# Branching ratio and *CP* violation of $B_s \to \pi K$ decays in the perturbative QCD approach

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In the framework of perturbative QCD approach, we calculate the branching ratio and *CP* asymmetry for $B_s^0(\bar{B}_s) \to \pi^\pm K^\mp$ and $B_s(\bar{B}_s) \to \pi^0 \bar{K}^0(K^0)$ decays. Besides the usual factorizable diagrams, both nonfactorizable and annihilation type contributions are taken into account. We find that (a) the branching ratio of $B_s^0(\bar{B}_s) \to \pi^\pm K^\mp$ is about $(6 - 10) \times 10^{-6}$; $Br(B_s(\bar{B}_s) \to \pi^0 \bar{K}^0(K^0))$ about $(1 - 3) \times 10^{-7}$; and (b) there are large *CP* asymmetries in the two processes, which can be tested in the near future LHC-b experiments at CERN and BTeV experiments at Fermilab.

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## I. INTRODUCTION

The rare charmless $B$ meson decays arouse more and more interest, since it is a good place for testing the Standard Model (SM), studying *CP* violation, and looking for possible new physics beyond the SM. Since 1999, the B factories in High Energy Accelerator Research Organization (KEK) and Stanford Linear Accelerator Center (SLAC) collect more and more data sample of rare $B$ decays. In the future CERN Large Hadron Collider beauty experiments (LHC-b), the heavier $B_s$ and $B_c$ mesons can also be produced. With the bright hope in LHC-b experiments and BTeV experiments at Fermilab, following a previous study of $B_s \to \pi^+\pi^-$ decay [1], we continue to investigate other $B_s$ rare decays.

The most difficult problem in theoretical calculation of nonleptonic $B$ decays is the calculation of hadronic matrix element. The widely used method is the factorization approach (FA) [2]. It is a great success in explaining the branching ratio of many decays [3,4], although it is a very simple method. In order to improve the theoretical precision, QCD factorization [5] and perturbative QCD approach (PQCD) [6] are developed. Perturbative QCD factorization theorem for exclusive heavy-meson decays has been proved some time ago, and applied to semileptonic $B \to D(\pi) l\nu$ decays [6], the nonleptonic $B \to K\pi$ [7], $\pi\pi$ [8] decays. PQCD is a method to factorize hard components from a QCD process, which can be treated by perturbation theory. Nonperturbative parts are organized in the form of universal hadron light-cone wave functions, which can be extracted from experiments or constrained by lattice calculations and QCD sum rules. More information about PQCD approach can be found in [6,9].

In this paper, we would like to study the $B_s^0(\bar{B}_s) \to \pi^\pm K^\mp$ and $B_s(\bar{B}_s) \to \pi^0 \bar{K}^0(K^0)$ decays in the perturbative QCD approach. In our calculation, we ignore the soft final state interaction because there are not many resonances near the energy region of $B_s$ mass. Our theoretical formulas for the decay $B_s \to \pi K$ in PQCD framework are given in the next section. In section III, we give the numerical results of the branching ratio of $B_s \to \pi K$ and discussions for *CP* asymmetries and the form factor of $B_s \to K$, etc. At last, we give a short summary in section IV.

## II. PERTURBATIVE CALCULATIONS

For decay $B_s \to \pi K$, the related effective Hamiltonian is given by [10]

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\left\{ V_{ud}V_{ub}^*[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] - V_{tb}^*V_{td}\sum_{i=3}^{10} C_i(\mu)O_i(\mu)\right\}, \quad (1)$$

where $C_i(\mu)(i = 1, \cdots, 10)$ are Wilson coefficients at the renormalization scale $\mu$ and $O_i(i = 1, \cdots, 10)$ are the four quark operators

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$$O_1 = (\bar{b}_i u_j)_{V-A}(\bar{u}_j d_i)_{V-A}, \qquad O_2 = (\bar{b}_i u_i)_{V-A}(\bar{u}_j d_j)_{V-A}, \qquad O_3 = (\bar{b}_i d_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V-A},$$
$$O_4 = (\bar{b}_i d_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V-A}, \qquad O_5 = (\bar{b}_i d_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V+A}, \qquad O_6 = (\bar{b}_i d_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V+A},$$
$$O_7 = \frac{3}{2}(\bar{b}_i d_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V+A}, \qquad O_8 = \frac{3}{2}(\bar{b}_i d_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V+A}, \qquad O_9 = \frac{3}{2}(\bar{b}_i d_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V-A},$$
$$O_{10} = \frac{3}{2}(\bar{b}_i d_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V-A}. \tag{2}$$

Here $i$ and $j$ are $SU(3)$ color indices; the sum over $q$ runs over the quark fields that are active at the scale $\mu = O(m_b)$, i.e., $q \in \{u, d, s, c, b\}$. Operators $O_1$, $O_2$ come from tree-level interaction, while $O_3$, $O_4$, $O_5$, $O_6$ are QCD-Penguins operators and $O_7$, $O_8$, $O_9$, $O_{10}$ come from electroweak-penguins.

Working at the rest frame of $B_s$ meson, we take kaon and pion masses $M_K \sim M_\pi \sim 0$, which are much smaller than $M_{B_s}$. In the light-cone coordinates, the momenta of the $B_s$, $K$, and $\pi$ can be written as:

$$P_1 = \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), \qquad P_2 = \frac{M_B}{\sqrt{2}}(0, 1, \mathbf{0}_T), \qquad P_3 = \frac{M_B}{\sqrt{2}}(1, 0, \mathbf{0}_T). \tag{3}$$

Denoting the light (anti-)quark momenta in $B$, $K$, and $\pi$ as $k_1$, $k_2$, and $k_3$, respectively, we can choose:

$$k_1 = (x_1 p_1^+, 0, \mathbf{k}_{1T}), \qquad k_2 = (0, x_2 p_2^-, \mathbf{k}_{2T}), \qquad k_3 = (x_3 p_3^+, 0, \mathbf{k}_{3T}). \tag{4}$$

In the following, we start to compute the decay amplitudes of $B_s \to \pi K$.

According to effective Hamiltonian (1), we draw the lowest order diagrams of $B_s \to \pi K$ in Fig. 1. Let us first look at the usual factorizable diagrams (a) and (b). They can give the $B_s \to K$ form factor if take away the Wilson coefficients. The operators $O_1$, $O_2$, $O_3$, $O_4$, $O_9$, and $O_{10}$ are $(V-A)(V-A)$ currents, and the sum of their contributions is given by

$$\begin{aligned} F_e[C] = {} & 16\pi C_F M_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1)\{[(2 - x_2)\phi_K^A(x_2) - r_K(1 - 2x_2)\phi_K^P(x_2) + r_K(1 - 2x_2)\phi_K^T(x_2)] \\ & \times \alpha_s(t_a^1) h_a(x_1, 1 - x_2, b_1, b_2) \exp[-S_B(t_a^1) - S_K(t_a^1)] C(t_a^1) + 2r_K \phi_K^P(x_2) \alpha_s(t_a^2) h_a(1 - x_2, x_1, b_2, b_1) \\ & \times \exp[-S_B(t_a^2) - S_K(t_a^2)] C(t_a^2)\}, \end{aligned} \tag{5}$$

where $r_\pi = m_{0\pi}/m_B = m_\pi^2/[m_B(m_u + m_d)]$, $r_K = m_{0K}/m_B = m_K^2/[m_B(m_s + m_u)]$. $C_F = 4/3$ is the group factor of the $SU(3)_c$ gauge group. The expressions of the meson distribution amplitudes $\phi_M$, the Sudakov factor $S_X(t_i)(X = B_s, K, \pi)$, and the functions $h_a$ are given in the appendix. In the above formula, the Wilson coefficients $C(t)$ of the corresponding operators are process dependent.

