Higgs bosons in the two-doublet model with *CP* violation

E. Akhmetzyanova,[1] M. Dolgopolov,[1] and M. Dubinin[2]

[1]*Physics Department, Samara State University, 443011 Samara, Russia*

[2]*Institute of Nuclear Physics, Moscow State University, 119899 Moscow, Russia*

(Received 6 October 2004; published 20 April 2005)

We consider the effective two-Higgs-doublet potential with complex parameters, when the *CP* invariance is broken both explicitly and spontaneously. The diagonal mass term in the local minimum of the potential is constructed for the physical basis of Higgs fields, keeping explicitly the limiting case of *CP* conservation, if the parameters are taken real. For the special case of the two-doublet Higgs sector of the minimal supersymmetric model, when *CP* invariance is violated by the Higgs bosons interaction with scalar quarks of the third generation, we calculate by means of the effective potential method the Higgs boson masses and evaluate the two-fermion Higgs boson decay widths and the widths of rare one-loop-mediated decays $H \to \gamma\gamma$, $H \to gg$.

DOI: 10.1103/PhysRevD.71.075008 PACS numbers: 14.80.Cp, 12.60.Jv

I. INTRODUCTION

It is well known that the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix originates from the standard model (SM) Lagrangian terms, describing the Higgs boson interaction with quarks (the Yukawa terms)

$$L = -g^u_{ij}\bar{\psi}^{i\prime}_L H u^{j\prime}_R - g^d_{ij}\bar{\psi}^{i\prime}_L \tilde{H} d^{j\prime}_R + \text{H.c.}, \qquad (1)$$

where $\bar{\psi}^{1\prime}_L = (\bar{u}', \bar{d}')_L$, $\bar{\psi}^{2\prime}_L = (\bar{c}', \bar{s}')_L$, $\bar{\psi}^{3\prime}_L = (\bar{t}', \bar{b}')_L$, $u^{1\prime}_R = u'_R$, $u^{2\prime}_R = c'_R$, $u^{3\prime}_R = t'_R$, $d^{1\prime}_R = d'_R$, $d^{2\prime}_R = s'_R$, $d^{3\prime}_R = b'_R$, and H denotes the scalar complex field doublet, $\tilde{H}_k = \epsilon_{kl}H^*_l$ and g^u_{ij}, g^d_{ij} are the 3×3 matrices with matrix elements that are generally speaking complex and defined with an uncertainty coming from the phases of *CP* transformation[1] for the quark spinor fields and the Higgs boson scalar field. In order to diagonalize the quark mass term after spontaneous symmetry breaking $H \to (0, v/\sqrt{2})$, the unitary transformations of the $u^{i\prime}$ and $d^{i\prime}$ quark fields $u^i_{L,R} = U_{L,R}u^{i\prime}_{L,R}$, $d^i_{L,R} = D_{L,R}d^{i\prime}_{L,R}$ are needed. After the diagonalization of the quark mass term the unitary matrices U_L and D_L do not appear neither in the Yukawa Lagrangian terms (1) nor in the quark neutral current interactions, but arise in the quark $u^{i\prime}$, $d^{i\prime}$ charged current interaction terms $g\bar{u}'_L\gamma_\mu d'_L W^\mu = g\bar{u}_L\gamma_\mu U_L D^\dagger_L d_L W^\mu$. The product $V_{\text{CKM}} = U_L D^\dagger_L$ defines the complex CKM matrix, which describes *CP* violation in the quark charged currents sector. In the framework of the SM the *CP* violation takes place since it is generally speaking not possible to get the mixing matrix with real matrix elements using *CP* transformations for six up- and down-quarks.

There are other sources of *CP* violation besides the CKM mechanism. It is possible to introduce explicitly *CP* noninvariant Hermitian Lagrangians [1] for the system of several scalar fields. For example, if we have three complex scalar fields φ_1, φ_2, φ_3

$$L = \lambda\varphi_1\varphi^*_2\varphi^*_3 + \lambda^*\varphi^*_1\varphi_2\varphi_3,$$
$$CPLP^\dagger C^\dagger = L^{CP} = \lambda e^{i\alpha}\varphi^*_1\varphi_2\varphi_3 + \lambda^* e^{-i\alpha}\varphi_1\varphi^*_2\varphi^*_3,$$

where λ is a complex parameter and α is the *CP* transformation phase, not essential in this case. It can be rotated away by the phase transformation of the fields, related to charge conservation. One can see that L and L^{CP} have different signs of the imaginary part of λ. In this simple example the difference in the sign does not lead to any observable consequences, because the phase of λ can be also rotated away by the $\text{U}(1)_Q$ transformation. However for the system with trilinear interactions of the four complex scalar fields it is generally speaking not possible to rotate away all phase factors. It is easy to show that the Lagrangian of such a system will be *CP* invariant only if the phases of the four parameters λ_i respect certain conditions, which ensure the possibility to remove them by U(1) rotations of the fields φ_i. From this point of view the models with extended Higgs sector, where *CP* invariance of the Higgs potential with complex parameters is explicitly broken, are of particular interest. The simplest example is represented by the two-doublet Higgs potential of the minimal supersymmetric standard model (MSSM), including (if the possibility of spontaneous *CP* violation [2] is not considered) ten parameters, four of which can be complex. In the framework of MSSM the dominant loop-mediated contributions from the third generation scalar quarks could lead to substantial violation of *CP* invariance of the two-doublet effective Higgs potential [3]. Various models with radiatively induced *CP* violation in the two-doublet Higgs sector have been studied [4,5].

[1]Let us remind one, for example, that from the definition of the P transformation $Pa^+_\sigma(\vec{p})P^\dagger = \eta_\sigma a^+_\sigma(-\vec{p})$, where the complex factor $|\eta_\sigma| = 1$ contains the P transformation phase, and $\sigma = 0$ or 1/2, it follows that $P\phi(x)P^\dagger = \eta^*{}_0\phi(x')$, $P\psi(x)P^\dagger = \eta^*{}_{1/2}\gamma_0\psi(x')$, where $x' = Px$.

In this paper we develop further on our approach to the Higgs boson phenomenology in the scenario with CP violation considered in [5]. In Sec. II, after brief introductory remarks, we calculate the effective λ_i parameters of the two-doublet MSSM Higgs potential at the $m_{\rm top}$ scale. In Sec. III we consider in detail the diagonalization of the mass term for the two-doublet Higgs potential with CP invariance broken both explicitly and spontaneously. In the Appendix some numerical results for the Higgs boson masses and the two-particle Higgs decay widths are presented. Our numerical results are compared with the output of other approaches.

II. THE EFFECTIVE TWO-DOUBLET HIGGS POTENTIAL WITH *CP* VIOLATION

In the general two-Higgs-doublet model (THDM) two SU(2) doublets of complex scalar fields are introduced:

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix}, \qquad (2)$$

$$\Phi_2 = e^{i\xi}\begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix} = e^{i\xi}\begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 e^{i\zeta} + \eta_2 + i\chi_2) \end{pmatrix}. \qquad (3)$$

Their vacuum expectation values (VEV's)

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{e^{i\xi}}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\zeta} \end{pmatrix} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}, \qquad (4)$$

where v_1 and v_2 are real. The phases ζ, relative phase of the VEV's, and ξ, relative phase of the SU(2) doublets, are introduced to consider the general case, their sum θ will be used for convenience of notations (Sec. III C). For the special case $\xi = 0$ the analysis of the Yukawa sector with the two-fermion generations can be found in [6], where a somewhat simpler form without the dimension 2 terms $\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1$ and real μ_{12}^2, $\lambda_{5,6,7}$ parameters of the THDM potential with spontaneous violation of CP invariance ($\zeta = \theta \neq 0$) has been considered in the context of superweak (i.e. flavor-changing Higgs boson exchange mediated) CP violation in meson decays.

The most general renormalizable Hermitian SU(2) $\times$ U(1) invariant Lagrangian for the system of scalar fields (2) and (3) can be written as

$$\begin{aligned}\mathcal{L}_H &= (\mathcal{D}_\nu\Phi_1)^\dagger\mathcal{D}^\nu\Phi_1 + (\mathcal{D}_\nu\Phi_2)^\dagger\mathcal{D}^\nu\Phi_2 \\ &\quad + \kappa(\mathcal{D}_\nu\Phi_1)^\dagger\mathcal{D}^\nu\Phi_2 + \overset{*}{\kappa}(\mathcal{D}_\nu\Phi_2)^\dagger\mathcal{D}^\nu\Phi_1 \\ &\quad - U(\Phi_1, \Phi_2), \end{aligned} \qquad (5)$$

where

$$\begin{aligned} U(\Phi_1,\Phi_2) &= -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \overset{*}{\mu}{}_{12}^2(\Phi_2^\dagger\Phi_1) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 \\ &\quad + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\lambda^*_5}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) \\ &\quad + \overset{*}{\lambda}_6(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_7(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1). \end{aligned} \qquad (6)$$

The parameters μ_{12}^2, λ_5, λ_6, and λ_7 are complex. Complex parameter κ could be introduced to describe an interesting possibility of a mixing in the kinetic term [7]. However, strong restrictions on the real part of κ are imposed by precise experimental data on the gauge boson masses $m_{W,Z}$. Moreover, mixing in the kinetic term does not allow one to construct the diagonal 4×4 matrix of the Higgs boson kinetic terms consistently with the diagonal matrix for their mass terms.[2] In the following we consider the case $\kappa = 0$.

The special case of the two-Higgs-doublet potential is the potential of the MSSM Higgs sector. At the energy scale $M_{\rm SUSY}$ (i.e. at the energy of the order of the sparticle masses) the tree-level parameters $\lambda_{1,\ldots,7}$ are real and can be expressed through the SU(2) $\times$ U(1) gauge couplings g_1 and g_2 [9]

$$\lambda_1(M_{\rm SUSY}) = \lambda_2(M_{\rm SUSY}) = \frac{1}{8}(g_2^2(M_{\rm SUSY}) + g_1^2(M_{\rm SUSY})), \qquad (7)$$

$$\lambda_3(M_{\rm SUSY}) = \frac{1}{4}(g_2^2(M_{\rm SUSY}) - g_1^2(M_{\rm SUSY})),$$

$$\lambda_4(M_{\rm SUSY}) = -\frac{1}{2}g_2^2(M_{\rm SUSY}),$$

$$\lambda_5(M_{\rm SUSY}) = \lambda_6(M_{\rm SUSY}) = \lambda_7(M_{\rm SUSY}) = 0.$$

At the scale $M_{\rm SUSY}$ the potential is CP invariant. However, the potential parameters of any model depend, generally speaking, on the energy scale where they are fixed or measured. The dependence is described by the renormalization group equations (RGE). The conditions (7) are the

[2]We analyzed these conditions written in the form of ten linear equations, having the solution practically only in the case $\kappa = 0$. The mixed term is not obligatory to ensure the renormalizability. It is shown below that the contributions of self-energy diagrams absorbed by the Higgs boson wave-function renormalization to the effective parameters $\lambda_{5,6,7}$ are zero; see also [8].

boundary conditions for the RGE. At the energies smaller than $M_{\rm SUSY}$ they are affected by large quantum corrections [10] where the main contribution is coming from the Higgs bosons—third generation quarks and scalar quarks interaction (the interactions with the first and second generations are suppressed). The potential of the Higgs bosons—scalar quarks interaction can be written in the form [8]

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\tilde{Q}}, \tag{8}$$

where

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\tilde{Q}}^2 (\tilde{Q}^\dagger \tilde{Q}) + M_{\tilde{U}}^2 \tilde{U}^* \tilde{U} + M_{\tilde{D}}^2 \tilde{D}^* \tilde{D}, \tag{9}$$

$$\mathcal{V}_\Gamma = \Gamma_i^D (\Phi_i^\dagger \tilde{Q}) \tilde{D} + \Gamma_i^U (i \Phi_i^T \sigma_2 \tilde{Q}) \tilde{U} + \overset{*D}{\Gamma_i} (\tilde{Q}^\dagger \Phi_i) \tilde{D}^* - \overset{*U}{\Gamma_i} (i \tilde{Q}^\dagger \sigma_2 \Phi_i^*) \tilde{U}^*, \tag{10}$$

$$\begin{aligned}\mathcal{V}_\Lambda = {} & \Lambda_{ik}^{jl} (\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l) + (\Phi_i^\dagger \Phi_j)[\Lambda_{ij}^Q (\tilde{Q}^\dagger \tilde{Q}) + \Lambda_{ij}^U \tilde{U}^* \tilde{U} + \Lambda_{ij}^D \tilde{D}^* \tilde{D}] + \overline{\Lambda}_{ij}^Q (\Phi_i^\dagger \tilde{Q})(\tilde{Q}^\dagger \Phi_j) \\ & + \frac{1}{2}[\Lambda \epsilon_{ij} (i \Phi_i^T \sigma_2 \Phi_j) \tilde{D}^* \tilde{U} + \text{H.c.}], \\ & i, j, k, l = 1, 2, \end{aligned} \tag{11}$$

$\mathcal{V}_{\tilde{Q}}$ denotes the four scalar quarks interaction terms, Pauli matrix $\sigma_2 \equiv \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$. The Yukawa couplings for the third generation of scalar quarks are defined in a standard way $h_t = \sqrt{2} m_t / v \sin\beta$, $h_b = \sqrt{2} m_b / v \cos\beta$. Following [11]:[3]

$$\Gamma_{\{1;2\}}^U = h_U \{-\mu^*; A_U\}, \qquad \Gamma_{\{1;2\}}^D = h_D \{A_D; -\mu^*\}, \tag{12}$$

they are complex in the case under consideration. One can observe CP violating terms of the structure similar to (1) in the sector of Higgs—scalar quark interactions, so complex mixing matrices are expected to appear there. The trilinear parameters A_t, A_b and the Higgs mass parameter μ should be taken complex, the imaginary parts of the mixing matrix elements could be large.

In the framework of the effective field theory approach [8] the MSSM potential (8) which explicitly describes sparticle interactions at the energy scale above $M_{\rm SUSY}$ is matched to an effective standard model-like Lagrangian at the energy scale below $M_{\rm SUSY}$, where the sparticles decouple. So the MSSM effective Higgs potential at the energy scale $m_{\rm top}$, much smaller than $M_{\rm SUSY}$, is represented by the general two-Higgs-doublet model potential (6), the parameters of the latter are expressed by means of the Higgs bosons—scalar quarks interaction parameters (12) and the scalar quark masses, playing the role of ultraviolet Pauli-Villars regulators. The RGE boundary conditions (7) modified by the interactions of the third generation squarks with the Higgs bosons (these modifications are sometimes called the "threshold" effects, since the stops decouple at the $M_{\rm SUSY}$ scale) are imposed at the energy scale $M_{\rm SUSY}$. They affect the evolution of λ_i parameters, the Yukawa couplings $h_{t,b}$, and the gauge couplings $g_{1,2}$. We calculated radiative corrections to the boundary conditions (7) for λ_i parameters at the scale $m_{\rm top}$ using the effective potential method [12]. The squark mass matrices $(\mathcal{M}_X^2)_{ab} \equiv \partial^2 \mathcal{V}_X / \partial \tilde{Q}_a \partial \tilde{Q}_b^*$ defined by (8) were calculated and then substituted to the one-loop effective potential

$$\mathcal{V} = \mathcal{V}^0 + \frac{N_C}{32\pi^2} \operatorname{tr} \mathcal{M}^4 \left[\ln\left(\frac{\mathcal{M}^2}{\sigma^2} \right) - \frac{3}{2} \right],$$

decomposed in the inverse powers of $M_{\rm SUSY}$. Taking into account the one-loop wave-function renormalization terms (i.e. terms introduced to absorb the contributions of self-energy diagrams to the Higgs bosons kinetic term, which are beyond the calculation by means of the effective potential method), the effective parameters can be evaluated as follows:

$$\begin{aligned}\lambda_1 = {} & \frac{g_2^2 + g_1^2}{8} + \frac{3}{32\pi^2} \left[h_b^4 \frac{|A_b|^2}{M_{\rm SUSY}^2} \left(2 - \frac{|A_b|^2}{6 M_{\rm SUSY}^2} \right) - h_t^4 \frac{|\mu|^4}{6 M_{\rm SUSY}^4} + 2 h_b^4 l + \frac{g_2^2 + g_1^2}{4 M_{\rm SUSY}^2} (h_t^2 |\mu|^2 - h_b^2 |A_b|^2) \right] + \Delta\lambda_1^{\rm field} \\ & + \frac{1}{768\pi^2} (11 g_1^4 + 9 g_2^4 - 36 (g_1^2 + g_2^2) h_b^2) l, \end{aligned} \tag{13}$$

$$\lambda_2 = \lambda_1 (t \leftrightarrow b),$$

$$\begin{aligned}\lambda_3 = {} & \frac{g_2^2 - g_1^2}{4} \left[1 - \frac{3}{16\pi^2} (h_t^2 + h_b^2) l \right] + \frac{3}{8\pi^2} h_t^2 h_b^2 \left[l + \frac{1}{2} X_{tb} \right] + \frac{3}{96\pi^2} \frac{|\mu|^2}{M_{\rm SUSY}^2} \left[h_t^4 \left(3 - \frac{|A_t|^2}{M_{\rm SUSY}^2} \right) + h_b^4 \left(3 - \frac{|A_b|^2}{M_{\rm SUSY}^2} \right) \right] \\ & + \frac{3 (g_2^2 - g_1^2) [h_b^2 (|\mu|^2 - |A_b|^2) + h_t^2 (|\mu|^2 - |A_t|^2)]}{128 \pi^2 M_{\rm SUSY}^2} + \Delta\lambda_3^{\rm field} + \frac{9 g_2^4 - 11 g_1^4}{384 \pi^2} l, \end{aligned} \tag{14}$$

[3]For the case of CP conservation, considered in [8], the trilinear parameters in (10) are real. Then $\Gamma_{\{1;2\}}^U \equiv h_U \{-\mu; A_U\}$, $\Gamma_{\{1;2\}}^D \equiv h_D \{A_D; -\mu\}$.

$$\lambda_4 = -\frac{g_2^2}{2}\left[1 - \frac{3}{16\pi^2}(h_t^2 + h_b^2)l\right] - \frac{3}{8\pi^2}h_t^2 h_b^2\left[l + \frac{1}{2}X_{tb}\right] + \frac{3}{96\pi^2}\frac{|\mu|^2}{M_{\rm SUSY}^2}\left[h_t^4\left(3 - \frac{|A_t|^2}{M_{\rm SUSY}^2}\right) + h_b^4\left(3 - \frac{|A_b|^2}{M_{\rm SUSY}^2}\right)\right] - \frac{3g_2^2[h_b^2(|\mu|^2 - |A_b|^2) + h_t^2(|\mu|^2 - |A_t|^2)]}{64\pi^2 M_{\rm SUSY}^2} + \Delta\lambda_4^{\rm field} - \frac{3g_2^4}{64\pi^2}l, \tag{15}$$

where

$$X_{tb} \equiv \frac{|A_t|^2 + |A_b|^2 + 2\,{\rm Re}(A_b^* A_t)}{2M_{\rm SUSY}^2} - \frac{|\mu|^2}{M_{\rm SUSY}^2} - \frac{|\,|\mu|^2 - A_b^* A_t|^2}{6M_{\rm SUSY}^4}. \tag{16}$$

The effective complex parameters $\lambda_{5,6,7}$

$$\lambda_5 = -\Delta\lambda_5 = -\frac{3}{96\pi^2}\left(h_t^4\left(\frac{\mu A_t}{M_{\rm SUSY}^2}\right)^2 + h_b^4\left(\frac{\mu A_b}{M_{\rm SUSY}^2}\right)^2\right), \tag{17}$$

$$\lambda_6 = -\Delta\lambda_6 = \frac{3}{96\pi^2}\left[h_t^4\frac{|\mu|^2\mu A_t}{M_{\rm SUSY}^4} - h_b^4\frac{\mu A_b}{M_{\rm SUSY}^2}\left(6 - \frac{|A_b|^2}{M_{\rm SUSY}^2}\right) + (h_b^2 A_b - h_t^2 A_t)\frac{3\mu}{M_{\rm SUSY}^2}\frac{g_2^2 + g_1^2}{4}\right], \tag{18}$$

$$\lambda_7 = -\Delta\lambda_7 = \frac{3}{96\pi^2}\left[h_b^4\frac{|\mu|^2\mu A_b}{M_{\rm SUSY}^4} - h_t^4\frac{\mu A_t}{M_{\rm SUSY}^2}\left(6 - \frac{|A_t|^2}{M_{\rm SUSY}^2}\right) + (h_t^2 A_t - h_b^2 A_b)\frac{3\mu}{M_{\rm SUSY}^2}\frac{g_2^2 + g_1^2}{4}\right]. \tag{19}$$

