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Black branes on the linear dilaton background

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We show that the complete static black p-brane supergravity solution with a single charge contains two and only two branches with respect to behavior at infinity in the transverse space. One branch is the standard family of asymptotically flat black branes, and another is the family of black branes which asymptotically approach the linear dilaton background (LDB) with antisymmetric form flux. Such configurations were previously obtained in the near-horizon near-extreme limit of the dilatonic asymptotically flat p-branes, and used to describe the thermal phase of field theories involved in the domain wall (DW)/quantum field theory (QFT) dualities and the thermodynamics of little string theory in the case of the NS5-brane. Here we show by direct integration of the Einstein equations that the asymptotically LDB p-branes are indeed exact supergravity solutions, and we prove a new uniqueness theorem for static p-brane solutions satisfying cosmic censorship. In the nondilatonic case, our general nonasymptotically flat p-branes are black branes on the background $AdS_{p+2} \times S^{D-p-2}$ supported by the form flux. We develop the general formalism of quasilocal quantities for nonasymptotically flat supergravity solutions with antisymmetric form fields, and show that our solutions satisfy the first law of thermodynamics. We also suggest a constructive procedure to derive rotating asymptotically LDB brane solutions.

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I. INTRODUCTION

The existence of two alternative descriptions of branes, the classical one within supergravity theories, and the quantum one in string theory, leads to various holographic dualities between classical supergravities and quantum field theories, from which the AdS/CFT correspondence [1] was originally discovered. This correspondence relates to nondilatonic branes, such as the M2 and M5 branes of the M-theory and the D3 brane of string theory, which have an AdS near-horizon structure. The asymptotic boundary of the AdS space is conformal to the Minkowski spacetime where the dual conformal field theory lives.

The AdS/CFT conjecture was extended later to the generic case of string-theoretical dilatonic branes, in which case the near-horizon geometry is either AdS or Minkowski with a nontrivial dilaton field depending linearly on an appropriate radial coordinate. Such configurations are also supersymmetric in the context of supergravities (though not maximally supersymmetric as in the case of nondilatonic branes), but the conformal symmetry is broken by the dilaton. These backgrounds are dual to nonconformal QFT-s with 16 supercharges living on their boundary [2]. In the case of the NS5 brane [3,4], the corresponding dual theory is not a local field theory, but the so-called little string theory [5] (LST) living on the flat six-dimensional world volume of the NS5-brane in the string frame (for a review and recent references see [6]).

More general considerations were presented in [7] (extending the previous work of [8]) for any dimensions and various fractions of supersymmetry. It was argued that although the near-horizon geometry of the extremal dilatonic brane is singular, by transforming it to the so-called "dual" frame (the Nambu-Goto frame for the dual brane probe) one obtains the product of an AdS space with a sphere. After reduction over the sphere one gets the domain wall (DW) solution, for which reason the corresponding duality was termed as the DW/QFT correspondence [8]. The near-horizon configuration of the generic dilatonic brane is the product of either AdS, or flat space-time with a sphere endowed with a nontrivial dilaton. In what follows we will call this field configuration the linear dilaton background (LDB) independently of any particular frame or coordinate system used.

By the standard argument, the *thermal* version of the dual quantum theory should have as a holographic dual the linear dilaton background endowed with an event horizon. Such a configuration was obtained for the NS5 case (dual to LST) by Maldacena and Strominger in the near-horizon limit of the near-extremal NS5-brane [9] and for a discrete family of rotating dilaton branes by Harmark and Obers [10]. A similar four-dimensional "horizon-plus-throat" geometry was presented earlier by Giddings and Strominger [11] (see also [12]) as a certain limit near the horizon of the near-extremal dilaton black hole [13]. The relation between the linear dilaton background and the horizon-plus-throat geometry is similar to the relation between the anti-de-Sitter space and the Schwarzschild-anti-de Sitter black hole. This configuration was shown to be a fully legitimate solution of the Einstein-Maxwell-dilaton four-dimensional theory, thus extending the family

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of asymptotically flat and asymptotically AdS/dS black holes to the asymptotically LDB solutions [14–16]. In addition, in Refs. [15,16] various generalizations (including rotation) of black holes with linear dilaton asymptotics were obtained. Similar solutions exist in presence of a dilaton potential [14,17].

The purpose of the present paper is to study systematically the *higher-dimensional* asymptotically LDB solutions to the supergravity theories. We reexamine the generic supergravity equations for the metric, dilaton, and the antisymmetric form in D dimensions for arbitrary values of the form rank and the dilaton coupling constant, assuming the $R \times ISO(p)$ symmetry of the world volume and the $SO(D-p-1)$ symmetry of the transverse space and imposing no further restrictions on the ansatz. The corresponding system of equations is fully integrable, and, following Ref. [18], we obtain the generic solution containing a number of integration constants. Assuming the existence and regularity of the (nondegenerate) event horizon, we reduce the number of free parameters to three and find that the metric functions become constant at infinity unless some special condition on the parameters is imposed. In this special case one obtains the solutions with linear dilaton asymptotics. In the first generic case, after trivial coordinate rescalings, one arrives at the usual asymptotically flat p-branes [19–21]. Thus the asymptotically LDB p-branes form a degenerate family of solutions in the full solution space. Remarkably, no other alternatives regarding the admissible asymptotic behavior exist for p-branes with regular horizons within the theory with no dilaton potential and no cosmological constant.

Our general family of nonasymptotically flat branes includes the nondilatonic case. In this case one deals with black p-branes on the background $AdS_{p+2} \times S^{D-p-2}$ supported by the flux of antisymmetric form. In particular, we obtain the $\tilde{\text{M}}2$ and $\tilde{\text{M}}5$ branes of M-theory approaching at infinity $AdS_4 \times S^7$ and $AdS_7 \times S^4$, respectively, and the $\tilde{\text{D}}3$-brane of IIB theory approaching $AdS_5 \times S^5$. These "tilde" p-branes interpolate between the product of flat space and a sphere at the horizon, and the product of the anti-de-Sitter space and a sphere at infinity. The tilde p-branes are not supersymmetric unless the mass parameter is set zero, in which case we obtain the linear dilaton background with a flux.

To calculate the brane tension and other physical characteristics of the asymptotically nonflat solutions one needs to generalize the formalism of quasilocal charges developed, in particular, in Refs. [22–24] to the case of an arbitrary number of space-time dimensions and to the presence of the antisymmetric form fields. We perform this and apply it to the case of branes on the linear dilaton background.

Summarizing our results, we formulate a generalized uniqueness theorem claiming that static brane solutions without naked singularities possessing the $R \times ISO(p) \times SO(D-p-1)$ isometries exist in two and only two realizations: one is the usual family of asymptotically flat p-branes and another is the family of branes which asymptotically approach the linear dilaton background. The first family has a Bogomolnyi-Prasad-Sommerfield (BPS) limit, while for the second one the BPS limit coincides with the linear dilaton background itself. We conjecture that the same should be true for the intersecting branes as well.

II. LINEAR DILATON BACKGROUND WITH FLUX

We consider the action containing the graviton, a q-form field strength, $F_{[\tilde{d}+1]}$, and a dilaton ϕ, coupled to a form field with the coupling constant a:

$$S = \int d^D x \sqrt{-g}\left(R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{e^{a\phi}}{2(\tilde{d}+1)!}F^2_{[\tilde{d}+1]}\right) \tag{1}$$

(throughout this paper, the Newton constant G is set to the value $1/16\pi$). In view of the electric-magnetic duality of the corresponding equations of motion

$$g_{\mu\nu} \to g_{\mu\nu}, \qquad F_{\tilde{d}+1} \to e^{a\phi} * F_{D-\tilde{d}-1}, \qquad \phi \to -\phi, \tag{2}$$

we will restrict ourselves here to the magnetic solutions.

The asymptotically flat static BPS solution with $\text{ISO}(p+1)$ symmetry of the world volume and $\text{SO}(\tilde{d}+2)$ symmetry of the transverse space, $(D = p + \tilde{d} + 3)$, reads [18,19]

$$ds^2 = f^{4\tilde{d}/\Delta(D-2)} ds^2_{p+1} + f^{2a^2/\Delta\tilde{d}}(f^{-2}dr^2 + r^2 d\Omega^2_{\tilde{d}+1}),$$
$$e^{\phi-\phi_\infty} = f^{2a/\Delta}, \qquad F_{[\tilde{d}+1]} = \frac{2\tilde{d}}{\sqrt{\Delta}} r_0^{\tilde{d}} e^{-a\phi_\infty/2} \text{vol}(\Omega_{\tilde{d}+1}), \tag{3}$$

where

$$ds^2_{p+1} = -dt^2 + d\mathbf{x}_p^2 \tag{4}$$

is the world-volume flat space-time interval,

$$\Delta = a^2 + \frac{2d\tilde{d}}{D-2}, \tag{5}$$

$(d = p + 1)$, and

$$f = 1 - \left(\frac{r_0}{r}\right)^{\tilde{d}}. \tag{6}$$

It depends on two real parameters r_0, ϕ_∞—the horizon radius and the asymptotic value of the dilaton. Shifting the horizon from $r = r_0$ to $r = 0$ via the coordinate transformation

$$r \to (r^{\tilde{d}} + r_0^{\tilde{d}})^{1/\tilde{d}} \tag{7}$$

one obtains

$$ds^2 = H^{-4\tilde{d}/\Delta(D-2)} ds_{p+1}^2 + H^{4d/\Delta(D-2)}(dr^2 + r^2 d\Omega_{\tilde{d}+1}^2), \tag{8}$$

$$\mathrm{e}^{\phi-\phi_\infty} = H^{-2a/\Delta}, \qquad H = 1 + \left(\frac{r_0}{r}\right)^{\tilde{d}}. \tag{9}$$

