Limits on deviations from the inverse-square law on megaparsec scales

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We present an attempt to constrain deviations from the gravitational inverse-square law on large-scale structure scales. A perturbed law modifies the Poisson equation, which implies a scale-dependent growth of overdensities in the linear regime and thus modifies the power spectrum shape. We use two large-scale structure surveys (the Sloan Digital Sky survey and the Anglo-Australian two-degree field galaxy redshift survey) to constrain the parameters of two simple modifications of the inverse-square law. We find no evidence for deviations from normal gravity on the scales probed by these surveys ($\sim 10^{23}$ m).

I. INTRODUCTION

Einstein's theory of general relativity, which generalizes Newton's law of gravity, has been extensively tested and verified. But precision tests of the inverse-square force law have been performed only on scales $\leq 10^{13}$ m ([1] and references therein). An enormous extrapolation is required to apply this law to cosmological scales $> 10^{22}$ m [2].

Some recent theories propose a deviation from Newtonian gravity at large distances, perhaps due to extra dimensions, branes, or new particles (e.g., [3]). These theories, mostly motivated by finding an alternative explanation to dark energy for the cosmic acceleration (e.g. [4–7]), modify gravity on scales comparable to the horizon scale $\sim 10^{26}$ m.

Here we set out to constrain theories in which gravity deviates from the inverse-square law on scales of ~ 10 Mpc or 10^{23} m. We evolve the power spectrum (and bispectrum) under a perturbed law of gravity in the linear (and mildly nonlinear) regimes, from recombination to the present day. The perturbed law modifies the evolution of density fluctuations, altering, in particular, the power spectrum shape. We compare the prediction for the modified power spectrum to that measured from two galaxy redshift surveys: the Anglo-Australian Two-Degree Field Galaxy Redshift Survey (2dFGRS; [8,9]) and the Sloan Digital Sky Survey (SDSS; [10–12]).

In the models we consider, gravity is consistent with the inverse-square law on scales smaller than ~ 1 Mpc, so the deviation does not affect early universe physics through recombination. Thus, we assume that the standard computation of the cosmic microwave background (CMB) anisotropies holds, and that the CMB observations provide the mass power spectrum (and bispectrum) at recombination. Moreover, as explained later, we marginalize over reasonable priors for the power spectrum amplitude and the primordial power spectrum slope, thus making our analysis insensitive to reasonable changes in the other cosmological parameters. We expect deviations from the inverse-square law to modify the integrated Sachs-Wolfe (ISW) effect. However, as this affects the CMB on large scales where cosmic variance is large, we do not discuss ISW here.

In the context of Newtonian gravity, a similar approach has been widely used in the literature to constrain cosmological parameters (e.g., [13]) and biasing, the relation between clustering of galaxies and clustering of mass (e.g., [14,15]). Here, we fix the background cosmology [13] and assume a scale-independent bias [16]. The deviation from Newtonian gravity is parametrized by a length-scale and a strength on which we place constraints. We find no compelling evidence for deviations from the inverse-square force law on large-scale structure scales. This null result leaves open the possibility that both Newton's gravity and our weak assumptions about scale-independent bias and the primordial power spectrum are incorrect in some contrived way so that their effects conspire to exactly cancel. We regard such a scenario as exceedingly unlikely. This is the first attempt to use cosmological data to constrain gravity on these scales, but for related theoretical work, see [19–22].

II. METHOD

We consider a force of gravity that deviates from an inverse-square law by a small perturbation on Mpc scales. We examine two possible functional forms for this perturbation, each parametrized by a strength and a length scale. Since Newtonian gravity has been tested on very small (mm) scales up to solar system scales [1], both models recover the inverse-square law at small scales. The expansion rate $a(t)$ is set by WMAP's best-fit ΛCDM background cosmology [13]. Note that these models differ from the modification to general relativity considered in [23]. Since we are testing a null hypothesis, the specific form of the parametrization does not matter; our choices are based loosely on parametrizations used in small-scale tests of gravity.

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In the first model, we consider a Yukawa-like contribution to the potential,

$$\Phi(\vec{r}) = -G \int d^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} [1 + \alpha(1 - e^{-|\vec{r} - \vec{r}'|/\lambda})], \quad (1)$$

where α is small. The Yukawa parametrization has been used in small-scale tests of gravity, where it is physically motivated by the exchange of virtual bosons [1]. This model nicely behaves like an inverse-square law at very large and very small length scales relative to the length scale λ.

