi'^2 + 2\cos\theta'd\psi'd\phi'), \quad (4)$$

where

$$f(r) = 1 - \frac{2M}{r^2}, \quad g(r) = 2jr, \\ h(r) = j^2(r^2 + 2M), \quad k(r) = \left(1 - \frac{2M}{r^2} + \frac{16j^2M^2}{r^2}\right)^{-1}. \quad (5)$$

The radius of the event horizon is also corrected by parameter  $j$ ,

$$r_{BH} = \sqrt{2M(1 - 8j^2M)}. \quad (6)$$

Note that the maximal value of the black hole mass  $M$  is  $1/8j^2$ . For a larger mass the horizon area vanishes and one has a naked singularity.

The above black hole metric keeps five of the nine isometries of the Gödel universe, generated by  $\partial_t$ , and by four generators of the  $SU(2) \times U(1)$  subgroup of the  $SO(4)$  isometry group acting on  $S^3$  [9].

In the limit  $j = 0$  we have the (4 + 1)-dimensional Schwarzschild solution, while in the limit of  $m = 0$  the pure Gödel space-time is recovered. To treat the scalar field perturbations around such a Schwarzschild-Gödel black hole, let us rewrite the metric in the bispherical coordinates  $(\phi, \psi, \theta)$ , which are connected with the Euler angles  $(\phi', \psi', \theta')$  by

$$\phi' = \psi + \phi, \quad \psi' = \psi - \phi, \quad \theta' = 2\theta. \quad (7)$$

Then, in the regime of small  $j$ , i.e. discarding terms of order  $O(j^2)$ , the metric takes the form

$$ds^2 \approx -f(r)dt^2 - 2g(r)r((\sin\theta)^2 d\phi + (\cos\theta)^2 d\psi)dt + k(r)dr^2 + r^2(d\theta^2 + (\cos\theta)^2 d\psi^2 + (\sin\theta)^2 d\phi^2). \quad (8)$$

Note that in the above equation  $k(r) = (1 - \frac{2M}{r^2})^{-1}$ .

Up to  $O(j^2)$ , the inverse metric  $g^{\mu\nu}$  has components

$$g^{11} = -\left(1 - \frac{2M}{r^2}\right)^{-1}, \quad g^{22} = 1 - \frac{2M}{r^2}, \quad g^{33} = \frac{1}{r^2} \quad (9)$$

$$g^{44} = r^{-2}(\cos\theta)^{-2}, \quad g^{55} = r^{-2}(\sin\theta)^{-2}, \\ g^{14} = g^{15} = -2j\left(1 - \frac{2M}{r^2}\right)^{-1}. \quad (10)$$

## III. SCALAR FIELD PERTURBATIONS OF THE SCHWARZSCHILD-GÖDEL SPACE-TIME

The scalar field perturbations in a curved background are governed by the Klein-Gordon equation

$$\square\Phi \equiv \frac{1}{\sqrt{-g}}(g^{\mu\nu}\sqrt{-g}\Phi_{,\mu})_{,\nu} = 0. \quad (11)$$

Since the background metric has the killing vectors  $\partial_t, \partial_{\psi}, \partial_{\phi}$ , the wave function  $\Psi$  can be represented in the form

$$\Phi \sim e^{i\omega t + ik\psi + im\phi} Y(\theta)R(r). \quad (12)$$

Unfortunately, variables in the Klein-Gordon equation are not separable, at least in the considered coordinates for the full Gimon-Hashimoto metric. The separability is connected with the existence of the killing tensor [12], and it is

possible that one can separate variables in some other privileged coordinate systems. Here we were limited to small values of  $j$ , for which the variables in the Klein-Gordon equation can be separated even in ordinary bi-spherical or Euler coordinates.