The desired near-horizon limit of this solution can be obtained by omitting the constant in the harmonic function H:

$$\begin{aligned} ds^2 &= \left(\frac{r}{r_0}\right)^{4\tilde{d}^2/\Delta(D-2)} ds_{p+1}^2 + \left(\frac{r_0}{r}\right)^{4d\tilde{d}/\Delta(D-2)} \\ &\quad \times (dr^2 + r^2 d\Omega_{\tilde{d}+1}^2), \\ \mathrm{e}^{(\phi-\phi_\infty)} &= \left(\frac{r}{r_0}\right)^{2a\tilde{d}/\Delta}, \\ F_{[\tilde{d}+1]} &= \frac{2\tilde{d}}{\sqrt{\Delta}} r_0^{\tilde{d}} \mathrm{e}^{-a\phi_\infty/2} \mathrm{vol}(\Omega_{\tilde{d}+1}). \end{aligned} \tag{10}$$

This is the solution which we will call LDB presented in the Einstein frame. For supergravity theories, admitting the $1/2$ BPS asymptotically flat p-brane solutions (in which case $\Delta = 4$), the linear dilaton background is $1/2$ supersymmetric as well, unless $a = 0$ in which case supersymmetry is fully restored in the near-horizon limit. This solution is supported both by the dilaton and the antisymmetric form flux.

In the Einstein frame the metric (10) does not have a clear geometric meaning as $r \to \infty$. However, as was clarified in [7,25], the space-time always has the AdS structure in the so-called "dual frame," which is the Nambu-Goto frame for the dual brane probe. Defining the dual frame [7] by the conformal transformation

$$ds_{\mathrm{dual}}^2 = \mathrm{e}^{-a(\phi-\phi_\infty)/\tilde{d}} ds^2, \tag{11}$$

and passing to a new radial coordinate

$$\lambda = r_0 \ln(r/r_0), \tag{12}$$

we find

$$ds_{\mathrm{dual}}^2 = \mathrm{e}^{2(2\tilde{d}-\Delta)\lambda/r_0\Delta} ds_{p+1}^2 + d\lambda^2 + r_0^2 d\Omega_{\tilde{d}+1}^2. \tag{13}$$

In terms of the new radial coordinate the dilaton is precisely linear:

$$\phi - \phi_\infty = \frac{2a\tilde{d}}{\Delta r_0}\lambda. \tag{14}$$

In the particular case

$$\Delta = 2\tilde{d}, \quad \text{or} \quad a^2 = \frac{2\tilde{d}^2}{D-2}, \tag{15}$$

the space-time (13) becomes the direct product of the $p+2$-dimensional Minkowski space and a sphere. In ten-dimensional supergravities (with $\Delta = 4$) this corresponds to fivebranes.

For $\Delta \neq 2\tilde{d}$ the metric (13) is the product $AdS_{p+2} \times S^{\tilde{d}+1}$. Introducing the horospherical coordinate

$$u = \frac{\mathrm{e}^{q\lambda/r_0}}{r_0 q}, \tag{16}$$

where

$$q = \frac{\Delta}{2\tilde{d} - \Delta}, \tag{17}$$

it can be cast into the more familiar form

$$ds_{\mathrm{dual}}^2 = r_0^2\left((qu)^2 ds_{p+1}^2 + \frac{du^2}{(qu)^2} + d\Omega_{\tilde{d}+1}^2\right). \tag{18}$$

It should be stressed that the LDB solution (10) is supported not only by the dilaton field, but by the field of the antisymmetric form as well. An important particular case of the above considerations is $a = 0$, when there is no dilaton at all. Then the dual frame coincides with the Einstein frame, so the solution (18) is an exact solution in the Einstein frame. In fact, in $D = 11$ supergravity this corresponds to $AdS_4 \times S^7$ and $AdS_7 \times S^4$, while in the D3 sector of type IIB theory—to $AdS_5 \times S^5$. These are fully supersymmetric solutions of the corresponding theories. Being supported by the form flux, strictly speaking they do not belong to the class of linear dilaton backgrounds. We still call the solution (10) "LDB" for any values of the parameters d, $\tilde{d}$, a simply because of its generality.

III. MULTICENTER GENERALIZATION

The transverse part of the Einstein frame LDB metric (10) is conformally flat, this suggests its multicenter generalization. To put the derivation into a constructive form, we invoke the sigma-model formulation of the problem [21].

Consider the class of metrics with the conformally flat world-volume part

$$ds^2 = \mathrm{e}^{(\psi-a\phi)/d} ds_{p+1}^2 + \mathrm{e}^{(a\phi-\psi)/\tilde{d}} h_{ij} dx^i dx^j, \tag{19}$$

where ds_{p+1}^2 is Minkowskian, and a new scalar ψ is introduced. It is also convenient to use as the second independent scalar the linear combination

$$\xi = \frac{D-2}{2}(a\psi - \Delta\phi) \tag{20}$$

instead of the dilaton ϕ. The antisymmetric form in the magnetic case can be parametrized by the scalar u via

$$F = \mathrm{e}^{-\psi} \star du, \tag{21}$$

where Hodge dualization is understood with respect to the transverse $\tilde{d}+2$-dimensional space. All scalar quantities ψ, ξ, u are assumed to depend only on coordinates x^i, parametrizing the transverse space.

Performing the dimensional reduction we obtain the $\tilde{d}+2$-dimensional gravitating sigma model

$$S_\sigma = \int\left(R(h) + \frac{1}{\Delta}h^{ij}\mathrm{Tr}(\partial_i M \partial_j M^{-1})\right)\sqrt{h}d^{\tilde{d}+2}x, \quad (22)$$

where the matrix M

$$M = \mathrm{e}^{-\psi/2}\begin{pmatrix} 2 & \frac{u\sqrt{\Delta}}{2} & 0 \\ \frac{u\sqrt{\Delta}}{2} & \frac{u^2\Delta}{8} - \frac{\mathrm{e}^{\psi}}{2} & 0 \\ 0 & 0 & \mathrm{e}^{\psi+\nu\xi/2} \end{pmatrix} \quad (23)$$

parametrizes the target space $SL(2,R)/SO(1,1)\times R$. In this formula $\nu = \frac{2}{\sqrt{(D-2)d\tilde{d}}}$. The corresponding equations of motion read

$$\frac{1}{\sqrt{h}}\partial_i(\sqrt{h}h^{ij}M^{-1}\partial_j M) = 0, \quad (24)$$

$$R_{ij}(h) = -\Delta^{-1}\mathrm{Tr}(\partial_i M \partial_j M^{-1}). \quad (25)$$

This representation is a convenient starting point for an application of the harmonic map technique.

It was noticed that the BPS solutions can be presented as null geodesics of the target space [26,27]. If the matrix M depends on x through a single function, $M = M(\sigma(x))$, with $\sigma(x)$ being a harmonic function on the curved space with metric h

$$\frac{1}{\sqrt{h}}\partial_i(\sqrt{h}h^{ij}\partial_j\sigma) = 0, \quad (26)$$

the Eq. (24) reduces to the matrix equation

$$\frac{d}{d\sigma}\left(M^{-1}\frac{dM}{d\sigma}\right) = 0, \quad (27)$$

whose solution can be expressed in the exponential form

$$M = M_0\mathrm{e}^{K\sigma}, \quad (28)$$

where K belongs to the Lie algebra of the group G ($SL(2,R)$ in the present case), and M_0 is a constant matrix corresponding to the value of M at some normalization point.

Substituting this into the Einstein equations (25) one gets

$$R_{ij}(h) = \Delta^{-1}\mathrm{Tr}(K^2)\partial_i\sigma\partial_j\sigma. \quad (29)$$

It follows that in the particular case $\mathrm{Tr}(K^2) = 0$ the metric h is Ricci-flat. This is a constructive way to build null-geodesic solutions to an arbitrary σ-model.