Some details of the calculation can be found in [13]. The one-loop wave-function renormalization terms in (13)–(15) are

$$\Delta\lambda_1^{\rm field} = \frac{1}{2}(g_1^2 + g_2^2)A'_{11}, \qquad \Delta\lambda_2^{\rm field} = \frac{1}{2}(g_1^2 + g_2^2)A'_{22}, \tag{20}$$

$$\Delta\lambda_3^{\rm field} = -\frac{1}{4}(g_1^2 - g_2^2)(A'_{11} + A'_{22}), \qquad \Delta\lambda_4^{\rm field} = -\frac{1}{2}g_2^2(A'_{11} + A'_{22}),$$

$$\Delta\lambda_5^{\rm field} = 0,$$

$$\Delta\lambda_6^{\rm field} = \frac{1}{8}(g_1^2 + g_2^2)(A'_{12} - A'_{21}) = 0, \qquad \Delta\lambda_7^{\rm field} = \frac{1}{8}(g_1^2 + g_2^2)(A'_{21} - A'_{12}) = 0.$$

They are similar to the case of *CP* conservation [8] containing the logarithmic contributions and imaginary parameters as a consequence of (12), and can be written as

$$A'_{ij} = -\frac{3}{96\pi^2 M_{\rm SUSY}^2}\left[h_t^2\begin{bmatrix} |\mu|^2 & -\mu^* A_t^* \\ -\mu A_t & |A_t|^2 \end{bmatrix} + h_b^2\begin{bmatrix} |A_b|^2 & -\mu^* A_b^* \\ -\mu A_b & |\mu|^2 \end{bmatrix}\right]\left(1 - \frac{1}{2}l\right). \tag{21}$$

Here and in the formulas given below $l \equiv \ln(M_{\rm SUSY}^2/\sigma^2)$, where $\sigma = m_{\rm top}$ is the renormalization scale. The one-loop wave-function renormalization does not yield a *CP* violating contribution to λ_i. For convenience we introduce the notation for the deviations of effective parameters λ_i from $\lambda_i^{\rm SUSY} = \lambda_i(M_{\rm SUSY})$ following [5]:

$$\lambda_{1,2} \equiv \lambda_{1,2}^{\rm SUSY} - \Delta\lambda_{1,2}/2, \qquad \lambda_{3,4} \equiv \lambda_{3,4}^{\rm SUSY} - \Delta\lambda_{3,4}, \qquad \lambda_{5,6,7} \equiv -\Delta\lambda_{5,6,7}, \tag{22}$$

where

$$\Delta\lambda_i \equiv \Delta\lambda_i^{\rm eff.pot.} - \Delta\lambda_i^{\rm field}, \qquad \Delta\lambda_i^{\{\rm eff.pot.;field\}} \equiv \Delta\lambda_i^{\rm log} + \Delta\lambda_i^{\rm finite}, \tag{23}$$

$$\Delta\lambda_{5,6,7}^{\rm log} = 0, \qquad \Delta\lambda_{5,6,7}^{\rm field} = 0. \tag{24}$$

At the end of this section we would like to make some general comments as well as some comments in connection with results obtained by other authors. Like in the existing effective field theory approach [8] we are using the standard scheme of leading logarithmic terms resummation by means of RGE, additionally taking into account in the boundary conditions at the scale $M_{\rm SUSY}$ the effects of Higgs bosons—the third generation of scalar quarks inter-

TABLE I. Numerical comparison of various corrections to the λ_i parameters at the scale $m_{\rm top}$. For convenience of the following Higgs boson masses comparison, the same parameter values as in the package CPsuperH [15] are chosen here: $m_Z = 91.19$ GeV, $m_b = 3$ GeV, $m_t = 175$ GeV, $m_W = 79.96$ GeV, $g_2 = 0.6517$, $g_1 = 0.3573$, $v = 245.4$ GeV, $G_F = 1.174 \times 10^{-5}$ GeV^{-2}, $\alpha_S(m_t) = 0.1072$, $\tan\beta = 5$, $M_{\rm SUSY} = 500$ GeV, $\sigma = m_t$, $m_{H^\pm} = 300$, $|A_t| = |A_b| = A = 1000$ GeV, $|\mu| = 2000$ GeV, and $\varphi \equiv \arg(\mu A_{t,b}) = 0$. The abbreviation "wfr" stands for the "wave-function renormalization."

i	1	2	3	4	5	6	7
Only $\mathcal{O}(h_t^4)$ terms [12]	0.907	−0.203	0.057	0.057	0.227	−0.453	0.057
$\Delta\lambda_i$	0.860	−0.182	0.054	0.072	0.227	−0.442	0.046
1-loop [3]	0.907	−0.191	0.064	0.043	0.227	−0.453	0.057
1-loop + 2-loop [3]	0.761	−0.152	0.052	0.032	0.135	−0.371	0.044
2-loop [3]	−0.146	0.039	−0.012	−0.011	−0.092	0.082	−0.013
1-loop (D + wfr)	−0.047	0.009	−0.010	0.028	0	0.011	−0.011
$\frac{\Delta\lambda(D)}{\Delta\lambda(\text{2-loop})}$	0.32	0.23	0.83	−2.55	0	0.13	0.85
1-loop + 2-loop + 1-loop (D + wfr)	0.715	−0.143	0.042	0.061	0.135	−0.360	0.033

action. The one-loop effective parameters (13)–(19) satisfy the boundary conditions defined by (7) and modified by the soft supersymmetry breaking potential terms (8) ("threshold effects"). The terms with the logarithmic factor l describe the parameters evolution from the energy scale $M_{\rm SUSY}$ down to the scale $\sigma = m_{\rm top}$. Finite power term threshold corrections to $\lambda_{1,\ldots,7}$ appear from the so-called F terms [the trilinear interaction terms in (10)] and D terms [contained in (11)]. The corrections to λ_5 come from the F terms only. Radiative corrections to the parameters $\lambda_{1,\ldots,7}$ of the effective two-Higgs-doublet potential have been considered earlier in [3] for the case of broken CP invariance and in [8,14] for the case of CP conservation. Phenomenological consequences of the two-doublet system are usually analyzed assuming for simplicity $A_t = A_b$ and introducing the universal phase $\arg(\mu A_{t,b})$, so that $\lambda_5 = |\lambda_5| \exp[i2\arg(\mu A)]$, $\lambda_6 = |\lambda_6| \exp[i\arg(\mu A)]$, $\lambda_7 = |\lambda_7| \exp[i\arg(\mu A)]$.

Only the leading D-term contributions were calculated in [3,14]. In our expressions for the effective parameters (13)–(19) the nonleading D-term contributions are represented by the power terms containing gauge couplings g_1^2, g_2^2. The one-loop contributions of the wave-function renormalization $\Delta\lambda^{\rm field}_{1,\ldots,4}$ are neglected in [3,14]. However, the QCD and weak corrections to Yukawa couplings up to two loops, not calculated in our case, have been included there. The expressions for $\lambda_{1,2,3,4}$ (13)–(19) do not contain imaginary parts up to the two-loop approximation and coincide with the results of [3,14] if we omit the contributions of nonleading D terms and $\Delta\lambda^{\rm field}_{1,\ldots,4}$ terms. If μ and A are real, the expressions (13)–(19) are consistent with the results of [8], where the D-term contribution was calculated.[4] Let us note that it is not possible to generalize the expressions for real $\lambda_{5,6,7}$ in the case of CP violating potential by the straightforward replacement of the real μ, A parameters to the complex ones.

If we neglect the contributions of D terms, the wave-function renormalization terms $\Delta\lambda^{\rm field}_{1,\ldots,4}$, and the terms of the order of h_b^2 for the b-quark couplings, only the one-loop corrections of the order of $\mathcal{O}(h_t^4)$ remain. This approximation was discussed in [12,14]. For example, λ_2 is given by

$$\lambda_2 \approx \frac{g_2^2 + g_1^2}{8} + \frac{3}{32\pi^2}\left[h_t^4 \frac{|A|^2}{M_{\rm SUSY}^2}\left(2 - \frac{|A|^2}{6M_{\rm SUSY}^2}\right) + 2h_t^4 l\right]. \quad (25)$$

The beta function for λ_2 contains large negative contribution $-6h_t^4$ [8], or equivalently, λ_2 (13) contains the large logarithmic term $6h_t^4 l/(32\pi^2)$ which was observed in the first calculations [10]. In the following the negative $\Delta\lambda_2$ defined by (22) gives a large positive contribution to the light Higgs boson mass in (38).

Numerical comparison of the λ_i parameters evaluated using different approximations is presented in Table I, where for our case in the second line of the Table

$$\Delta\lambda_i = \{\text{one-loop contribution}\} + \{\text{one-loop}(D \text{ terms} + \text{wave-func. renormalization})\}.$$

One can conclude that the one-loop corrections from D terms and wave-function renormalization can be of the order of the leading two-loop corrections. The difference of the effective λ_i of the order of 10^{-1} may result in the deviation of Higgs boson masses around 5 GeV and even more.

III. DIAGONALIZATION OF THE EFFECTIVE POTENTIAL MASS TERM IN THE LOCAL MINIMUM

A. Complex μ_{12}^2, $\lambda_{5,6,7}$ parameters, $\theta=0$

The components ω_i, η_i, χ_i of the SU(2) doublets (2) and (3) are not a physical Higgs fields (mass eigenstates). In order to extract the Higgs boson masses and the self-interaction of the physical fields from the potential (6) it

[4] In (13)–(15) we kept the terms of the order of $g_{1,2}^4$.

is necessary to diagonalize the mass term of the latter in the local minimum. This problem has been considered in [5] for the case of complex μ_{12}^2, $\lambda_{5,6,7}$ parameters and the zero phase of the Φ_2 VEV $\theta = 0$. As an example we derive an explicit representation for the triple Higgs boson vertex $H^+H^-h_i$, where the h_i mass eigenstates are defined by the formula (44) below, in the Appendix, see also Fig. 1. The diagonalization of the mass term is performed in two stages. First the CP-even fields h,H, the CP-odd field A ("pseudoscalar")[5], and the Goldstone field G^0 are defined by the linear transformation

$$h = -\eta_1 \sin\alpha + \eta_2 \cos\alpha, \tag{26}$$

$$H = \eta_1 \cos\alpha + \eta_2 \sin\alpha, \tag{27}$$

$$A = -\chi_1 \sin\beta + \chi_2 \cos\beta, \tag{28}$$

$$G^0 = \chi_1 \cos\beta + \chi_2 \sin\beta, \tag{29}$$

where $\mathrm{tg}\beta = v_2/v_1$ and (introducing compact notations $\sin\alpha = s_\alpha$, $\cos\beta = c_\beta$, etc.)

$$\mathrm{tg}\, 2\alpha = \frac{s_{2\beta}(m_A^2 + m_Z^2) + v^2((\Delta\lambda_3 + \Delta\lambda_4)s_{2\beta} + 2c_\beta^2\,\mathrm{Re}\Delta\lambda_6 + 2s_\beta^2\,\mathrm{Re}\Delta\lambda_7)}{c_{2\beta}(m_A^2 - m_Z^2) + v^2(\Delta\lambda_1 c_\beta^2 - \Delta\lambda_2 s_\beta^2 - \mathrm{Re}\Delta\lambda_5 c_{2\beta} + (\mathrm{Re}\Delta\lambda_6 - \mathrm{Re}\Delta\lambda_7)s_{2\beta})}. \tag{30}$$

Here the relations $g_1^2 + g_2^2 = g_2^2 m_Z^2/m_W^2$, $g_2^2 - g_1^2 = g_2^2(2 - m_Z^2/m_W^2)$ are used. Then we substitute to the effective potential the real parameters $\mu_{1,2}$, $\lambda_{1,2,3,4}$ and the real parts $\mathrm{Re}\mu_{12}^2$, $\mathrm{Re}\lambda_{5,6,7}$, which are related by linear transformation [5,16,17]:

$$\lambda_1 = \frac{1}{2v^2}\left[\left(\frac{s_\alpha}{c_\beta}\right)^2 m_h^2 + \left(\frac{c_\alpha}{c_\beta}\right)^2 m_H^2 - \frac{s_\beta}{c_\beta^3}\,\mathrm{Re}\mu_{12}^2\right] + \frac{1}{4}(\mathrm{Re}\lambda_7\,\mathrm{tg}^3\beta - 3\,\mathrm{Re}\lambda_6\,\mathrm{tg}\beta), \tag{31}$$

$$\lambda_2 = \frac{1}{2v^2}\left[\left(\frac{c_\alpha}{s_\beta}\right)^2 m_h^2 + \left(\frac{s_\alpha}{s_\beta}\right)^2 m_H^2 - \frac{c_\beta}{s_\beta^3}\,\mathrm{Re}\mu_{12}^2\right] + \frac{1}{4}(\mathrm{Re}\lambda_6\,\mathrm{ctg}^3\beta - 3\,\mathrm{Re}\lambda_7\,\mathrm{ctg}\beta), \tag{32}$$

$$\lambda_3 = \frac{1}{v^2}\left[2m_{H^\pm}^2 - \frac{\mathrm{Re}\mu_{12}^2}{s_\beta c_\beta} + \frac{s_{2\alpha}}{s_{2\beta}}(m_H^2 - m_h^2)\right] - \frac{\mathrm{Re}\lambda_6}{2}\,\mathrm{ctg}\beta - \frac{\mathrm{Re}\lambda_7}{2}\,\mathrm{tg}\beta, \tag{33}$$

$$\lambda_4 = \frac{1}{v^2}\left(\frac{\mathrm{Re}\mu_{12}^2}{s_\beta c_\beta} + m_A^2 - 2m_{H^\pm}^2\right) - \frac{\mathrm{Re}\lambda_6}{2}\,\mathrm{ctg}\beta - \frac{\mathrm{Re}\lambda_7}{2}\,\mathrm{tg}\beta, \tag{34}$$

$$\mathrm{Re}\,\lambda_5 = \frac{1}{v^2}\left(\frac{\mathrm{Re}\mu_{12}^2}{s_\beta c_\beta} - m_A^2\right) - \frac{\mathrm{Re}\lambda_6}{2}\,\mathrm{ctg}\beta - \frac{\mathrm{Re}\lambda_7}{2}\,\mathrm{tg}\beta, \tag{35}$$

$$\mu_1^2 = \lambda_1 v_1^2 + (\lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5)\frac{v_2^2}{2} - \mathrm{Re}\mu_{12}^2\,\mathrm{tg}\beta + \frac{v^2 s_\beta^2}{2}(3\,\mathrm{Re}\lambda_6\,\mathrm{ctg}\beta + \mathrm{Re}\lambda_7\,\mathrm{tg}\beta), \tag{36}$$

$$\mu_2^2 = \lambda_2 v_2^2 + (\lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5)\frac{v_1^2}{2} - \mathrm{Re}\mu_{12}^2\,\mathrm{ctg}\beta + \frac{v^2 c_\beta^2}{2}(\mathrm{Re}\lambda_6\,\mathrm{ctg}\beta + 3\,\mathrm{Re}\lambda_7\,\mathrm{tg}\beta). \tag{37}$$

At the purely real parameters (in the following we shall name this case of $\varphi = 0$ as the CP-conserving limit, $\mathrm{Re}\lambda_i = |\lambda_i|$, $\mathrm{Re}\Delta\lambda_i = |\Delta\lambda_i|$) the relations (36) and (37) set to zero the potential terms which are linear in the fields, so they are the minimization conditions. It follows from Eqs. (31)–(35) that in the CP-conserving limit the CP-even Higgs boson masses and the real part of the μ_{12}^2 parameter can be expressed as

$$m_h^2 = s_{\alpha+\beta}^2 m_Z^2 + c_{\alpha-\beta}^2 m_A^2 - v^2(\Delta\lambda_1 s_\alpha^2 c_\beta^2 + \Delta\lambda_2 c_\alpha^2 s_\beta^2 - 2(\Delta\lambda_3 + \Delta\lambda_4)c_\alpha c_\beta s_\alpha s_\beta + \mathrm{Re}\Delta\lambda_5(s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\beta^2) - 2c_{\alpha+\beta}(\mathrm{Re}\Delta\lambda_6 s_\alpha c_\beta - \mathrm{Re}\Delta\lambda_7 c_\alpha s_\beta)), \tag{38}$$

$$m_H^2 = c_{\alpha+\beta}^2 m_Z^2 + s_{\alpha-\beta}^2 m_A^2 - v^2(\Delta\lambda_1 c_\alpha^2 c_\beta^2 + \Delta\lambda_2 s_\alpha^2 s_\beta^2 + 2(\Delta\lambda_3 + \Delta\lambda_4)c_\alpha c_\beta s_\alpha s_\beta + \mathrm{Re}\Delta\lambda_5(c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2) + 2s_{\alpha+\beta}(\mathrm{Re}\Delta\lambda_6 c_\alpha c_\beta + \mathrm{Re}\Delta\lambda_7 s_\alpha s_\beta)), \tag{39}$$

$$m_{H^\pm}^2 = m_W^2 + m_A^2 - \frac{v^2}{2}(\mathrm{Re}\Delta\lambda_5 - \Delta\lambda_4),$$
$$\mathrm{Re}\mu_{12}^2 = s_\beta c_\beta\left[m_A^2 - \frac{v^2}{2}(2\,\mathrm{Re}\Delta\lambda_5 + \mathrm{Re}\Delta\lambda_6\,\mathrm{ctg}\beta + \mathrm{Re}\Delta\lambda_7\,\mathrm{tg}\beta)\right]. \tag{40}$$

After the substitution of (31)–(37) to (6) we find the mass term of the effective potential

[5]The fields h, H, and A are the physical fields at $\varphi = \arg(\mu A_{t,b}) = 0, n\pi$.

$$U_{\rm mass}(h, H, A, H^{\pm}) = c_0 A + c_1 hA + c_2 HA + \frac{m_h^2}{2} h^2 + \frac{m_H^2}{2} H^2 + \frac{m_A^2}{2} A^2 + m_{H^{\pm}}^2 H^+ H^-. \tag{41}$$

The minimization condition $c_0 = 0$ fixes the imaginary part of the μ_{12}^2 parameter

$$\mathrm{Im}\,\mu_{12}^2 = \frac{v^2}{2}(s_\beta c_\beta \,\mathrm{Im}\lambda_5 + c_\beta^2 \,\mathrm{Im}\lambda_6 + s_\beta^2 \,\mathrm{Im}\lambda_7), \tag{42}$$

and the factors in front of the nondiagonal terms hA and HA in the local minimum $c_0 = 0$ have the form

$$\begin{aligned} c_1 &= \frac{v^2}{2}(s_\alpha s_\beta - c_\alpha c_\beta)\mathrm{Im}\lambda_5 + v^2(s_\alpha c_\beta \,\mathrm{Im}\lambda_6 - c_\alpha s_\beta \,\mathrm{Im}\lambda_7), \\ c_2 &= -\frac{v^2}{2}(s_\alpha c_\beta + c_\alpha s_\beta)\mathrm{Im}\lambda_5 - v^2(c_\alpha c_\beta \,\mathrm{Im}\lambda_6 + s_\alpha s_\beta \,\mathrm{Im}\lambda_7). \end{aligned} \tag{43}$$

They include only the imaginary parts of the parameters $\mathrm{Im}\mu_{12}^2$, $\mathrm{Im}\lambda_{5,6,7}$. The nondiagonal term hH does not appear in (41), so in the mixing matrix (45) $M_{12} = M_{21} = 0$.

At the second stage in order to remove the nondiagonal terms hA and HA we perform the orthogonal transformation in the h, H, A sector

$$(h, H, A)M^2 \begin{pmatrix} h \\ H \\ A \end{pmatrix} = (h_1, h_2, h_3) a_{ik}^T M_{kl}^2 a_{lj} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}, \tag{44}$$

where the mass matrix is

$$M^2 = \frac{1}{2}\begin{pmatrix} m_h^2 & 0 & c_1 \\ 0 & m_H^2 & c_2 \\ c_1 & c_2 & m_A^2 \end{pmatrix}, \tag{45}$$

and get the physical Higgs bosons h_1, h_2, h_3 without a definite CP parity.[6] The eigenvalues of the M^2 matrix define their masses squared and the components of normalized eigenvectors are the matrix elements in the rows of the mixing matrix a_{ij}. The squared masses of Higgs bosons are ($m_{h_1}^2 \leq m_{h_2}^2 \leq m_{h_3}^2$)

[6]Note that this picture is different from the well-known description of weak CP violation in meson decays, when the mass splitting Δm of the states is given by $2\,\mathrm{Re}M_{12}$, M_{12} the off-diagonal elements of the complex 2×2 mass matrix, and the meson mixing ϵ parameter is $\mathrm{Im}M_{12}/(\sqrt{2}\Delta m)$. The meson decay formalism uses the non-Hermitian effective Hamiltonian and not precisely orthogonal mass "eigenstates."