Using the expressions for metric coefficients (8)–(10), the scalar field equation (11) takes the form

$$r^{-3} \frac{\partial}{\partial r} \left( \left( 1 - \frac{2M}{r^2} \right) r^3 \frac{\partial R(r)}{\partial r} \right) + (\omega^2 + 4j\omega(k+m)) f^{-1}(r) R(r) + \frac{\lambda}{r^2} R(r) = 0, \quad (13)$$

where the separation constant comes from the equation for angular variables,

$$\frac{1}{\cos\theta \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \cos\theta \frac{\partial Y}{\partial \theta} \right) - \left( \frac{k^2}{(\cos\theta)^2} + \frac{m^2}{(\sin\theta)^2} \right) Y = \lambda Y. \quad (14)$$

Going over to the tortoise coordinate  $dr^* = dr/f(r)$  and to the new wave function  $\Psi = R(r)r^{3/2}$ , the equation (13) can be reduced, after some algebra, to the wavelike form

$$\left( \frac{d^2}{dr^{*2}} + \omega^2 + 4j\omega(k+m) - V(r^*) \right) \Psi = 0. \quad (15)$$

The effective potential has the form

$$V(r) = f(r) \left( \frac{3}{4r^2} f(r) + \frac{3}{2r} f'(r) + \frac{(2l+k+m)(2l+k+m+2)}{r^2} \right). \quad (16)$$

Here  $l$ ,  $k$ , and  $m$  run over the values  $0, 1, 2, \dots$ . The tortoise coordinate  $r^*$  is defined on the interval  $(-\infty, +\infty)$  in such a way that the spatial infinity  $r = +\infty$  corresponds to  $r^* = \infty$ , while the event horizon corresponds to  $r^* = -\infty$ . The above effective potential is positively defined and has the form of the potential barrier which approaches constant values at both spatial infinity and event horizon. In fact, the potential  $V(r)$  coincides with that for the  $(4+1)$ -dimensional Schwarzschild black hole when taking the multipole number  $L$  to be  $2l+k+m$ , yet the spectrum is different, due to the term  $4j\omega(k+m)$  in (5), which depend not only on the final value  $L$ , but also on terms  $l$ ,  $k$ , and  $m$ .

#### IV. QUASINORMAL MODES OF THE SCHWARZSCHILD-GÖDEL BLACK HOLE

If choosing a positive sign for the real part of  $\omega$  ( $\omega = \text{Re } \omega - i \text{Im } \omega$ ), quasinormal modes (QNMs) satisfy the following boundary conditions:

$$\Psi(r^*) \sim C_{\pm} \exp(\pm i\omega r^*), \quad r \rightarrow \pm\infty, \quad (17)$$

corresponding to purely ingoing waves at the event horizon and purely outgoing waves at infinity.

In order to find quasinormal frequencies of the black hole with an effective potential in the form of the potential barrier (16), we use the WKB approach. The WKB approach was used for calculations of quasinormal modes in the first order beyond eikonal approximation by Schutz and Will [13], extended by Iyer and Will to the third order [14], and recently extended to the sixth WKB order [15]. The WKB approach up to the 6th WKB order has been used recently in a series of papers [16–18], where QN frequencies of different black holes were considered, and the comparison with accurate numerical values showed very good agreement. The accuracy of the WKB results is better the larger the multipole number  $L$ , and the less the overtone number  $n$ . In fact for  $n$  larger than  $L$  the WKB formula cannot be applied.

From here we shall use the units such that  $2M = 1$ .

The WKB formula has the form [15]

$$i \frac{\omega^2 - V_0}{\sqrt{-2V_0''}} - \Lambda_2 - \Lambda_3 - \Lambda_4 - \Lambda_5 - \Lambda_6 = n + \frac{1}{2}, \quad (18)$$

where  $V_0$  is the height and  $V_0''$  is the second derivative with respect to the tortoise coordinate of the potential at the maximum.  $\Lambda_2$  and  $\Lambda_3$  can be found in [14];  $\Lambda_4$ ,  $\Lambda_5$  and  $\Lambda_6$  are presented in [15]; the corrections depend on the value of the potential and higher derivatives of it at the maximum.