There are two distinct classes of solutions depending on whether $\det K$ is zero or not. In the first (degenerate) case, taking

$$K = \begin{pmatrix} 1 & 1/2 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad M_0 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (30)$$

one obtains

$$M = \begin{pmatrix} 2(1+\sigma) & \sigma & 0 \\ \sigma & -\frac{1}{2}(1-\sigma) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (31)$$

where σ is a harmonic function in $(\tilde{d}+2)$-dimensional Ricci-flat space. Comparing this with the Eq. (23) we get

$$\psi = -2\ln(1+\sigma), \qquad \xi = 0, \qquad u = \frac{2}{\sqrt{\Delta}}\frac{\sigma}{1+\sigma}, \quad (32)$$

which corresponds to the metric

$$ds^2 = (1+\sigma)^{-4\tilde{d}/\Delta(D-2)}ds^2_{p+1} + (1+\sigma)^{4d/\Delta(D-2)}h_{ij}dx^i dx^j. \quad (33)$$

This is the usual BPS p-brane solution with the harmonic function $H = 1+\sigma$ in the Ricci-flat transverse space. The corresponding dilaton field is given by ($\phi_\infty = 0$)

$$\mathrm{e}^{a\phi} = (1+\sigma)^{-2a^2/\Delta}. \quad (34)$$

The harmonic function has the Coulomb form once the transverse space is chosen flat $h_{ij} = \delta_{ij}$:

$$H = 1+\sigma = 1 + \left(\frac{r_0}{r}\right)^{\tilde{d}}. \quad (35)$$

The LDB solution (10) corresponds to replacing H by its limit for $r \to 0$:

$$H = 1+\sigma = \left(\frac{r_0}{r}\right)^{\tilde{d}}. \quad (36)$$

Clearly, it admits the following multicenter generalization

$$1+\sigma = \sum_n \frac{c_n}{|\mathbf{r}-\mathbf{r}_n|^{\tilde{d}}}, \quad (37)$$

where c_n is a set of real constants. Alternatively, we can express the multicenter LDB solution in the form

$$M = M_0'\mathrm{e}^{K\sigma'}, \quad \text{with} \quad \sigma' = \sum_n \frac{c_n}{|\mathbf{r}-\mathbf{r}_n|^{\tilde{d}}}, \quad (38)$$

i.e., $\sigma' = 1+\sigma$, leading to a nondiagonal $M_0' = M_0\mathrm{e}^{-K}$, such as used in [16].

In the case of a nondegenerate generating matrix $\det K \neq 0$ one gets solutions with naked singularities or geodesically complete solutions, for details see Refs. [16,21].

IV. GENERAL SUPERGRAVITY SOLUTION

Let us now pass to a more general formulation of the problem. We wish to study the p-brane solutions to the action (1) whose world volume is given by the $d = p + 1$-dimensional space with the isometries $ISO(p) \times R$ and whose transverse space is spherically symmetric. The line element

$$ds^2 = -e^{2B}dt^2 + e^{2D}d\mathbf{y}_p^2 + e^{2C}d\Omega_{\tilde{d}+1}^2 + e^{2A}d\rho^2, \quad (39)$$

is parametrized by four functions $A(\rho)$, $B(\rho)$, $C(\rho)$, and $D(\rho)$. Assuming this ansatz, the equations for the form field and the corresponding Bianchi identity

$$\partial_\mu(\sqrt{-g}e^{a\phi}F^{\mu\nu_1\cdots\nu_{\tilde{d}}}) = 0, \quad (40)$$

$$\partial_\mu(\sqrt{-g} * F^{\mu\nu_1\cdots\nu_d}) = 0, \quad (41)$$

(where dualization is understood with respect to the full D-dimensional space-time) can easily be solved in the magnetic sector as

$$F_{a_1,\ldots,a_{\tilde{d}+1}} = b\sqrt{\bar{g}}\epsilon_{a_1,\ldots,a_{\tilde{d}+1}}, \quad (42)$$

or in short notation

$$F_{[\tilde{d}+1]} = b\,\mathrm{vol}(\Omega_{\tilde{d}+1}), \quad (43)$$

where b is the constant field strength parameter and $\bar{g}_{ab}$ is the metric on the unit $\tilde{d}+1$-dimensional sphere.

The system of equations was derived in the previous papers [18,28], for convenience we present it here in the current notation. The Ricci tensor for the metric (39) has the following nonvanishing components

$$R_{tt} = e^{2B-2A}[B'' + B'(B' - A' + (\tilde{d}+1)C' + (d-1)D')], \quad (44)$$

$$R_{xx} = e^{2D-2A}[-D'' - D'(B' - A' + (\tilde{d}+1)C' + (d-1)D')], \quad (45)$$

$$R_{rr} = -B'' - B'(B' - A') - (\tilde{d}+1)(C'' + C'^2 - A'C') - (d-1)(D'' + D'^2 - A'D'), \quad (46)$$

$$R_{ab} = \{-e^{2C-2A}[C'' + C'(B' - A' + (\tilde{d}+1)C' + (d-1)D')] + \tilde{d}\}\bar{g}_{ab}, \quad (47)$$

where primes denote derivatives with respect to ρ. The integration of the Einstein equations is simplified by imposing the gauge condition

$$A = B + (\tilde{d}+1)C + pD. \quad (48)$$

Using the expressions for the Ricci tensor and substituting the form field (42), we then find three equations for B, C, and D with similar differential operators

$$B'' = \frac{\tilde{d}b^2}{2(D-2)}e^G, \quad (49)$$

$$C'' = -\frac{db^2}{2(D-2)}e^G + \tilde{d}e^{2(A-C)}, \quad (50)$$

$$D'' = \frac{\tilde{d}b^2}{2(D-2)}e^G, \quad (51)$$

where

$$G = a\phi + 2B + 2(d-1)D, \quad (52)$$

and the following constraint equation

$$-(B' + kC' + pD')^2 + B'^2 + kC'^2 + pD'^2 + \frac{1}{2}\phi'^2 = \frac{b^2}{2}e^G - \tilde{d}(\tilde{d}+1)e^{2(A-C)}. \quad (53)$$

The dilaton equation

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\phi) = \frac{a}{2(\tilde{d}+1)!}e^{a\phi}F^2_{[\tilde{d}+1]} \quad (54)$$

takes the following form

$$\phi'' = \frac{ab^2}{2}e^G. \quad (55)$$

From the field equations it is clear that the functions B, D, and $\tilde{d}\phi/(a(D-2))$ may differ only by a solution of the homogeneous equation, which is a linear function of ρ. Thus we have

$$D = B + d_1\rho + d_0, \quad (56)$$

$$\phi = \frac{a(D-2)}{\tilde{d}}B + \phi_1\rho + \phi_0, \quad (57)$$

where d_0, d_1, ϕ_0, ϕ_1 are free constant parameters. Substituting this into (52) one finds

$$G = \frac{\Delta(D-2)}{\tilde{d}}B + g_1\rho + g_0, \quad (58)$$

with

$$g_{0,1} = a\phi_{0,1} + 2(d-1)d_{0,1}, \quad (59)$$

so that the Eq. (49) becomes a decoupled equation for G:

$$G'' = \frac{b^2\Delta}{2}e^G. \quad (60)$$

Its general solution, depending on two integration constants α (real or imaginary) and ρ_0 reads:

$$G = \ln\left(\frac{\alpha^2}{\Delta b^2}\right) - \ln\left[\sinh^2\left(\frac{\alpha}{2}(\rho - \rho_0)\right)\right], \quad (61)$$

with α^2 being the first integral

$$G'^2 - b^2\Delta e^G = \alpha^2. \tag{62}$$

Combining the Eqs. (49), (50), and (51), one obtains for the linear combination

$$H = 2(A - C) = 2\tilde{d}C + 2B + 2(d-1)D \tag{63}$$

the second decoupled Liouville equation

$$H'' = 2\tilde{d}^2 e^H. \tag{64}$$

The general solution depending on two parameters β, ρ_1 reads

$$H = 2\ln\beta/2\tilde{d} - \ln(\sinh^2[\beta(\rho - \rho_1)/2]), \tag{65}$$

the first integral being

$$H'^2 - 4\tilde{d}^2 e^H = \beta^2. \tag{66}$$

Finally, expressing the metric functions A, C from (48),(63), one can write the full solution in terms of G, H as follows:

$$B = \frac{\tilde{d}}{\Delta(D-2)}(G - g_1\rho - g_0), \tag{67}$$

$$D = \frac{\tilde{d}}{\Delta(D-2)}(G - g_1\rho - g_0) + d_1\rho + d_0, \tag{68}$$

$$C = \frac{1}{2\tilde{d}}H - \frac{d}{\Delta(D-2)}G + c_1\rho + c_0, \tag{69}$$

$$A = \frac{(1+\tilde{d})}{2\tilde{d}}H - \frac{d}{\Delta(D-2)}G + c_1\rho + c_0, \tag{70}$$

$$\phi = \frac{a}{\Delta}G + f_1\rho + f_0, \tag{71}$$

where

$$\begin{aligned} c_{0,1} &= \frac{a}{\Delta}\left(\frac{d}{D-2}\phi_{0,1} - \frac{(d-1)a}{\tilde{d}}d_{0,1}\right), \\ f_{0,1} &= \phi_{0,1} - \frac{a}{\Delta}g_{0,1} = \frac{2\tilde{d}}{a}c_{0,1}. \end{aligned} \tag{72}$$

Our solution depends on nine parameters: b, d_0, d_1, ϕ_0, ϕ_1, ρ_0, ρ_1, α, β. There remains to enforce the constraint following from Eq. (53)

$$\begin{aligned} &\frac{(\tilde{d}+1)\beta^2}{4\tilde{d}} - \frac{\alpha^2}{2\Delta} - \frac{d\tilde{d}}{\Delta(D-2)}\phi_1^2 + \frac{2a(d-1)}{\Delta}\phi_1 d_1 \\ &\quad - \frac{d-1}{\Delta(D-2)\tilde{d}}\left[a^2(D-2)(D-3) + 2\tilde{d}^2\right]d_1^2 = 0, \end{aligned} \tag{73}$$

so that actually we have only eight independent parameters.