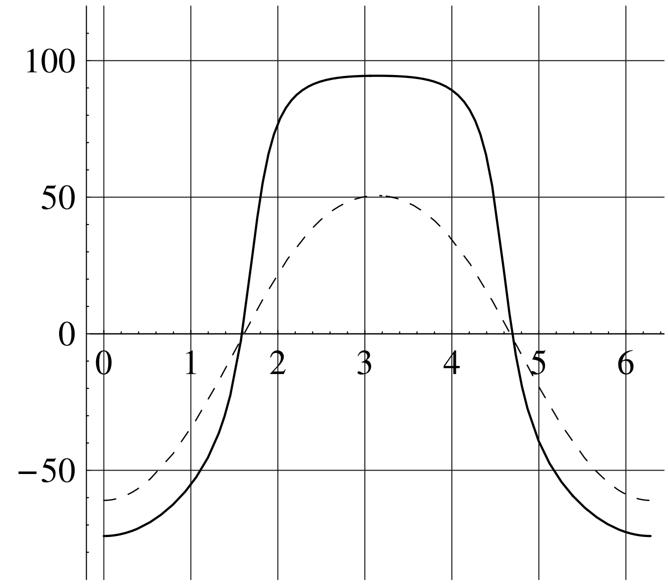

FIG. 1. Triple Higgs boson interaction vertex $g_{H^+H^-h_1}$ (GeV) calculated with the one-loop effective parameters λ_i vs the phase $\arg(\mu A)$ at the parameter values $M_{\rm SUSY} = 500$ GeV, $\mathrm{tg}\beta = 5$, $A_{t,b} = 1000$ GeV, and $\mu = 2000$ GeV. Dashed line $m_{H^{\pm}} = 300$ GeV; solid line $m_{H^{\pm}} = 200$ GeV.

$$\begin{aligned} m_{h_1}^2 &= 2\sqrt{(-q)}\cos\left(\frac{\Theta + 2\pi}{3}\right) - \frac{a_2}{3}, \\ m_{h_2}^2 &= 2\sqrt{(-q)}\cos\left(\frac{\Theta + 4\pi}{3}\right) - \frac{a_2}{3}, \\ m_{h_3}^2 &= 2\sqrt{(-q)}\cos\left(\frac{\Theta}{3}\right) - \frac{a_2}{3}, \end{aligned} \tag{46}$$

where

$$\Theta = \arccos\frac{r}{\sqrt{(-q^3)}}, \qquad r = \frac{1}{54}(9a_1 a_2 - 27a_0 - 2a_2^3),$$
$$q = \frac{1}{9}(3a_1 - a_2^2),$$

$$\begin{aligned} a_1 &= m_h^2 m_H^2 + m_h^2 m_A^2 + m_H^2 m_A^2 - c_1^2 - c_2^2, \\ a_2 &= -m_h^2 - m_H^2 - m_A^2, \end{aligned}$$

$$a_0 = c_1^2 m_H^2 + c_2^2 m_h^2 - m_h^2 m_H^2 m_A^2.$$

The normalized eigenvector components $(h, H, A) = a_{ij} h_j$, $a_{ij} = a'_{ij}/n_j$ are given by

$$\begin{aligned} a'_{11} &= ((m_H^2 - m_{h_1}^2)(m_A^2 - m_{h_1}^2) - c_2^2), \qquad a'_{21} = c_1 c_2, \\ a'_{31} &= -c_1(m_H^2 - m_{h_1}^2), \qquad a'_{12} = -c_1 c_2, \\ a'_{22} &= -((m_h^2 - m_{h_2}^2)(m_A^2 - m_{h_2}^2) - c_1^2), \\ a'_{32} &= c_2(m_h^2 - m_{h_2}^2), \end{aligned}$$

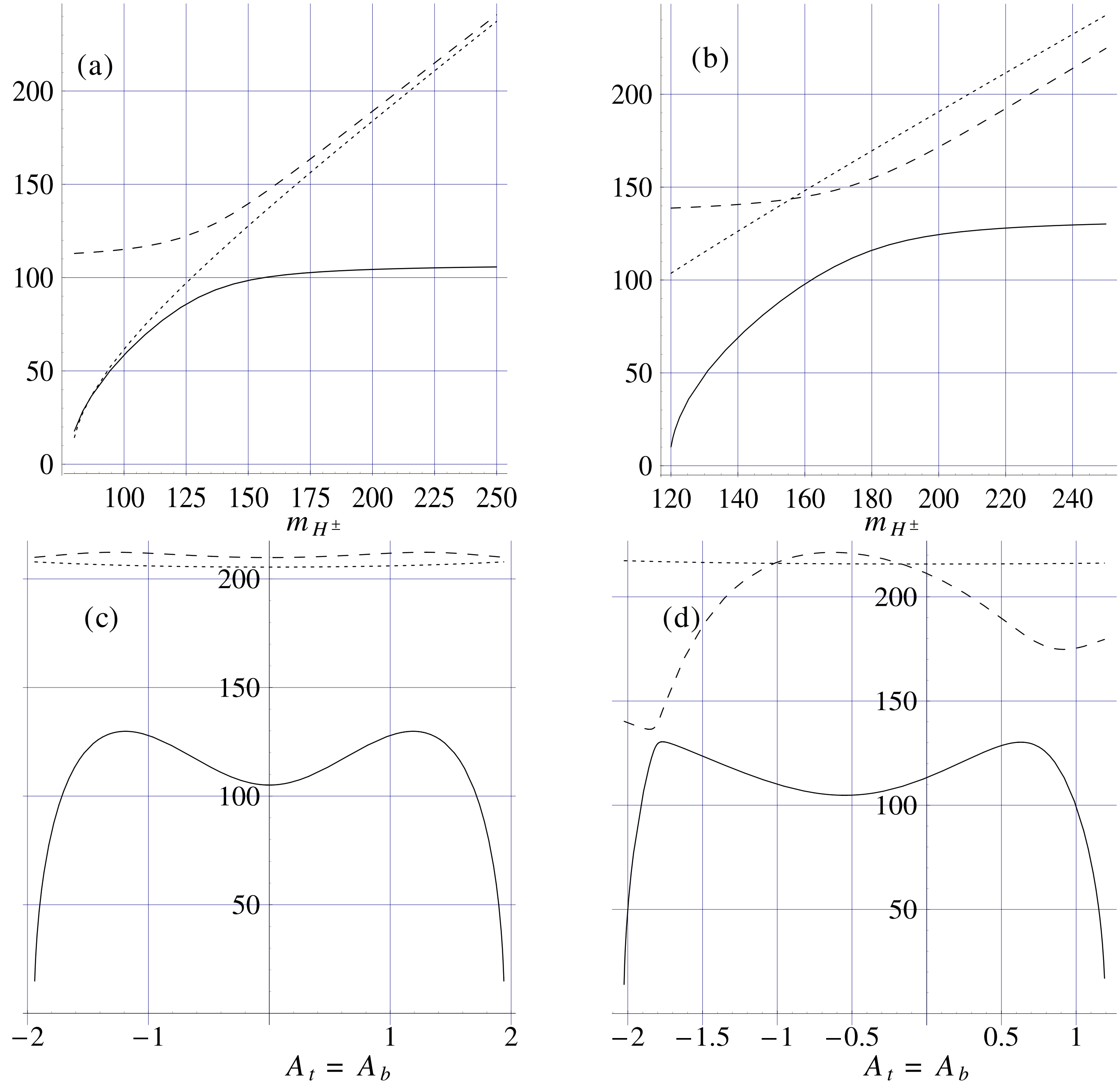

FIG. 2 (color online). Neutral Higgs boson masses h, H, A versus $m_{H^\pm}$ and the trilinear parameters A_t, A_b in the CP-conserving limit. Solid line denotes the m_h mass; short-dashed line m_A; long-dashed line m_H. (a) $\text{tg}\beta = 5$, $M_{\text{SUSY}} = 0.5$ TeV, and $A_t = A_b = \mu = 0$. (b) $\text{tg}\beta = 5$, $M_{\text{SUSY}} = 0.5$ TeV, $A_t = A_b = 0.9$ TeV, and $\mu = -1.5$ TeV. (c) $\text{tg}\beta = 5$, $M_{\text{SUSY}} = 0.5$ TeV, $m_{H^\pm} = 220$ GeV, $\mu = 0$, and $A_t = A_b$. (d) $\text{tg}\beta = 5$, $M_{\text{SUSY}} = 0.5$ TeV, $m_{H^\pm} = 220$ GeV, $\mu = -2$ TeV, and $A_t = A_b$.

$$a'_{13} = -c_1(m_H^2 - m_{h_3}^2), \qquad a'_{23} = -c_2(m_h^2 - m_{h_3}^2),$$
$$a'_{33} = (m_h^2 - m_{h_3}^2)(m_H^2 - m_{h_3}^2),$$

$n_i = \pm\sqrt{(a'^2_{1i} + a'^2_{2i} + a'^2_{3i})}$. The Higgs boson masses m_{h_1}, m_{h_2}, m_{h_3} and the mixing matrix elements a_{ij}, which describe the mixed states, are shown in Figs. 2–4 as a function of the $A_{t,b}$, μ parameters and/or the universal phase $\varphi = \arg(\mu A_{t,b})$. Different from the figures in [5], the $m_{H^\pm}$, $\text{tg}\beta$ parametrization is used for the convenience of comparison with [15,18]. The parameters c_1 and c_2 can change a sign with the variation of the phase φ; the ranges of positively or negatively defined c_1 and c_2 depend on the primary choice of the $m_{H^\pm}$, $\text{tg}\beta$, A, μ, and M_{SUSY} in the CP-conserving limit. When we pass the zeros of c_1 and c_2,

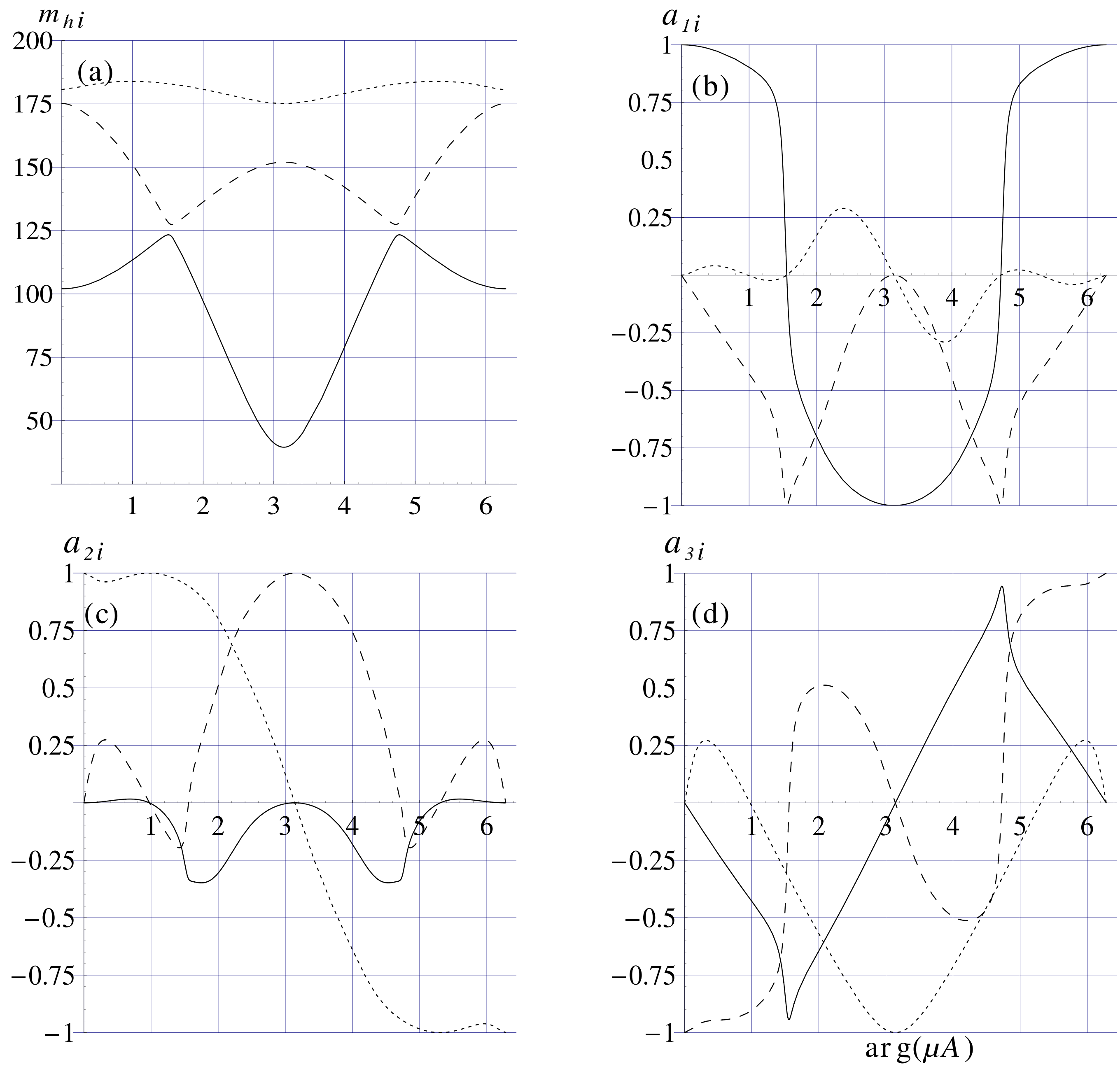

FIG. 3 (color online). (a) Neutral Higgs boson masses; (b)–(d) the matrix elements a_{ij} versus the phase $\varphi = \arg(\mu A)$ at the parameter values tg$\beta = 5$, $m_{H^\pm} = 180$ GeV, $M_{\rm SUSY} = 0.5$ TeV, $A_t = A_b = 1$ TeV, and $\mu = 2$ TeV. Solid line denotes $i = 1$; long-dashed line $i = 2$; and short-dashed line $i = 3$.

the matrix elements a_{ij} are expected to change their signs respecting the requirement of the left orthonormal basis for the eigenvectors. It is essential that m_{h_1}, m_{h_2}, and m_{h_3} are positioned in the mass matrix along the diagonal from the upper left to the lower right corner, satisfying in the limiting case $c_1 = c_2 = 0$ the correspondences $m_{h_1} \to \min(m_h, m_H, m_A)$, $m_{h_3} \to \max(m_h, m_H, m_A)$ (''the mass ordering''). Note also that as $\Delta\lambda_i$ increases, the denominator of (30) can change sign, so for the mass ordering one must define the angle $\alpha(\varphi)$ consistently with the boundary condition at the scale $M_{\rm SUSY}$, which has the known form $m_A^2 + m_Z^2 = -\sin 2\alpha/\sin 2\beta(m_H^2 - m_h^2)$, following from (31)–(35) and (7).

Some numerical values for the Higgs boson masses m_{h_1}, m_{h_2}, m_{h_3} as a function of the phase φ in our approach, and masses of the states H_1, H_2, and H_3 evaluated by means of CPsuperH [15] and FeynHiggs [18] packages are shown in Table II. These packages are using the renormalization group improved diagrammatic calculation that includes radiative corrections to Yukawa couplings up to two loops.

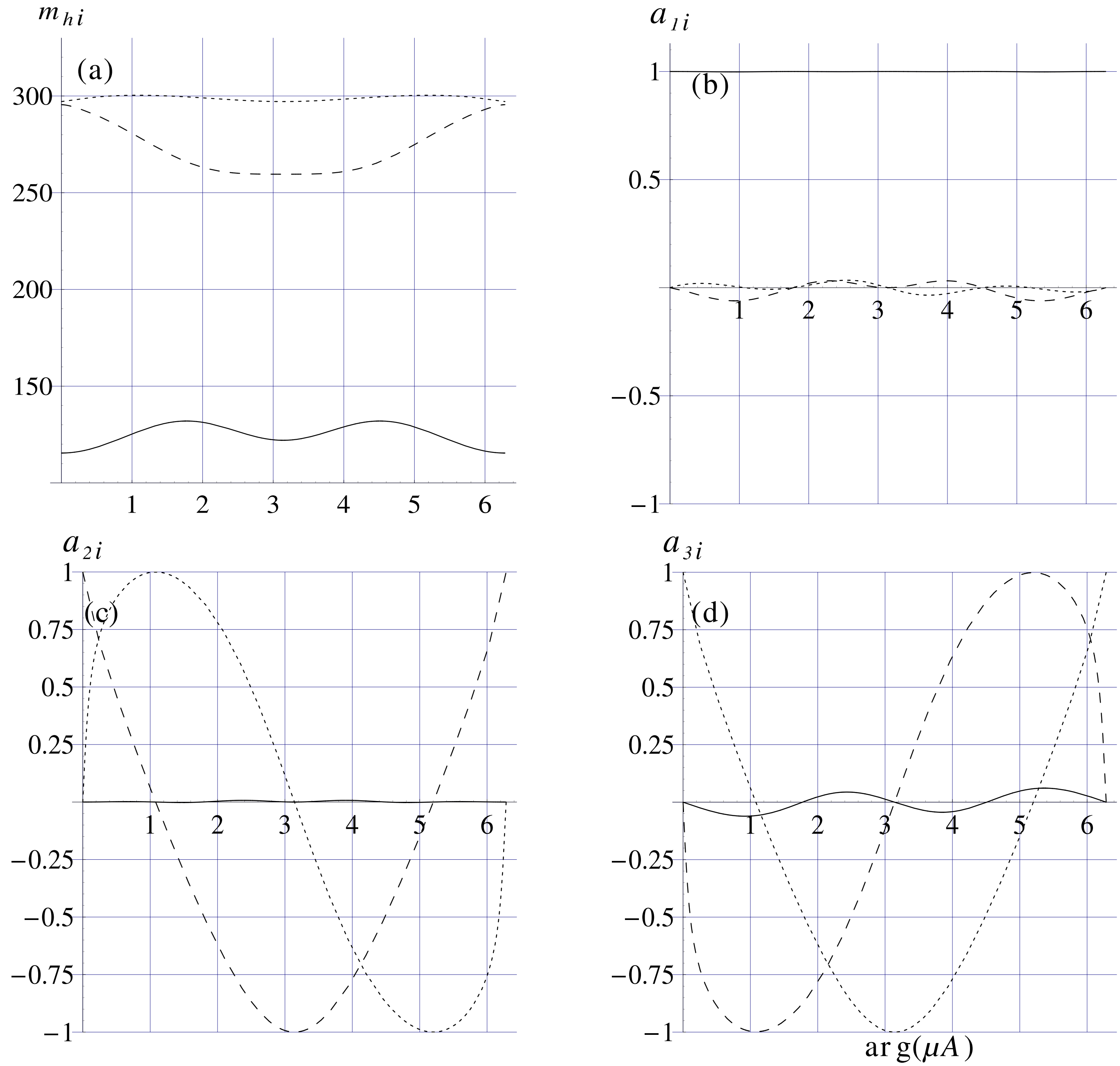

FIG. 4 (color online). (a) Neutral Higgs boson masses; (b)–(d) the matrix elements a_{ij} versus the phase $\varphi = \arg(\mu A)$ at the parameter values $\mathrm{tg}\beta = 5$, $m_{H^\pm} = 300$ GeV, $M_{\mathrm{SUSY}} = 0.5$ TeV, $A_t = A_b = 1$ TeV, and $\mu = 2$ TeV. Solid line denotes $i = 1$; long-dashed line $i = 2$; and short-dashed line $i = 3$.

A detailed general discussion on the conciliation of results obtained in the frameworks of the diagrammatic and the effective field theory approaches can be found in [19]. Different renormalization schemes in which calculations in the two approaches are performed may lead to the deviations of results evaluated with parameters taken at different renormalization scales, so the nontrivial reevaluation of parameters is needed for consistency. Besides this it is important to notice that in the CPsuperH and FeynHiggs packages the SU(2) eigenstates $\eta_{1,2}$ and $\xi_{1,2}$ are directly transformed to the Higgs boson mass eigenstates, which is different from our procedure, when we first transform to the states of the CP-conserving limit and then rotate to $h_{1,2,3}$. The "intermediate" Higgs boson states (h, H, A) of the CP-conserving limit are not used, so the η_1, η_2 mixing angle α is not introduced there. For this reason at $\varphi = 0$ the analog of the mixing matrix a_{ij}, see (44), has nonzero off-diagonal matrix elements $a_{12} = a_{21} \neq 0$, and in the analog of the mass matrix (45) M_{12} and M_{21} (the hH mixing terms in our notation) are also

TABLE II. The Higgs boson masses (GeV) in our case at the one-loop level and calculated by the packages CPsuperH [15] (at the two-loop) and FeynHiggs [18] (in the one-loop regime). See the Appendix for a more detailed comparison. The same parameter values were used: $\alpha_{\rm EM}(m_Z) = 0.7812 \times 10^{-2}$, $\alpha_S(m_Z) = 0.1172$, $G_F = 1.174 \times 10^{-5}$ GeV^{-2}, $\tan\beta = 5$, $M_{\rm SUSY} = 500$ GeV, $|A_t| = |A_b| = A$, $|\mu| = 2000$ GeV, $A = 1000$ GeV, and $m_{H^\pm} = 300$ GeV.