The 6th order WKB values of the quasinormal frequencies are shown in Figs. 1–4 and Table I. From Fig. 1 one can see that the real part of  $\omega$  is decreasing with growing of  $j$ , being roughly proportional to  $j$ , for a fixed black hole mass and fixed values of  $l$ ,  $m$ , and  $k$ . This can be easily explained in the following way: from the wave equation (15), one can learn that, if one discards small values of order  $O(j^2)$ , then  $\omega^2 + 4j(m+k)\omega = \omega_0^2$ , where  $\omega_0$  is the Schwarzschild value of  $\omega$  under some fixed  $M$ ,  $l$ ,  $m$ ,  $k$ ,  $n$ . Furthermore this can be represented as  $(\omega + 2j(k+m))^2 - 8j^2(k+m)^2 = \omega_0^2$ , i.e. the real part of  $\omega$  is roughly

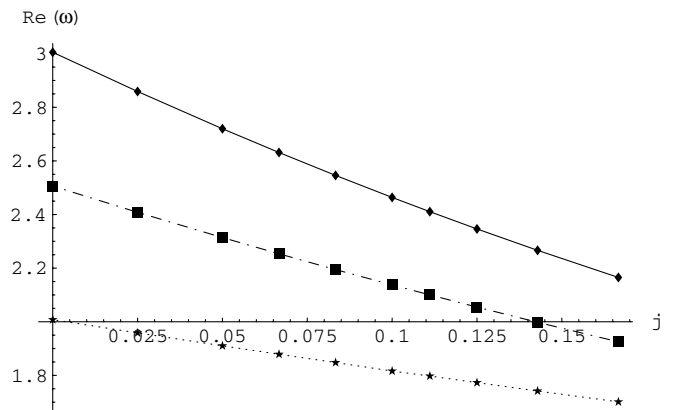


FIG. 1. Real part of  $\omega$  as a function of the Gödel background scale parameter  $j$  (diamond  $k = 2$ ,  $m = 1$ ,  $l = 1$ ), (box  $k = 1$ ,  $m = 1$ ,  $l = 1$ ), (star  $k = 0$ ,  $m = 1$ ,  $l = 1$ ).

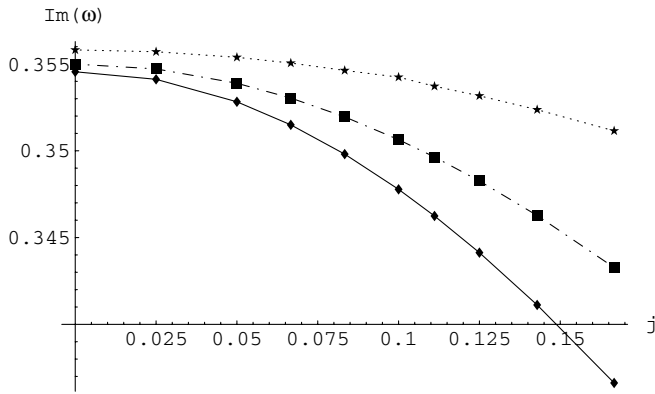


FIG. 2. Imaginary part of  $\omega$  as a function of the Gödel background scale parameter  $j$  (star  $k = 2, m = 1, l = 1$ ), (box  $k = 1, m = 1, l = 1$ ), (diamond  $k = 0, m = 1, l = 1$ ).

increased by  $2j(k + m)$ , while the change in the imaginary part comes from the term  $8j^2(k + m)^2$ . More accurately, Fig. 2 shows that the imaginary part is decreasing when  $j$  is increasing. Therefore, the influence of rotating cosmological background, represented by parameter  $j$ , gives rise to decreasing of the oscillation frequency and of the damping rate. Thus, in the rotating universe the QN modes damp more slowly, but, because of the considerable falling down of  $\text{Re } \omega$  and slight falling down of the  $\text{Im } \omega$ , the resulting quality factor  $\text{Re } \omega / 2\text{Im } \omega$  is decreasing and, thereby, the black hole in a rotating universe is a worse oscillator than in a nonrotating one.