The operators $O_5$, $O_6$, $O_7$, $O_8$ have the structure of $(V-A)(V+A)$, their amplitude is

$$\begin{aligned} F_e^P[C] = {} & 32\pi C_F M_B^2 r_\pi \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1)\{[\phi_K^A(x_2) - r_K(x_2 - 3)\phi_K^P(x_2) + r_K(1 - x_2)\phi_K^T(x_2)] \\ & \times \alpha_s(t_a^1) h_a(x_1, 1 - x_2, b_1, b_2) \exp[-S_B(t_a^1) - S_K(t_a^1)] C(t_a^1) + 2r_K \phi_K^P(x_2) \alpha_s(t_a^2) h_a(1 - x_2, x_1, b_2, b_1) \\ & \times \exp[-S_B(t_a^2) - S_K(t_a^2)] C(t_a^2)\}. \end{aligned} \tag{6}$$

For the nonfactorizable diagrams (c) and (d), all three meson wave functions are involved. Using $\delta$ function $\delta(b_1 - b_3)$, the integration of $b_1$ can be preformed easily. For the $(V-A)(V-A)$ operators the result is:

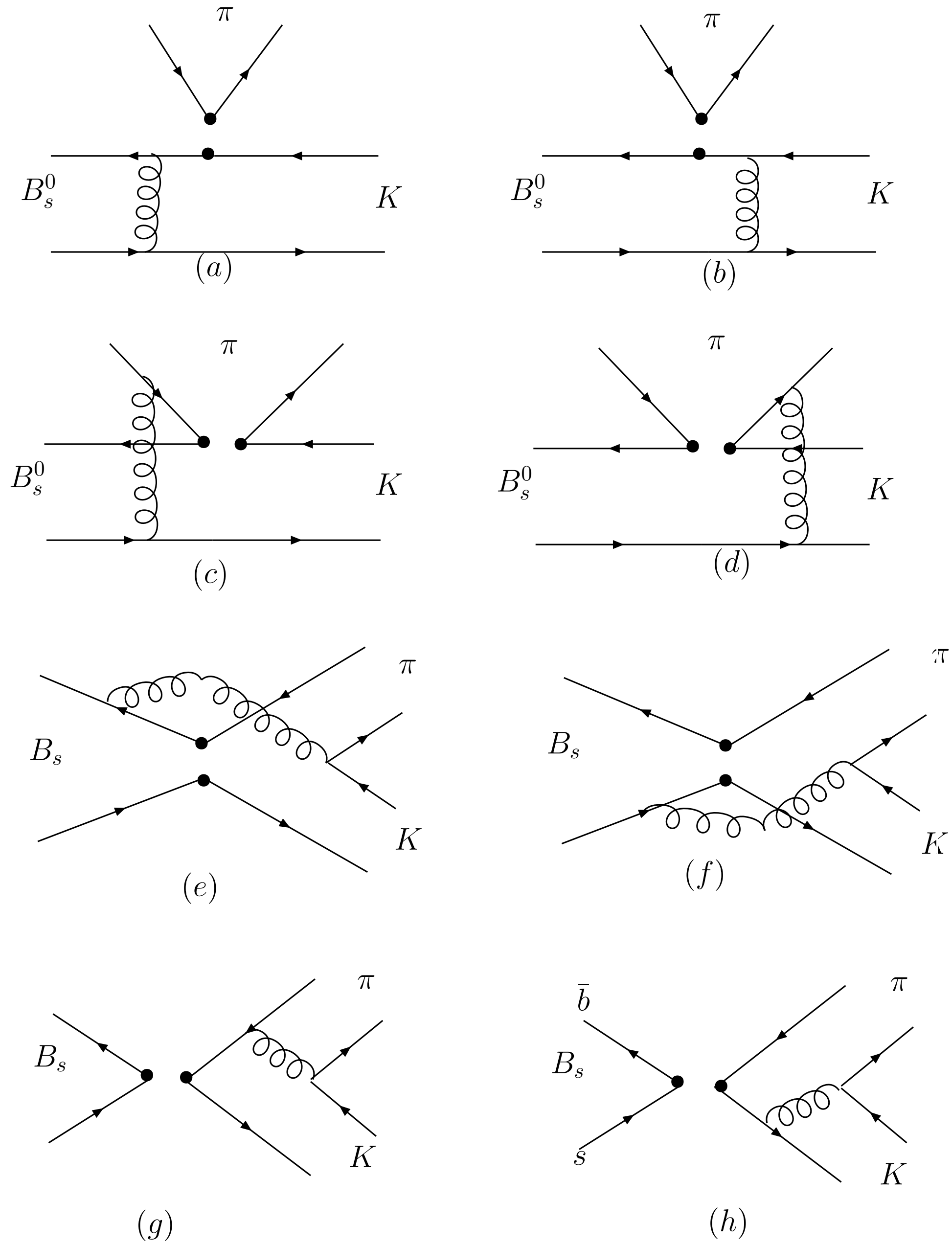


FIG. 1. The lowest order diagrams for $B_s^0 \to \pi K$ decay.

$$\begin{aligned} M_e[C] = \frac{32}{3}\pi C_F\sqrt{2N_c}M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_B(x_1, b_3)\{[(x_3 - 1)\phi_\pi^A(x_3)\phi_K^A(x_2) \\ + r_K(1 - x_2)\phi_\pi^A(x_3)\phi_K^P(x_2) + r_K(1 - x_2)\phi_\pi^A(x_3)\phi_K^T(x_2)]C(t_c^1)\alpha_s(t_c^1)h_c^{(1)}(x_1, x_2, x_3, b_2, b_3) \\ \times \exp[-S_B(t_c^1) - S_\pi(t_c^1) - S_K(t_c^1)] - [(x_2 - x_3 - 1)\phi_\pi^A(x_3)\phi_K^A(x_2) + r_K(1 - x_2)\phi_\pi^A(x_3)\phi_K^P(x_2) \\ - r_K(1 - x_2)\phi_\pi^A(x_3)\phi_K^T(x_2)]C(t_c^2)\alpha_s(t_c^2)h_c^{(2)}(x_1, x_2, x_3, b_2, b_3)\exp[-S_B(t_c^2) - S_\pi(t_c^2) - S_K(t_c^2)]\}. \end{aligned} \quad (7)$$