	$\varphi = 0$	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
m_{h_1}	115.4	118.7	125.9	131.4	130.7	125.2	122.0
m_{H_1} [15]	106.8	109.0	113.9	117.4	114.9	105.7	99.4
m_{H_1} [18]	115.8	118.8	125.5	130.2	123.2	98.2	78.0
m_{h_2}	295.5	289.6	279.7	269.3	262.2	259.8	259.6
m_{H_2} [15]	302.2	297.8	290.9	282.2	273.9	268.3	264.4
m_{H_2} [18]	295.6	290.0	279.1	264.3	249.2	239.7	236.9
m_{h_3}	297.1	299.5	300.4	299.9	298.8	297.6	297.1
m_{H_3} [15]	302.3	304.4	305.0	304.5	303.5	302.4	302.0
m_{H_3} [18]	297.6	300.0	301.1	301.3	300.9	300.4	300.2

nonzero. In the framework of the "direct" diagonalization procedure the matrix elements of (45) have the form

$$\begin{aligned}
M_{11} &= m_A^2 s_\beta^2 + v^2\,{\rm Re}\lambda_5 s_\beta^2 + v^2\,{\rm Re}\lambda_6 s_{2\beta} + 2v^2\lambda_1 c_\beta^2,\\
M_{22} &= m_A^2 c_\beta^2 + v^2\,{\rm Re}\lambda_5 c_\beta^2 + v^2\,{\rm Re}\lambda_7 s_{2\beta} + 2v^2\lambda_2 s_\beta^2,\\
M_{12} &= v^2\,{\rm Re}\lambda_6 c_\beta^2 + s_\beta(v^2\,{\rm Re}\lambda_7 s_\beta + c_\beta(-m_A^2 + v^2\lambda_3\\
&\quad + v^2\lambda_4)),\\
M_{13} &= -\frac{1}{2}v^2(2\,{\rm Im}\lambda_6 c_\beta + {\rm Im}\lambda_5 s_\beta),\\
M_{23} &= -\frac{1}{2}v^2({\rm Im}\lambda_5 c_\beta + 2\,{\rm Im}\lambda_7 s_\beta), \qquad M_{33} = m_A^2,
\end{aligned}$$

and the parameters a_0, a_1, a_2 in (46) should be redefined as follows:

$$\begin{aligned}
a_0 &= M_{12}^2 M_{33} + M_{23}^2 M_{11} + M_{13}^2 M_{22} - 2M_{12}M_{23}M_{13}\\
&\quad - M_{11}M_{22}M_{33},\\
a_1 &= M_{11}M_{22} + M_{11}M_{33} + M_{22}M_{33} - M_{12}^2 - M_{13}^2\\
&\quad - M_{23}^2,\\
a_2 &= -M_{11} - M_{22} - M_{33}.
\end{aligned}$$

We checked that both the "two-step" and the direct diagonalization methods lead within our procedure, as expected, to the same masses of Higgs states m_{h_1}, m_{h_2}, and m_{h_3} (see Table II). For the parameter values in the comparison, Table II, the benchmark point of the maximal CP violation "CPX scenario" [20] at $M_{\rm SUSY} = 500$ GeV was used. An extended list of numbers (Table V) including also the rare one-loop-mediated decay widths $h_1 \to \gamma\gamma$, $h_1 \to gg$ and the tree-level two-particle decays $h_1 \to f\bar{f}$ can be found in the Appendix. Good qualitative agreement of results is observed, but diversity of approaches to the calculation of radiative corrections makes precise numerical comparisons difficult.

B. Real μ_{12}^2, $\lambda_{5,6,7}$ parameters, $\theta \neq 0$

If the parameters μ_{12}^2 and $\lambda_{5,6,7}$ of the effective potential (6) are real, the latter is CP invariant. It is easy to show [3,5,17] that the phases of complex parameters μ_{12}^2, $\lambda_{5,6,7}$ can be rotated away by the $U(1)_Y$ hypercharge transformation if the conditions

$$\begin{aligned}
{\rm Im}(\mu_{12}^4 \overset{*}{\lambda}_5) = 0, \qquad {\rm Im}(\mu_{12}^2 \overset{*}{\lambda}_6) = 0,\\
{\rm Im}(\mu_{12}^2 \overset{*}{\lambda}_7) = 0
\end{aligned} \tag{47}$$

are satisfied. Insofar as the physical motivation of these "fine-tuning" conditions is not available, the case of real parameters and nonzero phase θ of the VEV, when CP is broken spontaneously, looks rather artificial. The local minimum of the effective potential (6) occurs at $\lambda_5 > 0$ [i.e. purely imaginary μA, see (17)] and

$$\cos\theta = \frac{\mu_{12}^2 - \frac{v_1^2}{2}\lambda_6 - \frac{v_2^2}{2}\lambda_7}{\lambda_5 v_1 v_2}. \tag{48}$$

Combining this equation with the diagonalization condition (35) we get

$$\cos\theta = \frac{m_A^2}{\lambda_5 v^2} + 1, \tag{49}$$

so there is no minimum if $m_A^2 > 0$. In the case $\lambda_5 < 0$ (48) corresponds to the maximum, the absolute minimum is achieved at the end points $\cos\theta = \pm 1$. For example, the absolute minimum at $\theta = 0$ [taking into account again the diagonalization condition (35)] is absent if

$$m_A^2 > 2|\lambda_5|v^2 \tag{50}$$

and it follows that for the case of real μ_{12}^2, $\lambda_{5,6,7}$, and CP broken spontaneously there are no mass eigenstates in the framework of our diagonalization procedure, at least if m_A is not extremely small.

C. Complex μ_{12}^2, $\lambda_{5,6,7}$ parameters, $\theta \neq 0$

In the case of complex parameters and the nonzero phase of Φ_2 vacuum expectation value,[7] the CP invariance of the potential is broken both explicitly and spontaneously. The condition to set to zero the derivative $\partial U/\partial\theta$ includes both the real and the imaginary parts of μ_{12}^2 and $\lambda_{5,6,7}$:

[7]The upper component of $\langle\Phi_2\rangle$ in (4) is taken to be zero. Otherwise additional constraint for the VEV components should be imposed to ensure the existence of the massless gauge field (photon) [21].

$$\cos\theta(2\,\mathrm{Im}\mu_{12}^2 - v_1^2\,\mathrm{Im}\lambda_6 - v_2^2\,\mathrm{Im}\lambda_7) - v_1 v_2\,\mathrm{Im}\lambda_5\cos 2\theta + \sin\theta(2\,\mathrm{Re}\mu_{12}^2 - v_1^2\,\mathrm{Re}\lambda_6 - v_2^2\,\mathrm{Re}\lambda_7) - v_1 v_2\,\mathrm{Re}\lambda_5 \sin 2\theta = 0. \quad (51)$$

The condition of the extremum for $\mathrm{Im}\mu_{12}^2$ depends on the phase between the VEV's θ, while the diagonalization condition for $\mathrm{Re}\mu_{12}^2$ depends also on the relative phase ξ [see (3) and (4)] of the SU(2) doublets. At the real μ_{12}^2, $\lambda_{5,6,7}$ and $\theta \neq 0$ the Eq. (51) is reduced to (48).

For convenience we present the extremum conditions $\partial U/\partial\eta = 0$, $\partial U/\partial\xi = 0$ in the cases of zero and nonzero θ in the form of Tables III and IV, where the factors in front of the potential parameters are shown. The bulky condition for the real part of μ_{12}^2 to define the pseudoscalar mass m_A for the general case of nonzero phases can explicitly be evaluated as follows:

$$\begin{aligned}
\mathrm{Re}\,\mu_{12}^2 = &-\lambda_2 \frac{v^2\cos\theta\sin^3(2\beta)\sin^2(\theta+\xi)}{3 + (1-\cos\theta\cos\xi)(\cos^4\beta - \frac{3}{2}\sin^2(2\beta)) + \sin^4\beta + \cos\theta\cos\xi(1-\sin^4\beta)} \\
&+ \mathrm{Re}\lambda_5 \frac{v^2(\cos^4\beta\cos^2\xi + \cos^2\theta\sin^4\beta + \cos\beta\cos(\theta-\xi)\sin\beta\sin(2\beta))}{\cos^2\beta\,\mathrm{ctg}\beta\sec\theta + \cos\xi\sin(2\beta) + \sec\theta\sin^2\beta\,\mathrm{tg}\beta} \\
&- \mathrm{Im}\lambda_5 \frac{v^2(\sin^2(2\beta)\sin(\theta-\xi) + \sin^4\beta(\sin(2\theta)+\mathrm{tg}\theta) + \cos^4\beta(\mathrm{tg}\theta - \sin(2\xi)))}{2(\cos^2\beta\,\mathrm{ctg}\beta\sec\theta + \cos\xi\sin(2\beta) + \sec\theta\sin^2\beta\,\mathrm{tg}\beta)} + \mathrm{Re}\lambda_6 \frac{1}{2} v^2\cos^2\beta \\
&+ \mathrm{Im}\lambda_6 \frac{v^2\cos^3\beta\sin\beta\sin\xi}{\cos^2\beta\,\mathrm{ctg}\beta\sec\theta + \cos\xi\sin(2\beta) + \sec\theta\sin^2\beta\,\mathrm{tg}\beta} \\
&+ \mathrm{Re}\lambda_7 \Big(\frac{v^2\cos^4\beta(4\cos(\theta+2\xi) - 2\cos(2\theta)\sec\theta)\mathrm{tg}\beta}{4(\cos^2\beta\,\mathrm{ctg}\beta\sec\theta + \cos\xi\sin(2\beta) + \sec\theta\sin^2\beta\,\mathrm{tg}\beta)} \\
&+ \frac{v^2(2\sin^2(2\beta)\cos\xi + 2\sec\theta\sin^4\beta - \cos(2\theta+\xi)\sin^2(2\beta))\mathrm{tg}\beta}{4(\cos^2\beta\,\mathrm{ctg}\beta\sec\theta + \cos\xi\sin(2\beta) + \sec\theta\sin^2\beta\,\mathrm{tg}\beta)} \Big) \\
&+ \mathrm{Im}\lambda_7 \frac{v^2\sin(2\beta)(2\cos^2\beta\cos\xi\sin(\theta+\xi) + \sin^2\beta(2\sin\xi + \sin(2\theta+\xi)))}{2(\cos^2\beta\,\mathrm{ctg}\beta\sec\theta + \cos\xi\sin(2\beta) + \sec\theta\sin^2\beta\,\mathrm{tg}\beta)} \\
&- \mathrm{Im}\mu_{12}^2 \frac{\sin(2\beta)\sin\xi}{\cos^2\beta\,\mathrm{ctg}\beta\sec\theta + \cos\xi\sin(2\beta) + \sec\theta\sin^2\beta\,\mathrm{tg}\beta} \\
&+ m_A^2 \frac{1}{\cos^2\beta\,\mathrm{ctg}\beta\sec\theta + \cos\xi\sin(2\beta) + \sec\theta\sin^2\beta\,\mathrm{tg}\beta}. \qquad (52)
\end{aligned}$$

If we set $\theta = 0$ and $\xi = 0$, the formulas coincide with the special case of only the explicit CP violation (35) and (42). The substitution of the extremum conditions corresponding to Tables III and IV to (51) gives an identity independently on the expression (52) for $\mathrm{Re}\mu_{12}^2$. The extremum is a minimum if the second derivative in θ is positively defined

TABLE III. The factors of the extremum conditions for μ_1^2 and μ_2^2 at zero and nonzero θ.

	μ_1^2 $\theta \neq 0$	$\theta = 0$	μ_2^2 $\theta \neq 0$	$\theta = 0$
λ_1	v_1^2	v_1^2	0	0
λ_2	0	0	v_2^2	v_2^2
λ_3	$\frac{1}{2}v_2^2$	$\frac{1}{2}v_2^2$	$\frac{1}{2}v_1^2$	$\frac{1}{2}v_1^2$
λ_4	$\frac{1}{2}v_2^2$	$\frac{1}{2}v_2^2$	$\frac{1}{2}v_1^2$	$\frac{1}{2}v_1^2$
$\mathrm{Re}\lambda_5$	$\frac{1}{2}v_2^2$	$\frac{1}{2}v_2^2$	$\frac{1}{2}v_1^2$	$\frac{1}{2}v_1^2$
$\mathrm{Im}\lambda_5$	$-\frac{1}{2}v_2^2\mathrm{tg}\theta$	0	$-\frac{1}{2}v_1^2\mathrm{tg}\theta$	0
$\mathrm{Re}\lambda_6$	$\frac{1}{2}v_1v_2(2+\cos 2\theta)\sec\theta$	$\frac{3}{2}v_1v_2$	$\frac{1}{2}v_1^2\sec\theta\,\mathrm{ctg}\beta$	$\frac{1}{2}v_1^2\,\mathrm{ctg}\beta$
$\mathrm{Im}\lambda_6$	$-v_1v_2\sin\theta$	0	0	0
$\mathrm{Re}\lambda_7$	$\frac{1}{2}v_2^2\sec\theta\,\mathrm{tg}\beta$	$\frac{1}{2}v_2^2\,\mathrm{tg}\beta$	$\frac{1}{2}v_1v_2(2+\cos 2\theta)\sec\theta$	$\frac{3}{2}v_1v_2$
$\mathrm{Im}\lambda_7$	0	0	$-v_1v_2\sin\theta$	0
$\mathrm{Re}\mu_{12}^2$	$-\mathrm{tg}\beta\sec\theta$	$-\mathrm{tg}\beta$	$-\mathrm{ctg}\beta\sec\theta$	$-\mathrm{ctg}\beta$

TABLE IV. The factors of the extremum condition for $\mathrm{Re}\mu_{12}^2$ at $\theta = 0$ and for $\mathrm{Im}\mu_{12}^2$ for zero and nonzero θ.

	$\mathrm{Re}\mu_{12}^2$ $\theta = 0$ $\xi = 0$	$\mathrm{Im}\mu_{12}^2$ $\theta \neq 0$	$\theta = 0$
λ_1	0	0	0
λ_2	0	0	0
λ_3	0	0	0
λ_4	0	0	0
$\mathrm{Re}\lambda_5$	$v_1 v_2$	$v_1 v_2 \sin\theta$	0
$\mathrm{Im}\lambda_5$	0	$\frac{1}{2} v_1 v_2 \cos 2\theta \sec\theta$	$\frac{1}{2} v_1 v_2$
$\mathrm{Re}\lambda_6$	$\frac{1}{2} v_1^2$	$\frac{1}{2} v_1^2 \,\mathrm{tg}\theta$	0
$\mathrm{Im}\lambda_6$	0	$\frac{1}{2} v_1^2$	$\frac{1}{2} v_1^2$
$\mathrm{Re}\lambda_7$	$\frac{1}{2} v_2^2$	$\frac{1}{2} v_2^2 \,\mathrm{tg}\theta$	0
$\mathrm{Im}\lambda_7$	0	$\frac{1}{2} v_2^2$	$\frac{1}{2} v_2^2$
m_A^2	$\sin\beta\cos\beta$	0	0
$\mathrm{Re}\mu_{12}^2$	$\cdots$	$-\mathrm{tg}\theta$	0

$$
\begin{aligned}
&- \sin\theta(2\,\mathrm{Im}\mu_{12}^2 - v_1^2\,\mathrm{Im}\lambda_6 - v_2^2\,\mathrm{Im}\lambda_7) \\
&+ 2 v_1 v_2\,\mathrm{Im}\lambda_5 \sin 2\theta + \cos\theta(2\,\mathrm{Re}\mu_{12}^2 - v_1^2\,\mathrm{Re}\lambda_6 \\
&\qquad - v_2^2\,\mathrm{Re}\lambda_7) - 2 v_1 v_2\,\mathrm{Re}\lambda_5 \cos 2\theta > 0. \quad (53)
\end{aligned}
$$

Numerical investigation shows that this condition is fulfilled in a rather wide range of the MSSM parameter space. If for simplicity we set $\xi = 0$ then the second derivative is positively defined in any region of the parameter space, so no restrictions on the phase of spontaneous *CP* breaking appear in this special case from the minimization.

The diagonalization of the effective potential mass term in the local minimum for the general case $\theta \neq 0$ and $\xi \neq 0$ is performed analogously to the procedure described in Sec. III A using the following scheme: (1) we define the four $\tilde{h}$, $\tilde{H}$, $\tilde{A}$, $\tilde{G}^0$ linear combinations of independent fields η_1, η_2, χ_1, χ_2 that are contained in the two-doublet system (2) and (3), where for the Goldstone field $\tilde{G}^0$ we define a zero row of matrix elements and a zero column of matrix elements in the symmetric mass matrix 4×4. In other words, the Goldstone mode is introduced as the linear combination, orthogonal to the plane defined by the "directions" in the complex scalar fields space, parallel to the VEV's v_1 and $v_2 \exp\{i(\xi + \zeta)\}$. Then the mass matrix 4×4 includes the symmetric 3×3 block with zero matrix elements in the power of the extremum conditions from Tables III and IV; (2) we define an orthogonal transformation for the 3×3 submatrix fixing the mixing angle $\tilde{\alpha}$ in the sector $\tilde{h} - \tilde{H}$ to set to zero the $\tilde{h}\tilde{H}$ nondiagonal term. In the framework of this procedure for the case of nonzero phases $\xi \neq 0$, $\theta \neq 0$ (when the fields are denoted by the symbol ~) the limiting cases of zero phases $\xi = \theta = 0$ (when the notation for the fields does not contain the symbol ~) and also the *CP*-conserving limit in the mass basis h, H, A, are clearly seen. For the physical Higgs fields in the case $\xi = 0$, $\theta \neq 0$ we finally obtain the representation

$$
\begin{aligned}
\tilde{h} &= -\eta_1 \sin\tilde{\alpha} + (\chi_2 \sin\theta + \eta_2 \cos\theta)\cos\tilde{\alpha}, \\
\tilde{H} &= \eta_1 \cos\tilde{\alpha} + (\chi_2 \sin\theta + \eta_2 \cos\theta)\sin\tilde{\alpha}, \\
\tilde{A} &= -\chi_1 \sin\beta + (\chi_2 \cos\theta - \eta_2 \sin\theta)\cos\beta, \\
\tilde{G}^0 &= \chi_1 \cos\beta + (\chi_2 \cos\theta - \chi_2 \sin\theta)\sin\beta.
\end{aligned} \quad (54)
$$

We checked explicitly, using the symbolic calculation packages, that direct substitution of these fields to the potential (6) gives the symmetric 4×4 squared mass matrix with zero row and column, corresponding to the Goldstone mode. The nondiagonal matrix elements of the 3×3 block, corresponding to the nondiagonal terms $\tilde{h}\tilde{A}$ $\tilde{H}\tilde{A}$ in the local minimum, can be written in the form

$$
\begin{aligned}
\tilde{c}_1 = -\frac{v^2}{2}(&\cos(\tilde{\alpha} + \beta)\cos(2\theta)\mathrm{Im}\lambda_5 \\
&- 2\sin\tilde{\alpha}\cos\beta\cos\theta\,\mathrm{Im}\lambda_6 + 2\cos\tilde{\alpha}\sin\beta\cos\theta\,\mathrm{Im}\lambda_7 \\
&- \cos(\tilde{\alpha} + \beta)\sin(2\theta)\mathrm{Re}\lambda_5 \\
&- 2\sin\tilde{\alpha}\cos\beta\sin\theta\,\mathrm{Re}\lambda_6 + 2\cos\tilde{\alpha}\sin\beta\sin\theta\,\mathrm{Re}\lambda_7),
\end{aligned} \quad (55)
$$

$$
\begin{aligned}
\tilde{c}_2 = -\frac{v^2}{2}(&\sin(\tilde{\alpha} + \beta)\cos(2\theta)\mathrm{Im}\lambda_5 \\
&- 2\cos\tilde{\alpha}\cos\beta\cos\theta\,\mathrm{Im}\lambda_6 + 2\sin\tilde{\alpha}\sin\beta\cos\theta\,\mathrm{Im}\lambda_7 \\
&+ \cos(\tilde{\alpha} + \beta)\sin(2\theta)\mathrm{Re}\lambda_5 \\
&- 2\cos\tilde{\alpha}\cos\beta\sin\theta\,\mathrm{Re}\lambda_6 + 2\sin\tilde{\alpha}\sin\beta\sin\theta\,\mathrm{Re}\lambda_7).
\end{aligned} \quad (56)
$$

In the case $\theta = 0$ they coincide with (43).