In Table I we put the low overtones of the QN spectrum for different values of  $k, m, l$ , and  $j$ . From that table we can learn that the higher any of the values  $k, l$ , or  $m$  and the lower the overtone  $n$ , the better the accuracy of the WKB formula, and, as a result, the less is the difference between the 6th and 3rd order WKB data. An intrinsic fact for any black hole quasinormal spectrum, when the overtone number grows  $\text{Re } \omega$  falls down, while the damping rate grows. We cannot judge what will happen with asymptotically

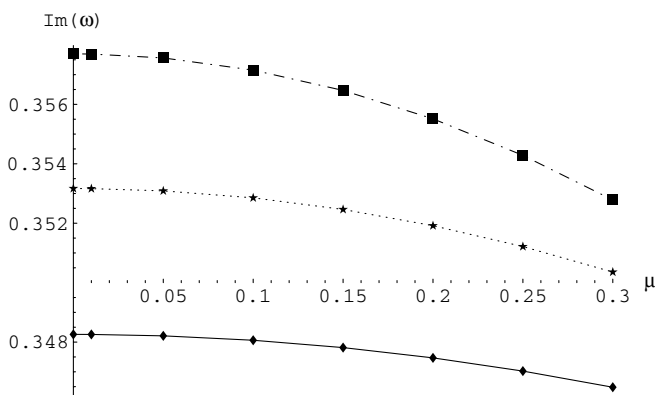


FIG. 3. Imaginary part of  $\omega$  as a function of the mass  $\mu$  for  $l = 1, k = 1, m = 1$  (box),  $l = 1, k = 0, m = 1$  (star),  $l = 1, k = 0, m = 0$  (diamond);  $j = 1/8$ .

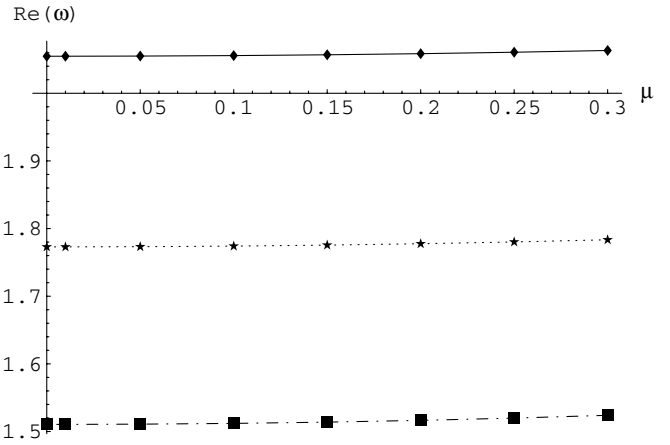


FIG. 4. Real part of  $\omega$  as a function of the mass  $\mu$  for  $l = 1, k = 1, m = 1$  (box),  $l = 1, k = 0, m = 1$  (star),  $l = 1, k = 0, m = 0$  (diamond);  $j = 1/8$ .

high overtones ( $n \rightarrow \infty$ ), since we analyze here only an approximate solution.

In the eikonal (high frequency) approximation, we can use the first order WKB formula for finding the lower overtones. Thus, for large  $l$ , and thereby for large  $L = 2l + k + m$ , in units  $2(2M)^{-1}$  we obtain

$$\omega = \frac{L + 1}{2} + 2j(k + m) - i \frac{2n + 1}{2\sqrt{2}}. \quad (19)$$

Note that there are two limitations on this formula. First, when  $k$  or  $m$  is also large, the general relation

$$\omega^2 - 4j(k + m)\omega = \left( \frac{L + 1}{2} - i \frac{2n + 1}{2\sqrt{2}} \right)^2 \quad (20)$$

holds. Moreover, we should be careful when interpreting this formula at asymptotically large  $L$ , since it uses an approximate metric and  $j^2$  corrections in metric may pro-

TABLE I. WKB values for QNMs at fixed  $j = 1/8, 2M = 1$ .

