

**Factorization fits to charmless strangeless  $B$  decays**

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We present fits to charmless strangeless hadronic  $B$  decay data for mean branching ratios and  $CP$ -violating asymmetries using the QCD factorization model of Beneke *et al.* Apart from one  $CP$ -violating parameter, the model gives a very good representation of 26 measured data. We find the CKM angle  $\alpha = (93.5 \pm 8.4 - 1.3)^\circ$  and to be quite stable to plausible “charming penguin” corrections.

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**I. INTRODUCTION**

A wealth of experimental data on hadronic charmless  $B$  decays has become available from the BaBar and Belle experiments. Many new branching ratios and  $CP$ -violating parameters have been measured within more precise error limits. These studies of the numerous  $B$  decay channels are designed to test the Cabbibo-Kobayashi-Maskawa (CKM) explanation of  $CP$  violation in the standard model.

We have made previous attempts [1,2] to understand charmless  $B$  meson decay data in terms of the QCD factorization model of Beneke *et al.* (BBNS) [3,4]. In fitting the data we found evidence for *charming penguinlike contributions* [5] to the decay amplitudes in addition to the BBNS amplitudes. The charming penguin contribution to the strangeless modes is suppressed relative to the strange modes by one power of the Wolfenstein parameter  $\lambda \approx 0.22$ . In [1] we attempted a simultaneous fit to both strange and nonstrange channels and, although we obtained a satisfactory fit to measured branching ratios and some  $CP$ -violating asymmetries, the predicted CKM angles deviated significantly from other analyses. The strange channels in isolation provide the best data for examining the phenomenology of charming penguins [6], with this information their influence on the strangeless modes should then be investigated.

We report here the results of our application of a BBNS analysis to charmless strangeless  $B$  decays. We use the method and notation of [1] and attempt to fit the data on the mean branching ratios and  $CP$ -violating asymmetries of the decays  $B \rightarrow \pi\pi, \pi\rho, \pi\omega$  and  $\rho\rho$ , together with  $\sin(2\beta)$ , 26 independent measurements in all. We include many more  $CP$  asymmetry measurements in our data set and also three new  $\rho\rho$  channels which exhibit the expected helicity-zero dominance. The paper is organized as follows. In Sec. II we briefly discuss the structure of the decay amplitude within QCD factorization. The weak annihila-

tion contributions are summarized in Sec. III and the method and results of our best fit are presented in Sec. IV. Sec. V contains our discussion and conclusions.

**II. DECAY AMPLITUDE IN QCD FACTORIZATION**

In QCD factorization, the matrix elements of the operators in the effective Hamiltonian  $\mathcal{H}_{\text{eff}}$  are separated into short distance contributions at scale  $O(1/m_b)$  that are perturbatively calculable and long distance contributions  $O(1/\Lambda_{\text{QCD}})$  that are parametrized. The amplitude for  $B$  decay into two light hadrons (mesons)  $M_{1,2}$  has the form, neglecting weak annihilation processes, [1–3]

$$\langle M_1 M_2 | \mathcal{H}_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \left\{ \left[ \sum_{i=1,2} \lambda_u a_i^u + \sum_{p=u,c} \sum_{i=3,\dots,6,9} \lambda_p a_i^p \right] \times \left[ T_i(M_1, M_2) + T_i(M_2, M_1) \right] \right\} \quad (1)$$

where  $\lambda_p = V_{pd}^* V_{pb}$  is a product of CKM matrix elements. In Eq. (1) we have included current-current tree processes represented by the Wilson coefficients  $C_{1,2}$ , QCD penguin processes represented by  $C_{3,\dots,6}$  and the dominant electro-weak penguin process represented by  $C_9$ . The factorization matrix elements  $T_i(M_1, M_2)$  involve products of two-quark current matrix elements and, neglecting factors  $(m_{P,V}/m_B)^2$  and assuming zero helicity dominance for  $B \rightarrow V_1 V_2$  decays, have the form [2]

$$T_i(M_1, M_2) = c m_B^2 f_{M_1} F_{M_2} \quad (2)$$

where  $f_M$  are the well determined electroweak decay constants  $\{f_\pi, f_\omega, f_\rho\}$ ,  $F_M$  are the  $B$  transition form factors  $\{F_\pi, A_\omega, A_\rho\}$  and the constant  $c$  is a product of factors  $1, \pm 1/\sqrt{2}$ , etc., from the flavor composition of the  $B$  and  $M_{1,2}$  mesons.