The same scheme is suitable for the case $\xi \neq 0$, $\theta \neq 0$ when the relative phase ξ between the SU(2) doublets appears in the mass eigenstates, which are obtained by the replacement $\theta \rightarrow \theta - \xi$:

$$
\begin{aligned}
\tilde{h} &= -\eta_1 \sin\tilde{\alpha} + (\chi_2 \sin(\theta - \xi) + \eta_2 \cos(\theta - \xi))\cos\tilde{\alpha}, \\
\tilde{H} &= \eta_1 \cos\tilde{\alpha} + (\chi_2 \sin(\theta - \xi) + \eta_2 \cos(\theta - \xi))\sin\tilde{\alpha}, \\
\tilde{A} &= -\chi_1 \sin\beta + (\chi_2 \cos(\theta - \xi) - \eta_2 \sin(\theta - \xi))\cos\beta, \\
\tilde{G}^0 &= \chi_1 \cos\beta + (\chi_2 \cos(\theta - \xi) - \chi_2 \sin(\theta - \xi))\sin\beta.
\end{aligned} \quad (57)
$$

IV. SUMMARY

The potential of a two-Higgs-doublet model in the general case is not *CP* invariant and the parameters μ_{12}^2 and $\lambda_{5,6,7}$ of the two-doublet MSSM Higgs sector should be taken complex. The choice of purely real parameters implicitly assumes that the fine-tuning conditions (47) are additionally imposed without clear physical motivation. In the MSSM the complex parameters naturally appear if we allow the *CP* invariance violating mixings in the squark-Higgs boson sector of the MSSM, analogous to the CKM mixings for the three quark generations in the charged current sector of the standard model. If these mixings lead to a strong *CP* parity violation[8] and the scalar sector of the MSSM is coupled strongly enough (i.e. large imaginary parts of the parameters μ_{12}^2 and $\lambda_{5,6,7}$ appear), the deviations of the observable effects in the scenario with *CP* violation from the phenomenology of the standard scenario can be substantial. The deviations are particularly strong if the power terms $A_{t,b}/M_{\rm SUSY}$, $\mu/M_{\rm SUSY}$ are large and the charged Higgs boson mass does not exceed 150–200 GeV, being rather weakly dependent on the value of tgβ. Such models could lead in principle to a reconsideration of the experimental priorities [23] for the signals of Higgs bosons production in the channels $\gamma\gamma$, $b\bar{b}$, W^+W^-, ZZ, ttH, bbH, etc. at the CERN LHC. The scenario with light Higgs boson $m_{h_1} \sim 70$–80 GeV that could escape the detection at CERN LEP2 [24], the analysis of the h_1 signal at Tevatron, and the high-luminosity linear colliders [25] demonstrate that physical possibilities in the framework of *CP* violating scenarios could be considerably modified in comparison with the traditional *CP*-conserving limit.

The comparison of our results for the masses of scalars m_{h_1}, m_{h_2}, and m_{h_3} and their two-particle decay widths with outputs of the CPsuperH [15] and the FeynHiggs [18] packages demonstrates rather good qualitative agreement. However, in some cases high sensitivity of the observables to the magnitude of radiatively induced correction terms in the effective two-Higgs-doublet potential shows up, so careful complementary analysis of the theoretical uncertainties is appropriate.

The relative phase of the SU(2) scalar doublet ζ and the VEV phase ξ, see (4), could be constrained on the basis of the conditions for the mass term diagonalization and the potential minimization (Sec. III C). In principle these conditions could lead to some nontrivial relations between the ζ, ξ and the variables of the MSSM parameter space. However, at the first sight it is questionable to expect some direct relations of this type connecting the CKM phase and the ζ, ξ phases of the THDM, which seem to describe the *CP* violation of a different origin. Returning to the notations of the Introduction, we can write the THDM type II Yukawa term as

$$-L = \eta_{ij}^u \bar{\psi}_L^{i\prime} \Phi_1 u_R^{j\prime} + \xi_{ij}^d \bar{\psi}_L^{i\prime} \tilde{\Phi}_2 d_R^{j\prime} + \text{H.c.}, \qquad (58)$$

where η_{ij}^u and ξ_{ij}^d are nondiagonal complex 3×3 matrices $(i, j = 1, 2, 3)$. As mentioned in the Introduction, in order to define the quark fields mass eigenstates the unitary mixing matrix V_{u^i,d^j} should be introduced in the Lagrangian terms of the charged Higgs boson interaction with quarks

$$\frac{M_d \,\mathrm{tg}\beta}{\sqrt{2}v} \bar{u}_L^i V_{u^i,d^j} d_R^j H^+ + \frac{M_u}{\sqrt{2}v\,\mathrm{tg}\beta} \bar{d}_L^i V_{u^i,d^j}^\dagger u_R^j H^-. \qquad (59)$$

If we extract the universal phase factor from the mixing matrix elements $V_{u^i,d^j} \to e^{i\varphi}|V_{u^i,d^j}|$, $V_{u^i,d^j}^\dagger \to e^{-i\varphi}|V_{u^i,d^j}|$, the Yukawa interaction terms take the form

$$\frac{M_d \,\mathrm{tg}\beta}{\sqrt{2}v} \bar{u}_L^i e^{i\varphi}|V_{u^i,d^j}| d_R^j H^+ + \frac{M_u}{\sqrt{2}v\,\mathrm{tg}\beta} \bar{d}_L^i e^{-i\varphi}|V_{u^i,d^j}| u_R^j H^-, \qquad (60)$$

so we can identify the universal phase φ as the relative phase ξ of the SU(2) doublets. The structure of this sort, however, does not look like the weak charged current sector mixing matrix, where the universal complex factor is not suitable to describe the effects of *CP* violation in meson decays.

ACKNOWLEDGMENTS

M. D. (MSU) is grateful to S. Heinemeyer, J. S. Lee, A. Pilaftsis, and G. Weiglein for useful discussions. He also thanks A. Semenov for help with LanHEP calculations. E. A. thanks the "Dynasty" Foundation and ICPPM for partial financial support. The work of M. D. and M. D. was partially supported by RFBR Grant No. 04-02-17448. The work of M. D. (MSU) was partially supported by INTAS 03-51-4007, UR 02.03.028, and NS 1685.2003.2.

APPENDIX

The decay width $h_i \to \gamma\gamma$, see Table V, can be written as

$$\Gamma(h_i \to \gamma\gamma) = \frac{M_{h_i}^3 \alpha^2}{256\pi^3 v^2}[|S_i^\gamma(M_{h_i})|^2 + |P_i^\gamma(M_{h_i})|^2], \qquad (A1)$$

where the scalar and the pseudoscalar factors are given by [15,26]

$$\begin{aligned} S_i^\gamma(M_{h_i}) = 2 \sum_{f=b,t,\tilde{\chi}_1^\pm,\tilde{\chi}_2^\pm} N_C Q_f^2 g_{h_i\bar{f}f}^S \frac{v}{m_f} F_{sf}(\tau_{if}) \\ - \sum_{\tilde{f}_j=\tilde{t}_1,\tilde{t}_2,\tilde{b}_1,\tilde{b}_2,\tilde{\tau}_1,\tilde{\tau}_2} N_C Q_f^2 g_{h_i\tilde{f}_j^*\tilde{f}_j} \frac{v^2}{2m_{\tilde{f}_j}^2} F_0(\tau_{i\tilde{f}_j}) \\ - g_{h_iVV} F_1(\tau_{iW}) - g_{h_iH^+H^-} \frac{v}{2M_{H^\pm}^2} F_0(\tau_{iH^\pm}), \end{aligned} \qquad (A2)$$

$$P_i^\gamma(M_{h_i}) = 2 \sum_{f=b,t,\tilde{\chi}_1^\pm,\tilde{\chi}_2^\pm} N_C Q_f^2 g_{h_i\bar{f}f}^P \frac{v}{m_f} F_{pf}(\tau_{if}).$$

$\tau_{ix} = M_{h_i}^2/4m_x^2$, $N_C = 3$ for squarks and $N_C = 1$ for stau and chargino, respectively. The vertex factors $g_{h_if\bar{f}}$ can be easily extracted from Table VI, where we list also the triple vertices with h_i and gauge bosons. The threshold corrections induced by the exchanges of gluinos and charginos [24,28] are not included in the following calculation.

[8] Recent discussion of the weak *CP* violation scenarios can be found in [22].

TABLE V. Higgs boson masses and their two-particle decay widths calculated in our approach and by means of CPsuperH and FeynHiggs packages. The parameter set $\alpha_{\rm EM}(m_Z) = 0.7812 \times 10^{-2}$, $\alpha_S(m_Z) = 0.1172$, $G_F = 1.174 \times 10^{-5}$ GeV^{-2}, $m_b = 3$ GeV, $\mathrm{tg}\beta = 5$, $M_{\rm SUSY} = 500$ GeV, $|A_t| = |A_b| = A = 1000$ GeV, $|\mu| = 2000$ GeV, and $m_{H^\pm} = 300$ GeV. Our results are calculated with $m_t = 175$ GeV. The choice of $A_{t,b}$, μ, and $M_{\rm SUSY}$ respects the constraints of the CPX scenario [20]. Our results at the one-loop, m_{h_i}, and with the two-loop QCD corrections to Yukawa terms [3], m'_{h_i}, are shown together with CPsuperH [15] and FeynHiggs [18] results. While the CPsuperH masses are calculated at the two-loop only, the one-loop FeynHiggs 2.2beta regime (program options 20030111) and the two-loop FeynHiggs 2.2beta regime (program options 20030211) are used. Higgs boson masses calculated in our approach at the two-loop omitting the nonleading D terms and the wave-function renormalization (wfr) in the effective λ_i are separately shown. In the calculation of decay widths Γ_{h_i} denotes our results with the λ_i taken at one-loop, sparticles contribution not included, Γ'_{h_i} denotes our results with the two-loop terms [3] introduced to λ_i, sparticles contribution again not included, Γ''_{h_i} denotes the decay widths in our case at the one-loop when sparticles contribution is included. The one-loop regime of FeynHiggs is used, while the CPsuperH numbers are at the two-loop.

	$\varphi = 0$	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
m_{h_1}, one-loop	115.4	118.7	125.9	131.4	130.7	125.2	122.0
m'_{h_1}, two-loop	111.8	113.9	118.4	121.8	121.4	118.3	116.5
m'_{h_1}, two-loop, no D and wfr	112.1	114.4	119.7	124.2	125.0	123.0	121.6
m_{H_1} [18], one-loop	115.8	118.8	125.5	130.2	123.2	98.2	78.0
m_{H_1} [18], two-loop	111.4	113.7	118.3	119.2	103.6	$\cdots$	$\cdots$
m_{H_1} [15], two-loop	106.8	109.0	113.9	117.4	114.9	105.7	99.4
m_{h_2}, one-loop	295.5	289.6	279.7	269.3	262.2	259.8	259.6
m'_{h_2}, two-loop	293.0	289.3	282.4	275.1	269.9	267.8	267.4
m'_{h_2}, two-loop, no D and wfr	294.4	291.0	283.9	276.2	270.6	268.1	267.6
m_{H_2} [18], one-loop	295.6	290.0	279.1	264.3	249.2	239.7	236.9
m_{H_2} [18], two-loop	290.4	286.5	275.8	260.7	246.1	$\cdots$	$\cdots$
m_{H_2} [15], two-loop	302.2	297.8	290.9	282.2	273.9	268.3	264.4
m_{h_3}, one-loop	297.1	299.5	300.4	299.9	298.8	297.6	297.1
m'_{h_3}, two-loop	296.1	297.3	297.5	296.7	295.2	293.6	293.0
m'_{h_3}, two-loop, no D and wfr	298.2	299.1	299.2	298.2	296.7	295.1	294.4
m_{H_3}, one-loop [18]	297.6	300.0	301.1	301.3	300.9	300.4	300.2
m_{H_3}, two-loop [18]	293.9	295.9	296.9	297.8	298.3	$\cdots$	$\cdots$
m_{H_3} [15], two-loop	302.3	304.4	305.0	304.5	303.5	302.4	302.0
$\Gamma_{h_1\to gg} \times 10^4$	1.378	1.529	1.907	2.220	2.101	1.707	1.516
$\Gamma'_{h_1\to gg} \times 10^4$	1.283	1.381	1.624	1.841	1.846	1.687	1.597
$\Gamma''_{h_1\to gg} \times 10^4$	2.103	2.355	3.024	3.643	3.397	2.412	1.889
$\Gamma_{H_1\to gg} \times 10^4$ [18]	2.040	2.187	2.462	2.225	0.863	0.037	0.110
$\Gamma_{H_1\to gg} \times 10^4$ [15]	1.878	1.964	2.107	1.961	1.262	0.503	0.263
$\Gamma_{h_1\to \gamma\gamma} \times 10^6$	7.703	8.593	10.981	13.313	12.953	10.645	9.508
$\Gamma'_{h_1\to \gamma\gamma} \times 10^6$	6.887	7.447	8.896	10.369	10.683	9.935	9.460
$\Gamma''_{h_1\to \gamma\gamma} \times 10^6$	7.470	8.371	10.832	13.321	12.945	10.274	8.887
$\Gamma_{H_1\to \gamma\gamma} \times 10^6$ [18]	6.373	7.058	9.038	11.217	9.983	5.336	3.021
$\Gamma_{H_1\to \gamma\gamma} \times 10^6$ [15]	5.796	6.287	7.605	8.996	8.969	7.223	6.101
$\Gamma_{h_1\to \mu\bar\mu} \times 10^5$	0.212	0.204	0.179	0.166	0.218	0.304	0.341
$\Gamma_{H_1\to \mu\bar\mu} \times 10^5$ [15]	0.157	0.152	0.141	0.137	0.175	0.240	0.269
$\Gamma_{h_1\to \tau\bar\tau} \times 10^3$	0.591	0.567	0.498	0.461	0.607	0.848	0.950
$\Gamma_{H_1\to \tau\bar\tau} \times 10^3$ [15]	0.435	0.423	0.391	0.382	0.485	0.668	0.746
$\Gamma_{h_1\to d\bar d} \times 10^{-7}$	0.202	0.194	0.170	0.158	0.208	0.290	0.325
$\Gamma_{H_1\to d\bar d} \times 10^7$ [15]	0.193	0.187	0.171	0.167	0.212	0.297	0.335
$\Gamma_{h_1\to s\bar s} \times 10^5$	0.744	0.713	0.626	0.580	0.764	1.066	1.195
$\Gamma_{H_1\to s\bar s} \times 10^5$ [15]	0.709	0.687	0.629	0.612	0.780	1.089	1.230
$\Gamma_{h_1\to c\bar c} \times 10^3$	0.083	0.086	0.093	0.097	0.095	0.088	0.083
$\Gamma_{H_1\to c\bar c} \times 10^3$ [15]	0.101	0.103	0.108	0.111	0.107	0.096	0.089
$\Gamma_{h_1\to b\bar b} \times 10^2$	0.504	0.483	0.424	0.393	0.518	0.724	0.810
$\Gamma_{H_1\to b\bar b} \times 10^2$ [15]	0.481	0.469	0.426	0.414	0.528	0.737	0.832

TABLE VI. Vertex factors of h_1, h_2, and h_3. This is a part of the complete set of vertices generated by LANHEP package [27].

Fields in the vertex	Vertex factor
$\bar{b}_{ap}\ b_{bq}\ h_1$	$-\frac{M_b}{c_\beta\cdot v}\delta_{pq}(c_\alpha\cdot a_{21}\cdot\delta_{ab}-s_\alpha\cdot a_{11}\cdot\delta_{ab}-s_\beta\cdot i\cdot a_{31}\cdot\gamma^5_{ab})$
$\bar{b}_{ap}\ b_{bq}\ h_2$	$-\frac{M_b}{c_\beta\cdot v}\delta_{pq}(c_\alpha\cdot a_{22}\cdot\delta_{ab}-s_\alpha\cdot a_{12}\cdot\delta_{ab}-s_\beta\cdot i\cdot a_{32}\cdot\gamma^5_{ab})$
$\bar{b}_{ap}\ b_{bq}\ h_3$	$-\frac{M_b}{c_\beta\cdot v}\delta_{pq}(c_\alpha\cdot a_{23}\cdot\delta_{ab}-s_\alpha\cdot a_{13}\cdot\delta_{ab}-s_\beta\cdot i\cdot a_{33}\cdot\gamma^5_{ab})$
$\bar{t}_{ap}\ b_{bq}\ H^+$	$-\frac{i\cdot\sqrt{2}\cdot Vtb}{s_{2\beta}\cdot v}\delta_{pq}(s^2_\beta\cdot M_b\cdot(1+\gamma^5)_{ab}+c^2_\beta\cdot M_t\cdot(1-\gamma^5)_{ab})$
$\bar{t}_{ap}\ t_{bq}\ h_1$	$-\frac{M_t}{s_\beta\cdot v}\delta_{pq}(s_\alpha\cdot a_{21}\cdot\delta_{ab}+c_\alpha\cdot a_{11}\cdot\delta_{ab}-c_\beta\cdot i\cdot a_{31}\cdot\gamma^5_{ab})$
$\bar{t}_{ap}\ t_{bq}\ h_2$	$-\frac{M_t}{s_\beta\cdot v}\delta_{pq}(s_\alpha\cdot a_{22}\cdot\delta_{ab}+c_\alpha\cdot a_{12}\cdot\delta_{ab}-c_\beta\cdot i\cdot a_{32}\cdot\gamma^5_{ab})$
$\bar{t}_{ap}\ t_{bq}\ h_3$	$-\frac{M_t}{s_\beta\cdot v}\delta_{pq}(s_\alpha\cdot a_{23}\cdot\delta_{ab}+c_\alpha\cdot a_{13}\cdot\delta_{ab}-c_\beta\cdot i\cdot a_{33}\cdot\gamma^5_{ab})$
$H^+\ W^-_\mu\ h_1$	$-\frac{1}{2}\frac{e}{s_w}(s_{\alpha-\beta}\cdot i\cdot a_{21}\cdot p^\mu_3+c_{\beta-\alpha}\cdot i\cdot a_{11}\cdot p^\mu_3-s_{\alpha-\beta}\cdot i\cdot a_{21}\cdot p^\mu_1-c_{\beta-\alpha}\cdot i\cdot a_{11}\cdot p^\mu_1+a_{31}\cdot p^\mu_3-a_{31}\cdot p^\mu_1)$
$H^+\ W^-_\mu\ h_2$	$-\frac{1}{2}\frac{e}{s_w}(s_{\alpha-\beta}\cdot i\cdot a_{22}\cdot p^\mu_3+c_{\beta-\alpha}\cdot i\cdot a_{12}\cdot p^\mu_3-s_{\alpha-\beta}\cdot i\cdot a_{22}\cdot p^\mu_1-c_{\beta-\alpha}\cdot i\cdot a_{12}\cdot p^\mu_1+a_{32}\cdot p^\mu_3-a_{32}\cdot p^\mu_1)$
$H^+\ W^-_\mu\ h_3$	$-\frac{1}{2}\frac{e}{s_w}(s_{\alpha-\beta}\cdot i\cdot a_{23}\cdot p^\mu_3+c_{\beta-\alpha}\cdot i\cdot a_{13}\cdot p^\mu_3-s_{\alpha-\beta}\cdot i\cdot a_{23}\cdot p^\mu_1-c_{\beta-\alpha}\cdot i\cdot a_{13}\cdot p^\mu_1+a_{33}\cdot p^\mu_3-a_{33}\cdot p^\mu_1)$
$W^+_\mu\ W^-_\nu\ h_1$	$\frac{1}{2}\frac{e^2\cdot v}{s^2_w}g^{\mu\nu}(c_{\beta-\alpha}a_{21}-s_{\alpha-\beta}a_{11})$
$W^+_\mu\ W^-_\nu\ h_2$	$\frac{1}{2}\frac{e^2\cdot v}{s^2_w}g^{\mu\nu}(c_{\beta-\alpha}a_{22}-s_{\alpha-\beta}a_{12})$
$W^+_\mu\ W^-_\nu\ h_3$	$\frac{1}{2}\frac{e^2\cdot v}{s^2_w}g^{\mu\nu}(c_{\beta-\alpha}a_{23}-s_{\alpha-\beta}a_{13})$
$Z_\mu\ Z_\nu\ h_1$	$2\frac{e^2\cdot v}{s^2_{2w}}g^{\mu\nu}(c_{\beta-\alpha}a_{21}-s_{\alpha-\beta}a_{11})$
$Z_\mu\ Z_\nu\ h_2$	$2\frac{e^2\cdot v}{s^2_{2w}}g^{\mu\nu}(c_{\beta-\alpha}a_{22}-s_{\alpha-\beta}a_{12})$
$Z_\mu\ Z_\nu\ h_3$	$2\frac{e^2\cdot v}{s^2_{2w}}g^{\mu\nu}(c_{\beta-\alpha}a_{23}-s_{\alpha-\beta}a_{13})$

