

**Effects of instanton induced interactions on pentaquarks**

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Roles of instanton induced interactions (III) in the masses of pentaquark baryons,  $\Theta^+$  ( $J = 1/2$  and  $3/2$ ) and  $\Xi^{--}$ , and a dibaryon,  $H$ , are discussed using the MIT bag model. It is shown that the two-body terms in III give a strong attraction mainly due to the increase of the number of flavor antisymmetric quark pairs in multi-quark systems. In contrast, the three-body  $u$ - $d$ - $s$  interaction is repulsive. It is found that III lowers the mass of negative-parity  $\Theta^+$  as much as 100 MeV from the mass predicted by the bag model without III. We also consider possible quark model configurations for positive-parity  $\Theta^+$  under III.

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**I. INTRODUCTION**

Reports of discoveries of exotic baryons,  $\Theta^+$  [1] and  $\Xi^{--}$  [2], started intensive discussions on various possibilities of pentaquark states. Existence of the  $\Theta^+$  has been confirmed by several groups [3], while some recent high energy experiments gave negative results [4]. Further experimental studies are being done in order to establish its existence. Meanwhile, it is a theoretical problem to reveal the properties of pentaquarks.

$\Theta^+$  is a baryon with strangeness +1 and therefore cannot be made of three quarks. The minimal quark content of  $\Theta^+$  is  $u^2 d^2 \bar{s}$ , and thus it is called ‘‘pentaquark’’. The most essential properties are spin and parity. Predictions by the chiral soliton model [5,6] and various quark models [7–15] claimed a  $1/2^+$  state of  $\Theta^+$ , while constituent quark model with all the five quarks sitting in the lowest energy level predicts a negative-parity ground state,  $1/2^-$  [16]. (The other possible spin is  $3/2$ , but most of the dynamical quark models, that can reproduce spectra of ordinary hadrons, predict that lower spin states have lower masses.) Furthermore, majority of QCD-based calculations, such as QCD sum rule [17–19] and lattice QCD [20–22], indicate that the positive-parity state has much higher mass.

The observed width of  $\Theta^+$  decaying into  $KN$  scattering states seems very small, i.e., likely to be less than 10 MeV. It is hard to understand this small width if  $\Theta^+$  is  $1/2^-$  state, because it then decays into  $KN$  S-wave states. Recent calculation [23] shows that  $1/2^-$  state should have a width of order a few hundred MeV, while  $1/2^+$  state may have much smaller width due mainly to the suppression by the centrifugal barrier and the small overlaps to  $KN$  state. Also the higher spin state,  $3/2^-$ , is favorable for a small width because the state decays to only the D-wave  $KN$  state.

Several possibilities to reduce the mass of the positive-parity state have been proposed in literatures [10,11,24]. Among them, proposals of exotic clustering of a diquark or triquark inside the pentaquark system are popular. For instance, Jaffe and Wilczek [11] proposed a structure of the pentaquark as a bound state of two scalar  $ud$  diquarks and  $\bar{s}$ . Because of the color structure and the Bose symmetry of diquarks, this bound state has to have a nonzero angular momentum,  $L = 1$ , between the diquarks and thus the resulting  $\Theta^+$  has positive parity.

Under these circumstances, it is important to study contributions of various different quark dynamics to the pentaquark states in detail. The constituent quark model has several important dynamical ingredients, among which we here consider confinement, perturbative one-gluon-exchange interaction and nonperturbative instanton induced interaction.

The first one has an obvious role, that is, to confine quarks inside a color singlet hadron. In the nonrelativistic quark models, a confinement potential of a linear form is often used. Its main role is to give localized wave functions of quarks in the hadron. However, the confinement mechanism for pentaquarks has not been fully understood in potential quark models, while the confinement potential was calculated in lattice QCD [25] in the heavy quark limit. The MIT bag model [26] gives another picture of confinement, that is, quarks are confined by the bag pressure at the bag surface. The bag is a cavity of perturbative vacuum in the nonperturbative vacuum of QCD and thus has a larger volume energy. This energy difference between the outside and the inside of the bag produces pressure at the bag surface and confines quarks. We consider that this confinement picture is valid for the pentaquark systems because it reflects only the nature of the QCD vacua.