For the $(V - A)(V + A)$ operators, the formula is:

$$\begin{aligned}M_e^P[C] = {}& \frac{32}{3}\pi C_F\sqrt{2N_c}M_B^2 r_\pi \int_0^1 dx_1dx_2dx_3\int_0^\infty b_2db_2b_3db_3\phi_B(x_1,b_3)\{[r_K(x_2+x_3-2)\phi_\pi^P(x_3)\phi_K^P(x_2)\\ &- r_K(x_2-x_3)\phi_\pi^P(x_3)\phi_K^T(x_2) - r_K(x_2-x_3)\phi_\pi^T(x_3)\phi_K^P(x_2) - r_K(2-x_2-x_3)\phi_\pi^T(x_3)\phi_K^T(x_2)\\ &- (1-x_3)\phi_\pi^P(x_3)\phi_K^A(x_2) - (1-x_3)\phi_\pi^T(x_3)\phi_K^A(x_2)]C(t_c^1)\alpha_s(t_c^1)h_c^{(1)}(x_1,x_2,x_3,b_2,b_3)\\ &\times\exp[-S_B(t_c^1)-S_\pi(t_c^1)-S_K(t_c^1)] + [r_K(1-x_2+x_3)\phi_\pi^P(x_3)\phi_K^P(x_2) + r_K(x_2+x_3-1)\phi_\pi^P(x_3)\phi_K^T(x_2)\\ &- r_K(x_2+x_3-1)\phi_\pi^T(x_3)\phi_K^P(x_2) - r_K(1-x_2+x_3)\phi_\pi^T(x_3)\phi_K^T(x_2) + x_3\phi_\pi^P(x_3)\phi_K^A(x_2)\\ &- x_3\phi_\pi^T(x_3)\phi_K^A(x_2)]C(t_c^2)\alpha_s(t_c^2)h_c^{(2)}(x_1,x_2,x_3,b_2,b_3)\exp[-S_B(t_c^2)-S_\pi(t_c^2)-S_K(t_c^2)]\}. \end{aligned} \tag{8}$$

Similar to (c) and (d), the annihilation diagrams (e) and (f) also involve all three meson wave functions. Here we have two kinds of amplitudes, $M_a$ is the contribution containing the operator of type $(V-A)(V-A)$, and $M_a^P$ is the contribution containing the operator of type $(V-A)(V+A)$.

$$\begin{aligned}M_a[C] = {}& \frac{32}{3}\pi C_F\sqrt{2N_c}M_B^2 \int_0^1 dx_1dx_2dx_3\int_0^\infty b_1db_1b_2db_2\phi_B(x_1,b_1)\{[x_3\phi_\pi^A(x_3)\phi_K^A(x_2)\\ &+ r_\pi r_K(2+x_2+x_3)\phi_\pi^P(x_3)\phi_K^P(x_2) - r_\pi r_K(x_2-x_3)\phi_\pi^P(x_3)\phi_K^T(x_2) - r_\pi r_K(x_2-x_3)\phi_\pi^T(x_3)\phi_K^P(x_2)\\ &- r_\pi r_K(2-x_2-x_3)\phi_\pi^T(x_3)\phi_K^T(x_2)]C(t_e^1)\alpha_s(t_e^1)h_e^{(1)}(x_1,x_2,x_3,b_1,b_2)\exp[-S_B(t_e^1)-S_\pi(t_e^1)-S_K(t_e^1)]\\ &- [x_2\phi_\pi^A(x_3)\phi_K^A(x_2) + r_\pi r_K(x_2+x_3)\phi_\pi^P(x_3)\phi_K^P(x_2) + r_\pi r_K(x_2-x_3)\phi_\pi^P(x_3)\phi_K^T(x_2) + r_\pi r_K(x_2-x_3)\\ &\times\phi_\pi^T(x_3)\phi_K^P(x_2) + r_\pi r_K(x_2+x_3)\phi_\pi^T(x_3)\phi_K^T(x_2)]C(t_e^2)\alpha_s(t_e^2)h_e^{(2)}(x_1,x_2,x_3,b_1,b_2)\\ &\times\exp[-S_B(t_e^2)-S_\pi(t_e^2)-S_K(t_e^2)]\}, \end{aligned} \tag{9}$$

$$\begin{aligned}M_a^P[C] = {}& \frac{32}{3}\pi C_F\sqrt{2N_c}M_B^2 \int_0^1 dx_1dx_2dx_3\int_0^\infty b_1db_1b_2db_2\phi_B(x_1,b_1)\{[r_K(2-x_2)\phi_\pi^A(x_3)\phi_K^P(x_2) + r_K(2-x_2)\\ &\times\phi_\pi^A(x_3)\phi_K^T(x_2) - r_\pi(2-x_3)\phi_\pi^P(x_3)\phi_K^A(x_2) - r_\pi(2-x_3)\phi_\pi^T(x_3)\phi_K^A(x_2)]C(t_e^1)\alpha_s(t_e^1)h_e^{(1)}(x_1,x_2,x_3,b_1,b_2)\\ &\times\exp[-S_B(t_e^1)-S_\pi(t_e^1)-S_K(t_e^1)] + [r_Kx_2\phi_\pi^A(x_3)\phi_K^P(x_2) + r_Kx_2\phi_\pi^A(x_3)\phi_K^T(x_2) - r_\pi x_3\phi_\pi^P(x_3)\phi_K^A(x_2)\\ &- r_\pi x_3\phi_\pi^T(x_3)\phi_K^A(x_2)]C(t_e^2)\alpha_s(t_e^2)h_e^{(2)}(x_1,x_2,x_3,b_1,b_2)\exp[-S_B(t_e^2)-S_\pi(t_e^2)-S_K(t_e^2)]\}. \end{aligned} \tag{10}$$

The factorizable annihilation diagrams (g) and (h) involve only two light mesons wave functions. $F_a$ is for $(V-A)\times(V-A)$ type operators, and $F_a^P$ is for $(V-A)(V+A)$ type operators:

$$\begin{aligned}F_a[C] = {}& 16\pi C_F M_B^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\{[-x_2\phi_\pi^A(x_3)\phi_K^A(x_2) - 2r_\pi r_K(1+x_2)\phi_\pi^P(x_3)\phi_K^P(x_2)\\ &+ 2r_\pi r_K(1-x_2)\phi_\pi^P(x_3)\phi_K^T(x_2)]\alpha_s(t_g^1)h_g(x_2,x_3,b_2,b_3)\exp[-S_\pi(t_g^1)-S_K(t_g^1)]C(t_g^1)\\ &+ [x_3\phi_\pi^A(x_3)\phi_K^A(x_2) + 2r_\pi r_K(1+x_3)\phi_\pi^P(x_3)\phi_K^P(x_2) - 2r_\pi r_K(1-x_3)\phi_\pi^T(x_3)\phi_K^P(x_2)]C(t_g^2)\alpha_s(t_g^2)\\ &\times h_g(x_3,x_2,b_3,b_2)\exp[-S_\pi(t_g^2)-S_K(t_g^2)]\}, \end{aligned} \tag{11}$$

$$\begin{aligned}F_a^P[C] = {}& 32\pi C_F M_B^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\{[r_Kx_2\phi_\pi^A(x_3)\phi_K^P(x_2) - r_Kx_2\phi_\pi^A(x_3)\phi_K^T(x_2) + 2r_\pi\phi_\pi^P(x_3)\phi_K^A(x_2)]\\ &\times\alpha_s(t_g^1)h_g(x_2,x_3,b_2,b_3)\exp[-S_\pi(t_g^1)-S_K(t_g^1)]C(t_g^1) + [2r_K\phi_\pi^A(x_3)\phi_K^P(x_2) + r_\pi x_3\phi_\pi^P(x_3)\phi_K^A(x_2)\\ &- r_\pi x_3\phi_\pi^T(x_3)\phi_K^A(x_2)]C(t_g^2)\alpha_s(t_g^2)h_g(x_3,x_2,b_3,b_2)\exp[-S_\pi(t_g^2)-S_K(t_g^2)]\}. \end{aligned} \tag{12}$$