The factors F_{sf}, F_{pf}, F_0, and F_1 [29] are expressed by means of the dimensionless function $f(\tau)$

$$\begin{aligned} F_{sf}(\tau) &= \tau^{-1}[1+(1-\tau^{-1})f(\tau)],\\ F_{pf}(\tau) &= \tau^{-1}f(\tau),\\ F_0(\tau) &= \tau^{-1}[-1+\tau^{-1}f(\tau)],\\ F_1(\tau) &= 2+3\tau^{-1}+3\tau^{-1}(2-\tau^{-1})f(\tau), \end{aligned} \tag{A3}$$

with an integral representation

$$\begin{aligned} f(\tau) &= -\frac{1}{2}\int_0^1\frac{dy}{y}\ln[1-4\tau y(1-y)]\\ &= \begin{cases} \arcsin^2(\sqrt{\tau}): & \tau\le 1,\\ -\frac{1}{4}[\ln(\frac{\sqrt{\tau}+\sqrt{\tau-1}}{\sqrt{\tau}-\sqrt{\tau-1}})-i\pi]^2: & \tau\ge 1. \end{cases} \end{aligned} \tag{A4}$$

QCD corrections in the large mass limit can be found in [30]

$$J^\gamma_q = 1-\frac{\alpha_s(M^2_{h_i})}{\pi},\qquad J^\gamma_{\tilde{q}} = 1+\frac{8\alpha_s(M^2_{h_i})}{3\pi}. \tag{A5}$$

Chargino contributions depend on the couplings

$$\begin{aligned} g^S_{h_1\tilde{\chi}^+_1\tilde{\chi}^-_1} &= V_{11}U_{12}GS_1+V_{12}U_{11}GS_2,\\ g^P_{h_1\tilde{\chi}^+_1\tilde{\chi}^-_1} &= V_{11}U_{12}GP_1+V_{12}U_{11}GP_2, \end{aligned} \tag{A6}$$

$$\begin{aligned} g^S_{h_1\tilde{\chi}^+_2\tilde{\chi}^-_2} &= V_{21}U_{22}GS_1+V_{22}U_{21}GS_2,\\ g^P_{h_1\tilde{\chi}^+_2\tilde{\chi}^-_2} &= V_{21}U_{22}GP_1+V_{22}U_{21}GP_2, \end{aligned} \tag{A7}$$

for h_1 we have $GS_1 = -\sin\alpha a_{11}+\cos\alpha a_{21}$, $GS_2 = \cos\alpha a_{11}+\sin\alpha a_{21}$, $GP_1 = \sin\beta a_{31}$, $GP_2 = \cos\beta a_{31}$, and the matrix elements U_{ij}

$$U_{12} = U_{21} = \frac{1}{\sqrt{2}}\sqrt{1+\frac{M^2_2-\mu^2-2m^2_W\cos2\beta}{W}}, \tag{A8}$$

$$U_{22} = -U_{11} = \frac{\varepsilon_B}{\sqrt{2}}\sqrt{1-\frac{M^2_2-\mu^2-2m^2_W\cos2\beta}{W}}, \tag{A9}$$

$$V_{21} = -V_{12} = \frac{\varepsilon_A}{\sqrt{2}}\sqrt{1+\frac{M^2_2-\mu^2+2m^2_W\cos2\beta}{W}}, \tag{A10}$$

$$V_{22} = V_{11} = \frac{4}{\sqrt{2}}\sqrt{1-\frac{M^2_2-\mu^2+2m^2_W\cos2\beta}{W}}, \tag{A11}$$

where

$$W = \sqrt{(M^2_2+\mu^2+2m^2_W)^2-4(M_2\cdot\mu-m^2_W\sin2\beta)^2}, \tag{A12}$$

$$\varepsilon_A = \mathrm{sgn}(M_2 \sin\beta + \mu \cos\beta),$$
$$\varepsilon_B = \mathrm{sgn}(M_2 \cos\beta + \mu \sin\beta). \tag{A13}$$

Chargino masses are given by

$$m^2_{\tilde{\chi}^+_1} = \frac{1}{2}\Big|\sqrt{(M_2^2 - \mu^2)^2 + 2m_W^2(1+\sin2\beta)} - \sqrt{(M_2^2 + \mu^2)^2 + 2m_W^2(1-\sin2\beta)}\Big|, \tag{A14}$$

$$m^2_{\tilde{\chi}^+_2} = \frac{1}{2}\Big(\sqrt{(M_2^2 - \mu^2)^2 + 2m_W^2(1+\sin2\beta)} + \sqrt{(M_2^2 + \mu^2)^2 + 2m_W^2(1-\sin2\beta)}\Big). \tag{A15}$$

Sfermion contributions depend on the couplings

$$g_{h_1\tilde{f}^*_j\tilde{f}_j} = \frac{1}{v}(\Gamma^{\alpha\tilde{f}^*\tilde{f}})_{\beta\gamma}a_{\alpha 1}U^{\tilde{f}*}_{\beta j}U^{\tilde{f}}_{\gamma j},$$

$\alpha = (1, 2, a)$, $\beta, \gamma = L, R$, $i = (h_1, h_2, h_3) = (1, 2, 3)$, $j, k = 1, 2$,

$$U^{\tilde{f}} = \begin{pmatrix} \cos\theta_{\tilde{f}} & -\sin\theta_{\tilde{f}}e^{-i\phi_{\tilde{f}}} \\ \sin\theta_{\tilde{f}}e^{+i\phi_{\tilde{f}}} & \cos\theta_{\tilde{f}} \end{pmatrix}, \tag{A16}$$

$$\Gamma^{1\tilde{f}^*\tilde{f}} = -\Gamma^{\phi_1\tilde{f}^*\tilde{f}}\sin\alpha + \Gamma^{\phi_2\tilde{f}^*\tilde{f}}\cos\alpha,$$

$$\Gamma^{2\tilde{f}^*\tilde{f}} = \Gamma^{\phi_1\tilde{f}^*\tilde{f}}\cos\alpha + \Gamma^{\phi_2\tilde{f}^*\tilde{f}}\sin\alpha,$$

where

$$\begin{aligned}
\Gamma^{a\tilde{b}^*\tilde{b}} &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & ih_b^*(s_\beta A_b^* + c_\beta\mu) \\ -ih_b(s_\beta A_b + c_\beta\mu^*) & 0 \end{pmatrix},\\
\Gamma^{\phi_1\tilde{b}^*\tilde{b}} &= \begin{pmatrix} -|h_b|^2 v c_\beta + \frac{1}{4}(g_2^2 + \frac{1}{3}g_1^2)vc_\beta & -\frac{1}{\sqrt{2}}h_b^*A_b^* \\ -\frac{1}{\sqrt{2}}h_bA_b & -|h_b|^2vc_\beta + \frac{1}{6}g_1^2vc_\beta \end{pmatrix},\\
\Gamma^{\phi_2\tilde{b}^*\tilde{b}} &= \begin{pmatrix} -\frac{1}{4}(g_2^2 + \frac{1}{3}g_1^2)vs_\beta & \frac{1}{\sqrt{2}}h_b^*\mu \\ \frac{1}{\sqrt{2}}h_b\mu^* & -\frac{1}{6}g_1^2vs_\beta \end{pmatrix},\\
\Gamma^{a\tilde{t}^*\tilde{t}} &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & ih_t^*(c_\beta A_t^* + s_\beta\mu) \\ -ih_t(c_\beta A_t + s_\beta\mu^*) & 0 \end{pmatrix},\\
\Gamma^{\phi_1\tilde{t}^*\tilde{t}} &= \begin{pmatrix} -\frac{1}{4}(g_2^2 - \frac{1}{3}g_1^2)vc_\beta & \frac{1}{\sqrt{2}}h_t^*\mu \\ \frac{1}{\sqrt{2}}h_t\mu^* & -\frac{1}{3}g_1^2vc_\beta \end{pmatrix},\\
\Gamma^{\phi_2\tilde{t}^*\tilde{t}} &= \begin{pmatrix} -|h_t|^2vs_\beta + \frac{1}{4}(g_2^2 - \frac{1}{3}g_1^2)vs_\beta & -\frac{1}{\sqrt{2}}h_t^*A_t^* \\ -\frac{1}{\sqrt{2}}h_tA_t & -|h_t|^2vs_\beta + \frac{1}{3}g_1^2vs_\beta \end{pmatrix},\\
\Gamma^{a\tilde{\tau}^*\tilde{\tau}} &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & ih_\tau^*(s_\beta A_\tau^* + c_\beta\mu) \\ -ih_\tau(s_\beta A_\tau + c_\beta\mu^*) & 0 \end{pmatrix},\\
\Gamma^{\phi_1\tilde{\tau}^*\tilde{\tau}} &= \begin{pmatrix} -|h_\tau|^2vc_\beta + \frac{1}{4}(g_2^2 - g_1^2)vc_\beta & -\frac{1}{\sqrt{2}}h_\tau^*A_\tau^* \\ -\frac{1}{\sqrt{2}}h_\tau A_\tau & -|h_\tau|^2vc_\beta + \frac{1}{2}g_1^2vc_\beta \end{pmatrix},\\
\Gamma^{\phi_2\tilde{\tau}^*\tilde{\tau}} &= \begin{pmatrix} -\frac{1}{4}(g_2^2 - g_1^2)vs_\beta & \frac{1}{\sqrt{2}}h_\tau^*\mu \\ \frac{1}{\sqrt{2}}h_\tau\mu^* & -\frac{1}{2}g_1^2vs_\beta \end{pmatrix}.
\end{aligned} \tag{A17}$$

In these formulas $h_{t,b,\tau}$ are real variables. Sfermion masses are given by

$$m^2_{\tilde{q}(\tilde{l})_{1,2}} = \frac{1}{2}\Big(m^2_{\tilde{q}(\tilde{l})L} + m^2_{\tilde{q}(\tilde{l})R} \mp \sqrt{(m^2_{\tilde{q}(\tilde{l})L} - m^2_{\tilde{q}(\tilde{l})R})^2 + 4|a_{q(l)}|^2m^2_{q(l)}}\Big), \tag{A18}$$

where

$$m^2_{\tilde{q}L} = M^2_{\tilde{Q}_3} + m_q^2 + c_{2\beta}m_Z^2(T_z^q - Q_qs_W^2),$$

$$m^2_{\tilde{q}R} = M^2_{\tilde{R}_3} + m^2_q + c_{2\beta} m^2_Z Q_q s^2_W,$$

$$a_q m_q = h_q v_q (A_q - \mu^* R_q)/\sqrt{2},$$

$$m^2_{\tilde{l}L} = M^2_{\tilde{L}_3} + m^2_\tau + c_{2\beta} m^2_Z (s^2_W - 1/2),$$

$$m^2_{\tilde{l}R} = M^2_{\tilde{E}_3} + m^2_\tau - c_{2\beta} m^2_Z s^2_W,$$

$$a_l m_l = h_\tau v_1 (A_\tau - \mu^* \tan\beta)/\sqrt{2}.$$

Here the Yukawa couplings of quarks h_q, $q = t, b$, $R = U, D$, $T^t_z = -T^b_z = 1/2$, $Q_t = 2/3$, $Q_b = -1/3$, $R_b = \mathrm{tg}\beta = v_2/v_1$, $R_t = \mathrm{ctg}\beta$, the mixing angles are

$$\cos\theta_{\tilde{q}(\tilde{l})} = \frac{-|a_{q(l)}| m_{q(l)}}{\sqrt{(m^2_{\tilde{q}(\tilde{l})L} - m^2_{\tilde{q}(\tilde{l})_1})^2 + |a_{q(l)}|^2 m^2_{q(l)}}}, \qquad \sin\theta_{\tilde{q}(\tilde{l})} = \frac{m^2_{\tilde{q}(\tilde{l})L} - m^2_{\tilde{q}(\tilde{l})_1}}{\sqrt{(m^2_{\tilde{q}(\tilde{l})L} - m^2_{\tilde{q}(\tilde{l})_1})^2 + |a_{q(l)}|^2 m^2_{q(l)}}}. \tag{A19}$$

A charged Higgs boson contribution depends on the effective triple self-couplings $g_{H^+H^-h_i}$ which can be written as

$$\begin{aligned} g_{H^+H^-h_1} = &-\frac{1}{2}\frac{1}{s_{2\beta}{}^2 \cdot v}(4(s_\alpha \cdot c_\beta{}^3 + c_\alpha \cdot s_\beta{}^3) s_{2\beta} m_H{}^2 a_{21} - 8 c_\beta{}^2 s_{\beta+\alpha} a_{21} \,\mathrm{Re}\mu^2_{12} - 8 s_{\beta+\alpha} s_\beta{}^4 a_{21} \,\mathrm{Re}\mu^2_{12} \\ &- 8 c_{\beta+\alpha} c_\beta{}^2 a_{11} \,\mathrm{Re}\mu^2_{12} - 8 c_{\beta+\alpha} s_\beta{}^4 a_{11} \,\mathrm{Re}\mu^2_{12} - c_\beta{}^2 s_{2\beta}{}^2 s_{\beta+\alpha} a_{21} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\beta{}^2 s_{\alpha-\beta} a_{21} \,\mathrm{Re}\lambda_6 v^2 \\ &+ 4 c_\beta{}^4 s_\alpha s_\beta{}^3 a_{11} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\alpha c_\beta{}^3 s_\beta{}^4 a_{11} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\beta{}^2 s_\alpha s_\beta a_{11} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\alpha c_\beta{}^5 a_{11} \,\mathrm{Re}\lambda_6 v^2 \\ &+ 4(c_\alpha \cdot c_\beta{}^3 - s_\alpha \cdot s_\beta{}^3) s_{2\beta} m_h{}^2 a_{11} - s_{2\beta}{}^2 s_{\beta+\alpha} s_\beta{}^2 a_{21} \,\mathrm{Re}\lambda_7 v^2 - 4 s_{\alpha-\beta} s_\beta{}^2 a_{21} \,\mathrm{Re}\lambda_7 v^2 + 4 c_\beta{}^2 s_\alpha s_\beta{}^5 a_{11} \,\mathrm{Re}\lambda_7 v^2 \\ &- 4 c_\alpha c_\beta s_\beta{}^2 a_{11} \,\mathrm{Re}\lambda_7 v^2 - 4 s_\alpha s_\beta{}^3 a_{11} \,\mathrm{Re}\lambda_7 v^2 - 4 c_\alpha c_\beta{}^3 s_\beta{}^4 a_{11} \,\mathrm{Re}\lambda_7 v^2 + 4 c_{\beta-\alpha} s_{2\beta}{}^2 m_{H^\pm}{}^2 a_{21} \\ &- 4 s_{2\beta}{}^2 s_{\alpha-\beta} m_{H^\pm}{}^2 a_{11} - s_{2\beta}{}^3 s_{\beta+\alpha} m_A{}^2 a_{21} - c_{\beta+\alpha} s_{2\beta}{}^3 m_A{}^2 a_{11} - s_{2\beta}{}^3 s_{\beta+\alpha} a_{21} \,\mathrm{Re}\,\lambda_5 v^2 - c_{\beta+\alpha} s_{2\beta}{}^3 a_{11} \,\mathrm{Re}\lambda_5 v^2 \\ &+ 8 c_\beta{}^3 s_\beta{}^3 a_{31} \,\mathrm{Im}\lambda_5 v^2 - 8 c_\beta{}^2 s_\beta{}^4 a_{31} \,\mathrm{Im}\lambda_6 v^2 - 8 c_\beta{}^4 s_\beta{}^2 a_{31} \,\mathrm{Im}\lambda_7 v^2), \end{aligned}$$

$$\begin{aligned} g_{H^+H^-h_2} = &-\frac{1}{2}\frac{1}{s_{2\beta}{}^2 \cdot v}(4(s_\alpha \cdot c_\beta{}^3 + c_\alpha \cdot s_\beta{}^3) s_{2\beta} m_H{}^2 a_{22} - 8 c_\beta{}^2 s_{\beta+\alpha} a_{22} \,\mathrm{Re}\mu^2_{12} - 8 s_{\beta+\alpha} s_\beta{}^4 a_{22} \,\mathrm{Re}\mu^2_{12} \\ &- 8 c_{\beta+\alpha} c_\beta{}^2 a_{12} \,\mathrm{Re}\mu^2_{12} - 8 c_{\beta+\alpha} s_\beta{}^4 a_{12} \,\mathrm{Re}\mu^2_{12} - c_\beta{}^2 s_{2\beta}{}^2 s_{\beta+\alpha} a_{22} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\beta{}^2 s_{\alpha-\beta} a_{22} \,\mathrm{Re}\lambda_6 v^2 \\ &+ 4 c_\beta{}^4 s_\alpha s_\beta{}^3 a_{12} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\alpha c_\beta{}^3 s_\beta{}^4 a_{12} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\beta{}^2 s_\alpha s_\beta a_{12} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\alpha c_\beta{}^5 a_{12} \,\mathrm{Re}\lambda_6 v^2 \\ &+ 4(c_\alpha \cdot c_\beta{}^3 - s_\alpha \cdot s_\beta{}^3) s_{2\beta} m_h{}^2 a_{12} - s_{2\beta}{}^2 s_{\beta+\alpha} s_\beta{}^2 a_{22} \,\mathrm{Re}\lambda_7 v^2 - 4 s_{\alpha-\beta} s_\beta{}^2 a_{22} \,\mathrm{Re}\lambda_7 v^2 + 4 c_\beta{}^2 s_\alpha s_\beta{}^5 a_{12} \,\mathrm{Re}\lambda_7 v^2 \\ &- 4 c_\alpha c_\beta s_\beta{}^2 a_{12} \,\mathrm{Re}\lambda_7 v^2 - 4 s_\alpha s_\beta{}^3 a_{12} \,\mathrm{Re}\lambda_7 v^2 - 4 c_\alpha c_\beta{}^3 s_\beta{}^4 a_{12} \,\mathrm{Re}\lambda_7 v^2 + 4 c_{\beta-\alpha} s_{2\beta}{}^2 m_{H^\pm}{}^2 a_{22} \\ &- 4 s_{2\beta}{}^2 s_{\alpha-\beta} m_{H^\pm}{}^2 a_{12} - s_{2\beta}{}^3 s_{\beta+\alpha} m_A{}^2 a_{22} - c_{\beta+\alpha} s_{2\beta}{}^3 m_A{}^2 a_{12} - s_{2\beta}{}^3 s_{\beta+\alpha} a_{22} \,\mathrm{Re}\lambda_5 v^2 - c_{\beta+\alpha} s_{2\beta}{}^3 a_{12} \,\mathrm{Re}\lambda_5 v^2 \\ &+ 8 c_\beta{}^3 s_\beta{}^3 a_{32} \,\mathrm{Im}\lambda_5 v^2 - 8 c_\beta{}^2 s_\beta{}^4 a_{32} \,\mathrm{Im}\lambda_6 v^2 - 8 c_\beta{}^4 s_\beta{}^2 a_{32} \,\mathrm{Im}\lambda_7 v^2), \end{aligned}$$

$$\begin{aligned} g_{H^+H^-h_3} = &-\frac{1}{2}\frac{1}{s_{2\beta}{}^2 \cdot v}(4(s_\alpha \cdot c_\beta{}^3 + c_\alpha \cdot s_\beta{}^3) s_{2\beta} m_H{}^2 a_{23} - 8 c_\beta{}^2 s_{\beta+\alpha} a_{23} \,\mathrm{Re}\mu^2_{12} - 8 s_{\beta+\alpha} s_\beta{}^4 a_{23} \,\mathrm{Re}\mu^2_{12} \\ &- 8 c_{\beta+\alpha} c_\beta{}^2 a_{13} \,\mathrm{Re}\mu^2_{12} - 8 c_{\beta+\alpha} s_\beta{}^4 a_{13} \,\mathrm{Re}\mu^2_{12} - c_\beta{}^2 s_{2\beta}{}^2 s_{\beta+\alpha} a_{23} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\beta{}^2 s_{\alpha-\beta} a_{23} \,\mathrm{Re}\lambda_6 v^2 \\ &+ 4 c_\beta{}^4 s_\alpha s_\beta{}^3 a_{13} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\alpha c_\beta{}^3 s_\beta{}^4 a_{13} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\beta{}^2 s_\alpha s_\beta a_{13} \,\mathrm{Re}\lambda_6 v^2 + 4 c_\alpha c_\beta{}^5 a_{13} \,\mathrm{Re}\lambda_6 v^2 \\ &+ 4(c_\alpha \cdot c_\beta{}^3 - s_\alpha \cdot s_\beta{}^3) s_{2\beta} m_h{}^2 a_{13} - s_{2\beta}{}^2 s_{\beta+\alpha} s_\beta{}^2 a_{23} \,\mathrm{Re}\lambda_7 v^2 - 4 s_{\alpha-\beta} s_\beta{}^2 a_{23} \,\mathrm{Re}\lambda_7 v^2 + 4 c_\beta{}^2 s_\alpha s_\beta{}^5 a_{13} \,\mathrm{Re}\lambda_7 v^2 \\ &- 4 c_\alpha c_\beta s_\beta{}^2 a_{13} \,\mathrm{Re}\lambda_7 v^2 - 4 s_\alpha s_\beta{}^3 a_{13} \,\mathrm{Re}\lambda_7 v^2 - 4 c_\alpha c_\beta{}^3 s_\beta{}^4 a_{13} \,\mathrm{Re}\lambda_7 v^2 + 4 c_{\beta-\alpha} s_{2\beta}{}^2 m_{H^\pm}{}^2 a_{23} \\ &- 4 s_{2\beta}{}^2 s_{\alpha-\beta} m_{H^\pm}{}^2 a_{13} - s_{2\beta}{}^3 s_{\beta+\alpha} m_A{}^2 a_{23} - c_{\beta+\alpha} s_{2\beta}{}^3 m_A{}^2 a_{13} - s_{2\beta}{}^3 s_{\beta+\alpha} a_{23} \,\mathrm{Re}\lambda_5 v^2 - c_{\beta+\alpha} s_{2\beta}{}^3 a_{13} \,\mathrm{Re}\lambda_5 v^2 \\ &+ 8 c_\beta{}^3 s_\beta{}^3 a_{33} \,\mathrm{Im}\lambda_5 v^2 - 8 c_\beta{}^2 s_\beta{}^4 a_{33} \,\mathrm{Im}\lambda_6 v^2 - 8 c_\beta{}^4 s_\beta{}^2 a_{33} \,\mathrm{Im}\lambda_7 v^2). \end{aligned}$$