Important roles of the one-gluon exchange (OgE) interactions for the meson and baryon spectra were first pointed

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out by DeRujula, Georgi and Glashow [27]. They showed, in particular, that the color-magnetic interaction is essential to explain the mass difference of the octet and decuplet baryons. They, however, noticed that the spectrum of the pseudoscalar mesons, the large mass differences among  $\pi$ ,  $\eta$  and  $\eta'$ , is not reproduced purely by OgE. This difficulty is known to be related to the  $U_A(1)$  anomaly, which was pointed out by Weinberg [28].

The instanton induced interaction (III) may solve the  $U_A(1)$  problem, as was shown by 't Hooft [29]. The instanton is a solution of the Yang Mills equation in the Euclidean 4-dimensional space [30]. Its coupling to light quarks has special properties that it occurs through zero modes of the quark localized around the instanton and that it changes chirality and thus breaks  $U_A(1)$  symmetry. In the meson spectrum, III gives significant mass splitting between  $\eta$  and  $\eta'$ .

Interesting roles of III in the baryon spectrum were studied by Shuryak-Rosner [31] and Oka-Takeuchi [32] in 1989. They pointed out that the spin-spin interaction comes also from III and III (with or without OgE) gives a similar spectrum of baryons. Therefore the baryon spectrum cannot be used to determine which of the two interactions are responsible for the mass splitting of hadrons. It is, of course, most likely that both of them contribute to the octet-decuplet splitting. Indeed, if we assume that the splitting comes only from the color-magnetic interaction of OgE, then the coupling strength required to reproduce empirical data is in general too large in the sense that the perturbative treatment is hardly justified. It is, therefore, favorable that III is responsible for a part of the octet-decuplet splitting and thus reduces the part given by OgE.

Other possibilities of interquark interactions include exchange of the Nambu-Goldstone bosons of chiral symmetry breaking [8,12,24]. The NG boson exchange gives a flavor-spin-dependent force, which contributes to the octet-decuplet mass splitting. It was also pointed out that such interaction may generate considerable diquark correlations in the pentaquark baryon [24]. We, however, do not consider the NG boson exchange in the present study in the context of the MIT bag model.

The purpose of this paper is to clarify roles of III in the pentaquark systems employing the model mentioned above [33,34]. We consider III in the context of the MIT bag model. Namely, we assume that III is applied to quarks inside a bag. This assumption may need some explanation. In the naive bag model, the ‘‘vacuum’’ inside the bag is identified as the perturbative vacuum where there is no gluon condensate. As a consequence, instanton densities inside the bag could be different from that employed in the instanton vacuum picture. It is, however, argued that such an approach is not able to reproduce appropriate spectrum of the pseudoscalar mesons. Therefore we consider a situation that the vacuum structure is minimally modified, while quantum excitation modes exist only in the bag.

On the other hand, it is natural to assume that the confinement mechanism that successfully describes three-quark baryon states is common to the pentaquark baryons. Also there is an advantage of the bag model, i.e., the quarks inside the baryon are treated relativistically.

III is well known to give strong correlation between  $ud$  quarks forming a scalar diquark with isospin  $I = 0$  and spin  $S = 0$ . It was also suggested that III is a main driving force of color superconducting phase that may appear at finite baryon density in QCD. Recent calculations indicated importance of  $u - d$  two-body correlation due to the OgE interaction [14,15,24]. A similar role of III was suggested by Kochelev *et al.* [13], where the authors considered very strong III model (4 times stronger than OGE) and proposed a diquark-triquark picture of  $\Theta^+$ . They evaluated contributions of III only within the diquark and triquark, while they neglect interaction between the clusters. In this study, we consider a ‘‘shell model’’ like structure of pentaquarks only and do not take into account two-body correlations. On the other hand, our calculation takes two-body and three-body III fully into account among all the five quarks.