In the second model (hereafter the "PL model"), we consider a power-law-like (PL) potential of the form,

$$\Phi(\vec{r}) = -G \int d^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} \left[1 + \tilde{\epsilon}(|\vec{r} - \vec{r}'|) \left(\frac{R}{|\vec{r} - \vec{r}'|} \right)^{N-1} \right], \quad (2)$$

where $\tilde{\epsilon}(r) = \Theta(r - R)$ is a step function, $\epsilon \ll 1$, R is a physical length scale, ϵ is the strength of the deviation, and we discuss $N = 2$. The r dependence of $\tilde{\epsilon}$ ensures agreement with standard gravity at small scales. Incidentally, this form of modification (with constant $\tilde{\epsilon}$) can be generated by the simultaneous exchange of two massless scalar particles ([1], and references therein). However, such effects would likely occur at much smaller scales than we are probing.

Let us consider the first model. In comoving coordinates, $\vec{x} \equiv [\vec{r}/a(t)]$, the comoving gravitational potential is given by $\phi(\vec{x}) = -G\bar{\rho}a^2\Delta(\vec{x})$, where

$$\Delta(\vec{x}) = \int d^3 x' \frac{1 + \delta(\vec{x}')}{|\vec{x} - \vec{x}'|} [1 + \alpha(1 - e^{-(a|\vec{x} - \vec{x}'|/\lambda)})], \quad (3)$$

$\delta(\vec{x}) = [\rho(\vec{x}) - \bar{\rho}]/\bar{\rho}$, $\bar{\rho}$ is the average density of the universe, and we scale $a(t)$ so $a = 1$ today.

Transforming Δ into Fourier space yields

$$\hat{\Delta}(\vec{k}) = \frac{4\pi\hat{\delta}(\vec{k})}{k^2} \left[1 + \alpha F_{\Delta_1} \left(\frac{a}{k\lambda} \right) \right] \quad (4)$$

with $F_{\Delta_1}(s) = s^2/(1 + s^2)$.

Following the same analysis as the first model, the PL model yields

$$\hat{\Delta}(\vec{k}) = \frac{4\pi\hat{\delta}(\vec{k})}{k^2} \left[1 + \epsilon F_{\Delta_2} \left(\frac{a}{kR} \right) \right], \quad (5)$$

with $F_{\Delta_2}(s) = \frac{1}{s}[\frac{\pi}{2} - \text{Si}(\frac{1}{s})]$, where $\text{Si}(x)$ denotes the sine integral.

A. Linear theory

In this section we compute the density fluctuations $\hat{\delta}$, the comoving velocity field, and the power spectrum for the Yukawa-like model. The results for the PL model are obtained simply by replacing α with ϵ, λ with R, and $F_{\Delta_1}(s)$ with $F_{\Delta_2}(s)$.

The universe's expansion rate is dominated by physics at horizon scales, beyond the scales we consider here. We thus use Friedman equations for the background cosmology. Although for the PL model we simply make this assumption, for the Yukawa-like model this assumption can be justified. The Yukawa-like model recovers the inverse-square law on horizon scales. A change in the value of Newton's constant G, as in the large-scale limit of the Yukawa-like model, has the effect of changing the overall amplitude of the growth factor, and thus of the power spectrum normalization, over which we marginalize. We are sensitive only to changes in the shape of the power spectrum on supercluser scales, which is most strongly affected by the growth of perturbations against the background cosmology, and not by the background cosmology itself.

The equation for $\hat{\delta}$ to first order in perturbation theory is

$$\ddot{\hat{\delta}} + 2\frac{\dot{a}}{a}\dot{\hat{\delta}} - G\bar{\rho}k^2\hat{\Delta} = 0. \quad (6)$$

For $\alpha = 0$ the equation is separable, with solution [24]

$$\hat{\delta}_A(\vec{k}, t) \equiv A(t)\hat{\delta}_o(\vec{k}). \quad (7)$$

We now look for a solution of the form

$$\hat{\delta}(\vec{k}, t) = \hat{\delta}_A(\vec{k}, t)[1 + \alpha\hat{d}(\vec{k}, t)] \quad (8)$$

to first order in α, and solve for $\hat{d}(\vec{k}, a(t))$ [25]. The initial conditions $\hat{d}(0) = 0$ and $\hat{d}'(0) = 0$ ensure that gravity behaves normally at small scales and that $\hat{\delta}$ is finite at $a = 0$. For the PL model, all solutions are undefined at $a = 0$. We set $\hat{d}$ and $\hat{d}'$ to zero at a sufficiently small scale that changing that scale by an order of magnitude changes $\hat{d}$ at $a = 1$ by significantly less than our error bars.