From Eq. (5)–(12), the total decay amplitude for $B_s\to\pi^+K^-$ can be written as

$$\begin{aligned} A(B_s^0 \to \pi^+ K^-) = {} & f_\pi F_e\left[V_{ud}V_{ub}^*\left(\frac{1}{3}C_1 + C_2\right) - V_{tb}^*V_{td}\left(\frac{1}{3}C_3 + C_4 + \frac{1}{3}C_9 + C_{10}\right)\right] \\ & - f_\pi V_{tb}^*V_{td}F_e^P\left[\frac{1}{3}C_5 + C_6 + \frac{1}{3}C_7 + C_8\right] + M_e[V_{ud}V_{ub}^*C_1 - V_{tb}^*V_{td}(C_3 + C_9)] \\ & - V_{tb}^*V_{td}M_e^P(C_5 + C_7) - V_{tb}^*V_{td}M_a\left(C_3 - \frac{1}{2}C_9\right) - V_{tb}^*V_{td}M_a^P\left(C_5 - \frac{1}{2}C_7\right) \\ & - f_B V_{tb}^*V_{td}F_a\left[\frac{1}{3}C_3 + C_4 - \frac{1}{6}C_9 - \frac{1}{2}C_{10}\right] - f_B V_{tb}^*V_{td}F_a^P\left[\frac{1}{3}C_5 + C_6 - \frac{1}{6}C_7 - \frac{1}{2}C_8\right], \end{aligned} \tag{13}$$

and the decay width is expressed as

$$\Gamma(B_s^0 \to \pi^+ K^-) = \frac{G_F^2 M_B^3}{128\pi}|A(B_s^0 \to \pi^+ K^-)|^2. \tag{14}$$

The Wilson coefficient $C_i's$ should be calculated at the appropriate scale $t$ which can be found in the Appendix of Ref. [8]. The decay amplitude of the charge conjugate channel $\bar{B}_s^0 \to \pi^- K^+$ can be obtained by replacing $V_{ud}V_{ub}^*$ to $V_{ud}^*V_{ub}$ and $V_{tb}^*V_{td}$ to $V_{tb}V_{td}^*$ in Eq. (13).

For the decay $B_s \to \pi^0 \bar{K}^0$, its amplitude can be written as

$$\begin{aligned} A(B_s^0 \to \pi^0 \bar{K}^0) = {} & f_\pi F_e\left[V_{ud}V_{ub}^*\left(C_1 + \frac{1}{3}C_2\right) - V_{tb}^*V_{td}\left(-\frac{1}{3}C_3 - C_4 + \frac{1}{6}C_9 + \frac{1}{2}C_{10}\right)\right] \\ & - f_\pi V_{tb}^*V_{td}F_e^p\left[-\frac{1}{3}C_5 - C_6 + \frac{1}{6}C_7 + \frac{1}{2}C_8\right] + M_e\left[V_{ud}V_{ub}^*C_2 - V_{tb}^*V_{td}\left(-C_3 + \frac{1}{2}C_9\right)\right] \\ & - V_{tb}^*V_{td}M_e^p\left(\frac{1}{2}C_7 - C_5\right) - V_{tb}^*V_{td}M_a\left(\frac{1}{2}C_9 - C_3\right) - V_{tb}^*V_{td}M_a^p\left(\frac{1}{2}C_7 - C_5\right) \\ & - f_B V_{tb}^*V_{td}F_a\left[-\frac{1}{3}C_3 - C_4 + \frac{1}{6}C_9 + \frac{1}{2}C_{10}\right] - f_B V_{tb}^*V_{td}F_a^p\left[-\frac{1}{3}C_5 - C_6 + \frac{1}{6}C_7 + \frac{1}{2}C_8\right] \end{aligned} \tag{15}$$

and the decay width is then expressed as

$$\Gamma(B_s^0 \to \pi^0 \bar{K}^0) = \frac{G_F^2 M_B^3}{256\pi}|A(B_s^0 \to \pi^0 \bar{K}^0)|^2. \tag{16}$$

## III. NUMERICAL EVALUATION

The following parameters have been used in our numerical calculation [11,12]:

$$\begin{aligned} & M_{B_s} = 5.37\ \text{GeV}, && m_{0\pi} = 1.4\ \text{GeV}, \\ & m_{0K} = 1.6\ \text{GeV}, && \Lambda_{QCD}^{f=4} = 0.25\ \text{GeV}, \\ & f_{B_s} = 230\ \text{MeV}, && f_\pi = 130\ \text{MeV}, \\ & f_K = 160\ \text{MeV}, && \tau_{B_s^0} = 1.46 \times 10^{-12}\ \text{s}, \\ & |V_{tb}^*V_{td}| = 0.0074, && |V_{ub}^*V_{ud}| = 0.0031. \end{aligned} \tag{17}$$

We leave the Cabibbo-Kobayashi-Maskawa (CKM) phase angle $\alpha = \phi_2$ as a free parameter, whose definition is

$$\alpha = \arg\left[-\frac{V_{tb}^*V_{td}}{V_{ud}V_{ub}^*}\right]. \tag{18}$$

In this language, the decay amplitude of $B_s \to \pi^+ K^-$ in Eq. (13) can be parametrized as

$$A = V_{ub}^*V_{ud}T - V_{tb}^*V_{td}P = V_{ub}^*V_{ud}T[1 + ze^{i(\alpha+\delta)}], \tag{19}$$

where $z = |V_{tb}^*V_{td}/V_{ub}^*V_{ud}||P/T|$, and $\delta$ is the relative strong phase between tree diagrams $T$ and penguin diagrams $P$. $z$ and $\delta$ can be calculated from PQCD. Using the above parameters in (17), we get $z = 21\%$ and $\delta = 134°$ from PQCD calculation, which shows the dominance of the tree contribution in this decay and a large strong phase calculated from PQCD.

Similarly, the decay amplitude for $\bar{B}_s \to \pi^- K^+$ can be parametrized as

$$\bar{A} = V_{ub}V_{ud}^*T - V_{tb}V_{td}^*P = V_{ub}V_{ud}^*T[1 + ze^{i(-\alpha+\delta)}]. \tag{20}$$

Therefore the averaged decay width for $B_s^0(\bar{B}_s^0) \to \pi^\pm K^\mp$ is

$$\begin{aligned} \Gamma(B_s^0(\bar{B}_s^0) \to \pi^\pm K^\mp) &= \frac{G_F^2 M_B^3}{128\pi}(|A|^2/2 + |\bar{A}|^2/2) \\ &= \frac{G_F^2 M_B^3}{128\pi}|V_{ub}^*V_{ud}T|^2 \\ &\quad \times [1 + 2z\cos\alpha\cos\delta + z^2]. \end{aligned} \tag{21}$$

It is a function of $\cos\alpha\cos\delta$.

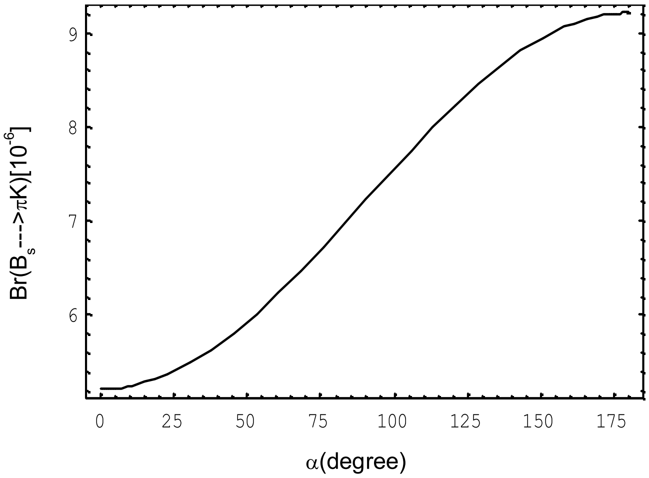


FIG. 2. The averaged branching ratio of $B_s^0(\bar{B}_s) \to \pi^\pm K^\mp$ decay as a function of CKM angle $\alpha$.