In Sec. II, we give formulations of the instanton induced interactions and the MIT bag model. In Sec. III, results for the masses and sizes of the pentaquarks and  $H$  dibaryon are presented. In Sec. IV, we discuss effects of III on positive parity  $\Theta^+$  states. A conclusion is given in Sec. V.

## II. FORMALISM

### A. Instanton Induced Interaction

The instanton induced interaction (III), introduced by 't Hooft [29], is an interaction among quarks of  $N_f (= 3)$  light flavors, which is induced by coupling of quarks to instantons (and anti-instantons) through zero modes. The instanton is a classical solution of the gauge field equation for the Euclidean 4-dimensional space, which is one of the most important nonperturbative effects in QCD. The main difference from the perturbative gluon-exchange interactions is that III is not chirally invariant and applies only on flavor singlet states of quarks. 't Hooft pointed out that III solves the  $U_A(1)$  problem, namely, the origin of  $\eta - \eta'$  mass splitting. We employ a long wave length approximation so that the size of the instanton ( $\approx 1/3$  fm) is neglected. Then the interaction is written as a contact interaction [35].

$$H^{(3)} = G^{(3)} \epsilon_{ijk} \epsilon_{i'j'k'} \bar{\psi}_{R,i}(1) \bar{\psi}_{R,j}(2) \bar{\psi}_{R,k}(3) \\ \times \left( 1 - \frac{1}{7} \sum_{i < j} \sigma_i \cdot \sigma_j \right) \psi_{L,k'}(3) \psi_{L,j'}(2) \psi_{L,i'}(1) \\ + (\text{h.c.}), \quad (1)$$

$$H^{(2)} = G^{(2)} \epsilon_{ij} \epsilon_{i'j'} \bar{\psi}_{R,i}(1) \bar{\psi}_{R,j}(2) \\ \times \left( 1 - \frac{1}{5} \sigma_1 \cdot \sigma_2 \right) \psi_{L,j'}(2) \psi_{L,i'}(1) + (\text{h.c.}), \quad (2)$$

where  $\psi$  is the field operator of the quark,  $\epsilon_{ijk}$  is totally asymmetric tensor, and  $i, j$  and  $k$  represent flavor.  $R$  and  $L$  are chiral indices.  $H^{(3)}$  is the 3-body Hamiltonian and  $H^{(2)}$  is the 2-body Hamiltonian obtained by contracting a quark pair in the 3-body III into a quark condensate or the quark mass term. Then the relation between the strengths of the 2-body and 3-body III is given by

$$G_{ud}^{(2)} \simeq \frac{25}{7} \frac{\langle \bar{u}u \rangle}{2} \frac{m_s^{\text{eff}}}{m_u^{\text{eff}}} G^{(3)}, \quad (3)$$

where we neglect the current quark mass,  $m^{\text{eff}}$  is the constituent mass of the quark. We use  $m_u^{\text{eff}}/m_s^{\text{eff}} \simeq 0.6$ . The  $\langle \bar{u}u \rangle$  is the quark condensate. We use  $\langle \bar{u}u \rangle \simeq (-225 \text{ MeV})^3$ . The 3-body interaction  $H^{(3)}$  is repulsive, while the 2-body interaction  $H^{(2)}$  is attractive because the quark condensate is negative. The 2-body III has three flavor types,  $(H_{ud}^{(2)}, H_{us}^{(2)}, H_{ds}^{(2)})$ .  $H_{ud}^{(2)}$ , for instance, is an interaction between  $u$  and  $d$  quarks that is obtained by contracting  $s\bar{s}$  quark pair from the 3-body Hamiltonian. As a result,  $H_{ud}^{(2)}$  is proportional to the effective quark mass of strangeness,  $m_s$ . Thus we obtain

$$G_{us}^{(2)} = \frac{m_u^{\text{eff}}}{m_s^{\text{eff}}} G_{ud}^{(2)}. \quad (4)$$