Figure 1 shows $\hat{d}(1/k\lambda)$ for the Yukawa-like model (dashed line) and $\hat{d}(1/kR)$ for the PL model (solid line) for a ΛCDM universe at $a = 1$. Since a positive value of α

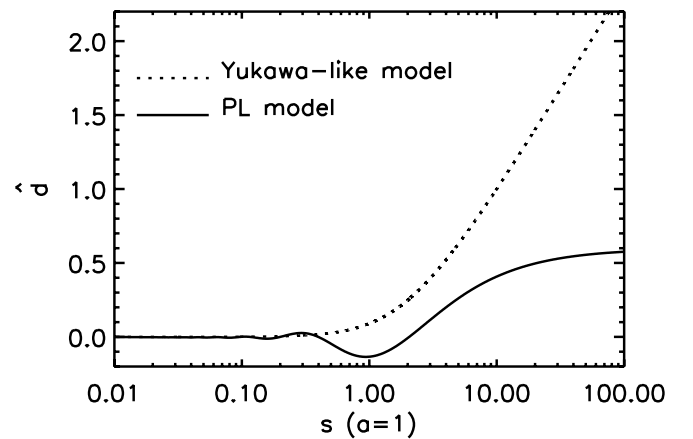FIG. 1. $\hat{d}(1/k\lambda)$ and $\hat{d}(1/kR)$ for the Yukawa-like and PL models, respectively, at $a = 1$ for a ΛCDM cosmological model ($\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$).

makes gravity stronger, we expect more clustering ($\hat{d} > 0$). The step function in the PL model's potential causes some ringing in the Fourier transform, so the change in the comoving potential and $\hat{d}$ both oscillate as functions of k.

We find that Ω_Λ weakens the effect of α or ϵ at large scales.

We can now compute the comoving velocity to first order in perturbation theory using the continuity equation and the equation of motion in Fourier space [26]. For simplicity, we will work in an Einstein–de Sitter (EdS) universe. We only use the velocity to compute the bispectrum, and the bispectrum is robust to changes in cosmology (see, e.g., Fig. 1 in [27]). To first order in α, we find

$$\hat{v} = ia\frac{\vec{k}}{k^2}\dot{\hat{\delta}} = ia\frac{\vec{k}}{k^2}A\hat{\delta}_o\left[1 + \alpha F_v\left(\frac{a}{k\lambda}\right)\right], \quad (9)$$

where

$$F_v(s) \equiv \hat{d}(s) + s\hat{d}'(s). \quad (10)$$

The power spectrum is given by

$$(2\pi)^3 P(\vec{k}, t)\delta^D(\vec{k} + \vec{k}') = \langle\hat{\delta}(\vec{k}, t)\hat{\delta}(\vec{k}', t)\rangle, \quad (11)$$

where

$$\langle|\hat{\delta}(\vec{k}, t)|^2\rangle = \langle|\hat{\delta}_A(\vec{k}, t)(1 + \alpha\hat{d}(\vec{k}, t))|^2\rangle \quad (12)$$

and $\langle\ \rangle$ denote an ensemble average over all realizations of the universe. The present-day power spectrum derived using our model of modified gravity, to first order in α, is then

$$P(k) = P_o(k)\left[1 + 2\alpha\hat{d}\left(\frac{1}{k\lambda}\right)\right], \quad (13)$$

where P_o is the power spectrum expected for Newtonian gravity.

To test the validity of our first-order approximation in α, we also compared $\hat{d}$ for an EdS universe to the Yukawa-like model's exact solution. The growing EdS solution for the density fluctuation is a hypergeometric function,

$$\hat{\delta}(s) = {}_2F_1\left(\frac{5 - \sqrt{25 + 24\alpha}}{8}, \frac{5 + \sqrt{25 + 24\alpha}}{8}, \frac{9}{4}; -s^2\right)$$
$$\times s\hat{\delta}_0(k), \quad (14)$$

where $s \equiv (a/k\lambda)$. We find that the higher-order α corrections contribute a small fraction most values of $s = (a/k\lambda)$ in our analysis. For example, for $s = 100$ (the largest s in our analysis), we get $\sim 20\%$ error for $|\alpha| \sim 0.2$; for small values of s—and the same $|\alpha|$ value—the error is less than a percent. We are only concerned with effects that change the result by orders of magnitude, not factors of a few, since gravity has never been tested on these scales before. Thus, we believe the Taylor expansion in α and ϵ is sufficient for our first search for constraints.