In Fig. 2, we plot the averaged branching ratio of the decay $B_s^0(\bar{B}_s^0) \to \pi^\pm K^\mp$ with respect to the parameter $\alpha$. Since the latest experiment constraint upon the CKM angle $\alpha$ from Belle and *BABAR* is $\alpha$ around 100° [13], we can arrive from Fig. 2:

$$6.5 \times 10^{-6} < Br(B_s^0(\bar{B}_s^0) \to \pi^\pm K^\mp) < 8.5 \times 10^{-6}, \quad \text{for} \quad 70° < \alpha < 130°. \tag{22}$$

Previous naive and generalized factorization approach gives a similar branching ratios at $6 - 9 \times 10^{-6}$ with the form factor $F^{B_s \to K} \simeq 0.27$ [14]. In paper [15], Beneke *et al.* also calculate this decay mode using QCD improved factorization approach (BBNS). It is based on naive factorization approach. The dominant contribution is still proportional to $B_s \to K$ form factor, which is introduced as an input parameter. In principal, the decay amplitude expand as series of $\alpha_s$ and $\Lambda/m_B$. But in practice, only the first order of $\alpha_s$ corrections is calculated, including the so-called nonfactorizable contributions. The annihilation type contribution is power ($\Lambda/m_B$) suppressed in BBNS approach. Therefore, the branching ratio predicted in QCD factorization and PQCD should not differ too much; but the *CP* violation in these two approaches will be different, since it depends on many nonleading order contributions (see below for discussion). In Ref. [15], the branching ratio is about $10 \times 10^{-6}$, which is larger than our PQCD result and previous FA method [14], because their form factor $F^{B_s \to K}(0) = 0.31$ [15] is larger than the previous factorization approach and our calculation below.

The diagrams (a) and (b) in Fig. 1 correspond to the $B_s \to K$ transition form factor $F^{B_s \to K}(q^2 = m_\pi^2 \simeq 0)$, where $q = P_1 - P_2$ is the momentum transfer. The sum of their amplitudes have been given by Eq. (5), so we can use PQCD approach to compute this form factor. Our result is $F^{B_s \to K}(0) = 0.27$, if $\omega_b = 0.5$; and $F^{B_s \to K}(0) = 0.32$, if $\omega_b = 0.45$. In our approach, this form factor is sensitive to the decay constant and wave function of $B_s$ meson, where there is large uncertainty; but not sensitive to the $K$ meson wave function. Eventually this form factor can be extracted from semileptonic experiments $B_s \to K^- l^+ \nu_l$ in the future.

In our calculation, the only input parameters are wave functions, which stand for the nonperturbative contributions. Up to now, no exact solution is made for them. So the main uncertainty in PQCD approach comes from $B_s$, $K$, $\pi$ wave functions. In this paper, we choose the light-cone wave functions which are obtained from QCD Sum Rules [16,17]. For $\pi$ meson, the distribution amplitude of light-cone wave function should take asymptotic form if the energy scale $\mu \to \infty$. But in our case, the scale is not more than 5 GeV, so we choose the corrected asymptotic form for twist 2 distribution amplitude $\phi_\pi^A$, and other twist 3 distribution amplitudes derived using equation of motion by neglecting three particle wave functions [17]. These functions are listed in the Appendix, which are also used in decay mode $B \to K\pi$ [7] and $B \to \pi\pi$ [8], etc.

We also try to use the asymptotic form for $\pi$ meson, for all the three distribution amplitudes $\phi_\pi^A$, $\phi_\pi^P$, and $\phi_\pi^T$, since we have very poor knowledge about twist 3 distribution amplitudes [18]. The branching ratio of $B_s \to \pi^+ K^-$ is nearly unchanged (only 4%), because the branching ratio of $B_s \to \pi^+ K^-$ is mainly determined by the form factor $F^{B_s \to K}(0)$ [see Fig. 1(a) and 1(b)] which is not dependent on $\pi$ wave function. However, the *CP* asymmetry changes from $-27\%$ to $-14\%$ by $-48\%$, when $\alpha = 100°$. This is because the direct *CP* asymmetry depend on the strong phase (see discussion below), which comes from nonfactorizable and annihilation diagrams, where all three meson wave functions are involved. The *CP* asymmetry predicted here should be used with great care, since it depends on too many uncertainties.

For heavy $B$ and $B_s$ meson, its wave function is still under discussion using different approaches [19]. In this paper, we find the branching ratio of $B_s^0(\bar{B}_s) \to \pi^\pm K^\mp$ is sensitive to the wave function parameter $\omega_b$. For $0.45 < \omega_b < 0.5$, the resulted branching ratio will decrease from about $11 \times 10^{-6}$ to about $8 \times 10^{-6}$. When we set $\omega_b = 0.45$, our result is more close to that of QCD factorization [15]. This sensitive dependence should be fixed by the $B_s \to K$ form factors from the semileptonic $B_s$ decays. Other uncertainties in our calculation include the next-to-leading order $\alpha_s$ QCD corrections and higher twist contributions, which need more complicated calculations.

From our calculation, we find that the dominant contribution comes from tree-level diagrams [see Fig. 1(a) and 1(b)] in this decay. If $SU(3)$ symmetry is good, the branching ratio of $B_s \to \pi^+ K^-$ should be equal to that of $B^0 \to \pi^+\pi^-$. The experimental result of $B^0 \to \pi^+\pi^-$ is $Br(B \to \pi^+\pi^-) = (4.3^{+1.6}_{-1.4} \pm 0.5) \times 10^{-6}$ [20]. The predicted branching ratio of $B_s \to \pi K$ is about 1.7 times that of $B_d \to \pi^+\pi^-$, where the difference comes mainly from

$SU(3)$ symmetry breaking: the decay constant $f_{B_s}$ larger than $f_B$ and $f_K$ larger than $f_\pi$. In the calculation, we also find that the electroweak-penguins contribution is negligibly small as 0.002% in branching ratio.

For the experimental side, there is recent upper limit on the decay $B_s^0 \to \pi^+ K^-$ [21],

$$Br(B_s^0 \to \pi^+ K^-) < 7.5 \times 10^{-6}, \tag{23}$$

at 90% C.L. Our predicted result is consistent with this upper limit.

For the decays of $B_s(\bar{B}_s) \to \pi^0 \bar{K}^0(K^0)$, the tree-level contribution is suppressed due to the small Wilson coefficients $C_1 + C_2/3$. Thus the penguin diagram contribution is comparable with the tree contribution. We study the averaged branching ratio of the decay $B_s(\bar{B}_s) \to \pi^0 \bar{K}^0(K^0)$ as a function of $\alpha$ in Fig. 3. It is similar with Fig. 2. We find that the branching ratio of $B_s(\bar{B}_s) \to \pi^0 \bar{K}^0(K^0)$ is about $2.5 \times 10^{-7}$ when $\alpha$ is near 100°, it is a little smaller than the result of Ref. [15].