We concentrate on the ground state baryons. Both the 2-body III and the 3-body III do not affect the decuplet baryons,  $\Delta, \Sigma^*, \Xi^*$  and  $\Omega$ , because III is applied only to flavor antisymmetric quarks. Likewise for the octet baryon,  $N, \Lambda, \Sigma$  and  $\Xi$ , the 3-body III does not exert. On the other hand, the 2-body III gives attractive force. But the effects of the 2-body III are similar to the one gluon exchange (OgE) regarding the spin structure [31]. Thus the baryon spectrum can be reproduced by any combination OgE and the 2-body III, that is, we can not determine the strength of III from the baryon spectrum.

Takeuchi and Oka [36] pointed out that the 3-body III plays an important role in the  $H$  dibaryon, a flavor singlet system of  $u^2 d^2 s^2$ . The fact that a deeply bound  $H$  dibaryon does not exist favors a strong 3-body III, while is repulsive in the  $H$  system. The pseudoscalar and scalar mesons are also known to be sensitive to III [37–41].

Since  $\Theta^+$  has only one strange antiquark, the repulsive force from the 3 body III is expected to be weaker than that for the  $H$  dibaryon.

## B. MIT Bag Model

We introduce the instanton induced interaction in the MIT bag model. This effect for the baryon and meson is shown by Kochelev *et al.* [42]. The mass splitting between the octet baryons and the decuplet baryons comes from the hyperfine interaction. The origin of the hyperfine interaction can be either the one gluon exchange (OgE) or the instanton induced interaction (III).

The mass of a hadron in the MIT Bag Model [43] is given by

$$M(R) = n_u w(m_u, R) + n_s w(m_s, R) + \frac{4\pi}{3} B R^3 - \frac{Z_0}{R} + (1 - P_{III}) \sum_{i>j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) (\vec{\lambda}_i \cdot \vec{\lambda}_j) M_{ij}(R) + P_{III} (H^{(3)}(R) + H^{(2)}(R)) + E_0, \quad (5)$$

where  $R$  is the bag radius,  $n_u$  is the number of the  $u, d$  quarks, and  $n_s$  is the number of the  $s$  and  $\bar{s}$  quarks. The function  $w(m, R)$  denotes the single particle energy obtained as an eigenvalue of the Dirac equation for the bag.  $-Z_0/R$  is the zero point energy as well as corrections due to the center of mass motion of the bag.  $B$  is the bag constant. The fifth term is the color-magnetic part from OgE.  $M_{ij}(R)$  is the strength including the spatial contribution, which is given in Appendix A. We neglect the color-electric interaction. The  $H^{(3)}$  and  $H^{(2)}$  are contributions of III introduced in Sec. II A.

$P_{III}$  is a parameter which represents the portion of the hyperfine splitting induced by III. By multiplying  $P_{III}$  to III and  $(1 - P_{III})$  to OgE, we keep the mass splitting of  $N - \Delta$  to 300 MeV. If  $P_{III} = 0$ , the mass splitting of  $N - \Delta$  comes purely from OgE, while for  $P_{III} = 1$  it comes purely from III. Another way of determining  $P_{III}$  is to reproduce the  $\eta - \eta'$  mass difference. Although the method suffers from model dependence, we estimated  $P_{III} = 0.25 - 0.6$ , and in the case of the MIT Bag model,  $P_{III} = 0.31$  [44]. In the present analysis, we allow  $P_{III}$  from 0 to 1 and study effects of III compared with OgE.