B. Mildly nonlinear theory

The equation for $\hat{\delta}$ to second order in perturbation theory is

$$\ddot{\hat{\delta}} + 2\frac{\dot{a}}{a}\dot{\hat{\delta}} + \frac{k^2}{a^2}\hat{\phi} = -\frac{1}{a^2}[k_i\hat{\phi} * k^i\hat{\delta} + \hat{\delta} * k^2\hat{\phi}$$
$$+ k_ik_j(\hat{v}^i * \hat{v}^j)], \quad (15)$$

where we use the Einstein summation convention over spacial indices i, j. The symbol "$*$" denotes a convolution in k space,

$$f(\vec{k}) * g(\vec{k}) \equiv \int d^3l \int d^3m\,\delta^D(\vec{l} + \vec{m} - \vec{k})f(\vec{l})g(\vec{m}), \quad (16)$$

where $\delta^D(\vec{k})$ is the three-dimensional Dirac delta function. We use the solution for $\vec{v}$ from first-order perturbation theory.

Let $\hat{\delta}^{(1)}$ represent the solution to $\hat{\delta}$ in linear theory, and $\hat{\delta}^{(2)}$ represent the second-order term (of order $\hat{\delta}_A^2$), so $\hat{\delta} = \hat{\delta}^{(1)} + \hat{\delta}^{(2)}$ plus smaller, higher-order terms.

For $\alpha = 0$ (or $\epsilon = 0$), the solution for the second-order term is

$$\hat{\delta}_A^{(2)} \equiv A^2(t)\hat{\delta}_o^{(2)}(\vec{k})$$
$$= A^2\left(\frac{5}{7}\hat{\delta}_o * \hat{\delta}_o + k_i\hat{\delta}_o * \frac{k^i}{k^2}\hat{\delta}_o + \frac{2}{7}\frac{k_ik_j}{k^2}\hat{\delta}_o * \frac{k^ik^j}{k^2}\hat{\delta}_o\right). \quad (17)$$

We then follow the same approach as in the linear theory. Using the first model as our example, we look for a solution to first order in α of the form

$$\hat{\delta}^{(2)} = \hat{\delta}_A^{(2)}[1 + \alpha\hat{g}(\vec{k}, t)]. \quad (18)$$

Switching the time variable to a, the resulting differential equation for $\hat{g}(\vec{k}, a)$ in an EdS universe is

$$\frac{\partial^2\hat{g}}{\partial a^2} + \frac{11}{2a}\frac{\partial\hat{g}}{\partial a} + \frac{7}{2a^2}\hat{g} = \frac{1}{\hat{\delta}_o^{(2)}}\frac{3}{2a^2}N(\vec{k}, a), \quad (19)$$

where

$$N(\vec{k}, a) = F_{\Delta_1}\left(\frac{a}{k\lambda}\right)\hat{\delta}_o^{(2)}$$
$$+ \frac{k_i}{k^2}\hat{\delta}_o\left(F_{\Delta_1}\left(\frac{a}{k\lambda}\right) + \hat{d}\left(\frac{a}{k\lambda}\right) + \frac{4}{3}F_v\left(\frac{a}{k\lambda}\right)\right) * k^i\hat{\delta}_o$$
$$+ \frac{k_i}{k^2}\hat{\delta}_o * k^i\hat{\delta}_o\left(\hat{d}\left(\frac{a}{k\lambda}\right) + \frac{4}{3}F_v\left(\frac{a}{k\lambda}\right)\right)$$
$$+ \hat{\delta}_o * \hat{\delta}_o\left(F_{\Delta_1}\left(\frac{a}{k\lambda}\right) + 2\hat{d}\left(\frac{a}{k\lambda}\right) + \frac{4}{3}F_v\left(\frac{a}{k\lambda}\right)\right)$$
$$+ \frac{4}{3}\frac{k_ik_j}{k^2}\hat{\delta}_oF_v\left(\frac{a}{k\lambda}\right) * \frac{k^ik^j}{k^2}\hat{\delta}_o, \quad (20)$$

with initial conditions $\hat{g}(\vec{k}, 0) = 0$ and $\frac{\partial}{\partial a}\hat{g}(\vec{k}, 0) = 0$. The

bispectrum is given by

$$B(\vec{k}_1, \vec{k}_2)\delta^D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \equiv \langle \hat{\delta}(\vec{k}_1)\hat{\delta}(\vec{k}_2)\hat{\delta}(\vec{k}_3) \rangle$$

$$= \langle [\hat{\delta}^{(1)}(\vec{k}_1) + \hat{\delta}^{(2)}(\vec{k}_2)]$$

$$\times [\hat{\delta}^{(1)}(\vec{k}_2) + \hat{\delta}^{(2)}(\vec{k}_2)]$$

$$\times [\hat{\delta}^{(1)}(\vec{k}_3) + \hat{\delta}^{(2)}(\vec{k}_3)] \rangle. \tag{21}$$