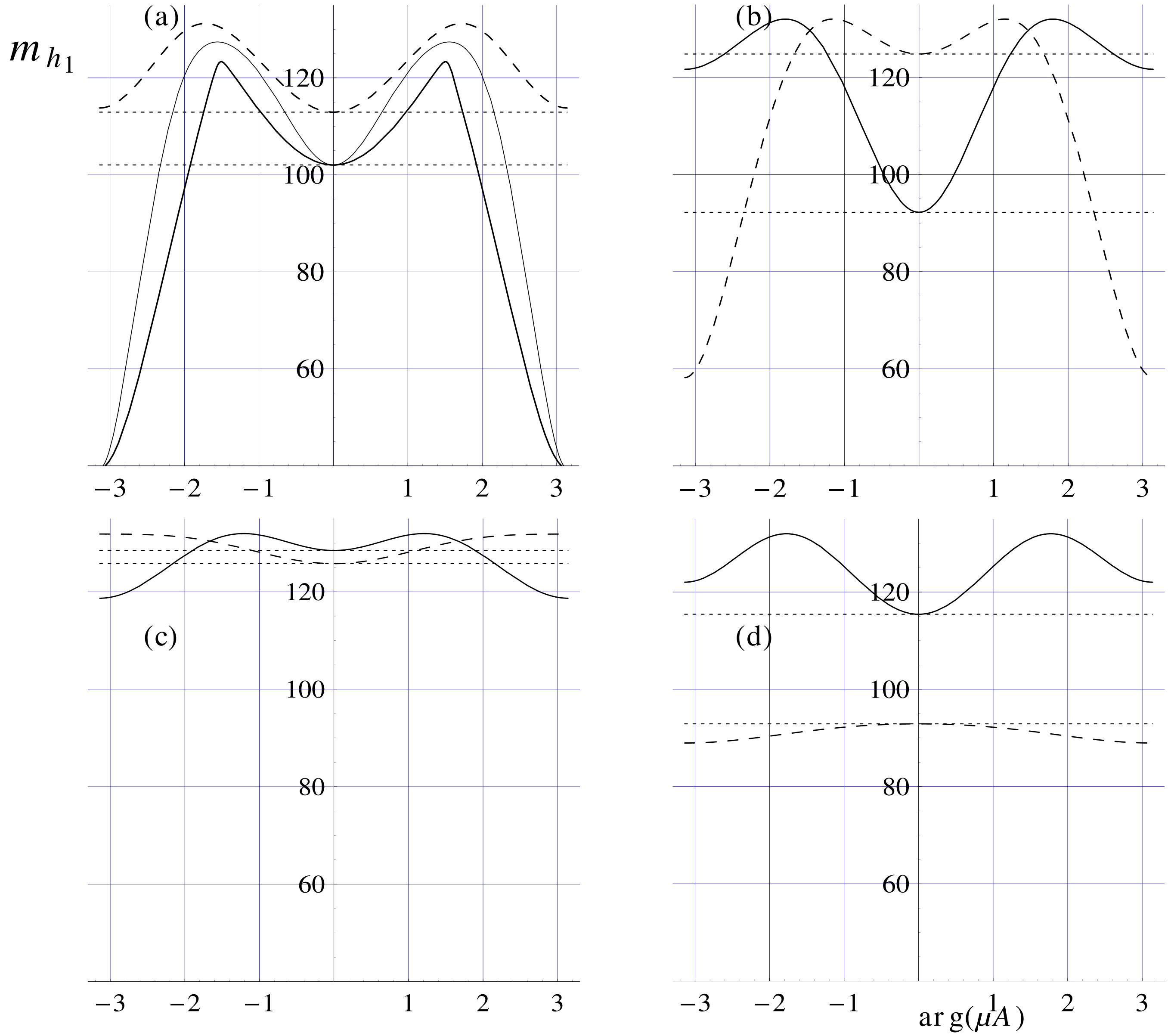

FIG. 5 (color online). Light Higgs boson mass $m_{h_1}(\varphi)$ (GeV) vs arg(μA) in various regions of the MSSM parameter space. Horizontal dotted lines indicate the h_1 mass in the CP-conserving limit ($m_{h_1} = m_h$). (a) tg$\beta = 5$, $M_{\mathrm{SUSY}} = 0.5$ TeV, $A_t = A_b = 1$ TeV, and $\mu = 2$ TeV. Solid line $m_{H^\pm} = 180$ GeV; dashed line $m_{H^\pm} = 250$ GeV. Thin solid line denotes $m_h(\varphi)$. (b) tg$\beta = 5$, $m_{H^\pm} = 300$ GeV, and $\mu = 2$ TeV; solid line $A_t = A_b = -1.2$ TeV; dashed line $A_t = A_b = 1.3$ TeV. (c) tg$\beta = 5$, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line $\mu = -1.6$ TeV; dashed line $\mu = 0.7$ TeV. (d) $\mu = 2$ TeV, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line tg$\beta = 5$; dashed line tg$\beta = 40$.

This representation uses the mass basis for CP-even/odd Higgs fields (h, H, A), then rotated by matrix a_{ij} in the three-dimensional (h, H, A) isospace, and for this reason includes m_h, m_H, m_A, and $m_{H^\pm}$ of the CP-conserving limit, calculated with one-loop MSSM corrections from the squark sector. In this sense the vertices above are MSSM effective one-loop Higgs self-interaction vertices. If the imaginary parts in these vertices are set to zero they are reduced to the self-interaction vertices of the CP-conserving limit, when m_h, m_H, m_A, and $m_{H\pm}$ are the masses of physical states. Various extremal cases (decoupling limits) are clearly seen. Equivalent representation of the triple couplings can be written in the λ_i basis (see details on the representations in mass and λ_i basis in [5]). For example,

$$g_{h_1 H^+ H^-} = -v \sum_{\alpha=1}^{3} a_{\alpha 1} g_{\alpha H^+ H^-},$$

where

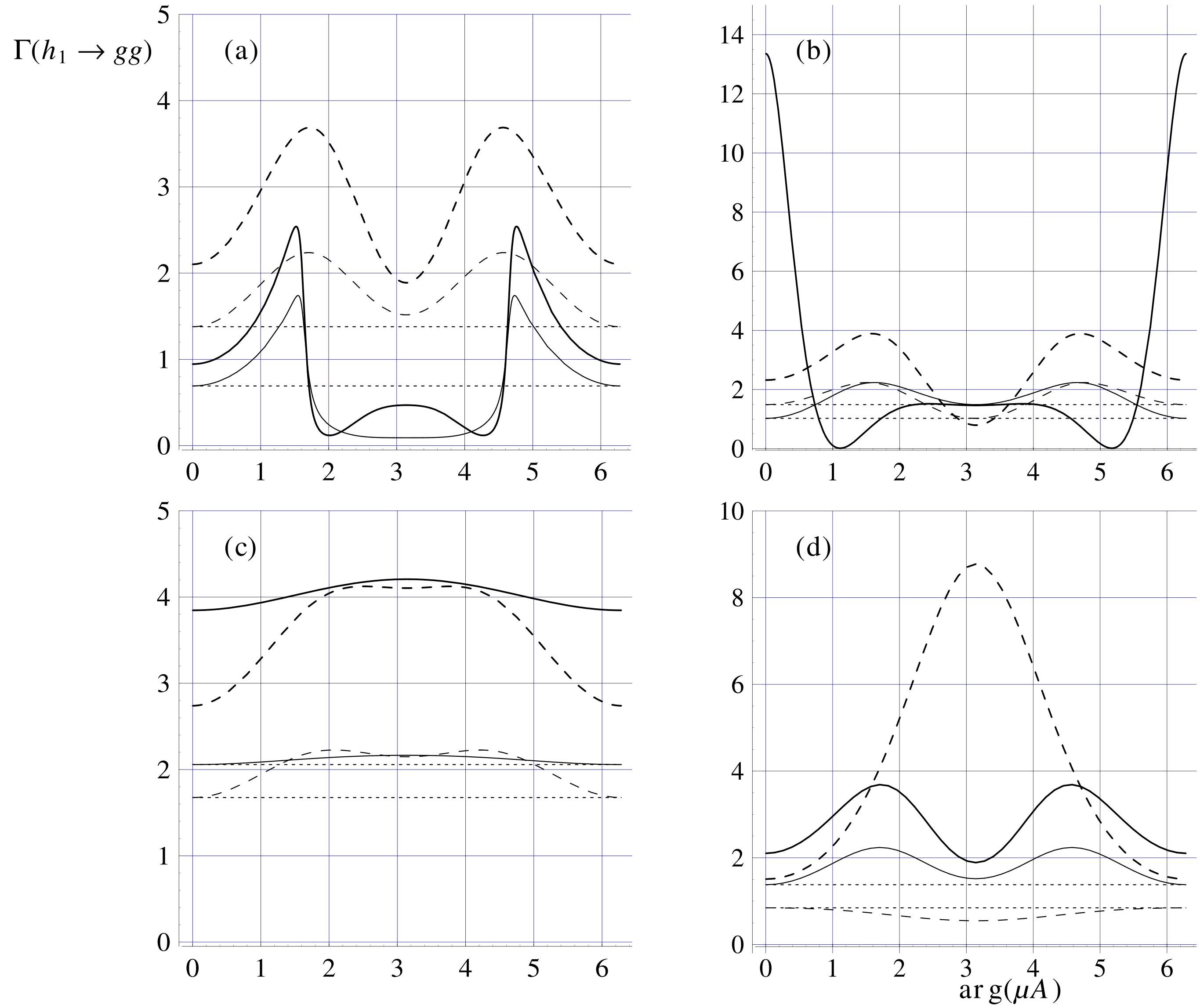

FIG. 6 (color online). The decay width $\Gamma(h_1 \to gg) \times 10^4$ (GeV) at $M_{\mathrm{SUSY}} = 500$ GeV. Dotted lines show Γ_h in the CP-conserving limit $\varphi = 0$. Thin solid or dashed lines are only for SM contributions, thick solid or dashed lines show SM and sparticle contributions with the K factor included. (a) $\mathrm{tg}\beta = 5$, $A_t = A_b = 1$ TeV, and $\mu = 2$ TeV; solid line $m_{H^\pm} = 190$ GeV; dashed line $m_{H^\pm} = 300$ GeV. (b) $\mathrm{tg}\beta = 5$, $m_{H^\pm} = 300$ GeV, and $\mu = 2$ TeV; solid line $A_t = A_b = -1.1$ TeV; dashed line $A_t = A_b = 1.1$ TeV. (c) $\mathrm{tg}\beta = 5$, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line $\mu = 0.2$ TeV; dashed line $\mu = 1.2$ TeV. (d) $\mu = 2$ TeV, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line $\mathrm{tg}\beta = 5$; dashed line $\mathrm{tg}\beta = 40$.

$$\begin{aligned} g_{1H^+H^-} ={}& \mathrm{Re}\Delta\lambda_5 s_\beta c_\beta c_{\alpha+\beta} - \mathrm{Re}\Delta\lambda_6 c_\alpha s_\beta^2 c_\beta + \mathrm{Re}\Delta\lambda_6 s_\alpha s_\beta^3 \\ &+ \mathrm{Re}\Delta\lambda_7 c_\beta (s_\alpha s_\beta c_\beta - c_\alpha(c_\beta^2 - 2s_\beta^2)) \\ &- \mathrm{Re}\Delta\lambda_6 s_\alpha s_{2\beta} c_\beta - 2s_\alpha s_\beta^2 c_\beta \lambda_1 + 2c_\alpha s_\beta c_\beta^2 \lambda_2 \\ &- c_\beta^3 s_\alpha \lambda_3 + c_\alpha s_\beta^3 \lambda_3 - c_\alpha c_\beta^2 s_\beta \lambda_4 + c_\beta s_\alpha s_\beta^2 \lambda_4, \end{aligned}$$

$$\begin{aligned} g_{2H^+H^-} ={}& \mathrm{Re}\Delta\lambda_5 s_\beta c_\beta s_{\alpha+\beta} + 2\,\mathrm{Re}\Delta\lambda_6 c_\alpha s_\beta c_\beta^2 \\ &- \mathrm{Re}\Delta\lambda_6 c_\alpha s_\beta^3 - \mathrm{Re}\Delta\lambda_6 s_\alpha s_\beta^2 c_\beta \\ &- \mathrm{Re}\Delta\lambda_7 c_\beta (c_\alpha s_\beta c_\beta + s_\alpha(c_\beta^2 - 2s_\beta^2)) \\ &+ 2c_\alpha s_\beta^2 c_\beta \lambda_1 + 2s_\alpha s_\beta c_\beta^2 \lambda_2 + c_\alpha c_\beta^3 \lambda_3 \\ &+ s_\alpha s_\beta^3 \lambda_3 - c_\alpha s_\beta^2 c_\beta \lambda_4 - s_\alpha s_\beta c_\beta^2 \lambda_4, \end{aligned}$$

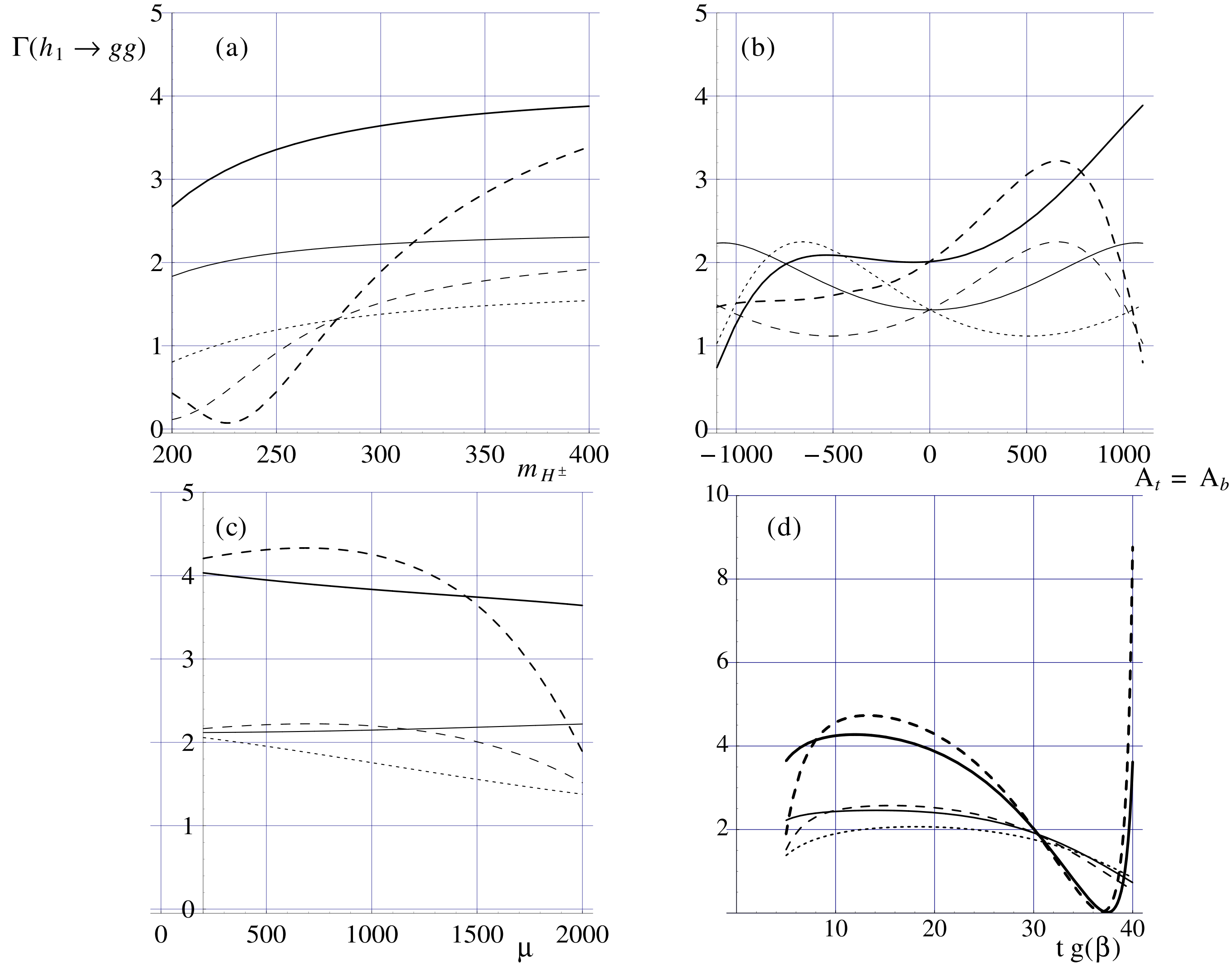

FIG. 7 (color online). The decay width $\Gamma(h_1 \to gg) \times 10^4$ (GeV) at $M_{\mathrm{SUSY}} = 500$ GeV. Dotted lines show Γ_h in the CP-conserving limit $\varphi = 0$. Thin solid or dashed lines are only for SM contributions, thick solid or dashed lines show SM and sparticle contributions with the K factor included. (a) $\mathrm{tg}\beta = 5$, $A_t = A_b = 1$ TeV, and $\mu = 2$ TeV; solid line $\varphi = \pi/2$; dashed line $\varphi = \pi$. (b) $\mathrm{tg}\beta = 5$, $m_{H^\pm} = 300$ GeV, and $\mu = 2$ TeV; solid line $\varphi = \pi/2$; dashed line $\varphi = \pi$. (c) $\mathrm{tg}\beta = 5$, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line $\varphi = \pi/2$; dashed line $\varphi = \pi$. (d) $\mu = 2$ TeV, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line $\varphi = \pi/2$; dashed line $\varphi = \pi$.

$$g_{3H^+H^-} = c_\beta^2 \,\mathrm{Im}\Delta\lambda_7 - s_\beta c_\beta \,\mathrm{Im}\Delta\lambda_5 + s_\beta^2 \,\mathrm{Im}\Delta\lambda_6.$$

In this representation the scalar masses of the CP-conserving limit do not explicitly participate. The magnitude of the coupling $g_{H^+H^-h_1}$ is shown in Fig. 1.