$E_0$  in Eq. (5) is a parameter introduced to reproduce the mass of the nucleon. If we ignore the changes of the radii of  $N$  and  $\Delta$  under the variation of  $P_{III}$ ,  $E_0$  is given roughly by  $E_0 = 150 \text{ MeV} \times P_{III}$ . The effect of  $E_0$  can be taken into account by changing  $Z_0$  and  $B$  accordingly, but here for simplicity we fix  $Z_0$  and  $B$  and change  $E_0$  as a function of  $P_{III}$ . In the present study, we leave  $P_{III}$  to be a free parameter within  $0 < P_{III} < 1$  and investigate its effects.

We obtain the masses and the radii of the pentaquarks by minimizing  $M(R)$ ,

$$\frac{dM(R)}{dR} = 0. \quad (6)$$

The parameters of the bag model are taken from the original MIT bag model, DeGrand *et al.* [43],  $B = (0.145 \text{ GeV})^4$ ,  $Z_0 = 1.84$ ,  $\alpha_c = \alpha_s/4 = 0.55$ ,  $m_s = 0.279 \text{ GeV}$  and  $m_u = 0 \text{ GeV}$ .

## III. NEGATIVE PARITY STATES

We consider the pentaquarks  $\Theta^+$  composed of  $uudd\bar{s}$  with isospin 0, spin 1/2, and  $\Xi^{--}$ , a partner within the flavor  $\bar{10}$  with isospin 3/2. First an analysis is shown for negative-parity states, where all the quarks are placed in the lowest single particle state,  $1S_{1/2}$ . We also consider

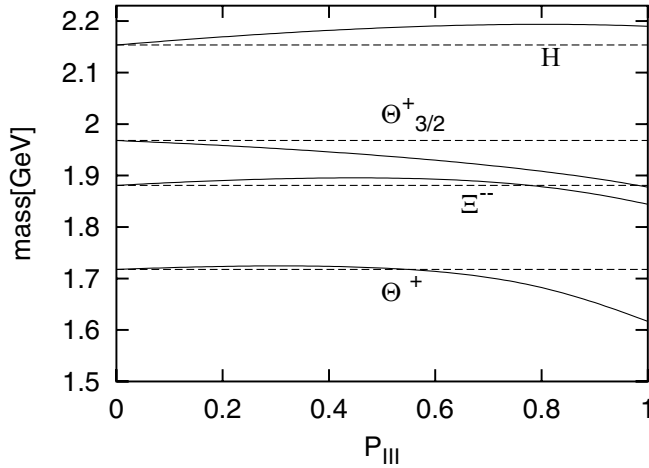


FIG. 1. The masses of negative-parity pentaquarks,  $\Theta^+$ ,  $\Xi^{--}$ ,  $\Theta^+_{S=3/2}$ , and the dibaryon  $H$ . The dashed lines are the values at  $P_{III} = 0$ .

$\Theta^+_{S=3/2}$ , which is the spin partner of  $\Theta^+$ . Note that in the negative-parity case, the only possible spin partner is a spin 3/2 state, and no spin 5/2 state is allowed due to permutation symmetry.

In Fig. 1 and Table I, we show the masses of the pentaquarks as functions of  $P_{III}$ . The dashed lines are the values at  $P_{III} = 0$ , which correspond to the masses under the influence only of OgE. The right end,  $P_{III} = 1$ , gives the masses when the  $N - \Delta$  splitting is purely due to III. First, we point out that the pure OgE lowers the masses of  $\Theta^+$  from the noninteracting 5 quark state. One sees that the  $\Theta^+$  is affected by III most strongly among these states. At  $P_{III} = 1$ , the mass of  $\Theta^+$  is 100 MeV smaller than that at  $P_{III} = 0$ . It is, however, not sensitive to  $P_{III}$  in the region,  $P_{III} = 0.25 \sim 0.6$ , which is considered to be realistic. In contrast,  $\Theta^+_{S=3/2}$  changes significantly in this region. For all  $P_{III}$ , the mass of  $\Xi^{--}$  is almost constant. The mass of the  $H$  dibaryon [45] (positive-parity ground state) is calculated in the same model and parameters and it is found that it grows monotonically as  $P_{III}$  increases. A paper by Dorokov *et al.* [46] calculated the  $H$  dibaryon mass using the bag model including the III contribution. Their results are essentially the same as ours except for the choice of the parameters.