$k$	$m$	$l$	$n$	3rd WKB order	6th WKB order
0	0	0	0	0.491 23 - 0.411 00i	0.546 33 - 0.360 87i
1	0	0	0	0.780 80 - 0.353 84i	0.791 51 - 0.355 75i
1	0	0	1	0.597 94 - 1.151 13i	0.618 46 - 1.149 02i
0	0	1	0	1.507 07 - 0.357 87i	1.510 50 - 0.357 70i
0	0	1	1	1.385 24 - 1.107 54i	1.392 49 - 1.105 37i
0	0	1	2	1.194 42 - 1.906 90i	1.186 39 - 1.948 29i
1	0	1	0	1.771 55 - 0.353 30i	1.772 93 - 0.353 17i
1	0	1	1	1.676 18 - 1.079 42i	1.679 27 - 1.078 38i
1	0	1	2	1.514 26 - 1.844 24i	1.505 11 - 1.860 38i
1	0	1	3	1.307 66 - 2.639 20i	1.276 39 - 2.732 56i
1	1	1	0	2.054 08 - 0.348 32i	2.054 75 - 0.348 26i
1	1	1	1	1.973 96 - 1.058 21i	1.975 52 - 1.057 74i
1	1	1	2	1.831 10 - 1.798 90i	1.824 49 - 1.806 21i
1	1	1	3	1.645 11 - 2.570 51i	1.617 26 - 2.617 62i
1	1	1	4	1.425 47 - 3.364 34i	1.373 79 - 3.510 96i

duce different asymptotic values. Yet we expect it should be correct for moderately large values of  $L$ . Note that the above formulas are accurate enough even for not very large values of  $L$ , for instance, for  $L = 4$  the relative error is about several percents.

The massless scalar QNMs analysis can easily be extended to the massive case, in which one has the same wavelike equation but with the effective potential

$$V(r) = f(r) \left( \frac{3}{4r^2} f(r) + \frac{3}{2r} f'(r) + \frac{(2l + k + m)(2l + k + m + 2)}{r^2} + \mu^2 \right). \quad (21)$$

Yet we can use the above WKB formula only for small values of the field mass  $\mu$ , since for large  $\mu$  the effective potential has three turning points. The 6th order WKB frequencies are presented in Figs. 3 and 4 as functions of  $\mu$ . Thus, we see that the larger the field mass, the larger is the real part of  $\omega$ , and the smaller the imaginary part. In other words, the “massive” QN modes decay more slowly and have greater real frequency of oscillation. It is known that at asymptotically high overtones the mass of the field does not affect the QN modes [19]. Note that all the above features were observed for the massive scalar field of the Schwarzschild [19] and Reissner-Nordstrom black holes [20,21]. Since these features were found also for the massive Dirac field (see [22] and references therein), it is

possible that they are generic for massive fields of arbitrary spin.

## V. DISCUSSIONS

We have investigated the decay of the (generally speaking, massive) scalar field around a Schwarzschild black hole immersed in a rotating cosmological background. In the limit of the small cosmological parameter  $j$ , the QNMs, which govern the decay of the scalar field at late times, have been found. It was found that the cosmological rotation gives rise to the decreasing of the real frequencies of oscillations (proportional to the cosmological parameter) and of damping rates. The quality factor of the black hole as an oscillator is smaller in the presence of cosmological rotation. The massive scalar field damps more slowly and has greater oscillation frequency. All found modes are damping what supports the stability of the Schwarzschild-Gödel space-time against scalar field perturbations. Yet, within the approximate solution we analyzed, one cannot judge about stability eventually. Note also that the stability of the metric as such is determined by the gravitational perturbations, although the scalar field perturbations may coincide with tensor-type gravitational perturbations [23] which are decisive in gravitational stability [24]. The present analysis can also be extended to the case of scalar field interacting electromagnetically with the charge of the black hole [25], i.e. to the case of the decay of charged scalar field around a Reissner-Nordstrom-Gödel black hole.