The leading term, $\langle \hat{\delta}^{(1)}(\vec{k}_1)\hat{\delta}^{(1)}(\vec{k}_2)\hat{\delta}^{(1)}(\vec{k}_3) \rangle$, vanishes under Gaussian initial conditions. Modified gravity still preserves Gaussianity in the linear regime, and introduces a correction to the bispectrum in second-order perturbation theory. To first nonvanishing order in $\hat{\delta}_A$ and α, we get

$$\langle \hat{\delta}(\vec{k}_1)\hat{\delta}(\vec{k}_2)\hat{\delta}(\vec{k}_3) \rangle = A^4 \langle \hat{\delta}_o(\vec{k}_1)\hat{\delta}_o(\vec{k}_2)\hat{\delta}_o^{(2)}(\vec{k}_3) \rangle$$

$$\times \left[1 + \alpha \left(\hat{d}\left(\frac{a}{k_1\lambda}\right) + \hat{d}\left(\frac{a}{k_2\lambda}\right) \right. \right.$$

$$\left. \left. + \hat{g}(\vec{k}_3, a) \right) \right] + \text{cyc} \tag{22}$$

where again we can pull $\hat{d}$ and $\hat{g}$ out of the ensemble average, $\vec{k}_3 = -\vec{k}_1 - \vec{k}_2$, and cyc denotes cyclic permutations of the subscripts 1,2,3.

Since we already know $\hat{d}$, we need to solve only for the last term. To do so, let us examine the term $X \equiv \langle \hat{\delta}_o(\vec{k}_1)\hat{\delta}_o(\vec{k}_2)\hat{\delta}_o^{(2)}(\vec{k}_3)\hat{g}(\vec{k}_3, a) \rangle$. From Eq. (19) we can write,

$$\frac{\partial^2 X}{\partial a^2} + \frac{11}{2a}\frac{\partial X}{\partial a} + \frac{7}{2a^2}X = \left\langle \hat{\delta}_o(\vec{k}_1)\hat{\delta}_o(\vec{k}_2)\frac{3}{2a^2}N(\vec{k}_3, a) \right\rangle. \tag{23}$$

$N(\vec{k}, a)$ is a sum of terms with k-space convolutions and a dependence. We can thus rewrite Eq. (20) as

$$N(\vec{k}, a) = \sum_n \hat{\delta}_o(\vec{k})J_n(\vec{k}, a) * \hat{\delta}_o(\vec{k})K_n(\vec{k}, a), \tag{24}$$

where the expressions for J_n and K_n are easily obtained by equating Eq. (24) with Eq. (20). Using Eq. (16), we rewrite the right-hand side of Eq. (23) as

$$\frac{3}{2a^2} \left\langle \int dl^3 dm^3 \delta^D(\vec{l} + \vec{m} - \vec{k}_3)\hat{\delta}_o(\vec{k}_1)\hat{\delta}_o(\vec{k}_2)\hat{\delta}_o(\vec{l})\hat{\delta}_o(\vec{m}) \right.$$

$$\left. \times \sum_n J_n(\vec{l}, a)K_n(\vec{m}, a) \right\rangle \tag{25}$$

which becomes

$$\frac{3}{2a^2}\frac{P_o}{A^2}(k_1)\frac{P_o}{A^2}(k_2)\sum_n J_n(\vec{k}_1, a)K_n(\vec{k}_2, a)$$

$$+ J_n(\vec{k}_2, a)K_n(\vec{k}_1, a). \tag{26}$$

We can then solve Eq. (23) for each function of a that appears in the above sum, and get an expression for the bispectrum.