TABLE I. The masses and radii of the negative-parity pentaquarks and the  $H$  dibaryon.

	$\Theta^+$	$\Theta^+_{S=3/2}$	$\Xi^{--}$	$H$
$M(P_{III} = 0)$ [MeV]	1717	1968	1881	2154
$M(P_{III} = 1)$ [MeV]	1617	1877	1844	2190
$M(P_{III} = 1) - M(P_{III} = 0)$ [MeV]	-101	-91	-37	+36
$R(P_{III} = 0)$ [fm]	1.14	1.22	1.13	1.20
$R(P_{III} = 1)$ [fm]	0.76	0.98	0.83	1.00

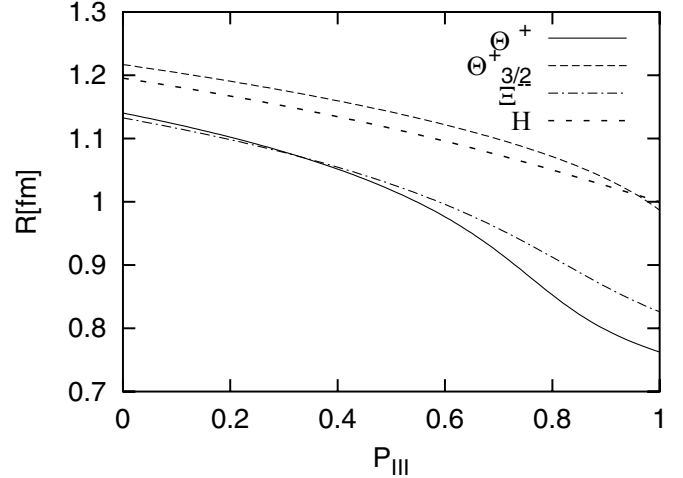


FIG. 2. The bag radii for the pentaquarks.

It is found that the mass of  $\Theta^+$  does not agree with the experimental value (1540 MeV) even if the full III is introduced. On the other hand, the model reproduces the mass of  $\Xi^{--}$  at  $P_{III} = 0.25 \sim 0.6$ . In order to improve this situation we may further take into account an attractive interaction or correlation.

In Fig. 2 and the last two rows of Table I, the radii of the considered baryons are given. They show that  $\Theta^+$  shrinks as  $P_{III}$  increases.  $\Theta^+$  and  $\Xi^{--}$  have almost the same radii at  $P_{III} = 0$ , which are larger than the radius of the nucleon  $5 \text{ GeV}^{-1}$ . At the realistic region of  $P_{III}$ , the radii of pentaquarks are about 0% ~ 20% larger than the radius of the nucleon. We find that the strongly attractive force of III does not reduce the masses, but rather shrinks the radii for the all states.

Contributions of each term of Eq. (5) are listed for  $P_{III} = 1$  in Table II. The contribution of the 3 body III is roughly 10% of that of the 2 body III for the pentaquarks. We find that contribution of OgE is very small for  $\Theta^+_{S=3/2}$ , and thus the effect of III must be most prominent in  $\Theta^+_{S=3/2}$ . We also find that the difference between  $\Theta^+$  and  $\Xi^{--}$  is roughly equal to the mass difference of the  $u$  and  $d$  quarks and the  $s$  quark. The 3 body interaction is stronger in the  $H$  dibaryon than the other cases. The main reason is that the  $H$  dibaryon has more ( $uds$ ) combinations.