The factorization coefficients  $a_i$  are calculated from the Wilson coefficients  $C_i(\mu)$  at a scale  $\mu$  of  $O(m_b)$  and have the form [3]

$$a_i(M_1, M_2) = a_{i,I}(M_2) + a_{i,II}(M_1, M_2) \quad (3)$$

where  $M_1$  is the recoil meson containing the spectator (anti) quark and  $M_2$  is the emitted meson. The complex quantities  $a_{i,I}$  describe the formation of  $M_2$ , including nonfactorizable corrections from hard gluon exchange or light quark loops in penguins. The hard gluon exchanges with the spectator quark are described by the (possibly) complex quantities  $a_{i,II}$ . In these correction terms the leading-twist light cone distribution functions for both pseudoscalar and vector mesons are expanded in the first few terms of a Gegenbauer expansion

$$\Phi_M(x, \mu) = 6x(1-x) \left[ 1 + \sum_n \alpha_n^M C_n^{3/2}(2x-1) \right]. \quad (4)$$

The light cone corrections  $\alpha_n^M$  are anticipated to be small but they are not well established. Consequently it is common to use the asymptotic form  $\Phi_M(x, \mu) = 6x(1-x)$ , valid for the mass scale  $\mu \rightarrow \infty$ , in applications of QCD factorization. All coefficients  $a_{i,I}$  except  $a_{6,I}$  are then the same for all decays. If the corrections  $\alpha_{1,2}^M$  are included, the coefficients most affected are  $a_{2,I}$  and  $a_{4,I}$  (see Table IV of [1]).

The contributions of the  $a_{i,II}$  coefficients to the decay amplitudes have the form

$$f_{M_2} F_{M_1} a_{i,II} = \frac{4\pi}{9} \epsilon_i C_i \alpha_s \beta_i \quad (5)$$

where  $i' = i - (-1)^i$ ,  $\epsilon_i = +1 (i = 1, \dots, 4, 9)$ ,  $\epsilon_5 = -1$ ,  $\epsilon_6 = 0$ , and

$$\beta_i = \frac{f_B f_{M_2} f_{M_1}}{m_B \lambda_B} [3(1 + \epsilon_i \alpha_1^{M_2} + \alpha_2^{M_2})(1 + \alpha_1^{M_1} + \alpha_2^{M_1}) + r_X^{M_1} (1 - \epsilon_i \alpha_1^{M_2} + \alpha_2^{M_2}) X_H^{M_1}]. \quad (6)$$

Here  $r_X^{M_1}$  are chiral factors,  $X_H^P = X_H$  and [4]

$$X_H^V = 3(\alpha_1^V + \alpha_2^V) X_H - (6 + 9\alpha_1^V + 11\alpha_2^V). \quad (7)$$

The nonperturbative complex parameter  $X_H$  is the contribution of a logarithmic end-point divergence in the integration over the twist-3 light cone distribution function

$$X_H = \int_0^1 \frac{dx}{1-x}, \quad (8)$$

and is usually parametrized as

$$X_H = \ln\left(\frac{m_B}{\Lambda_{\text{QCD}}}\right) + \rho_H e^{i\phi_H} \quad (9)$$

where  $\ln(m_B/\Lambda_{\text{QCD}}) = 3.03$ . The coefficients  $a_{i,II}$  are not universal even when light cone corrections are neglected and they contain the parameter  $X_H$  which is only loosely constrained by model estimations. These  $a_{i,II}$  contributions to the decay amplitudes are independent of the  $B$  transition form factors but do involve the poorly determined parameter  $f_B/\lambda_B$  where  $f_B$  is the  $B$  leptonic decay constant and  $\lambda_B \approx 0.35$  GeV is related to the  $B$  light cone distribution function. We note that substantial light cone corrections  $\alpha_{1,2}^M$  can significantly enhance the  $a_{i,II}$  coefficients.