V. SOLUTIONS WITH REGULAR EVENT HORIZON

The ansatz (39) is invariant under translations of ρ, so that without loss of generality we can choose $\rho_1 = 0$. There remain seven parameters. Also, the results (61) and (65) do not depend on the sign of ρ, which we choose so that the horizon $e^{2B} \to 0$ corresponds to $\rho \to +\infty$, with $\alpha > 0$, $\beta > 0$. The main divergent terms in the functions involved are

$$G \sim -\alpha\rho, \qquad H \sim -\beta\rho, \tag{74}$$

so B tends to

$$B \simeq \frac{\tilde{d}}{\Delta(D-2)}(-\alpha\rho - g_1\rho). \tag{75}$$

We are interested in solutions possessing an event horizon, that is a zero of

$$g_{tt} = e^{2B}.$$

This may happen while $\rho \to \infty$ provided

$$\alpha + g_1 > 0. \tag{76}$$

In addition, we have to ensure regularity of the horizon. A sufficient condition is that D and ϕ be finite on the horizon (it can be shown, along the lines followed in [18], that this condition is also necessary). When $\rho \to \infty$ we have

$$D \simeq -\frac{\tilde{d}}{\Delta(D-2)}(\alpha + g_1)\rho + d_1\rho \tag{77}$$

$$\phi \simeq -\frac{a}{\Delta}\alpha\rho + f_1\rho, \tag{78}$$

so the coefficients of ρ in D and ϕ must vanish, which gives the following two relations

$$\alpha = \frac{\Delta}{a}f_1 \tag{79}$$

$$d_1 = \frac{\tilde{d}}{a(D-2)}\phi_1. \tag{80}$$

Thus using the shift of ρ and imposing the condition of the regularity of the horizon we have fixed three parameters, and five parameters still remain free. But we can rescale ρ, t, et x without changing the physical meaning of the solution, this allows to fix two more parameters, namely,

$$d_0 = 0, \qquad d_1 = 1. \tag{81}$$

Combining all the preceding conditions (79)–(81), and the constraint (53), we obtain

$$\begin{aligned} &\alpha = \beta = 2, \qquad f_1 = \frac{2a}{\Delta}, \qquad \phi_1 = \frac{a}{\tilde{d}}, \\ &g_1 = \frac{\Delta(D-2)}{\tilde{d}} - 2, \qquad c_1 = \frac{a^2}{\Delta\tilde{d}}, \end{aligned} \tag{82}$$

so only three parameters remain free: ρ_0, c_0, et b. After suitable rescaling of the brane world volume, the resulting metric reads

$$ds^2 = \left(\frac{e^{\rho}}{2\sinh(\rho-\rho_0)}\right)^{4\tilde{d}/\Delta(D-2)}(-e^{-2\rho}dt^2 + d\mathbf{x}_p^2) + \mu^2\left(\frac{e^{\rho}}{2\sinh\rho}\right)^{2/\tilde{d}}\left(\frac{2\sinh(\rho-\rho_0)}{e^{\rho}}\right)^{4d/\Delta(D-2)} \times\left(d\Sigma^2 + \frac{d\rho^2}{\tilde{d}^2\sinh^2\rho}\right), \tag{83}$$

where μ is defined as

$$\ln\mu = c_0 + \frac{a^2}{\Delta\tilde{d}}\ln 2 - \frac{1}{\tilde{d}}\ln\tilde{d} - \frac{d}{\Delta(D-2)}\ln\left(\frac{4}{\Delta b^2}\right). \tag{84}$$

The corresponding dilaton function is

$$e^{a\phi} = \frac{4\tilde{d}^2}{\Delta b^2}\mu^{2\tilde{d}}\left(\frac{e^{\rho}}{2\sinh(\rho-\rho_0)}\right)^{2a^2/\Delta}. \tag{85}$$

Provided $\rho_0 < 0$, the range of ρ is the positive semiaxis, with $\rho \to +\infty$ corresponding to the regular horizon, $\rho = 0$ to spacelike infinity.

VI. *p*-BRANES WITH LDB ASYMPTOTICS

For the subsequent analysis it is convenient to make the transformation of the radial coordinate

$$e^{2\rho} = \frac{r}{r-\mu}, \tag{86}$$

with $r = \mu > 0$ being the horizon radius, so that

$$\sinh^2\rho = \frac{\mu^2}{4r(r-\mu)}, \qquad \frac{d\rho^2}{\sinh^2\rho} = \frac{dr^2}{r(r-\mu)}. \tag{87}$$

Also, putting

$$e^{2\rho_0} = \frac{r_*}{\mu - r_*}, \tag{88}$$

with $0 < r_* < \mu$ (note that r_* is not an image of ρ_0 with respect to the map (86)), we obtain for g_{tt} the following expression:

$$g_{tt} = -\left(\frac{r_*(\mu-r_*)r^2}{[(\mu-2r_*)r + r_*\mu]^2}\right)^{2\tilde{d}/\Delta(D-2)}\frac{r-\mu}{r}. \tag{89}$$

An examination of this formula shows that there are two and only two possibilities. In the generic case $r_* \neq \mu/2$ ($\rho_0 \neq 0$), one obtains the usual asymptotically flat black brane solutions [19–21]. In the special case $r_* = \mu/2$ ($\rho_0 = 0$), the solution is no longer asymptotically flat, and reduces to the two-parameter (μ, b) configuration

$$ds^2 = \left(\frac{r}{\mu}\right)^{4\tilde{d}/\Delta(D-2)}\left(-\frac{r-\mu}{r}dt^2 + d\mathbf{y}_p^2\right) + \mu^2\left(\frac{r}{\mu}\right)^{2a^2/\Delta\tilde{d}}\left(d\Omega^2_{\tilde{d}+1} + \frac{dr^2}{\tilde{d}^2 r(r-\mu)}\right) \tag{90}$$

$$e^{a\phi} = \frac{4\tilde{d}^2}{\Delta b^2}\mu^{2\tilde{d}}\left(\frac{r}{\mu}\right)^{2a^2/\Delta}, \qquad F_{[\tilde{d}+1]} = b\,\mathrm{vol}(\Omega_{\tilde{d}+1}). \tag{91}$$

The regular event horizon is at $r = \mu$, and if $\mu = 0$ the coordinates systems is not well-behaved. To improve this one has to rescale the radial variable,

$$r = \frac{\mu}{c}\bar{r}^{\tilde{d}}, \tag{92}$$

to introduce instead of μ a new parameter c as follows

$$\mu = b^{2d/\Delta(D-2)}c^{a^2/\Delta\tilde{d}} \tag{93}$$

and to rescale the coordinates t and x as follows

$$t \to \left(\frac{b}{c}\right)^{-2\tilde{d}/\Delta(D-2)}t, \qquad x \to \left(\frac{b}{c}\right)^{-2\tilde{d}/\Delta(D-2)}x. \tag{94}$$

In terms of the new coordinates the solution reads after relabelling $\bar{r} \to r$:

$$ds^2 = \left(\frac{r^{\tilde{d}}}{b}\right)^{4\tilde{d}/\Delta(D-2)}\left[-\left(1-\frac{c}{r^{\tilde{d}}}\right)dt^2 + d\mathbf{y}_p^2\right] + \left(\frac{b}{r^{\tilde{d}}}\right)^{4d/\Delta(D-2)}\left[\left(1-\frac{c}{r^{\tilde{d}}}\right)^{-1}dr^2 + r^2 d\Omega^2_{\tilde{d}+1}\right],$$
$$e^{a\phi} = \frac{4\tilde{d}^2}{\Delta}\left(\frac{r^{\tilde{d}}}{b}\right)^{2a^2/\Delta}, \qquad F_{[\tilde{d}+1]} = b\,\mathrm{vol}(\Omega_{\tilde{d}+1}). \tag{95}$$

This is the two-parameter family of asymptotically nonflat p-branes. When $r \to \infty$ the solution approaches the linear dilaton background (10) with the following identification of parameters

$$r_0 = b^{1/\tilde{d}}, \qquad e^{a\phi_\infty} = \frac{4\tilde{d}^2}{\Delta}. \tag{96}$$

The quantity ϕ_∞ is no longer the asymptotic value of the dilaton, but rather an "inherited value" from the asymptotically flat p-brane (3). In terms of it the dilaton function in (95) reads

$$e^{(\phi-\phi_\infty)} = \left(\frac{r}{r_0}\right)^{2a\tilde{d}/\Delta}, \tag{97}$$

which coincides with (9). The parameter c measures the strength of the singularity at $r = 0$ and so is presumably proportional to the mass of the black brane, as will be checked by a quasilocal computation in Sect. VIII (Eq. (137)). The parameter b from (95) is proportional to the form flux, or "magnetic charge" associated with the solution. It is important to note [15,16], however, that this charge is the same for all the members of the black brane family (b, c) living on the linear dilaton background $(b, 0)$. Besides this charge associated with the background, it therefore makes sense to define a "proper brane charge" as the difference between the total charge of an asymptotically LDB black brane solution and that of the associated LDB vacuum. This proper charge is identically zero in the present case (see also [10]).

