PHYSICAL REVIEW D **71,** 083003 (2005)

Type-I superconductivity and neutron star precession

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(Received 25 August 2004; published 7 April 2005)

Type-I proton superconducting cores of neutron stars break up in a magnetic field into alternating domains of superconducting and normal fluids. We examine two channels of superfluid-normal fluid friction where (i) rotational vortices are decoupled from the nonsuperconducting domains and the interaction is due to the strong force between protons and neutrons; (ii) the nonsuperconducting domains are dynamically coupled to the vortices and the vortex motion generates transverse electric fields within them, causing electronic current flow and Ohmic dissipation. The obtained dissipation coefficients are consistent with the Eulerian precession of neutron stars.

DOI: 10.1103/PhysRevD.71.083003 PACS numbers: 97.60.Jd, 26.60.+c, 74.25.−q

I. INTRODUCTION

The timing observations of radio pulsars provide a unique tool to study the properties of superdense matter in compact stars. While pulsars are known to be perfect clocks over long periods of time, the timing observations of past few decades revealed several types of timing "irregularities" in a subclass of isolated compact objects. Glitches (or macrojumps)—sudden increases in the pulsar rotation frequency and its derivative—are the most spectacular examples of timing anomalies. The slow relaxation of their spin and its derivative following a glitch has been interpreted as an evidence for superfluidity of compact star interiors [1]. At temperatures prevailing in an evolved compact star the dense hadronic matter is expected to be in the superfluid state due to the attractive component of the nuclear force which binds neutrons into Cooper pairs either in the relative ${}^1S_0$ state (at low densities) or ${}^3P_2 - {}^3F_2$ state (at high densities) [2]. Other members of the spin-1/2 octet of baryons (protons and strangeness $S = 1$ particles Σ^{-0+}, Λ) are expected to pair in the S-wave channel due to their low concentration. If the central densities of compact stars exceed the density of deconfinement phase transition to a quark matter phase, the deconfined quark matter will be in one of the many possible color superconducting states [3]. Since the glitches are related to the axisymmetric perturbations from the state of uniform rotation there is a twofold degeneracy in the interpretation of the data on both the jumps and the post-jump relaxations; the time-scales of these processes can be associated either with the weak or strong coupling between the superfluid and normal fluid in the star's interiors, and it is impossible to distinguish between these regimes on the basis of glitch observations alone. During the recent years it became increasingly clear that another type of timing anomaly—the long-term periodic variations superimposed on the spin-down of the star—can provide an additional and independent information on the dynamical coupling between the superfluid and the normal fluid in compact stars. If these irregularities are interpreted in terms of the precession of the star (a motion which involves nonaxisymmetric perturbations from the rotational state) the degeneracy inherent to the interpretation of glitches is lifted. It turns out that the free precession is possible only in the weak coupling limit and it is damped in the strong coupling case. Interpretations of the timing anomalies in pulsars in terms of the friction between the superfluid and the normal fluid require a model of the friction between the superfluid and normal components of the star on mesoscopic scales characteristic for the vorticity. This paper discusses two new mechanisms of mutual friction in the core of a neutron star in the case where protons form a type-I superconductor. The remainder of the introduction sets the stage by briefly reviewing the relevant physics. Section II studies the dynamics of a type-I superconducting model where there is a single normal domain per rotational vortex. In Section III we discuss the dissipation in an alternative picture where there is a large number of rotational vortices associated with a single normal domain. Section IV is devoted to the implications of the dissipative dynamics of type-I superconductors for the free precession of compact stars and contains a brief summary of the results.