The energies involved in the calculation of  $a_{i,II}$  imply that the appropriate scale is not that of the scale  $\mu$  used in calculating the  $a_{i,I}$  but  $\mu_h = \sqrt{\Lambda_h \mu}$  where [3]  $\Lambda_h = 0.5$  GeV. For the choice  $\mu = m_b/2$  this gives  $\alpha_s f_B / (m_B \lambda_B) = 0.0325$ .

### III. ANNIHILATION CONTRIBUTIONS

$B$  meson decay can also be initiated by  $b$  quark annihilation with its partner. Although the annihilation contributions to the decay amplitude are formally of  $O(\Lambda_{\text{QCD}}/m_b)$  and power suppressed, they violate QCD factorization because of end-point divergences. However these weak annihilation contributions can be included into the decay amplitudes by treating the end-point divergences as phenomenological complex parameters  $X_A$ .

The annihilation contribution to the decay amplitude is

$$\begin{aligned} \langle M_1 M_2 | \mathcal{H}_{\text{eff}}^{\text{ann}} | B \rangle &= \frac{G_F}{\sqrt{2}} B_{M_1 M_2} \{ \lambda_u [d_1 C_1 + d_2 C_2] A_1^i \\ &\quad - \lambda_t \{ d_3 [C_3 A_1^i + C_5 A_3^i] \\ &\quad + [d_4 C_4 + d_6 C_6] A_1^i \\ &\quad + d_5 [C_5 + N_c C_6] A_3^f \} \} \end{aligned} \quad (10)$$

where

$$B_{M_1 M_2} = \frac{C_F}{N_c^2} f_B f_{M_1} f_{M_2}, \quad (11)$$

$C_F = (N_c^2 - 1)/2N_c$  and  $N_c$  is the number of colors. The quantities  $A_{1,3}^{i,f}(M_1, M_2)$ , where the superscript  $i(f)$  denotes gluon emission from initial (final) state quarks, evaluated using the asymptotic form of Eq. (4) are given in [4] The coefficients  $d_i(M_1, M_2)$  are Clebsch-Gordan type factors and are tabulated in [1] for all the decays studied here apart from  $B \rightarrow \rho^+ \rho^-$  for which the required  $d$ -coefficients are  $(1, 0, -1, 2, -1, 2)$ .

### IV. FITTING METHOD

We attempt to fit the experimental data on the mean branching ratios and  $CP$ -violating asymmetries of the decays  $B \rightarrow \pi\pi, \pi\rho, \pi\omega$  and  $\rho\rho$ , together with  $\sin(2\beta)$ . This gives 26 independent measurements in all. There are many parameters in the equations of the BBNS theory that

are not precisely known, we choose the  $B$  semileptonic transition form factors  $F_\pi, A_\rho$  and  $A_\omega$ , the Wolfenstein CKM parameters  $\rho$  and  $\eta$  and the complex nonperturbative hard scattering parameter  $X_H$  and annihilation parameter  $X_A$  as our nine fitting parameters.

We assign to each of the 26 independent data a number  $\alpha$  and construct a  $\chi^2$  function of the nine fitting parameters

$$\chi^2 = \sum_{\alpha=1}^{26} [((\text{Theory})_\alpha - (\text{mean data value})_\alpha) / \sigma_\alpha]^2. \quad (12)$$

$\sigma_\alpha$  is an experimental error formed by amalgamating statistical and systematic errors. We then use a minimization procedure based upon the program MRQMIN from [7] to try to obtain acceptable fits of the theory to experiment. An acceptable fit has a low value of  $\chi^2$  at the minimum with parameter values that lie within known acceptable limits. Other parameters in the analysis such as the correction factors  $\alpha_{1,2}^M$  to the light cone distribution functions of the participating mesons are poorly known and not easy to incorporate as variables in the minimization procedure. In this paper we use three sets of values for  $\alpha_{1,2}^M$ , the zero set and two sets suggested by theory (those of [4,8]) to examine the sensitivity of the results to the choice of these correction factors. We also examine how the results depend on our choice of the chiral factor  $r_\chi^\pi$  and the factor  $\xi_B \equiv \alpha_s f_B / (m_B \lambda_B)$  which enters into the coefficients  $a_{i,H}$  representing the spectator quark interactions.