Near the horizon the space-time asymptotes to the product $M_{p+2} \times S^{\tilde{d}+1}$, similarly to the case of the Schwarzschild geometry, this can be easily seen by introducing the tortoise radial coordinate. Thus our black branes on the linear dilaton background interpolate between the product of a flat space and a sphere near the horizon (with fixed value of the dilaton) and the linear dilaton background at infinity. This is somehow inverse to the situation with BPS asymptotically flat p-branes which interpolate between the linear dilaton background at the horizon and flat space (with constant dilaton) at infinity.

Computing the scalar curvature for the solution (95) one gets

$$R = \frac{4d\tilde{d}^2}{\Delta^2(D-2)r^2}\left(\frac{r^{\tilde{d}}}{b}\right)^{4d/\Delta(D-2)}\left(\Delta - \tilde{d} - \frac{(D-2)a^2c}{2dr^{\tilde{d}}}\right). \tag{98}$$

At infinity $r \to \infty$

$$R \sim r^{-2a^2/\Delta}, \tag{99}$$

so in the dilatonic case ($a \neq 0$) the scalar curvature vanishes. In the nondilatonic case $a = 0$ one finds a constant value throughout the whole space-time

$$R = \frac{4d\tilde{d}^2(\Delta - \tilde{d})}{\Delta^2(D-2)} \tag{100}$$

independently of whether c is zero or not (for $c = 0$ the space-time coincides with (13) and is the product $AdS_{p+2} \times S^{\tilde{d}+1}$). Note that for $2d = (D-2)$ (even D) and $a = 0$ the Lagrangian possesses a conformal symmetry and thus $R = 0$ identically.

Using the relations (96) one can pass to the dual frame via the conformal transformation (11):

$$ds^2_{\text{dual}} = \mathrm{e}^{2(2\tilde{d}-\Delta)\lambda/r_0\Delta}\left[-\left(1 - \frac{c}{r^{\tilde{d}}}\right)dt^2 + d\mathbf{y}_p^2\right] + \left(1 - \frac{c}{r^{\tilde{d}}}\right)^{-1}d\lambda^2 + r_0^2 d\Omega^2_{\tilde{d}+1}. \tag{101}$$

This metric approaches the product space $AdS_{p+2} \times S^{\tilde{d}+1}$ as $r \to \infty$, unless $2\tilde{d} = \Delta$. In this latter case the asymptotic space is the product of the $(p+2)$-dimensional Minkowski space with a sphere, $M_{p+2} \times S^{\tilde{d}+1}$.

A. NS5

In the particular case $D = 10$, $a = -1$, $p = 5$ the solution (95) reads

$$ds^2 = \left(\frac{r}{r_0}\right)^{1/2}\left[-\left(1 - \frac{c}{r^2}\right)dt^2 + d\mathbf{y}_5^2\right] + \left(\frac{r_0}{r}\right)^{3/2}\left[\left(1 - \frac{c}{r^2}\right)^{-1}dr^2 + r^2 d\Omega_3^2\right], \tag{102}$$
$$\mathrm{e}^{-(\phi-\phi_\infty)} = \frac{r}{r_0}, \qquad F_{[3]} = r_0^2 \,\mathrm{vol}(\Omega_3)$$

($r_0^2 = b$). Passing to the string frame

$$ds^2_{\text{string}} = \mathrm{e}^{(\phi-\phi_\infty)/2}ds^2, \tag{103}$$

and changing the radial variable

$$r = \sqrt{c}\cosh\sigma, \tag{104}$$

we obtain

$$ds^2_{\text{string}} = -\tanh^2\sigma dt^2 + r_0^2 d\sigma^2 + d\mathbf{y}_5^2 + r_0^2 d\Omega_3, \tag{105}$$

$$\mathrm{e}^{2(\phi-\phi_\infty)} = \frac{r_0^2}{c\cosh\sigma}, \qquad F_{[3]} = r_0^2\,\mathrm{vol}(\Omega_3). \tag{106}$$

This is the product of the two-dimensional black hole, a three-sphere, and a five-dimensional flat space, as found earlier in Ref. [9] and used as a holographic counterpart to the little string theory.

B. Black holes

Specifying $d = 1$, $p = 0$, $\tilde{d} = D - 3$, we obtain the following two-parametric family of multidimensional (magnetic) black holes asymptotically approaching the linear dilaton background in the Einstein frame

$$ds^2 = -\left(\frac{r^{D-3}}{b}\right)^{4(D-3)/\Delta(D-2)}\left(1 - \frac{c}{r^{D-3}}\right)dt^2 + \left(\frac{b}{r^{D-3}}\right)^{4/\Delta(D-2)} \times \left[\left(1 - \frac{c}{r^{D-3}}\right)^{-1}dr^2 + r^2 d\Omega^2_{D-2}\right], \tag{107}$$
$$\mathrm{e}^{a\phi} = \frac{4(D-3)^2}{\Delta}\left(\frac{r^{D-3}}{b}\right)^{2a^2/\Delta},$$
$$F_{[D-2]} = b\,\mathrm{vol}(\Omega_{D-2}).$$

Transforming to the "Schwarzschild" radial coordinate

$$\tilde{r} = r^\nu \nu^{-1} b^{2(D-4)/\Delta(D-2)}, \qquad \nu = 1 + \frac{2(D-4)(D-3)}{\Delta(D-2)}, \tag{108}$$

one obtains the metric

$$ds^2 = -Udt^2 + \frac{d\tilde{r}^2}{U} + R^2 d\Omega^2_{D-3}, \tag{109}$$

with

$$U = \left(\frac{r^{D-3}}{b}\right)^{4(D-3)/\Delta(D-2)}\left(1 - \frac{c}{r^{D-3}}\right),$$
$$R^2 = r^2\left(\frac{b}{r^{D-3}}\right)^{4/\Delta(D-2)}, \qquad (110)$$

which was found earlier by Chan, Horne, and Mann [14] and interpreted as "charged dilaton black hole with unusual asymptotic."

In the nondilatonic case $a = 0$ one obtains

$$ds^2 = -\left(\frac{r^{D-3}}{b}\right)^2\left(1 - \frac{c}{r^{D-3}}\right)dt^2 + b^{2/(D-3)}\left[\left(1 - \frac{c}{r^{D-3}}\right)^{-1}\frac{dr^2}{r^2} + d\Omega^2_{D-2}\right]. \qquad (111)$$

Contrary to expectations, this is not a black hole. Changing the coordinates $\hat{r} = r^{D-3} - c/2$ one can rewrite this as the product space $AdS_2 \times S^{D-2}$. Thus the asymptotically LDB black holes exist only in the dilatonic version. This is not true, however, for higher p-branes.

VII. p-BRANES ON $AdS_{p+2} \times S^{\tilde{d}+1}$

For theories without the dilaton, $a = 0$, our general solution (94) describes black branes on the background $AdS_{p+2} \times S^{\tilde{d}+1}$. The background is supported by the antisymmetric form flux

$$F_{[\tilde{d}+1]} = r_0^{\tilde{d}}\,\mathrm{vol}(\Omega_{\tilde{d}+1}). \qquad (112)$$

In this case the Einstein and dual frames coincide and we obtain

$$ds^2 = \left(\frac{r}{r_0}\right)^{2\tilde{d}/d}\left[-\left(1 - \frac{c}{r^{\tilde{d}}}\right)dt^2 + d\mathbf{y}_p^2\right] + \left(1 - \frac{c}{r^{\tilde{d}}}\right)^{-1}\left(\frac{r_0}{r}\right)^2 dr^2 + r_0^2 d\Omega^2_{\tilde{d}+1}. \qquad (113)$$

This metric describes a black p-brane on the fluxed background $AdS_{p+2} \times S^{\tilde{d}+1}$ for $p \geq 1$ (for $p = 0$ as we mentioned above the metric is gauge equivalent to the background itself). The event horizon is at $r = c^{1/\tilde{d}}$ and in the near-horizon limit the geometry is the product space $M_{p+2} \times S^{\tilde{d}+1}$. Therefore the solution (113) interpolates between the product space $M_{p+2} \times S^{\tilde{d}+1}$ at the horizon and $AdS_{p+2} \times S^{\tilde{d}+1}$ at infinity. Recall that the usual asymptotically flat extremal nondilatonic branes interpolate between $AdS_{p+2} \times S^{\tilde{d}+1}$ at the horizon (throat) and M_D at infinity [25].