A. No-go theorems for precession

To see how the superfluidity of neutron stars changes their Eulerian precession (which would be intact if the neutron stars were nonsuperfluid) let us begin with the equation of motion of approximately massless neutron vortex

$$\rho_S \kappa(\boldsymbol{v}_S - \boldsymbol{v}_L) \times \boldsymbol{n} - \eta \boldsymbol{u} - \eta'(\boldsymbol{n} \times \boldsymbol{u}) = 0, \qquad (1)$$

where $\boldsymbol{u} \equiv \boldsymbol{v}_N - \boldsymbol{v}_L$, and $\boldsymbol{v}_S$, $\boldsymbol{v}_N$ and $\boldsymbol{v}_L$ are the velocities of the superfluid, the normal fluid and the vortex; ρ_S is the effective neutron density, κ is the quantum of circulation, $\boldsymbol{n} = \vec{\kappa}/\kappa$, and the coefficients η and η' are the measure of the friction between the neutron vortex and the ambient normal fluid. Here we work within the two-fluid superfluid hydrodynamics, where it is assumed that the hydrodynamic forces are linear functions of the velocities, which guarantees that the energy variation is always a quadratic form. Shaham first observed that the long-term Eulerian preces-

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sion is impossible if the neutron vortices are strongly pinned [4]. In terms of the mesoscopic parameters in Eq. (1), his observation is equivalent to the statement that precession is absent in the limit $\zeta \to \infty$, $\zeta' \to 0$ where $\zeta = \eta/\rho_S\kappa$ and $\zeta' = \eta'/\rho_S\kappa$ are the drag-to-lift ratios. Since in the frictionless limit a star must precess at the classical frequency $\epsilon\Omega$, where ϵ is the eccentricity and Ω is the rotation frequency, it is clear that there exists a crossover from the damped to the free precession as ζ is decreased. The crossover is determined by the dimensionless parameters $(I_S/I_N)\beta$ and $(I_S/I_N)\beta'$ where I_S is the moment of inertia of the superfluid and I_N is the moment of inertia of the crust plus any component coupled to it on time-scales much shorter than the precession time scale and $\beta = \zeta/[(1-\zeta')^2 + \zeta^2]$, $\beta' = 1 - \beta(1-\zeta')/\zeta$. The precession frequency is [5] (hereafter SWC)

$$\Omega_P = \epsilon\Omega_S\left[\left(1 + \beta'\frac{I_S}{I_N}\right) + i\beta\frac{I_S}{I_N}\right], \tag{2}$$

where Ω_S is the spin frequency and ϵ is the eccentricity. The result of SWC can be cast in a no-go theorem that states that the Eulerian precession in a superfluid neutron star is impossible if $(I_S/I_N)\zeta > 1$ (assuming as before $\zeta' \to 0$). There is a subtlety to this result: the precession is impossible because the precession mode, apart from being damped, is renormalized by the nondissipative component of superfluid-normal fluid interaction ($\propto \beta'$). In effect the value of the precession eigen-frequency drops below the damping frequency for any ζ larger than the crossover value. Note that this counter-intuitive result can not be obtained from the arguments based solely on dissipation: in fact, according to Eq. (2) the damping time-scale for precession increases linearly with ζ and in the limit $\zeta \to \infty$ one would predict (wrongly) undamped precession. If a neutron star contains multiple layers of superfluids the picture is more complex, but the generic features of the crossover are the same [5].

B. Previous work

Long-term variabilities were observed in a number of pulsars and have been attributed phenomenologically to precession of the neutron star (see Ref. [6] and references therein). A strong case for long-term variability (again attributed to precession) was made recently by Stairs *et al.* [7]. While it is common to study perturbations from the state of uniform rotation, Wasserman [8] demonstrated that the precessional state may correspond to the local energy minimum of an inclined rotator if there is a large enough magnetic stress on the star's core. This type of precession is likely to be damped away by the superfluid-normal friction. Link [9] argued that the long-term variations, which can be fitted by assuming Eulerian precession of the pulsar, are incompatible with type-II superconductivity of neutron stars. Type-I superconductivity was proposed to resolve the discrepancy [9]. Jones [10] argued that the friction of vortices in the crusts against the nuclear lattice will give rise to a dissipation which will damp the free precession; thus, the free precession (even in absence of pinning) would be incompatible with the known properties of matter at subnuclear densities.