Table I shows the best fit values for the parameters, together with the statistical error from the  $\chi^2$  and a systematic error from the different light cone correction factors, etc. This fit has a minimum  $\chi^2$  of 9.2 and is for the light cone distribution functions of Lü and Yang [8], the chiral factors  $r_\chi^\pi = 1.0$  and  $r_\chi^\rho = 0.1$ , and  $\xi_B = 0.0488$ . An unexpected feature of this fit is the large value of the spectator quark interaction term  $\rho_H$  which was not expected to be greater than 3.0. It can be seen however that its statistical error is large, the  $\chi^2$  function has a very shallow worm hole through parameter space. Constraining  $\rho_H$  to be 3.0 gives a minimum  $\chi^2$  of 12.5 and a quite acceptable fit. The parameter changes this induces, the changes due to the different light cone distribution functions, and those from taking  $r_\chi^\pi = 0.9$  or  $\xi_B = 0.0325$ , are summarized in the column of systematic errors. These errors, like the statistical errors, are of course highly correlated but they indicate the sensitivity of the parameters to different choices. Our fit also implies that  $V_{ub} = (3.89 \pm 0.24) \times 10^{-3}$ , to be compared with  $(3.7 \pm 0.8) \times 10^{-3}$ .

The predicted mean branching ratios and  $CP$  asymmetries for the parameter values of Table I are shown in Table II, together with the individual contributions to the total minimum  $\chi^2$  of 9.2. Also shown are the estimated theoretical errors, the statistical errors are the  $1\sigma$  standard deviations from the  $9 \times 9$  error matrix of the  $\chi^2$  minimization and the systematic errors arise from changes in the light cone distribution functions, etc.

If significant contributions, such as charming penguins, are needed to understand the charmless strange  $B$  decays then it is to be expected that similar but smaller contributions should be added to the BBNS analysis. In the spirit of [1] we have investigated the influence of charming penguins, as will be discussed in the next section.

## V. DISCUSSION AND CONCLUSIONS

Apart from the value of  $\rho_H$ , which has been discussed above, the fitting parameters of Table I are within their expected ranges. In particular, the CKM parameters  $V_{ub}$  and the angles  $\alpha_{\text{CKM}}$  and  $\beta_{\text{CKM}}$  have very acceptable values within quite tight bounds. All the experimental data, apart from  $C_{\pi^\mp \rho^\pm}$  are well fitted by the analysis. The largest individual  $\chi^2$  of 3.76 is for this  $CP$ -violating parameter and it can be seen that our analysis gives a value much smaller than the measured value, which is from the BaBar experiment. The theoretical result is in much better accord with the Belle measurement  $C_{\pi^\mp \rho^\pm} = 0.25 \pm 0.17_{-0.06}^{+0.02}$  [9] (talk by M. Giorgi). A remeasurement of this parameter would be of interest.