The full expression for the bispectrum given by

$$B(\vec{k}_1, \vec{k}_2)\delta^D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) = J(\vec{k}_1, \vec{k}_2)P_o(k_1)P_o(k_2)$$

$$+ \text{cyc}, \tag{27}$$

where

$$J(\vec{k}_1, \vec{k}_2) = J_o(\vec{k}_1, \vec{k}_2) + \alpha \left(J_o(\vec{k}_1, \vec{k}_2)\left(\hat{d}\left(\frac{a}{k_1\lambda}\right) + \hat{d}\left(\frac{a}{k_2\lambda}\right) \right) + \left[G_1\left(\frac{a}{k_3\lambda}\right)\left(\frac{5}{7} + \frac{\vec{k}_1 \cdot \vec{k}_2}{k_2^2} + \frac{2}{7}\left(\frac{\vec{k}_1 \cdot \vec{k}_2}{k_1 k_2}\right)^2 \right) \right.$$

$$+ \frac{\vec{k}_1 \cdot \vec{k}_2}{k_1^2}\left(G_1\left(\frac{a}{k_1\lambda}\right) + G_2\left(\frac{a}{k_1\lambda}\right) + \frac{4}{3}G_3\left(\frac{a}{k_1\lambda}\right) + G_2\left(\frac{a}{k_2\lambda}\right) + \frac{4}{3}G_3\left(\frac{a}{k_2\lambda}\right) \right) + G_1\left(\frac{a}{k_2\lambda}\right) + 2G_2\left(\frac{a}{k_2\lambda}\right)$$

$$+ \frac{4}{3}G_3\left(\frac{a}{k_2\lambda}\right) + \frac{4}{3}G_3\left(\frac{a}{k_2\lambda}\right)\left(\frac{\vec{k}_1 \cdot \vec{k}_2}{k_1 k_2}\right)^2 + (\vec{k}_1 \leftrightarrow \vec{k}_2) \right] \right), \tag{28}$$

$J_o(\vec{k}_1, \vec{k}_2)P_o(k_1)P_o(k_2)$ is the Newtonian gravity result given by Eq. (40) of [28], and

$$G_1(s) = \frac{3}{7} + \frac{2}{5s^2} + \frac{3\sqrt{2}}{10}\frac{1}{s^{7/2}}\arctan\left(\frac{\sqrt{2s}}{s-1}\right) - \frac{3}{5s}\arctan(s) + \frac{3\sqrt{2}}{20}\frac{1}{s^{7/2}}\ln\left(\frac{1+s+\sqrt{2s}}{1+s-\sqrt{2s}}\right),$$

$$G_2(s) = \frac{9}{50s^{7/2}}\int_0^s dx\, x^{5/2}\left({}_2F_1^{(1,0,0,0)}\left(0, \frac{5}{4}, \frac{9}{4}; -x^2\right) - {}_2F_1^{(0,1,0,0)}\left(0, \frac{5}{4}, \frac{9}{4}; -x^2\right) \right)$$

$$- \frac{9}{50s}\int_0^s dx\left({}_2F_1^{(1,0,0,0)}\left(0, \frac{5}{4}, \frac{9}{4}; -x^2\right) - {}_2F_1^{(0,1,0,0)}\left(0, \frac{5}{4}, \frac{9}{4}; -x^2\right) \right),$$

$$G_3(s) = \frac{1}{s}\int_0^s dx \frac{x^2}{5}{}_2F_1\left(1, \frac{9}{4}, \frac{13}{4}; -x^2\right) + \frac{1}{s^{7/2}}\int_0^s dx \frac{-x^{9/2}}{5}{}_2F_1\left(1, \frac{9}{4}, \frac{13}{4}; -x^2\right) + G_2(s), \tag{29}$$

Here, $_2F_1^{(1,0,0,0)}$ denotes the partial derivative of the hypergeometric function with respect to the first argument, and $_2F_1^{(0,1,0,0)}$ denotes the partial derivative with respect to the second argument.

Since the bispectrum contains complementary information from the power spectrum, we anticipate that combining the two should produce stronger constraints. However, we leave the bispectrum analysis to future work.

III. ANALYSIS

We can use the power spectrum measurements from the 2dFGRS [9] and the SDSS [12] to constrain the parameters of our models. The bispectrum derivation shows that the bispectrum should help disentangle the effects of biasing from possible modifications to gravity, but the analysis will follow in a subsequent paper.

The $z = 0$ power spectrum for normal gravity can be theoretically calculated using the transfer functions computed with CMBFAST for the WMAP best-fit parameters. The theoretical power spectrum for given values of α and λ (or ϵ and R) can then be obtained from Eq. (13). Assuming Gaussian likelihood, we computed the likelihood for each survey's power spectrum measurements as a function of α and λ (or ϵ and R).