C. Type-I superconducting neutron stars

As is well known, type-I superconductivity arises when the Ginzburg-Landau parameter satisfies the condition $\kappa_{GL} = \lambda/\xi \leq 1/\sqrt{2}$, where λ is the magnetic field penetration depth and ξ is the coherence length. Type-I superconductivity can arise locally within the current models based on the BCS theory [11], with domain structures analogous to those observed in laboratory experiments. In Ref. [11] the theory of these structures was constructed along the lines of the theories developed for laboratory superconductors, where the magnetic fields are generated by normal currents driven around a cylindrical cavity by temperature gradients [12]. However, global type-I superconductivity would require a suppression of the proton pairing gap Δ_p (due to the scaling $\kappa_{GL} \propto \xi^{-1} \propto \Delta_p$) by polarization or related effects. Buckley *et al.* [13] studied the effect the interactions between the neutron and proton Cooper pairs would have on the type of the proton superconductivity. Their results suggest that type-I superconductivity can be enforced within the entire core without the suppression of the pairing gap if the strength of the yet unknown interaction between Cooper pairs will turn out to be significant.

The equilibrium structure of the alternating superconducting and normal domains in a type-I superconductor is a complicated problem and depends, among other things, on the nucleation history of the superfluid phase. The equilibrium dimension of a layer is of the order of magnitude $d \sim \sqrt{L\xi}$ where L is the size of the core; (for typical parameter values $L \simeq 5 \times 10^5$ cm and $\xi \simeq 200$ fm, $d \sim 3.2 \times 10^{-3}$ cm). By flux conservation, the ratio of the sizes of the superfluid and normal domains is given by the relation $d_S/d_N = \sqrt{H_{\rm cm}/B} \sim 10$, where $B \sim 10^{12}$ G is the average value of the magnetic induction, $H_{\rm cm} \sim 10^{14}$ G is the thermodynamic magnetic field. The dimensional analysis above suggests that there is roughly a single normal domain per neutron vortex. We shall consider below the dynamics of two distinct models where (i) a neutron vortex features a single coaxial normal domain of a smaller size according to Ref. [11] (hereafter $1 + 1$ model) and (ii) a large fraction of neutron vortices is accommodated by a single normal domain, as described in Ref. [13] ($1 + N$ model). The difference between these models is not simply the number of the neutron vortices accommodated by a single nonsuperconducting domain; in the $1 + 1$ model the magnetic fields are generated dynamically (this is explained in more detail below) and hence the normal domains are tied to the neutron vortices on dynamical times scales. In effect the motion of a neutron vortex

requires the motion of the normal domain attached to it on the dynamical time scale. Therefore the dissipation is due to the interaction between the combined structure (a vortex plus a normal domain) with the background electron liquid. While it is conceivable that the normal domains are tied up to the macroscopic neutron vortex lattice on certain time scales in the $1 + N$ model, we shall assume that the neutron vortex lattice sweeps through the nonsuperconducting domains and the dissipation arises from the scattering of the normal protons off the cores of rotational vortices [14].

II. DYNAMICS OF 1+1 STRUCTURES

In two component superfluids the supercurrent of any given component transports mass of both components; this is the essence of the entrainment effect first studied in the context of charge-neutral and nonrotating superfluid mixtures of ^{3}He–^{4}He [15]. If one of the components is charged—a case first studied in Refs. [16,17]—the neutral superfluid which rotates by forming a lattice of charge-neutral vortices generates magnetic fields because neutral supercurrent carries along a finite mass of the charged component. The mass currents of neutrons and protons $\boldsymbol{p}_{p/n}$ (here and below indices p and n refer to the protons and neutrons), are related to their velocities $\boldsymbol{v}_n$ and $\boldsymbol{v}_p$ by a density matrix in the isospin space [15–17]