The decay width $h_i \to gg$ has the form

$$\Gamma(h_i \to gg) = \frac{M_{h_i}^3 \alpha_S^2}{32\pi^3 v^2}[K_H^g |S_i^g(M_{h_i})|^2 + K_A^g |P_i^g(M_{h_i})|^2], \tag{A20}$$

where

$$S_i^g(M_{h_i}) = \sum_{f=b,t} g^S_{h_i f\bar{f}} \frac{v}{m_f} F_{sf}(\tau_{if}) - \sum_{\tilde{f}_j=\tilde{t}_1,\tilde{t}_2,\tilde{b}_1,\tilde{b}_2} g_{h_i \tilde{f}_j^* \tilde{f}_j} \frac{v^2}{4m_{\tilde{f}_j}^2} F_0(\tau_{i\tilde{f}_j}), \tag{A21}$$

$$P_i^g(M_{h_i}) = \sum_{f=b,t} g^P_{h_i f\bar{f}} \frac{v}{m_f} F_{pf}(\tau_{if})$$

and QCD K factors are

$$K_H^g = 1 + \frac{\alpha_S(M_{h_i}^2)}{\pi}\left(\frac{95}{4} - \frac{7}{6}N_F\right), \qquad K_A^g = 1 + \frac{\alpha_S(M_{h_i}^2)}{\pi}\left(\frac{97}{4} - \frac{7}{6}N_F\right), \tag{A22}$$

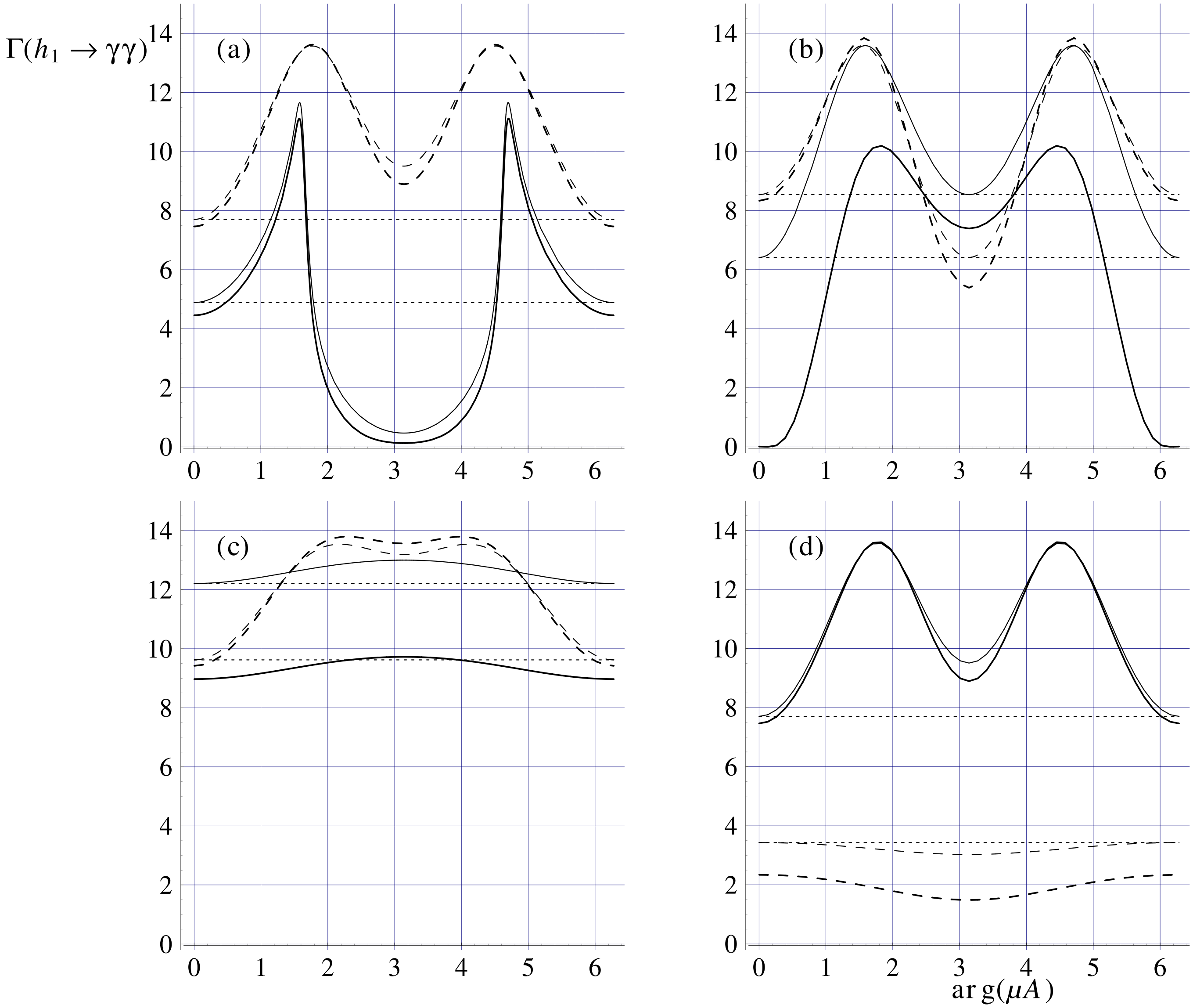

FIG. 8 (color online). The decay width $\Gamma(h_1 \to \gamma\gamma) \times 10^6$ (GeV) at $M_{\mathrm{SUSY}} = 500$ GeV. Dotted lines show Γ_h in the CP-conserving limit $\varphi = 0$. Thin solid or dashed lines are only for SM contributions, thick solid or dashed lines show SM and sparticle contributions with the J factor included. (a) tg$\beta = 5$, $A_t = A_b = 1$ TeV, and $\mu = 2$ TeV; solid line $m_{H^\pm} = 190$ GeV; dashed line $m_{H^\pm} = 300$ GeV. (b) tg$\beta = 5$, $m_{H^\pm} = 300$ GeV, and $\mu = 2$ TeV; solid line $A_t = A_b = -1.1$ TeV; dashed line $A_t = A_b = 1.1$ TeV. (c) tg$\beta = 5$, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line $\mu = 0.2$ TeV; dashed line $\mu = 1.2$ TeV. (d) $\mu = 2$ TeV, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line tg$\beta = 5$; dashed line tg$\beta = 40$.

$N_F = 5$ is the number of quark flavors with masses less than m_{h_1}.

The decay width of Higgs boson to the two fermions $h_1 \to f\bar{f}$ can be written as

$$\Gamma_{h_1\to f\bar{f}} = \frac{N_C g_f^2 m_{h_1} \beta_k^{3/2}}{8\pi}\begin{cases}(s_\alpha a_{21} - c_\alpha a_{11})^2 \frac{1}{s_\beta^2} + \mathrm{ctg}^2\beta a_{31}^2, & f \equiv u, c, t,\\ (c_\alpha a_{21} - s_\alpha a_{11})^2 \frac{1}{c_\beta^2} + \mathrm{tg}^2\beta a_{31}^2, & f \equiv b, d, s, e, \mu, \tau,\end{cases} \tag{A23}$$

where $\beta_k = 1 - 4k$, $k = m_f^2/m_{h_1}^2$, $g_f = g m_f/2m_W$, and $N_C = 3$ (1) for quarks (leptons).

In the following Table V we list the Higgs boson masses m_{h_1}, m_{h_2}, m_{h_3} which are calculated using the effective λ_i parameters (13)–(19), Sec. II, and the mass term diagonalization method described in Sec. III A. The decay widths $\Gamma_{h_1\to gg}$, $\Gamma_{h_1\to\gamma\gamma}$ (unprimed) include only the leading one-loop contributions of t, b quarks and $W^\pm$ bosons. For an

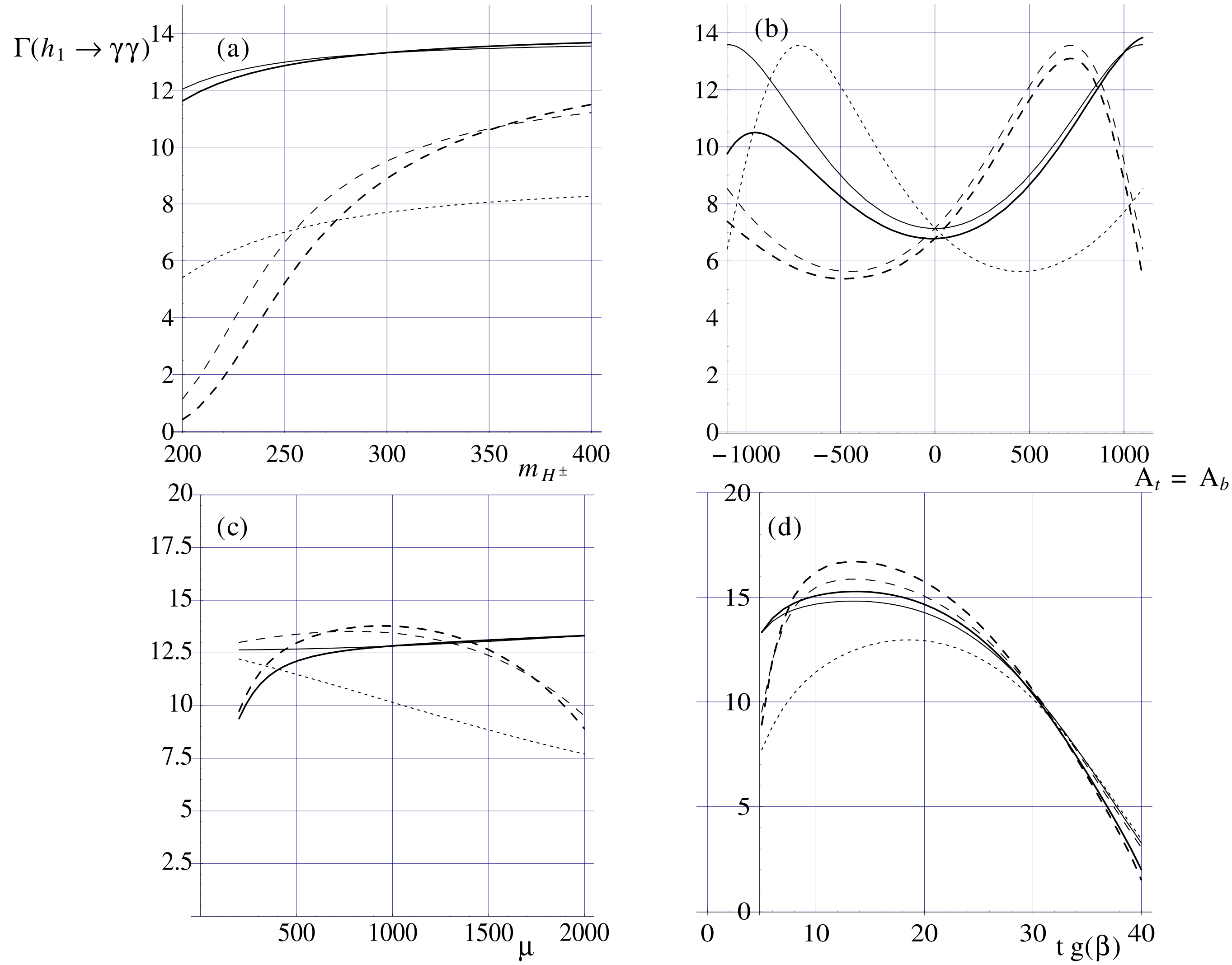

FIG. 9 (color online). The decay width $\Gamma(h_1 \to \gamma\gamma) \times 10^6$ (GeV) at $M_{\rm SUSY} = 500$ GeV. Dotted lines show Γ_h in the CP-conserving limit $\varphi = 0$. Thin solid or dashed lines denote the SM contributions, thick solid or dashed lines show SM and sparticle contributions with the J factor included. (a) tg$\beta = 5$, $A_t = A_b = 1$ TeV, and $\mu = 2$ TeV; solid line $\varphi = \pi/2$; dashed line $\varphi = \pi$. (b) tg$\beta = 5$, $m_{H^\pm} = 300$ GeV, and $\mu = 2$ TeV; solid line $\varphi = \pi/2$; dashed line $\varphi = \pi$. (c) tg$\beta = 5$, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line $\varphi = \pi/2$; dashed line $\varphi = \pi$. (d) $\mu = 2$ TeV, $m_{H^\pm} = 300$ GeV, and $A_t = A_b = 1$ TeV; solid line $\varphi = \pi/2$; dashed line $\varphi = \pi$.

illustration of the sensitivity of m_{h_1}, m_{h_2}, m_{h_3} and their decay widths to the values of λ_i we computed Higgs boson masses m'_{h_1,h_2,h_3} and the leading one-loop decay widths $\Gamma'_{h_1\to gg}$, $\Gamma'_{h_1\to\gamma\gamma}$ (include t, b and W contributions only) using the effective potential parametrization with both the one-loop and two-loop contributions to λ_i from the paper [3]. Finally, the decay widths $\Gamma''_{h_1\to gg}$, $\Gamma''_{h_1\to\gamma\gamma}$ are found using the effective parameters (13)–(19) and taking into account all possible one-loop fermion (t, b), gauge boson $W^\pm$, sfermion ($\tilde{t}$, $\tilde{b}$), chargino, and charged Higgs boson contributions, with K factors introduced in the expressions for decay widths.

Table V contains also the output of the CPsuperH [15] package and the FeynHiggs [18] package with the input parameter values taken the same as used in our parameter set. The two-loop evaluation in the CPsuperH and the one-loop evaluation in the FeynHiggs 2.1beta has been performed. Note that physical Higgs bosons H_1, H_2, and H_3 of the CPsuperH and FeynHiggs are evaluated in the way that is technically different from the construction of our mixed states h_1, h_2, h_3, however a difference of numbers (which is from several percent to 40% in the majority of cases) is caused mainly by theoretical uncertainties of the effective two-doublet potential representation, not by different definitions of the Higgs boson eigenstates in the generic basis of scalar doublets, as demonstrated explicitly in Sec. III A.

In Figs. 5–9 we show the variation of the light Higgs boson mass and the variations of $\Gamma(h_1 \to gg)$, $\Gamma(h_1 \to \gamma\gamma)$ decay widths in different regions of the parameter space

(φ, $m_{H^{\pm}}$, $A_{t,b}$, μ, tgβ). At the parameter set (0, 300 GeV, 1000 GeV, 2000 GeV, 5) the decay widths of h_1 to $\gamma\gamma$ and gg are not far from the decay widths of the SM Higgs boson with $m_H = 120$ GeV. The largest sensitivity of the widths to the charged Higgs mass is observed. At $m_{H^{\pm}}$ around 200 GeV [Figs. 6(a) and 8(a)] we observe the suppression of the branchings of h_1 to gg and $\gamma\gamma$ of more than 10 times at $\varphi \sim \pi$, which takes place in CPsuperH and FeynHiggs at higher masses of $m_{H^{\pm}}$ around 300 GeV.

Our approach is algorithmized in the form of the model in CompHEP 41.10 format [31], where the symbolic expressions for vertices are a starting level for the calculation of the complete tree-level sets of diagrams with the following cross section/decay width calculations and the generation of unweighted events.

[1] H. Georgi, Hadr. J. Phys. **1**, 155 (1978).
[2] T.D. Lee, Phys. Rev. D **8**, 1226 (1973).
[3] A. Pilaftsis and C.E.M. Wagner, Nucl. Phys. **B553**, 3 (1999).
[4] M. Carena, J. Ellis, A. Pilaftsis, and C.E.M. Wagner, Nucl. Phys. **B625**, 345 (2002); S. Heinemeyer, Eur. Phys. J. C **22**, 521 (2001); S.Y. Choi and J.S. Lee, Phys. Rev. D **61**, 015003 (2000); S.Y. Choi, M. Drees, and J.S. Lee, Phys. Lett. B **481**, 57 (2000); D.A. Demir, Phys. Rev. D **60**, 055006 (1999).
[5] M.N. Dubinin and A.V. Semenov, Eur. Phys. J. C **28**, 223 (2003).
[6] J. Liu and L. Wolfenstein, Nucl. Phys. **B289**, 1 (1987); Y.L. Wu and L. Wolfenstein, Phys. Rev. Lett. **73**, 1762 (1994); **73**, 2809 (1994).
[7] I. Ginzburg and M. Krawczyk, hep-ph/0408011.
[8] H.E. Haber and R. Hempfling, Phys. Rev. D **48**, 4280 (1993).
[9] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **67**, 1889 (1982); **68**, 927 (1982); R.A. Flores and M. Sher, Ann. Phys. (N.Y.) **148**, 95 (1983).
[10] Y. Okada, M. Yamaguchi, and T. Yanagida, Phys. Lett. B **262**, 54 (1991); J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. B **257**, 83 (1991); H.E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991); R. Barbieri, M. Frigeni, and F. Caravaglios, Phys. Lett. B **258**, 167 (1991).
[11] M. Carena, J.R. Ellis, A. Pilaftsis, and C.E.M. Wagner, Nucl. Phys. **B586**, 92 (2000).
[12] M. Quiros, in *Perspectives on Higgs Physics II*, edited by G.L. Kane (World Scientific, Singapore, 1998), p. 148.
[13] E. Akhmetzyanova, M. Dolgopolov, and M. Dubinin, in *Proceedings of the XVII Workshop on High Energy Physics and Quantum Field Theory (QFTHEP 2003), Samara-Saratov, 2003* (Moscow State University Press, Moscow, 2004).
[14] M. Carena, J.R. Espinosa, M. Quiros, and C.E.M. Wagner, Phys. Lett. B **355**, 209 (1995).
[15] J.S. Lee, A. Pilaftsis, M. Carena, S.Y. Choi, M. Drees, J.R. Ellis, and C.E.M. Wagner, Comput. Phys. Commun. **156**, 283 (2004).
[16] M. Dubinin and A. Semenov, SNUTP Report No. 98-140, 1998; F. Boudjema and A. Semenov, Phys. Rev. D **66**, 095007 (2002).
[17] J.F. Gunion and H.E. Haber, Phys. Rev. D **67**, 075019 (2003).
[18] M. Frank, S. Heinemeyer, W. Hollik, and G. Weiglein, in *Proceedings of the Xth International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY02)*, edited by P. Nath, P.M. Zerwas, and C. Grosche (DESY, Hamburg, 2002), Vol. 2, p. 637.
[19] M. Carena, H.E. Haber, S. Heinemeyer, W. Hollik, C.E.M. Wagner, and G. Weiglein, Nucl. Phys. **B580**, 29 (2000); J.R. Espinosa and R.J. Zhang, J. High Energy Phys. 03 (2000) 026.
[20] M. Carena, J. Ellis, A. Pilaftsis, and C. Wagner, Phys. Lett. B **495**, 155 (2000).
[21] M.J.G. Veltman, Acta Phys. Pol. B **25**, 1627 (1994); J.L. Diaz-Cruz and A. Mendez, Nucl. Phys. **B380**, 39 (1992).
[22] I.F. Ginzburg and M.V. Vychugin, in *Proceedings of the XVI Workshop on High Energy Physics and Quantum Field Theory (QFTHEP 2001)* (Moscow State University Press, Moscow, 2002), p. 64.
[23] S. Abdullin *et al.*, CMS Note No. 2003/033, 2003.
[24] M. Carena, J. Ellis, S. Mrenna, A. Pilaftsis, and C.E.M. Wagner, Nucl. Phys. **B659**, 145 (2003).
[25] P. Niezurawski, A.F. Zarnecki, and M. Krawczyk, J. High Energy Phys. 02 (2005) 041; R.M. Godbole, S.D. Rindani, and R.K. Singh, Phys. Rev. D **67**, 095009 (2003); **71**, 039902(E) (2005); A. Dedes and S. Moretti, Nucl. Phys. **B576**, 29 (2000); Phys. Rev. Lett. **84**, 22 (2000); B. Grzadkowski, J.F. Gunion, and J. Kalinowski, Phys. Rev. D **60**, 075011 (1999).
[26] J. Gunion, G. Gamberini, and S. Novaes, Phys. Rev. D **38**, 3481 (1988).
[27] A. Semenov, Nucl. Instrum. Methods Phys. Res., Sect. A **389**, 293 (1997).
[28] H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, and Y. Yamada, Phys. Rev. D **62**, 055006 (2000).
[29] J. Ellis, M.K. Gaillard, and D.V. Nanopoulos, Nucl. Phys. **B106**, 292 (1976); A.I. Vainshtein, M.B. Voloshin, V.I. Zakharov, and M.A. Shifman, Yad. Fiz. **30**, 1368 (1979); [Sov. J. Nucl. Phys. **30**, 711 (1979)].
[30] M. Spira, A. Djouadi, D. Graudenz, and P.M. Zerwas, Nucl. Phys. **B453**, 17 (1995); M. Spira, Fortschr. Phys. **46**, 203 (1998).
[31] E. Boos, V. Bunichev, M. Dubinin, L. Dudko, V. Edneral, V. Ilyin, A. Kryukov, V. Savrin, A. Semenov, and A. Sherstnev, Nucl. Instrum. Methods Phys. Res., Sect. A **534**, 250 (2004); A. Pukhov *et al.*, hep-ph/9908288.