To make our analysis insensitive to assumptions about the background cosmology, we marginalized over reasonable priors for the power spectrum amplitude and the primordial power-law power spectrum slope, n. For the amplitude, we find that the ranges 0.6 to 1.4 (for the first model) and 0.4 to 1.4 (for the PL model) times the best-fit power spectrum amplitude for normal gravity are sufficient to allow for changes in the expansion rate of the universe. The values of Ω_m, Ω_Λ, and the dark-energy equation of state parameter (assumed to be constant) affect primarily the power spectrum amplitude in linear theory and for the

scales of interest. We marginalized over n using the WMAP prior ($n = 0.99 \pm 0.04$ at the one-sigma level [13]). The large-scale structure power spectrum shape has a weak dependence on $\Gamma \simeq \Omega_m h$, which shows up only on the largest scales and is strongly degenerate with Ω_{baryon} and n [9,29]. The large-scale structure power spectrum has a very weak sensitivity to Ω_{baryon} via two effects. One is the appearance of the so-called baryonic wiggles; as this effect has not yet been detected from these data sets, it cannot yet give sensitivity to Ω_{baryon}. The other effect is a weak change in power spectrum slope, degenerate with n. We are also somewhat insensitive to the uncertainty in neutrino mass. Increasing neutrino mass affects first the amplitude and then the shape of the galaxy power spectrum in the scales of interest (see, e.g., Fig. 8 of [30]). Present constraints would be weakened if a running of $P(k)$ and/or a redshift dependence of the dark-energy equation of state were explored.

Last, since we are mainly interested in a constraint on α and/or ϵ rather than on the scale λ and/or R, we marginalized over λ (or R) from 10 to 100 Mpc/h.

IV. RESULTS AND CONCLUSIONS

Figure 2 shows the Yukawa-like model's parameter constraints for the 2dFGRS and SDSS data, and Fig. 3 shows the PL model's constraints. The "ringing" in the $\hat{d}$ function in the PL model, creates "wiggles" in the large-scale structure power spectrum which are responsible for the geometry of the isolikelihood contours in Fig. 3. The left plots show the likelihood contours for one-sigma marginalized, one-sigma joint, and two-sigma joint confidence levels for one survey in λ, α (or R, ϵ) space. The center plots show the difference in chi square ($\Delta \chi^2$) as a function of α or ϵ, marginalized over λ or R. We used data up to $k \sim 0.15$ h/Mpc for both surveys. The results are
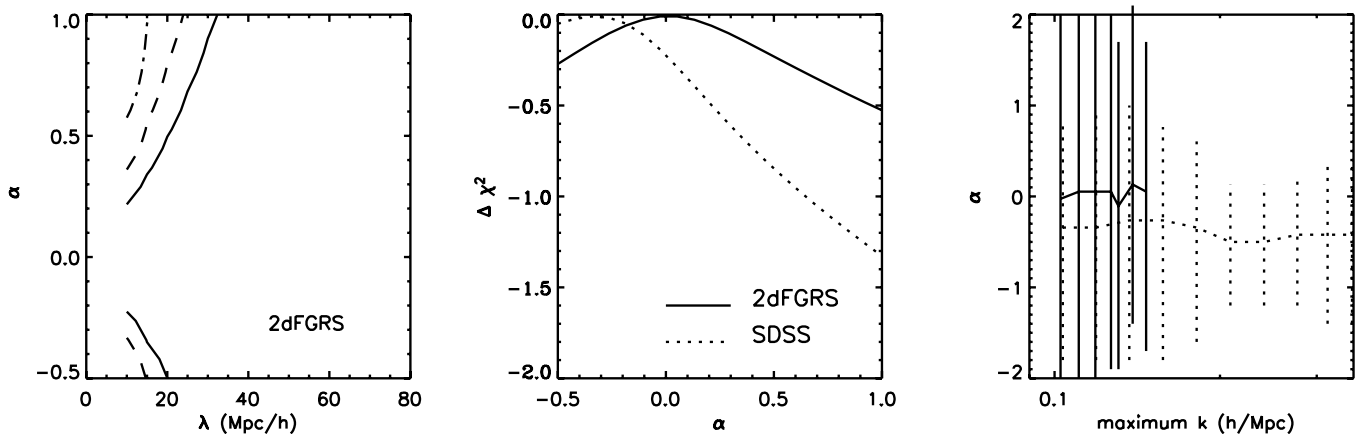