The angles  $\alpha_{\text{CKM}}$  and  $\beta_{\text{CKM}}$  are the most important parameters to come out of the analysis. It is of interest to compare our value of  $\alpha_{\text{CKM}}$  with those from isospin analyses and the Grossman-Quinn bound. These analyses are independent of the details of the QCD penguin contributions. The small  $\rho^0 \rho^0$  branching fraction makes the  $\rho\rho$  analysis the most precise and yields  $\alpha_{\text{CKM}} = 96^\circ \pm 10^\circ \pm 4^\circ \pm 11^\circ$  [9] (talk by M. Giorgi) where the last error is from the Grossman-Quinn bound. However our neglect, in particular, of charming penguin contributions to the decay amplitudes will bias our estimates. We have only made a preliminary assessment of the influence of charming penguins, our conclusions so far are that the small branching fractions like  $\text{Br}(\pi^0 \pi^0)$  are somewhat sensitive but the  $CP$ -violating data can be more affected, especially those like  $C_{\pi^\mp \rho^\pm}$  which our analysis predicts to be very small. So far we have not found the angles  $\alpha_{\text{CKM}}$  and  $\beta_{\text{CKM}}$  to be

TABLE I. Best fit values and errors for the  $B$  transition form factors  $F_\pi, A_\rho$  and  $A_\omega$ , the complex hard scattering  $X_H = 3.03 + \rho_H \exp(i\phi_H)$  and annihilation  $X_A = x_A + iy_A$  parameters, and the CKM angles  $\alpha_{\text{CKM}}$  and  $\beta_{\text{CKM}}$ .

| Parameter             | Mean Value   | Statistical Error | Systematic Error         |
|-----------------------|--------------|-------------------|--------------------------|
| $F_\pi$               | 0.218        | $\pm 0.022$       | $+0.015, -0.003$         |
| $A_\rho$              | 0.317        | $\pm 0.027$       | $-0.021$                 |
| $A_\omega$            | 0.372        | $\pm 0.050$       | $-0.03$                  |
| $\rho_H$              | 8.73         | $\pm 2.82$        | $+3.1, -5.26$            |
| $\phi_H$              | $-2.05$      | $\pm 0.15$        | $+0.49, -0.16$           |
| $x_A$                 | $-1.01$      | $\pm 0.50$        | $+0.02, -0.34$           |
| $y_A$                 | 2.89         | $\pm 0.85$        | $+0.73, -0.39$           |
| $\alpha_{\text{CKM}}$ | $93.5^\circ$ | $\pm 8.4^\circ$   | $-1.3^\circ$             |
| $\beta_{\text{CKM}}$  | $24.2^\circ$ | $\pm 2.3^\circ$   | $+0.2^\circ, -0.4^\circ$ |

TABLE II. Measured branching ratios and  $CP$  asymmetries (Data), experimental error ( $\sigma$ ), best fit theoretical values (Theory), estimated theoretical errors (Statistical) and (Systematic), contribution  $\chi^2_\alpha$  to the total minimum  $\chi^2$  of 9.2, and the reference (Ref.) for the experimental data. All branching ratios are in units of  $10^{-6}$ .