Denoting our brane solutions by tilde, we list the following particular cases

$$\tilde{\mathrm{M}}2: \quad ds^2 = \left(\frac{r}{r_0}\right)^4\left[-\left(1 - \frac{c}{r^6}\right)dt^2 + d\mathbf{y}_2^2\right] + \left(1 - \frac{c}{r^6}\right)^{-1}\left(\frac{r_0}{r}\right)^2 dr^2 + r_0^2 d\Omega_7^2,$$
$$r_0 = b^{1/6}, \qquad (114)$$

$$\tilde{\mathrm{M}}5: \quad ds^2 = \left(\frac{r}{r_0}\right)\left[-\left(1 - \frac{c}{r^3}\right)dt^2 + d\mathbf{y}_5^2\right] + \left(1 - \frac{c}{r^3}\right)^{-1}\left(\frac{r_0}{r}\right)^2 dr^2 + r_0^2 d\Omega_4^2,$$
$$r_0 = b^{1/3}, \qquad (115)$$

$$\tilde{\mathrm{D}}3: \quad ds^2 = \left(\frac{r}{r_0}\right)^2\left[-\left(1 - \frac{c}{r^4}\right)dt^2 + d\mathbf{y}_3^2\right] + \left(1 - \frac{c}{r^4}\right)^{-1}\left(\frac{r_0}{r}\right)^2 dr^2 + r_0^2 d\Omega_5^2,$$
$$r_0 = b^{1/4}. \qquad (116)$$

These solutions are not supersymmetric unless $c = 0$, in which case they become $AdS_4 \times S^7$, $AdS_7 \times S^4$, and $AdS_5 \times S^5$, respectively.

VIII. QUASILOCAL CHARGES

In order to find the physical characteristics of the nonasymptotically flat solutions, one can use the Hamiltonian formulation of the problem similar to that applied earlier to four-dimensional linear dilaton black holes [15,16]. Our approach closely follows that of Brown and York [22] and Hawking and Horowitz [23] (for a recent review see [29]).

The space-time metric is decomposed à la Arnowitt-Deser-Misner (ADM)

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \qquad (117)$$

where N is called the lapse function and N^i the shift vector. This decomposition means geometrically that the space-time is foliated by spacelike surfaces Σ_t, of metric $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$, labeled by a time coordinate t with the unit normal vector $u^\mu = -N\delta_0^\mu$. It follows that the timelike vector field t^μ, satisfying $t^\mu \nabla_\mu t = 1$, is decomposed into the lapse function and the shift vector as $t^\mu = Nu^\mu + N^\mu$. The space-time boundary ∂_M consists of initial and final surfaces Σ_t, $t = t_i, t_f$ and a timelike surface B to which the vector u^μ is tangent. This latter surface is foliated by the $(D-2)$-dimensional surfaces S_t^r, of metric $\sigma_{\mu\nu} = h_{\mu\nu} - n_\mu n_\nu$, which are intersections of Σ_t and B. The unit spacelike (outward) normal to S_t^r, n^μ, is orthogonal to u^μ.

In the following, we generalize the treatment of the four-dimensional Einstein-Maxwell-dilaton theory [29] to the case of the Einstein-dilaton-antisymmetric form theory in D dimensions.

The starting point is the D-dimensional Einstein-dilaton-antisymmetric form action (1) supplemented by a boundary term [30] necessary to have a well-defined variational principle ($q = \tilde{d} + 1$)

$$S = \int_M d^D x \sqrt{-g}\Big(R_D - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2q!} e^{a\phi} F^2_{[q]}\Big) + 2\int_{\Sigma_{t_i}}^{\Sigma_{t_f}} K\sqrt{h} d^{D-1}x - 2\int_B \Theta \sqrt{\gamma} d^{D-1}x. \tag{118}$$

K is the trace of the extrinsic curvature $K^{\mu\nu}$ of $\Sigma_{t_{i,f}}$, while Θ is the trace of the extrinsic curvature $\Theta^{\mu\nu}$ of B, defined as

$$K_{\mu\nu} = -h^\alpha_\mu \nabla_\alpha u_\nu = -\frac{1}{2N}(\dot{h}_{\mu\nu} - 2D_{(\mu}N_{\nu)}), \tag{119}$$

$$\Theta_{\mu\nu} = -\gamma^\alpha_\mu \nabla_\alpha n_\nu, \tag{120}$$

where ∇ and D are the covariant derivatives compatible with the metric $g_{\mu\nu}$ and h_{ij}, respectively.

The momentum variables conjugate to ϕ, h_{ij}, and $A_{i_1\ldots i_{q-1}}$ are

$$p_\phi = -\sqrt{-g}\partial^0\phi, \qquad p^{ij} = \sqrt{h}(h^{ij}K - K^{ij}), \tag{121}$$

$$\Pi^{i_1\ldots i_{q-1}} = -\sqrt{-g} e^{a\phi} F^{0i_1\ldots i_{q-1}} \equiv \sqrt{-g} E^{i_1\ldots i_{q-1}}. \tag{122}$$

First, consider the gravitational part of the action (118). To obtain the corresponding gravitational contribution to the Hamiltonian,

$$H_G = \int_{\Sigma_t} (p^{ij}\dot{h}_{ij} - L_G)\sqrt{h} d^{D-1}x, \tag{123}$$

we have used the well-known result expressing the D-dimensional scalar curvature in terms of the $(D-1)$-dimensional scalar curvature and the extrinsic curvature of Σ_t, together with total derivative terms [23],

$$R_D = R_{D-1} - K^2 + K_{\mu\nu}K^{\mu\nu} + 2\nabla_\mu(u^\mu \nabla_\nu u^\nu) - 2\nabla_\nu(u^\mu \nabla_\mu u^\nu), \tag{124}$$

and the following relation

$$p^{ij}\dot{h}_{ij} = -2N\sqrt{h}(K^2 - K^{ij}K_{ij}) + 2p^{ij}D_{(i}N_{j)} \tag{125}$$

obtained by combining the relations (119) and (121).

Using these two results and the Eqs. (117), (119), the gravitational part of the Hamiltonian reads

$$H_G = \int_{\Sigma_t} \sqrt{h}\Big[N(-R_{D-1} + K_{\mu\nu}K^{\mu\nu} - K^2) - N^i D_\mu\Big(\frac{p^\mu_i}{\sqrt{h}}\Big)\Big] + 2\int_{S^r_t} \sqrt{\sigma}\Big(Nk + \frac{n_\mu p^{\mu\nu} N_\nu}{\sqrt{h}}\Big) d^{D-2}x, \tag{126}$$

where $k = -\sigma^{\mu\alpha} D_\alpha n_\mu$ is the extrinsic curvature of S^r_t embedded in Σ_t. In the same way, the matter contribution to the Hamiltonian is obtained using the following relations

$$\partial_0 \phi = \frac{N^2}{\sqrt{-g}} p_\phi + N^i \partial_i \phi,$$

$$\partial^i \phi = h^{ij}\partial_j \phi + \frac{N^i}{\sqrt{-g}} p_\phi, \tag{127}$$

$$F_{0i_2\cdots i_q} = N^j F_{ji_2\cdots i_q} + e^{-a\phi}\frac{N^2}{\sqrt{-g}} \Pi_{i_2\cdots i_q},$$

$$F^{i_1\cdots i_q} = \bar{F}^{i_1\cdots i_q} + e^{-a\phi}\Big(N^{i_1}\frac{\Pi^{i_2\cdots i_q}}{\sqrt{-g}} - N^{i_2}\frac{\Pi^{i_1 i_3\cdots i_q}}{\sqrt{-g}} + \cdots - (-1)^q N^{i_q}\frac{\Pi^{i_1\cdots i_{q-1}}}{\sqrt{-g}}\Big), \tag{128}$$

where $\bar{F}$ is a $(D-1)$-dimensional tensor which indices are raised and lowered by h_{ij}.

Finally, collecting the gravitational and matter parts, one obtains the following expression for the Hamiltonian

$$H = \int_{\Sigma_t} \sqrt{h}(N\mathcal{H} + N^i \mathcal{H}_i + A_{0i_2\ldots i_{q-1}} \mathcal{H}_A^{i_2\ldots i_{q-1}}) d^{D-1}x + 2\int_{S^r_t} \sqrt{\sigma}\Big(Nk + \frac{n_\mu p^{\mu\nu}N_\nu}{\sqrt{h}}\Big) d^{D-2}x + (q-1)\int_{S^r_t} N\sqrt{\sigma} A_{0i_2\ldots i_{q-1}} E^{ji_2\ldots i_{q-1}} n_j d^{D-2}x, \tag{129}$$

where the constraints read

$$\mathcal{H} = -R_{D-1} + K_{\mu\nu}K^{\mu\nu} - K^2 + \frac{p^2_\phi}{2\sqrt{h}} + \frac{\sqrt{h}}{2}(\partial\phi)^2 + \frac{e^{-a\phi}}{2(q-1)!\sqrt{h}}\Pi^2 + \frac{\sqrt{h}}{2q!} e^{a\phi} F^2, \tag{130}$$

$$\mathcal{H}_j = -D_\mu\Big(\frac{p^\mu_j}{\sqrt{h}}\Big) + p_\phi \partial_j \phi + \frac{1}{(q-1)!}\Pi^{i_1\ldots i_{q-1}} F_{ji_1\ldots i_{q-1}}, \tag{131}$$

$$\mathcal{H}_A^{i_2\ldots i_{q-1}} = -(q-1)\partial_j \Pi^{ji_2\ldots i_{q-1}}. \tag{132}$$