Figure 3 shows the mass splitting between  $\Theta^+$  and the other pentaquarks. The mass splitting between  $\Xi^{--}$  and

TABLE II. Contributions of each term of  $M(R)$  at  $P_{III} = 1$  in units of GeV.

	Total	B	$Z_0$	W	$H^{(2)}$	$H^{(3)}$	$E_0$
$\Theta^+$	1.617	0.110	-0.475	2.795	-1.304	0.351	0.140
$\Theta^+_{S=3/2}$	1.877	0.239	-0.367	2.202	-0.358	0.022	0.140
$\Xi^{--}$	1.844	0.140	-0.439	2.751	-0.984	0.235	0.140
$H$	2.190	0.249	-0.362	2.739	-0.727	0.152	0.140

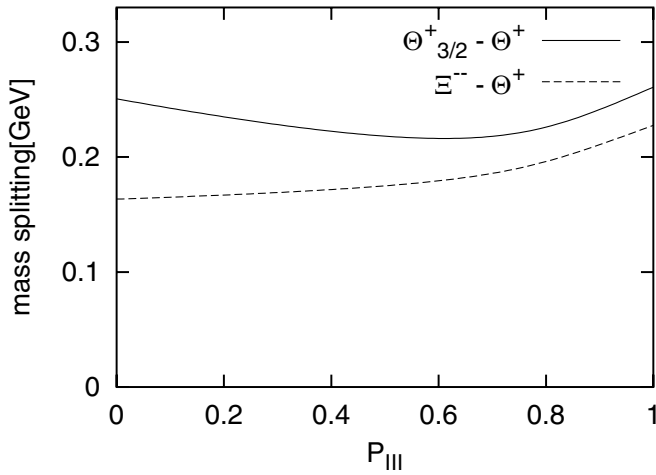


FIG. 3. The mass splittings of  $\Theta_{S=3/2}^+ - \Theta^+$  and  $\Xi^{--} - \Theta^+$ .

$\Theta^+$  increases as  $P_{III}$  because the single particle energy in the bag behaves as  $\sim \frac{1}{R}$ . On the other hand, the mass splitting between  $\Theta_{S=3/2}^+$  and  $\Theta^+$  comes from the spin-spin interaction between the four quarks state with spin 1 and the  $\bar{s}$  quark, and its dependence on  $P_{III}$  is weak.

Finally, we show the contribution of each term of III. III in Eqs. (1) and (2) have two types of interactions, which are the term proportional to identity and the term dependent on the spin. We show the numbers for the  $\Theta^+$  with its bag radius same as the nucleon,  $R = 5 \text{ GeV}^{-1} = 0.985 \text{ fm}$ .

$$\langle \phi_{\Theta^+} | H^{(2)} | \phi_{\Theta^+} \rangle = G_{ud}^{(2)} (18.6_1 + 1.9_\sigma) \times 10^{-3}, \quad (7)$$

$$\langle \phi_{\Theta^+} | H^{(3)} | \phi_{\Theta^+} \rangle = G^{(3)} (12.2_1 - 3.2_\sigma) \times 10^{-5}. \quad (8)$$

The total contribution, which is  $H^{(2)}$  and  $H^{(3)}$ , is

$$\langle \phi_{\Theta^+} | H^{(2)} + H^{(3)} | \phi_{\Theta^+} \rangle = G_{ud}^{(2)} (14.5_1 + 2.8_\sigma) \times 10^{-3}. \quad (9)$$

The numbers suffixed by 1 ( $\sigma$ ) denote the contributions from the identity (spin) term of III. We find that the main contribution of III in the pentaquark comes from the identity term, which gives  $\simeq 83\%$  of the total value. The corresponding value for the nucleon is  $5/8 \simeq 63\%$ . The contribution of the identity term is so large because its effect is essentially proportional to the number of quark pairs in the pentaquark, namely, 10, while it is only 3 for the ordinary baryon.