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- [1] R.H. Price, Phys. Rev. D **5**, 2419 (1972); V. Cardoso, S. Yoshida, O. J. C. Dias, and J. P. S. Lemos, Phys. Rev. D **68**, 061503 (2003); R. A. Konoplya, Phys. Rev. D **68**, 124017 (2003); Jia-Feng Chang, and You-Gen Shen, Nucl. Phys. **B712**, 347 (2005); Jose Natario and Ricardo Schiappa, hep-th/0411267.
- [2] P.R. Brady, C.M. Chambers, W. Krivan, and P. Laguna, Phys. Rev. D **55**, 7538 (1997); R. A. Konoplya and A. Zhidenko, J. High Energy Phys. **06** (2004) 037; C. Molina, D. Giugno, E. Abdalla, and A. Saa, Phys. Rev. D **69**, 104013 (2004); Jose Natario and Ricardo Schiappa, hep-th/0411267.
- [3] G. T. Horowitz and V. Hubeny, Phys. Rev. D **62**, 024027 (2000); Bin Wang, C. Molina, and Elcio Abdalla, Phys. Rev. D **63**, 084001 (2001); R. A. Konoplya, Phys. Rev. D **66**, 044009 (2002); V. Cardoso, R. Konoplya, and J. P. S. Lemos, Phys. Rev. D **68**, 044024 (2003); R. A. Konoplya, Phys. Rev. D **70**, 047503 (2004); E. Berti and K. D. Kokkotas, Phys. Rev. D **67**, 064020 (2003); G. Siopsis, Phys. Lett. **B 590**, 105 (2004); Bin Wang, Chi Young Lin, and Elcio Abdalla, Phys. Lett. **B 481**, 79 (2000).
- [4] Yu.N. Obukhov, Gen. Relativ. Gravit. **24**, 121 (1992).
- [5] P. Brich, Nature (London) **298**, 451 (1982); **301**, 736 (1983).
- [6] Yu. N. Obukhov, astro-ph/0008106.
- [7] K. Gödel, Rev. Mod. Phys. **21**, 447 (1949).
- [8] T. Ortin, Classical Quantum Gravity **22**, 939 (2005); Saurya Das and Jack Gegenberg, hep-th/0407053; D. Israel, J. High Energy Phys. **01** (2004) 042; N. Drukker, B. Fiol, and J. Simon, J. Cosmol. Astropart. Phys. **10** (2004) 012; M. P. Dabrowski and J. Garecki, Phys. Rev. D **70**, 043511 (2004); D. Brace, C. A. R. Herdeiro, and S. Hirano, Phys. Rev. D **69**, 066010 (2004); M. S. Modgil and S. Panda, gr-qc/0501051.
- [9] Eric G. Gimon and Akikazu Hashimoto, Phys. Rev. Lett. **91**, 021601 (2003).
- [10] D. Klemm and L. Vanzo, hep-th/0411234 [Fortsch. Phys. (to be published)]; G. Barnich and G. Compere, hep-th/0501102; Wung-Hong Huang, hep-th/0412096.
- [11] C. A. R. Herdeiro, Classical Quantum Gravity **20**, 4891 (2003); D. Brecher, U. H. Danielsson, J. P. Gregory, and M. E. Olsson, J. High Energy Phys. **11** (2003) 033; K. Behrndt and D. Klemm, Classical Quantum Gravity **21**, 4107 (2004); M. M. Caldarelli and D. Klemm, Classical Quantum Gravity **21**, L17 (2004).

- [12] V.G. Bagrov and V.V. Obukhov, *Classical Quantum Gravity* **7**, 19 (1990).
- [13] B.F. Schutz and C.M. Will, *Astrophys. J.* **291**, L33 (1985).
- [14] S. Iyer and C.M. Will, *Phys. Rev. D* **35**, 3621 (1987).
- [15] R. A. Konoplya, *Phys. Rev. D* **68**, 024018 (2003).
- [16] Emanuele Berti, Vitor Cardoso, and Jose P.S. Lemos, *Phys. Rev. D* **70**, 124006 (2004).
- [17] E. Berti and K. D. Kokkotas, gr-qc/0502065.
- [18] R. Konoplya, *Phys. Rev. D* **71**, 024038 (2005).
- [19] R. A. Konoplya and A. V. Zhidenko, *Phys. Lett. B* **609**, 377 (2005).
- [20] R. A. Konoplya, *Phys. Lett. B* **550**, 117 (2002).
- [21] L.H. Xue, B. Wang, and R.K. Su, *Phys. Rev. D* **66**, 024032 (2002).
- [22] K.H.C. Castello-Branco, R. A. Konoplya, and A. Zhidenko, *Phys. Rev. D* **71**, 047502 (2005).
- [23] Hideo Kodama and Akihiro Ishibashi, gr-qc/0312012.
- [24] G. Gibbons and S. A. Hartnoll, *Phys. Rev. D* **66**, 064024 (2002).
- [25] R. A. Konoplya, *Phys. Rev. D* **66**, 084007 (2002).