|                               | Data  | $\sigma$ | Theory | Statistical Error | Systematic Error | $\chi^2_\alpha$ | Ref. |
|-------------------------------|-------|----------|--------|-------------------|------------------|-----------------|------|
| $\text{Br}(\pi^+ \pi^-)$      | 4.6   | 0.4      | 4.73   | $\pm 0.45$        | +0.22, -0.07     | 0.11            | [8]  |
| $\text{Br}(\pi^0 \pi^0)$      | 1.2   | 0.4      | 1.15   | $\pm 0.33$        | +0.0, -0.38      | 0.02            | [9]  |
| $\text{Br}(\omega \pi^0)$     | <1.0  |          | 0.48   | $\pm 0.30$        | +0.02, -0.23     | 0.27            | [8]  |
| $\text{Br}(\pi^\pm \rho^\mp)$ | 24.0  | 2.5      | 23.6   | $\pm 1.54$        | +0.20, -0.06     | 0.03            | [8]  |
| $\text{Br}(\pi^0 \rho^0)$     | 5.1   | 2.4      | 2.90   | $\pm 0.44$        | +0.0, -1.08      | 0.84            | [8]  |
| $\text{Br}(\rho^+ \rho^-)$    | 30.0  | 6.0      | 31.9   | $\pm 4.02$        | +0.0, -1.90      | 0.10            | [9]  |
| $\text{Br}(\rho^0 \rho^0)$    | <1    |          | 0.66   | $\pm 0.23$        | +0.0, -0.33      | 0.12            | [9]  |
| $\text{Br}(\pi^\pm \pi^0)$    | 5.2   | 0.8      | 4.51   | $\pm 0.65$        | +0.71, -0.24     | 0.74            | [8]  |
| $\text{Br}(\omega \pi^\pm)$   | 5.9   | 0.8      | 5.90   | $\pm 1.00$        | $\pm 0.0$        | 0.00            | [8]  |
| $\text{Br}(\pi^\pm \rho^0)$   | 9.2   | 1.2      | 10.1   | $\pm 1.03$        | +0.40, -0.0      | 0.61            | [10] |
| $\text{Br}(\pi^0 \rho^\pm)$   | 12.0  | 1.9      | 10.8   | $\pm 0.80$        | +0.30, -1.70     | 0.42            | [8]  |
| $\text{Br}(\rho^0 \rho^\pm)$  | 26.4  | 6.4      | 23.0   | $\pm 2.19$        | +0.0, -3.10      | 0.28            | [11] |
| $C_{\pi^+ \pi^-}$             | -0.09 | 0.19     | -0.13  | $\pm 0.03$        | $\pm 0.01$       | 0.04            | [9]  |
| $S_{\pi^+ \pi^-}$             | -0.30 | 0.21     | -0.21  | $\pm 0.14$        | +0.0, -0.06      | 0.17            | [9]  |
| $A_{\pi^\pm \pi^0}$           | 0.05  | 0.15     | -0.02  | $\pm 0.0$         | +0.01, -0.0      | 0.23            | [9]  |
| $A_{\pi^\pm \rho^0}$          | -0.17 | 0.13     | -0.15  | $\pm 0.08$        | +0.0, -0.03      | 0.03            | [10] |
| $A_{\pi^0 \rho^\pm}$          | 0.23  | 0.22     | 0.15   | $\pm 0.06$        | +0.03, -0.0      | 0.13            | [10] |
| $C_{\pi^\mp \rho^\pm}$        | 0.34  | 0.16     | 0.03   | $\pm 0.01$        | +0.0, -0.01      | 3.76            | [9]  |
| $\Delta C_{\pi^\mp \rho^\pm}$ | 0.15  | 0.14     | 0.18   | $\pm 0.08$        | +0.0, -0.01      | 0.25            | [9]  |
| $S_{\pi^\mp \rho^\pm}$        | -0.10 | 0.18     | -0.15  | $\pm 0.15$        | $\pm 0.01$       | 0.08            | [9]  |
| $\Delta S_{\pi^\mp \rho^\pm}$ | 0.22  | 0.18     | 0.15   | $\pm 0.05$        | +0.02, -0.11     | 0.15            | [9]  |
| $A_{\pi^\mp \rho^\pm}$        | -0.09 | 0.06     | -0.10  | $\pm 0.03$        | $\pm 0.0$        | 0.04            | [9]  |
| $C_{\rho^\mp \rho^\pm}$       | -0.23 | 0.38     | 0.03   | $\pm 0.09$        | $\pm 0.0$        | 0.47            | [9]  |
| $S_{\rho^\mp \rho^\pm}$       | -0.19 | 0.44     | -0.27  | $\pm 0.02$        | +0.04, -0.0      | 0.03            | [9]  |
| $A_{\rho^\mp \rho^0}$         | -0.09 | 0.16     | -0.01  | $\pm 0.0$         | $\pm 0.0$        | 0.27            | [11] |
| $\sin(2\beta)$                | 0.734 | 0.054    | 0.747  | $\pm 0.046$       | $\pm 0.005$      | 0.06            | [8]  |

particularly sensitive to charming penguins but more work is required to get reliable estimates of the systematic errors.

In conclusion, we find that the BBNS analysis so far does well in fitting the data. We see no evidence in charm-

less strangeless decays for any physics beyond the Standard Model and the analysis holds the promise of giving tight constraints on the values of the CKM parameters.

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