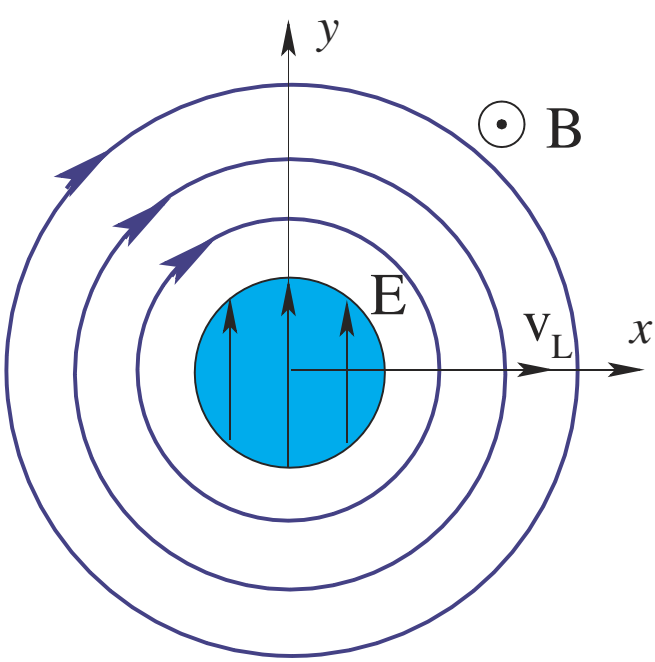

FIG. 1 (color online). An illustration of the structure of a rotational vortex placed in a type-I superconductor. The vortex velocity field is shown by the concentric circles. The nonsuperconducting domain (shaded region) of radius a is coaxial with the vortex and carries a magnetic field $H_{cm} \sim 10^{14}$ G. The vortex motion along the x-axis generates a transverse electric field, which drives the electron current through the domain and leads to Ohmic dissipation.

$$\begin{pmatrix} \boldsymbol{p}_p \\ \boldsymbol{p}_n \end{pmatrix} = \begin{pmatrix} \rho_{pp} & \rho_{pn} \\ \rho_{np} & \rho_{nn} \end{pmatrix} \begin{pmatrix} \boldsymbol{v}_p \\ \boldsymbol{v}_n \end{pmatrix}, \tag{3}$$

where in the mean-field approximation the elements of the density matrix can be expressed through the effective masses of neutrons and protons. Mendell obtained previously the general form of mutual friction damping from vortices, which incorporated the entrainment effect, in the case of type-II superconductivity [18].

If the proton superconductor is type-I, nonsuperconducting domains coaxial with the neutron vortices nucleate in response to the entrainment current set up by the vortex circulation [11]. Consider a cylindrical domain of radius a coaxial with a vortex (see Fig. 1 for an illustration). In the cylindrical coordinates (r, ϕ, z) with the symmetry axis at the center of the vortex the magnetic field induction is [Ref. [11], Eq. (20)]

$$B_z(r) = \frac{\Phi_1}{2\pi\lambda^2} \ln\left(\frac{b}{a}\right)\frac{N(r)}{N(a)}, \quad B_x = B_y = 0, \tag{4}$$

where $\Phi_1 = (\rho_{pn}/\rho_{pp})\Phi_0$, Φ_0 is the flux quantum, λ is the magnetic field penetration depth, b is the vortex (outer) radius and

$$N(r) = I_0\left(\frac{b}{\lambda}\right)K_0\left(\frac{r}{\lambda}\right) - K_0\left(\frac{b}{\lambda}\right)I_0\left(\frac{r}{\lambda}\right), \tag{5}$$

where $I_0(z)$ and $K_0(z)$ are the modified Bessel functions. In the mean-field approximation the magnitude of the (nonquantized) flux Φ_1 is determined by the effective mass of a proton quasiparticle $\rho_{pn}/\rho_{pp} \equiv |m_p^*/m_p - 1|$.