The quasilocal energy is defined as the "on-shell" value of the Hamiltonian. Since the volume terms in the Hamiltonian are proportional to constraints which vanish for a solution of the theory, the quasilocal energy of a solution is simply given by the surface terms in the Hamiltonian, i.e. by quantities evaluated on the 2-surface S^r_t. We define the quasilocal mass as the quasilocal energy evaluated in the limit $r \to \infty$. However, it is known that the quasilocal energy is generally divergent at infinity. This divergence may be regularized by subtracting the contribution of a background solution, provided one can impose the same Dirichlet boundary conditions on S^r_t for the solution under consideration and for the background solution. Finally, the quasilocal angular momentum of a solution may be obtained by carrying out an infinitesimal gauge transformation $\delta\varphi = \delta\Omega t$ and evaluating the response

$$J = \frac{\delta H}{\delta \Omega}. \qquad (133)$$

The resulting quasilocal energy and quasilocal angular momentum are given by

$$E = 2\int_{S_t^r} \sqrt{\sigma}\Big(N(k - k_0) + \frac{n_\mu p^{\mu\nu} N_\nu}{\sqrt{h}}\Big) d^{D-2}x + (q-1)\int_{S_t^r} A_{0i_2\ldots i_{q-1}}(\bar{\Pi}^{ji_2\ldots i_{q-1}} - \bar{\Pi}_0^{ji_2\ldots i_{q-1}}) n_j d^{D-2}x, \qquad (134)$$

$$J_i = -2\int_{S_t^r} \frac{n_\mu p_i^\mu}{\sqrt{h}}\sqrt{\sigma} d^{D-2}x - (q-1)\int_{S_t^r} A_{ii_2\ldots i_{q-1}} \bar{\Pi}^{ji_2\ldots i_{q-1}} n_j d^{D-2}x, \qquad (135)$$

where $\bar{\Pi}^{ji_2\ldots i_{q-1}} = (\sqrt{\sigma}/\sqrt{h})\Pi^{ji_2\ldots i_{q-1}}$. The quantities with the subscript 0 are those associated with the background solution. Here, we have written the formulas of the quasilocal quantities for a static background solution ($N_0^i = 0$ and $J_0 = 0$). Notice that the dilaton does not contribute directly to the quasilocal energy and quasilocal angular momentum.

Now, we are able to compute the mass and the angular momentum of the solution (95). Since the solution is static, the angular momentum density $N^i = 0$ is zero. Also, for our purely magnetic solution $A_{0i_2\ldots i_{q-1}} = 0$. So, only the first term in (134) contributes to the quasilocal mass. The extrinsic curvature of S_t^r (a section $t = r =$ constant of (95) reads

$$k = -\Big(\frac{(1+\tilde{d})a^2}{\Delta} + \frac{2\tilde{d}^2(d-1)}{\Delta(D-2)}\Big) b^{-2d/\Delta(D-2)} r^{-a^2/\Delta} \times \sqrt{1 - \frac{c}{r^{\tilde{d}}}}, \qquad (136)$$

The natural candidate for the background solution is the vacuum of the black branes, i.e. the LDB background obtained by taking the parameter c equal to zero. Subtracting its contribution leads, for $r \to \infty$, to

$$\frac{\mathcal{M}}{\text{vol}(p\text{-brane})} = \Big(\frac{(1+\tilde{d})a^2}{\Delta} + \frac{2\tilde{d}^2(d-1)}{\Delta(D-2)}\Big) c\, \text{vol}(\Omega_{\tilde{d}+1}). \qquad (137)$$

This expression is always positive in view of the inequalities $d \geq 1$, $\Delta > 0$, and $D > 2$. The first term may be interpreted as the dilaton contribution to the mass, and the second as the proper brane contribution. Note that for nondilatonic branes $a = 0$, one has a nonzero mass density only for $p \geq 1$; the fact that in this case the would-be black holes ($d = 1$) are actually massless is due to the gauge equivalence, previously pointed out, between the solution (111) with $c \neq 0$ and the background $c = 0$.

Obviously, the solution (95) being static, the quasilocal angular momentum is equal to zero.

Now we address the question of the thermodynamics of the LDB black branes (95). In the case of the standard asymptotically flat magnetostatic black branes, the first law is

$$d\mathcal{M} = TdS, \qquad (138)$$

where the temperature is given by the inverse period of imaginary time (or equivalently by the horizon surface gravity over 2π) and the entropy by the quarter of the horizon area in Planckian units. A simple calculation gives for the temperature and the entropy of (95),

$$T = \frac{\tilde{d}}{4\pi} b^{-2/\Delta} c^{2/\Delta - 1/\tilde{d}}, \qquad (139)$$

$$\frac{S}{\text{vol}(p\text{-brane})} = 4\pi b^{2/\Delta} c^{-2/\Delta + 1/\tilde{d} + 1} \text{vol}(\Omega_{\tilde{d}+1}) \qquad (140)$$

(recall that we use the value of the Newton constant $G = 1/16\pi$). Then, using (137) and (139) and (140), the left-hand side and the right-hand side of Eq. (138) read

$$\frac{d\mathcal{M}}{\text{vol}(p\text{-brane})\text{vol}(\Omega_{\tilde{d}+1})} = \Big(\frac{(1+\tilde{d})a^2}{\Delta} + \frac{2\tilde{d}^2(d-1)}{\Delta(D-2)}\Big) dc, \qquad (141)$$

$$\frac{TdS}{\text{vol}(p\text{-brane})\text{vol}(\Omega_{\tilde{d}+1})} = \Big(\frac{(1+\tilde{d})a^2}{\Delta} + \frac{2\tilde{d}^2(d-1)}{\Delta(D-2)}\Big) dc + \frac{2\tilde{d}}{\Delta}\frac{c}{b} db. \qquad (142)$$

So, we see that the first law is satisfied only if the parameter b, related to the magnetic charge of the solution according to (91), is not varied. This is consistent with our observation that this charge is associated not with a specific black brane, but rather with the LDB background. Since b is not a parameter of the black branes, it should not be varied [15,16]. Then, we conclude that the asymptotically LDB black branes satisfy the first law of thermodynamics.

IX. KALUZA-KLEIN INTERPRETATION AND ROTATION

Recently the near-horizon limit of near-extremal rotating branes was discussed by Harmark and Obers [10]. The field configurations they derived have to be regarded as rotating counterparts of the static asymptotically LDB p-branes discussed here. It would be interesting to give a constructive derivation of these solutions. However it seems difficult to obtain them via the direct integration of the Einstein equations. Also, owing to the lack of supersymmetry, they cannot be found via the Bogomolny equations. Here we suggest a transparent Kaluza-Klein procedure which could be used to perform this goal in the case of asymptotically LDB electric p-branes.

Consider the D-dimensional action

$$S_D = \int d^D x \sqrt{-g_D} \Big(R_D - \frac{1}{2}(\partial\phi)^2 - \frac{e^{a\phi}}{2(p+2)!} F^2_{(p+2)} \Big) \tag{143}$$

and specialize to the electric p-brane sector ($p = d - 1$)

$$ds^2_{(D)} = e^{-2p\beta/(\tilde d+1)} g_{\mu\nu} dx^\mu dx^\nu + e^{2\beta} d\mathbf{y}^2, \tag{144}$$

$$F_{(p+2)} = F_{\mu\nu} dx^\mu \wedge dx^\nu \wedge dy^1 \wedge \cdots \wedge dy^p, \tag{145}$$

where all fields depend only on the x^μ ($\mu = 0, \cdots, \tilde d + 2$). The dimensional reduction to $D - p = \tilde d + 3$ dimensions leads to the action

$$S_{\tilde d+3} = \int d^{\tilde d+3} x \sqrt{-g} \Big(R - \frac{p(D-2)}{\tilde d + 1}(\partial\beta)^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4} e^{a\phi - (2p\tilde d/(\tilde d+1))\beta} F^2_{(2)} \Big). \tag{146}$$

The two scalar fields are decoupled by

$$\beta = \alpha^{-1}\Big(-\frac{\tilde d}{D-2}\varphi + a\eta \Big), \tag{147}$$

$$\phi = \alpha^{-1}\Big(a\varphi + \frac{2p\tilde d}{\tilde d+1}\eta \Big), \tag{148}$$

with

$$\alpha^2 = a^2 + \frac{2p\tilde d^2}{(D-2)(\tilde d+1)}, \tag{149}$$

leading to

$$S_{\tilde d+3} = \int d^{\tilde d+3} x \sqrt{-g} \Big(R - \frac{p(D-2)}{\tilde d + 1}(\partial\eta)^2 - \frac{1}{2}(\partial\varphi)^2 - \frac{1}{4} e^{\alpha\varphi} F^2_{(2)} \Big). \tag{150}$$

For the value

$$\alpha^2 = 2\frac{\tilde d+2}{\tilde d+1} \Longleftrightarrow a^2 = 4 - \frac{2d\tilde d}{D-2} \quad (\Delta = 4), \tag{151}$$

this action is the sum of the action for the harmonic field η in $\tilde d + 3$ dimensions and of the Kaluza-Klein dimensional reduction of the $\tilde d + 4$ Einstein-Hilbert action

$$S_{\tilde d+4} = \int d^{\tilde d+4} x \sqrt{-g_{(\tilde d+4)}} R_{(\tilde d+4)}. \tag{152}$$

This leads to a simple procedure to construct rotating asymptotically LDB electric p-brane solutions of the action (1) for the values of a^2 satisfying (151): (1) Start from the trivial embedding in $\tilde d + 4$ dimensions of the $\tilde d + 3 = D - p$ Myers-Perry solution [31]; (2) carry out a twisted dimensional reduction to $\tilde d + 3$ dimensions; (3) oxidize the resulting solution to D dimensions, taking into account an arbitrary harmonic function η in (147) and (148). The associated rotating magnetic p-brane solution may be derived from this by the electric-magnetic duality transformation (2). In the case $p = D - 4$, rotating asymptotically LDB dyonic p-branes have also been generated [32] by a procedure generalizing that used for $D = 4$ in [16].