In SM, the CKM phase angle is the origin of $CP$ violation. Using Eqs. (19) and (20), the direct $CP$ violation parameter can be derived as

$$A_{CP}^{\rm dir} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2z \sin\alpha \sin\delta}{1 + 2z \cos\alpha \cos\delta + z^2}. \tag{24}$$

It is approximately proportional to CKM angle $\sin\alpha$, strong phase $\sin\delta$, and the relative size $z$ between penguin contribution and tree contribution. We show the direct $CP$ violation parameters as a function of CKM angle $\alpha$ in Fig. 4. From this figure one can see that the direct $CP$ asymmetry parameter of $B_s^0(\bar{B}_s^0) \to \pi^\pm K^\mp$ and $\pi^0 \bar{K}^0(K^0)$ can be as large as $-30\%$ and $-50\%$ when $\alpha$ is near 75°. The larger direct $CP$ asymmetry of $B_s^0(\bar{B}_s^0) \to \pi^0 \bar{K}^0(K^0)$ decay is mainly due to a larger $z$ in $B_s^0(\bar{B}_s^0) \to \pi^0 \bar{K}^0(K^0)$ than in $B_s^0(\bar{B}_s^0) \to \pi^\pm K^\mp$.

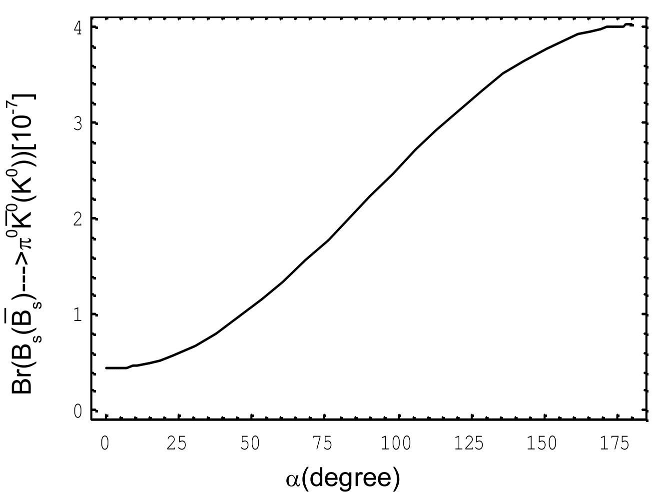


FIG. 3. The averaged branching ratio of $B_s(\bar{B}_s) \to \pi^0 \bar{K}^0(K^0)$ decay as a function of CKM angle $\alpha$.

The direct $CP$ asymmetry predicted in QCD factorization approach is quite different from our result, due to the different source of strong phases. In QCD factorization approach, the strong phase mainly comes from the perturbative charm quark loop diagram, which is $\alpha_s$ suppressed [15]. While the strong phase in PQCD comes mainly from nonfactorizable and annihilation type diagrams. The sign of the direct $CP$ asymmetry is different for these two approaches in $B_s^0(\bar{B}_s^0) \to \pi^\pm K^\mp$ decay, and the magnitude of $CP$ asymmetry in QCD factorization (about 5%) is also smaller than PQCD. The future LHC-b experiments can make a test for the two methods.

For the decays of $B_s(\bar{B}_s) \to \pi^0 \bar{K}^0(K^0)$, the final $\bar{K}^0(K^0)$ mesons cannot be detected directly. What the experiments measured are their mixtures $K_s$ and $K_L$, thus a mixing induced $CP$ violation is involved. Following notations in the previous literature [22], we define the mixing induced $CP$ violation parameter as

$$a_{\epsilon+\epsilon'} = \frac{-2Im(\lambda_{CP})}{1 + |\lambda_{CP}|^2}, \tag{25}$$

where

$$\lambda_{CP} = \frac{V_{tb}^* V_{ts} \langle \pi^0 K^0 | H_{eff} | \bar{B}_s^0 \rangle}{V_{tb} V_{ts}^* \langle \pi^0 \bar{K}^0 | H_{eff} | B_s^0 \rangle}. \tag{26}$$

Using unitarity condition of the CKM matrix $V_{tb} V_{td}^* = -V_{ub} V_{ud}^* - V_{cb} V_{cd}^*$, and Eqs. (19) and (20), we can get

$$\lambda_{CP} = \frac{e^{-i\gamma} + x}{e^{i\gamma} + x}, \tag{27}$$

where $x = \frac{V_{cb} V_{cd}^*}{|V_{ub} V_{ud}^*|} \frac{P}{T+P}$. Combining Eq. (27) and (25), we can get

$$a_{\epsilon+\epsilon'} = \frac{\sin 2\gamma + 2Re(x) \sin\gamma}{1 + |x|^2 + 2Re(x) \cos\gamma}. \tag{28}$$

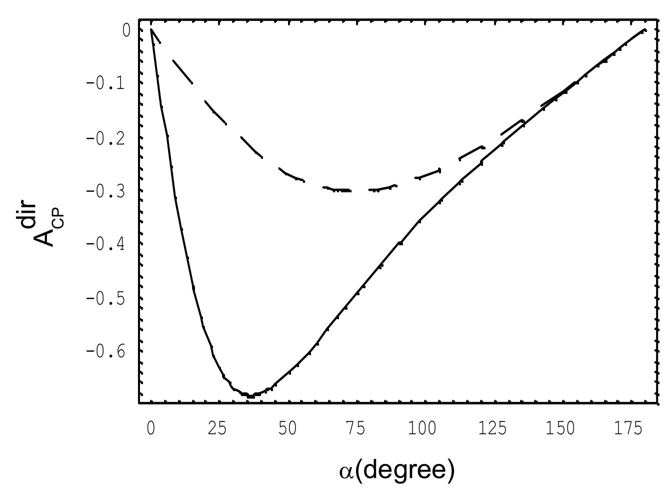


FIG. 4. Direct $CP$ violation parameters of $B_s^0(\bar{B}_s) \to \pi^\pm K^\mp$ (dashed line) and $B_s(\bar{B}_s) \to \pi^0 \bar{K}^0(K^0)$ (solid line) as a function of CKM angle $\alpha$.

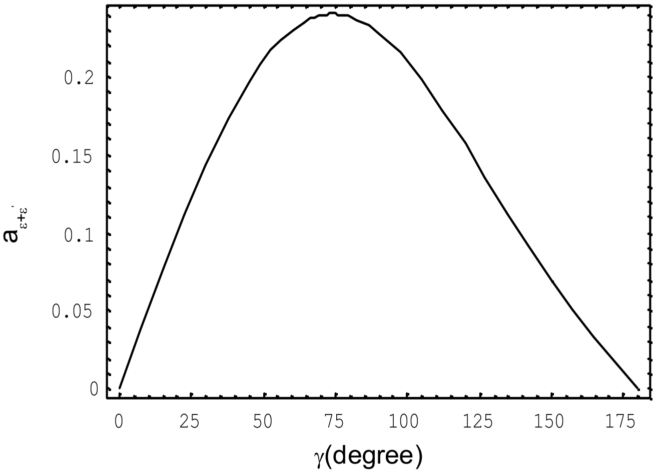


FIG. 5. Mixing induced $CP$ violation parameter of $B_s(\bar{B}_s) \to \pi^0\bar{K}^0(K^0)$ as a function of CKM angle $\gamma$.