FIG. 2.   Results from the first (Yukawa-like) model. Left: Likelihood as a function of α and λ using data up to $k \sim 0.15$ h/Mpc using 2dF data. Contours denote one sigma marginalized (solid line), one sigma joint (dashed line), and two sigma joint (dashed-dotted line). Middle: Chi square as function of α, marginalized over λ, using data up to $k \sim 0.15$. Right: Values of α corresponding to the maximum chi square with error bars denoting one sigma, as a function of the maximum k included in the analysis.
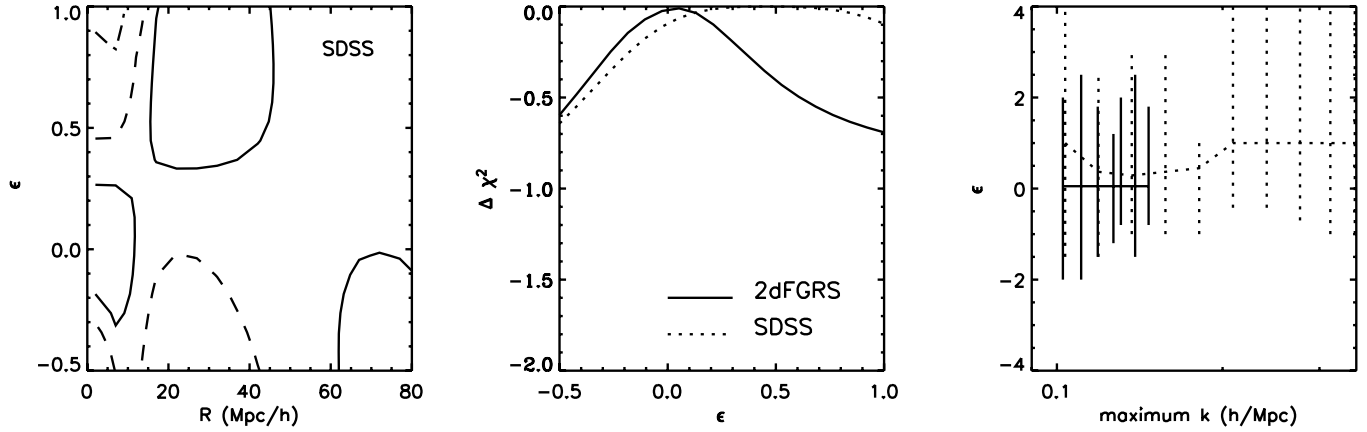
FIG. 3.   Results from the PL model. Left: Likelihood as a function of ϵ and R using data up to $k \sim 0.15$ h/Mpc using SDSS data. Inner contours denote one sigma marginalized ($\sim \Delta\chi^2 = 1$ solid line), bottom contour denotes one sigma joint ($\sim \Delta\chi^2 = 2.3$ dashed line). Right: Chi square as function of ϵ, marginalized over R, using data up to $k \sim 0.15$.

consistent with each other and with a strict inverse-square law. Since the constraints are weak and the Taylor expansion analysis breaks down at large values of $|\alpha|$ and $|\epsilon|$, we extrapolated the error bars from the graphs. We find $\alpha = 0.025^{+1.7}_{-1.7}$, $\epsilon = 0.05^{+1.3}_{-0.4}$ for 2dFGRS and $\alpha = -0.35^{+0.9}_{-0.9}$, $\epsilon = 0.45^{+1.4}_{-1.4}$ for SDSS, at the one-sigma level. Nonlinearities become increasingly important as k increases and might introduce systematic errors, but the signal-to-noise for the power spectrum increases with increasing k. Therefore we explore the dependence of the result as a function of the maximum k included in the analysis (k_{max}) on the right panel of the figures.

Since our analysis probes deviations in the *shape* of today's power spectrum, the strongest constraints on α and ϵ occur at scales where the *slope* of $\hat{d}$ (with $a = 1$) is greatest. This typically corresponds to λ and R towards the smallest scales of the data, so the smallest comoving scales probed have just reached the physical scale of the force-law deviation today, and the larger comoving scales have passed it. (A deviation near a physical scale λ or R has

only affected comoving scales that have grown larger than λ or R.) The weakness of our constraints arises primarily from the need to marginalize over a wide range of power spectrum amplitudes and the spectral slope. Future weak lensing surveys will measure the power spectrum for dark matter directly. If $a(t)$ can also be measured independently, this data will eliminate the need to marginalize over the amplitude and shrink our error bars for the PL model. We forecast that this would roughly halve the error on ϵ. The constraint does not improve significantly for the Yukawa-like model.

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