Let us check that, in the static case, this procedure reproduces the asymptotically LDB electrostatic p-brane already found. The $\tilde d + 3$-dimensional Tangherlini metric embedded (with a twist $dt \to dt - d\chi$) in $\tilde d + 4$ dimensions is

$$\begin{aligned} ds^2_{\tilde d+4} &= d\chi^2 - \Big(1 - \frac{\mu}{\rho^{\tilde d}}\Big)(dt - d\chi)^2 + \Big(1 - \frac{\mu}{\rho^{\tilde d}}\Big)^{-1} d\rho^2 + \rho^2 d\Omega^2_{\tilde d+1} \\ &= d\chi^2 - \Big(1 - \frac{\mu}{r}\Big)(dt - d\chi)^2 + r^{2/\tilde d}\Big(\frac{1}{\tilde d^2}\frac{dr^2}{r(r-\mu)} + d\Omega^2_{\tilde d+1}\Big) \\ &= e^{(\tilde d+1)/(\tilde d+2)\alpha\varphi}(d\chi + A_0 dt)^2 + e^{-1/(\tilde d+2)\alpha\varphi} ds^2_{\tilde d+3}, \end{aligned} \tag{153}$$

with

$$e^{\varphi/\alpha} = \Big(\frac{\mu}{r}\Big)^{1/2}, \qquad A_0 = -\frac{r-\mu}{\mu}, \tag{154}$$

$$ds^2_{\tilde d+3} = -\Big(\frac{r}{\mu}\Big)^{\tilde d/(\tilde d+1)} \frac{r-\mu}{r} dt^2 + \mu^{2/\tilde d}\Big(\frac{r}{\mu}\Big)^{(\tilde d+2)/\tilde d(\tilde d+1)} \times \Big(\frac{1}{\tilde d^2}\frac{dr^2}{r(r-\mu)} + d\Omega^2_{\tilde d+1}\Big) \tag{155}$$

(the twisted dimensional reduction also involves a scale r_0 which we have omitted). Carrying out the third step (oxidization) then leads, for the choice $\eta = 0$, to

$$ds^2_{(D)} = \Big(\frac{r}{\mu}\Big)^{\tilde d/(D-2)}\Big(-\frac{r-\mu}{r}dt^2 + d\mathbf{y}^2\Big) + \mu^{2/\tilde d}\Big(\frac{r}{\mu}\Big)^{a^2/2\tilde d}\Big(\frac{1}{\tilde d^2}\frac{dr^2}{r(r-\mu)} + d\Omega^2_{\tilde d+1}\Big), \tag{156}$$

$$e^{a\phi} = \Big(\frac{r}{\mu}\Big)^{\tilde d/(D-2)}. \tag{157}$$

This is essentially the asymptotically LDB black p-brane.[1]

[1]For the reduced ($\tilde d + 3$)-metric above, the generic spherically symmetric harmonic function η is $\eta = c\ln((r-\mu)/r) + d$. It follows that for the generic ($\eta \neq$ constant) asymptotically LDB p-brane solution, the dilaton ϕ and the p-brane metric function β are singular on the horizon.

X. UNIQUENESS, COSMIC CENSORSHIP, AND SUPERSYMMETRY

Combining the results of the previous analysis we can formulate the following uniqueness theorem for static p-branes with a spherical transverse space:

Theorem: *A singly charged p-brane solution of the Einstein equations with the dilaton and antisymmetric form sources possessing a regular event horizon and the* $R \times ISO(p) \times SO(D - p - 1)$ *isometries is either asymptotically flat, in which case it coincides with the standard black brane solution, or asymptotically LDB and then it is given by Eq.* (95).

These two families are geometrically dual in the following sense. The standard asymptotically flat dilatonic p-branes possess the BPS limit in which the solutions have a null singularity. The BPS dilatonic branes interpolate between the LDB at the horizon and Minkowski space with a constant dilaton at infinity. The second family interpolates between the product of flat space with a sphere (with fixed dilaton) at the horizon and LDB at infinity. For the second family the BPS limit coincides with the LDB itself.

Our proof was based on a complete integration of the corresponding system of equations and subsequent determination of the free parameters from physical requirements. It turns out that if one imposes first the condition of existence of a regular event horizon, there remain two and only two options for the asymptotic behavior: either the solution is asymptotically flat, or it is asymptotically LDB. A nontrivial feature of this situation is that both asymptotic configurations are supersymmetric in ten and 11-dimensional supergravities and their toroidal dimensional reductions. Therefore, demanding that the curvature singularity be hidden behind the event horizon, i.e. imposing the cosmic censorship requirement, we find that nonsupersymmetric black brane solutions always "choose" a supersymmetric asymptotic. This "choice" is another aspect of the relationship which was called in Ref. [33] "supersymmetry as a cosmic censor." Though this has been proven here under the assumption of $R \times ISO(p) \times SO(D - p - 1)$ isometries (i.e. for singly charged branes), we expect this to be also true for intersecting branes [34] (for other early references see [35], intersecting solutions with extra parameters were also found recently [36]).

It is worth noting that various attempts [37] were made recently to interpret the supergravity p-brane solution (obtained under the same ansatz as here) with other free parameters than the mass, the charge and the asymptotic value of the dilaton as describing non-BPS string theory branes (fractional branes, brane-antibrane systems, etc.). Some of these solutions were used in a formal way without careful checking of regularity and asymptotic behavior. In view of the results of Ref. [18], all solutions with extra parameters other than the mass, the form charge, and the asymptotic value of the dilaton are either asymptotically nonflat or contain naked singularities. Here we have proved in addition that the two-parameter family of asymptotically nonflat p-branes without naked singularities is necessarily a p-brane with LDB asymptotics. We realize, however, that if cosmic censorship is not imposed (which may have some justification in the string theory context) the solutions with extra parameters can be useful.

No uniqueness of this kind is expected for the stationary solutions. Indeed, as was extensively discussed recently, the higher-dimensional Kerr solution is by no means unique within the class of regular asymptotically flat stationary solutions, and other asymptotically flat solutions such as rotating rings [38] and their generalizations (for recent references see [39]) including supersymmetric configurations [40] exist with horizon topologies other than a sphere. Whether these solutions admit asymptotically nonflat counterparts as in the case of the asymptotically LDB branes discussed here remains an open question.

XI. CONCLUSIONS

In this paper we have constructively derived the static black p-brane solutions to supergravity theories which asymptotically approach the linear dilaton background. The latter is the bulk configuration which was interpreted as a holographic dual to certain nonconformal quantum field theories (DW/QFT correspondence) or (in the case of the NS5 brane) to little string theory.

In the static case the LDB space-time in the dual frame factorizes into the product of an AdS space and a sphere. There are two substantially different cases: dilatonic, in which the dilaton is linear in terms of a special radial variable, and nondilatonic, when the asymptotic space-time has the above factorization property also in the Einstein frame. This second case is fully supersymmetric, while in the first one the supersymmetry is partially broken by the dilaton. p-brane solutions asymptotically approaching the LDB space-time exist only in the black version and are not supersymmetric; their BPS limit coincides with the linear dilaton background itself. In the nondilatonic case the lower member of the family $p = 0$ (black hole) does not exist, while in presence of the dilaton all p are possible (though we did not study explicitly the domain wall and instanton cases).

We have presented the generalization of the Brown-York-Hawking-Horowitz formalism of quasilocal charges to the case of arbitrary dimensions and the presence of antisymmetric form fields. Using it we have shown that the asymptotically LDB p-branes satisfy the first law of thermodynamics. In this derivation a finite value for the asymptotic mass was obtained by subtracting the infinite contribution of the background LDB. It is worth noting that the charge parameter associated with the solution should not be varied, being a property of the background rather than of a specific brane. Therefore one may consider the asymptotically LDB branes to be essentially uncharged with respect to the form field (this nicely fits with the fact that their BPS limit is the LDB itself).

Some stationary solutions of the type discussed here were obtained previously as near-horizon limits of near-extremal spinning black branes, and here we have suggested a constructive procedure for their derivations as solutions to the supergravity field equations. This also opens an interesting question about the possible existence of asymptotically nonflat rotating rings. A more obvious possible generalization could be to intersecting branes of the type considered here.

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