If $|x|$ is a very small number, the mixing induced $CP$ asymmetry is proportional to $\sin 2\gamma$, which will be a good place for the CKM angle $\gamma$ measurement. However as we already mentioned, the tree contribution in this channel is suppressed, $|x| = 3.7$ is a large number so that the $\sin\gamma$ behavior is dominant in the Eq. (28). The result of mixing induced $CP$ violation is shown in Fig. 5, which is indeed a roughly $\sin\gamma$ behavior. The tail near $\gamma \sim 180°$ also shows the contribution from $\sin 2\gamma$ in Eq. (28).

## IV. SUMMARY

In this work, we study the branching ratio and $CP$ asymmetry of the decays $B_s^0(\bar{B}_s^0) \to \pi^\pm K^\mp$ and $B_s(\bar{B}_s) \to \pi^0\bar{K}^0(K^0)$ in PQCD approach. From our calculation, we find that the branching ratio of $B_s^0(\bar{B}_s^0) \to \pi^\pm K^\mp$ is about $(6 \sim 10) \times 10^{-6}$; $Br(B_s(\bar{B}_s) \to \pi^0\bar{K}^0(K^0))$ around $2 \times 10^{-7}$ and there are large $CP$ violations in the processes, which may be measured in the future LHC-b experiments and BTeV experiments at Fermilab.

## ACKNOWLEDGMENTS

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## APPENDIX: FORMULAS FOR THE CALCULATIONS USED IN THE TEXT

In the appendix we present the explicit expressions of the formulas used in section II. First, we give the expressions of the meson distribution amplitudes $\phi_M$. For $B_s$ meson wave function, we use the similar wave function as $B$ meson [7,8]:

$$\phi_{B_s}(x, b) = N_{B_s} x^2(1-x)^2 \exp\left[-\frac{M_{B_s}^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right]. \tag{A1}$$

We set the central value of parameter $\omega_b = 0.5$ GeV in our numerical calculation, and $N_{B_s} = 63.7$ GeV is the normalization constant using $f_{B_s} = 230$ MeV.

The $\pi$ meson's distribution amplitudes are given by light-cone QCD sum rules [17]:

$$\begin{aligned}
\phi_\pi^A(x) &= \frac{3f_\pi}{\sqrt{2N_c}} x(1-x)\{1 + 0.44C_2^{3/2}(t) + 0.25C_4^{3/2}(t)\},\\
\phi_\pi^P(x) &= \frac{f_\pi}{2\sqrt{2N_c}}\{1 + 0.43C_2^{1/2}(t) + 0.09C_4^{1/2}(t)\},\\
\phi_\pi^T(x) &= \frac{f_\pi}{2\sqrt{2N_c}}(1-2x)\{1 + 0.55(10x^2 - 10x + 1)\},
\end{aligned} \tag{A2}$$

where $t = 1 - 2x$. The Gegenbauer polynomials are defined by:

$$\begin{aligned}
C_2^{1/2}(t) &= \frac{1}{2}(3t^2 - 1), \qquad C_4^{1/2}(t) = \frac{1}{8}(35t^4 - 30t^2 + 3),\\
C_2^{3/2}(t) &= \frac{3}{2}(5t^2 - 1), \qquad C_4^{3/2}(t) = \frac{15}{8}(21t^4 - 14t^2 + 1).
\end{aligned} \tag{A3}$$

We use the distribution amplitude $\phi_K^{A,P,T}$ of the $K$ meson from Ref. [16]:

$$\begin{aligned}
\phi_K^A(x) &= \frac{6f_K}{2\sqrt{2N_c}} x(1-x)[1 + 0.15t + 0.405(5t^2 - 1)],\\
\phi_K^P(x) &= \frac{f_K}{2\sqrt{2N_c}}[1 + 0.106(3t^2 - 1)\\
&\quad - 0.148(3 - 30t^2 + 35t^4)/8],\\
\phi_K^T(x) &= \frac{f_K}{2\sqrt{2N_c}} t[1 + 0.1581(5t^2 - 3)],
\end{aligned} \tag{A4}$$

whose coefficients correspond to $m_{0K} = 1.6$ GeV.

In our numerical analysis, we use the one-loop expression for the strong running coupling constant,

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}, \tag{A5}$$

where $\beta_0 = (33 - 2n_f)/3$ and $n_f$ is the number of active quark flavor at the appropriate scale $\mu$. $\Lambda$ is the QCD scale, which we take $\Lambda = 250$ MeV at $n_f = 4$.

$S_{B_s}$, $S_{\pi^+}$, $S_{k^-}$ used in the decay amplitudes are defined as

$$S_{B_s(t)} = s(x_1 P_1^+, b_1) + 2\int_{1/b_1}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \tag{A6}$$

$$S_{\pi^+}(t) = s(x_3 P_3^+, b_3) + s((1-x_3)P_3^+, b_3) + 2\int_{1/b_3}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \quad \text{(A7)}$$

$$S_{K^-}(t) = s(x_2 P_2^-, b_2) + s((1-x_2)P_2^-, b_2) + 2\int_{1/b_2}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \quad \text{(A8)}$$

where the so-called Sudakov factor $s(Q, b)$ resulting from the resummation of double logarithms is given as [23,24]

$$s(Q, b) = \int_{1/b}^{Q} \frac{d\mu}{\mu}\left[\ln\left(\frac{Q}{\mu}\right) A(\alpha(\bar{\mu})) + B(\alpha_s(\bar{\mu}))\right] \quad \text{(A9)}$$

with

$$A = C_F \frac{\alpha_s}{\pi} + \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{2}{3}\beta_0 \ln\left(\frac{e^{\gamma E}}{2}\right)\right]\left(\frac{\alpha_s}{\pi}\right)^2, \quad \text{(A10)}$$

$$B = \frac{2}{3} \frac{\alpha_s}{\pi} \ln\left(\frac{e^{2\gamma_{E-1}}}{2}\right). \quad \text{(A11)}$$

Here $\gamma_E = 0.57722\cdots$ is the Euler constant, $n_f$ is the active quark flavor number. For the detailed derivation of the Sudakov factors, see Refs. [6,25].

The functions $h_i (i = a, c, e, g)$ come from the Fourier transformation of propagators of virtual quark and gluon in the hard part calculations. They are given as

$$h_a(x_1, x_2, b_1, b_2) = S_t(x_2) K_0(M_B\sqrt{x_1 x_2} b_1) \times [\theta(b_2 - b_1) I_0(M_B\sqrt{x_2} b_1) K_0(M_B\sqrt{x_2} b_2) + (b_1 \leftrightarrow b_2)], \quad \text{(A12)}$$

$$h_c^{(j)}(x_1, x_2, x_3, b_2, b_3) = \{\theta(b_2 - b_3) I_0(M_B\sqrt{x_1(1-x_2)} b_3) K_0(M_B\sqrt{x_1(1-x_2)} b_2) + (b_2 \leftrightarrow b_3)\} \times \begin{pmatrix} \mathrm{K}_0(M_B F_{(j)} b_3), & \text{for} \quad F_{(j)}^2 > 0 \\ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{|F_{(j)}^2|} b_3), & \text{for} \quad F_{(j)}^2 < 0 \end{pmatrix}, \quad \text{(A13)}$$

where $\mathrm{H}_0^{(1)}(z) = \mathrm{J}_0(z) + i\mathrm{Y}_0(z)$, and $F_{(j)}$'s are defined by