Now we can write down the proton supercurrent $j_\phi = (c/4\pi)(\vec{\nabla} \times \boldsymbol{B})_\phi$ by substituting the B-field from Eq. (4). We need, however, the velocity of superfluid protons, which is the difference between the net supercurrent and the supercurrent moving with the neutron superfluid velocity; we find

$$v_{p\phi} = \frac{k\hbar}{2mr}\left[\frac{r}{\lambda} \ln\left(\frac{b}{a}\right) \coth\left(\frac{b-r}{a}\right) - 1\right]. \tag{6}$$

Equation (6) keeps the leading order term of the expansion of the Bessel functions with respect to large arguments r/λ, where r is a mesoscopic scale $\geq a$.

Consider now a vortex which moves at a constant velocity $\boldsymbol{v}_L$, and carries a coaxial normal domain of protonic fluid with respect to the background electron liquid (see Fig. 1). The equation of motion of the superfluid protons, written in the reference frame where the vortex is at rest, acquires an additional term $(\boldsymbol{v}_L \cdot \boldsymbol{\nabla}) \cdot \boldsymbol{v}_p$. The continuity of the electro-chemical potentials of the superfluid and normal phases across the boundary of the normal and superconducting phases gives $\mu_S = \mu_N$, where

$$\mu_S = \mu - m_p^* v_L v_p(r)\frac{y}{r}, \quad \mu_N = \mu - e\phi, \tag{7}$$

here μ is the proton chemical potential in equilibrium, ϕ is

the scalar electric potential and we use 2d Cartesian coordinates with vortex circulation along z-axis and vortex velocity along the x-axis, see Fig. 1; small terms $O(\Delta^2/\mu)$ are neglected. Thus, the motion of the vortex generates a constant transverse electric field across the normal domain

$$E_y = -\nabla_y \phi = -\frac{m_p^* v_L v_p(a)}{ea}, \quad E_x = E_z = 0. \tag{8}$$

The power dissipated per unit length of a vortex is $W = \sigma E^2 (a/b)^2$, where σ is the electrical conductivity and the factor $(a/b)^2$ is the fractional 2d volume occupied by the domain. Upon substituting Eq. (8) in this relation, we obtain an alternative form of dissipation $W = \eta v_L^2$, which identifies the friction coefficient

$$\eta = \frac{\sigma}{c^2}\left(\frac{\Phi_1}{2\pi ab}\right)^2 \left[\frac{a}{\lambda}\ln\left(\frac{b}{a}\right)\coth\left(\frac{b-a}{a}\right) - 1\right]^2. \tag{9}$$

Equation (9) is our central result, which defines the friction coefficient for a vortex featuring a single coaxial domain of a type-I proton superconductor in terms of the electrical conductivity of the electron Fermi-liquid in normal proton matter.

Since the mean-free path of electrons is much smaller than the size of a single normal domain, we can neglect the finite-size effects and use the result for the bulk normal matter [19]. The zero-field conductivity of ultrarelativistic electrons is $\sigma_0 = n_e e^2 c \tau_c/(\hbar k_F)$, where the relaxation time for the Coulomb scattering of electrons off the protons in the normal domains is [19]

$$\tau_c = \frac{12}{\pi^2}\left(\frac{\hbar c}{e^2}\right)^2 \left(\frac{\epsilon_F}{T}\right)^2 \frac{k_{FT}}{c k_F^2}, \tag{10}$$

where ϵ_F and k_F are the electron Fermi-energy and Fermi-wavenumber, and $k_{TF} = [4k_F m_p^* e^2/\pi\hbar^2]^{1/2}$ is the Thomas-Fermi-wave-length. Since the Larmor radius of an electron moving in a normal domain carrying a field $H_{\rm cm} \sim 10^{14}$ G is much smaller than the linear size of the domain, the conductivity becomes $\sigma = \sigma_0/(\omega_c \tau_c)^2$, where $\omega_c = (eH_{cm})/(\hbar k_F)$, where ω_c is the electron cyclotron frequency. Table I lists some of the relevant parameters of the problem, including transport quantities and the drag-to-lift ratio at the temperature $T = 10^8$ K. The kinetic coefficients can be rescaled to other temperatures by using the scalings $\tau \propto T^{-2}$, $\sigma \propto T^2$ and $\zeta \propto T^2$. The drag-to-lift ratio satisfies the condition $\zeta \ll 1$ at all temperatures (the largest values correspond to temperatures just below the critical temperature $T_c \sim 10^9$ K). We shall return to the implications of these results for the neutron star precession in the closing section.