$$F_{(1)}^2 = x_1 + x_2 + x_3 - x_1 x_2 - x_2 x_3 - 1, \qquad F_{(2)}^2 = x_1 - x_3 - x_1 x_2 + x_2 x_3; \quad \text{(A14)}$$

$$h_e^{(j)}(x_1, x_2, x_3, b_1, b_2) = \left\{\theta(b_2 - b_1) \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{x_2 x_3} b_2) \mathrm{J}_0(M_B\sqrt{x_2 x_3} b_1) + (b_1 \leftrightarrow b_2)\right\} \times \begin{pmatrix} \mathrm{K}_0(M_B F_{e(j)} b_1), & \text{for} \quad F_{e(j)}^2 > 0 \\ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{|F_{e(j)}^2|} b_1), & \text{for} \quad F_{e(j)}^2 < 0 \end{pmatrix}, \quad \text{(A15)}$$

where $F_{e(j)}$'s are defined by

$$F_{e(1)}^2 = x_1 + x_2 + x_3 - x_1 x_2 - x_2 x_3, \qquad F_{e(2)}^2 = x_1 x_2 - x_2 x_3; \quad \text{(A16)}$$

$$h_g(x_2, x_3, b_2, b_3) = S_t(x_2) \frac{\pi i}{2} H_0^{(1)}(M_B\sqrt{x_2 x_3} b_3) \times \left[\theta(b_3 - b_2) J_0(M_B\sqrt{x_2} b_2) \frac{\pi i}{2} H_0^{(1)}(M_B\sqrt{x_2} b_3) + (b_2 \leftrightarrow b_3)\right]. \quad \text{(A17)}$$

We adopt the parametrization for $S_t(x)$ contributing to the factorizable diagrams [26],

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)} [x(1-x)]^c, \qquad c = 0.3. \quad \text{(A18)}$$

The hard scale $t_i' s$ in Eqs. (5)–(12) are chosen as

$$
\begin{aligned}
&t_a^1 = \max(M_B\sqrt{1-x_2}, 1/b_1, 1/b_2), \qquad t_a^2 = \max(M_B\sqrt{x_1}, 1/b_1, 1/b_2),\\
&t_c^1 = \max(M_B\sqrt{|F_{(1)}^2|}, M_B\sqrt{x_1(1-x_2)}, 1/b_2, 1/b_3), \qquad t_c^2 = \max(M_B\sqrt{|F_{(2)}^2|}, M_B\sqrt{x_1(1-x_2)}, 1/b_2, 1/b_3),\\
&t_e^1 = \max(M_B\sqrt{|F_{e(1)}^2|}, M_B\sqrt{x_2x_3}, 1/b_1, 1/b_2), \qquad t_e^2 = \max(M_B\sqrt{|F_{e(2)}^2|}, M_B\sqrt{x_2x_3}, 1/b_1, 1/b_2),\\
&t_g^1 = \max(M_B\sqrt{x_2}, 1/b_2, 1/b_3), \qquad t_g^2 = \max(M_B\sqrt{x_3}, 1/b_2, 1/b_3).
\end{aligned} \tag{A19}
$$

They are given as the maximum energy scale appearing in each diagram to kill the large logarithmic radiative corrections.

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[1] Y. Li, C.-D. Lü, Z. J. Xiao, and X.-Q. Yu, Phys. Rev. D **70**, 034009 (2004).

[2] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985); **34**, 103 (1987); L.-L. Chau, H.-Y. Cheng, W. K. Sze, H. Yao, and B. Tseng, Phys. Rev. D **43**, 2176 (1991); **58**, 019902(E) (1998).

[3] A. Ali, G. Kramer, and C. D. Lü, Phys. Rev. D **58**, 094009 (1998); **59**, 014005 (1999); C. D. Lü, Nucl. Phys. B, Proc. Suppl. **74**, 227 (1999).

[4] Y.-H. Chen, H.-Y. Cheng, B. Tseng, and K.-C. Yang, Phys. Rev. D **60**, 094014 (1999); **62**, 054029 (2000).

[5] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. B **591**, 313 (2000).

[6] H.-n. Li and H. L. Yu, Phys. Rev. Lett. **74**, 4388 (1995); Phys. Lett. B **353**, 301 (1995); **348**, 597 (1995); H.-n. Li and H. L. Yu, Phys. Rev. D **53**, 2480 (1996).

[7] Y.-Y. Keum, H.-n. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).

[8] C.-D. Lü, K. Ukai, and M.-Z. Yang, Phys. Rev. D **63**, 074009 (2001).

[9] H.-n. Li, Prog. Part. Nucl. Phys. **51**, 85 (2003).

[10] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[11] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).

[12] S. Narison, Phys. Lett. B **520**, 115 (2001); S. Hashimoto, hep-ph/0411126.

[13] Belle Collaboration, A. J. Schwartz, hep-ex/0411075; *BABAR* collaboration, A. Bevan, hep-ex/0411090.

[14] D. S. Du and Z. Z. Xing, Phys. Rev. D **48**, 3400 (1993); D. S. Du and M. Z. Yang, Phys. Lett. B **358**, 123 (1995); Y. H. Chen, H. Y. Cheng, and B. Tseng, Phys. Rev. D **59**, 074003 (1999).

[15] M. Beneke and M. Neubert, Nucl. Phys. B **675**, 333 (2003); J.-F.Sun, G. H. Zhu, and D.-S. Du, Phys. Rev. D **68**, 054003 (2003).

[16] A. Khodjamirian, T. Mannel, and M. Melcher, Phys. Rev. D **70**, 094002 (2004); V. M. Braun and A. Lenz, Phys. Rev. D **70**, 074020 (2004).

[17] V. M. Braun and I. E. Filyanov, Z. Phys. C **44**, 157 (1989); **48**, 239 (1990); P. Ball, J. High Energy Phys. 01 (1999) 010.

[18] T. Huang, X.-H. Wu, and M. Z. Zhou, Phys. Rev. D **70**, 014013 (2004); T. Huang, X.-G. Wu, and X.-H. Wu, Phys. Rev. D **70**, 053007 (2004); T. Huang and X.-G. Wu, Phys. Rev. D **70**, 093013 (2004); T. Huang, M.-Z. Zhou, and X.-H. Wu, hep-ph/0501032.

[19] H. Kawamura, J. Kodaira, C.-F. Qiao, and K. Tanaka, Nucl. Phys. B, Proc. Suppl. **116**, 269 (2003); H.-n. Li and H.-S. Liao, Phys. Rev. D **70**, 074030 (2004); T. Huang, X.-G. Wu, and M.-Z. Zhou, Phys. Lett. B **611**, 260 (2005); B. Geyer and O. Witzel, hep-ph/0502239.

[20] CLEO Collaboration, D. Cronin-Hennessy *et al.*, hep-ex/0001010.

[21] A. Warburton, hep-ex/0411079.

[22] G. Kramer, W. F. Palmer, and Y. L. Wu, Commun. Theor. Phys. **27**, 457 (1997).

[23] H.-n. Li and K. Ukai, Phys. Lett. B **555**, 197 (2003).

[24] H.-n. Li and B. Melic, Eur. Phys. J. C **11**, 695 (1999).

[25] H.-n. Li, Phys. Rev. D **52**, 3958 (1995).

[26] T. Kurimoto, H.-n. Li, and A. I. Sanda, Phys. Rev. D **65**, 014007 (2002).