TABLE I. Listed are the protons density (column 1), the Fermi-wave number (2), the effective mass of protons (3), the magnetic field penetration depth (4), the critical thermodynamic field (5), the electron relaxation time (6), the electrical conductivity (7), and the drag-to-lift ratio (8). The short-hand notions $\rho_{14} = \rho/10^{14}$, etc., are used.

ρ_{14} g cm^{-3}	k_{Fp} fm^{-1}	m_p^*/m_p	λ fm	$H_{\rm cm,14}$ G	$\tau_{c,-13}$ s	σ_{16} s^{-1}	ζ_{-4}
7.91	0.85	0.69	41.58	11.62	10.94	3.76	0.39
8.31	0.88	0.68	39.20	7.82	12.20	8.56	0.36
8.56	0.90	0.68	37.87	3.88	12.90	35.94	0.88

III. DYNAMICS OF 1+N STRUCTURES

Buckley *et al.* [13] argued (qualitatively) that the size of the normal domains could be large enough to accommodate about $N = 10$ neutron vortices across a single normal domain of protonic fluid. Since there is no dynamical coupling (in the sense of the entrainment) between the vortices and the normal domains the damping of the differential rotation between electron-proton plasma and the neutron superfluid is due to the interaction of domain (nonsuperconducting) protons with the core quasiparticles confined in the neutron vortex core. The relaxation process is thus the same as for the case where the proton fluid is nonsuperconducting over the entire bulk of the core, but the final result needs to be rescaled by the ratio of the areas occupied by the normal and superconducting layers. The relaxation time per single vortex is [14]

$$\tau_{np} = 6\left(\frac{k_{Fp}}{k_{Fn}}\right)^4 \frac{m_n \mu_{pn}^*}{\hbar m_p^* T \sigma_{np}} \exp\left(0.02\frac{\Delta_n^2}{\epsilon_{Fn} T}\right), \tag{11}$$

where k_{Fp} and k_{Fn} are the Fermi-wave-numbers of protons and neutrons, $\mu_{pn}^* = m_p^* m_n^*/(m_p^* + m_n^*)$ is the reduced effective mass, with m_n^* begin the neutron effective mass, σ is the total in-medium neutron-proton scattering cross-section, Δ_n is the gap in the neutron quasiparticle spectrum, ϵ_{Fn} is the neutron Fermi-energy. [Eq. (11) differs from the analogous expression in Ref. [14] by the factor $4m_n/\hbar P$; here P is the pulsar period, m_n—free-space neutron mass].

In the relaxation time-approximation, the force exerted by a normal proton on a single vortex is given by a phase-space integral

TABLE II. The columns 3–5 list the relaxation time for proton scattering off the neutron quasiparticles in the neutron vortex cores (3), the vortex friction coefficient (4), and the drag-to-lift ratio (5) for the temperature $T = 10^8$ K. The columns 6–9 list the same parameters for $T = 10^7$ K.

ρ_{14} g/cm^3	k_{Fp} fm^{-1}	τ_{np} s	η_{10} g/cm/s	ζ_{-2}	τ_{np} s	η_{10} g/cm/s	ζ_{-2}
		$T_8 = 1$			$T_8 = 0.1$		
0.40	0.89	71.48	10.48	7.95	285.9	2.62	19.87
0.60	1.08	428.9	3.68	1.86	1072.2	1.47	7.45
0.80	1.26	786.3	3.76	1.42	1572.6	1.88	7.13

$$\boldsymbol{f} = \frac{1}{\tau_{np}} \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3} f_F(\boldsymbol{p}, \boldsymbol{u}) = \eta \boldsymbol{u}, \tag{12}$$

over the proton Fermi-distribution function, $f_F(\boldsymbol{p}, \boldsymbol{u}) = [\exp(\epsilon_p - \epsilon_{Fp} + \boldsymbol{p} \cdot \boldsymbol{u})/T + 1]^{-1}$, where the quasiparticle energy is shifted due to the motion with a velocity $\boldsymbol{u}$. Here ϵ_p is the dispersion relation of normal protons, ϵ_{Fp} is their Fermi-energy. The integral (12) is straightforward in the $T = 0$ limit and we obtain

$$\eta = \frac{\hbar k_{Fp} n_p}{c \tau_{np}}, \tag{13}$$

where n_p is the proton number density. The result for the friction coefficient η and the corresponding drag-to-lift ratio for several densities are listed in Table II for the case where the proton fluid is nonsuperconducting. For a given model of the type-I superconducting structure, the friction coefficient η must be rescaled by a factor $(d_N/d_S)^2$.

IV. IMPLICATIONS FOR PRECESSION AND CONCLUSIONS

The SWC no-go theorem requires the condition $(I_S/I_N)\zeta < 1$ to be fulfilled for precession to occur; otherwise the precession is damped. The magnitude of the ratio I_S/I_N depends on the superfluid-normal fluid friction within all superfluid regions of a neutron star and is difficult to access. Glitches and post-glitch relaxation provide a model independent lower bound on $I_S/I_N \geq 0.1$. An upper bound is difficult to place, since the deep interiors of neutrons stars, if superfluid, could be decoupled from the observable parts of the star on evolutionary time scales without any effect on short time-scale physics [20] (but one needs $\zeta \to 0$, rather than $\zeta \to \infty$, to prevent the damping of the precession). However, it is rather unlikely that this ratio exceeds unity by many orders of magnitude. For the first dissipation channel studied $\zeta \sim 10^{-4} - 10^{-3}$ (Table I) and this clearly suggests an undamped precession. The second chanel is more effective, $\zeta \sim 10^{-2}$ (Table II), but these numbers must be reduce by a factor $(d_S/d_N)^2 \simeq 100$. On account of the lower bound on the ratio of the moments of inertia, one can conclude that the precession is undamped for both dissipation mechanism.

We have provided a first discussion of the dynamics of the type-I superconducting domains in neutron star interiors. Although the details of the coupling of the superconducting and normal components of the star depend on the form of the superconducting-normal structures that nucleate and, in particular, whether the nonsuperconducting domains are dynamically coupled to the rotational vortices or not, in all cases we find friction coefficients that imply undamped precession.

The results above by no means suggest that type-I superconductivity is the only resolution to the precession puzzle and alternatives should be searched for. An alternative to free, Eulerian precession is the forced precession due to time-dependent periodic torques [5]. Other periodic motions, for example, Tkachenko oscillations of the vortex lattice could generate the observed timing features [21–23]. While the eigen-frequencies of the Tkachenko modes are of correct order of magnitude and could explain long-term periodicities it remains to be studied whether these modes will be undamped by the mutual friction between the superfluid and the normal fluid.

The fact that statistically insignificant number of pulsars show long-term variabilities, indicates that a subtle tuning is needed for the underlying mechanisms to work. On the other hand, the Eulerian precession, if undamped, should be a common place in the pulsar population, since neutron stars frequently undergo nonaxisymmetric perturbations such as glitches and quakes.

ACKNOWLEDGMENTS

I am grateful to A. R. Zhitnitsky for discussions and Ira Wasserman for reading the draft and comments. I would like to thank the Institute for Nuclear Theory at the University of Washington for its hospitality and the Department of Energy for the partial support of the stay at the UW.

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