#### IV. POSITIVE PARITY $\Theta^+$

Narrowness of the pentaquark resonances suggests that they may have positive parity and decay to  $L = 1$  meson-baryon scattering states. However, it is also noted that the centrifugal barrier may not make the width as narrow as a few MeV [11] and a suppression of the overlap of the pentaquark and  $KN$  wave functions, or the spectroscopic factor, is necessary [23,47]. For instance, a narrow width

will be achieved by a large distance between the  $\bar{s}$  and  $ud$  quarks so that the overlap to the kaon wave function is suppressed.

In this section, we investigate the effects of three-body repulsion of III on the  $P$ -wave pentaquarks. We employ the nonrelativistic quark model since the MIT bag model is notorious to have difficulty in describing excited baryon spectrum [48,49]. Thus we do not discuss the absolute mass, but concentrate on identifying which of possible excited states is favored by III, and whether such state is same as the one favored by OgE. They may depend on the choices of details of the wave functions. Here we use the  $SU(3)_F$  harmonic oscillator wave function with  $1\hbar\omega$ .

In the quark model, positive-parity states require orbital excitation of at least one quark from the ground state. Thus we concentrate on the lowest states for the  $P$ -wave pentaquark with  $I = 0$  and  $J = 1/2$ . Possible total spins,  $S_{5q}$ , which can be combined with the orbital angular momentum,  $L = 1$ , are  $S_{5q} = 1/2$  and  $3/2$ . They do not mix with each other unless a tensor force is applied. In the current study, we neglect tensor forces among quarks. Then both OgE and the two-body interaction of III favor lower spin state,  $S_{5q} = 1/2$ .

Among the five possible  $S_{5q} = 1/2$  states, one of them should correspond to the  $KN$  state with relative  $L = 1$ , while the other four states have no overlap with  $KN$ . Thus the pentaquark is identified as the lowest energy state in the latter four states and the state is connected to the  $KN$  state by the hyperfine interaction. By diagonalizing the OgE plus III Hamiltonian in the subspace of the four non  $KN$  states, we find a pentaquark state and the strength to coupling the  $KN$  state.

We summarize the results: (i) First, we find that the three-body term of III mixes the four non  $KN$  states. As a result, for  $P_{III} = 0.6 - 0.9$ , the strength to coupling to the  $KN$  state is significantly small. (ii) We find that a diquark-diquark configuration of  $S_{4q} = 0$  proposed by Ref. [11] is favored and occupies 50% of the ground state at  $P_{III} = 0.6 - 0.9$ . The next favorable state is that the orbital excitation is placed in the relative motion  $\bar{s}$  and  $4q$  system, which accounts for 30%.

We conclude that the three-body repulsion of III significantly changes the ground state of the  $P$  wave pentaquark. The results reflect the property of the three-body repulsion, which applies only to flavor singlet states. The three-body term tends to separate  $\bar{s}$  from  $ud$  quarks and the decay width expected to be reduced. Furthermore, if we introduces tensor interactions, which are neglected here, we expect that the decay width is further reduced because it mixes  $S_{5q} = 3/2$  states that has no  $KN$  overlap.

#### V. CONCLUSION

Effects of the instanton induced interaction on the spectrum of  $\Theta^+$ ,  $\Theta_{S=3/2}^+$ ,  $\Xi^{--}$  and  $H$  dibaryon have been

studied using the MIT bag model. The bag provides a permanent confinement and the mass splittings are attributed to spin-dependent interactions induced by the one-gluon exchange as well as the instanton-quark couplings.

We have found that III lowers the mass of  $\Theta^+$  and  $\Theta_{S=3/2}^+$ , while the mass of  $H$  increases as the strength of III increases. The  $\Xi^{--}$  mass is insensitive to the strength of III. We have also found that the strongly attractive force of III makes the bag radii shrink. In fact, the radii of the pentaquarks happen to be as small as the radius of the 3-quark baryons. The attraction comes mainly from the spin-independent part of III, which is proportional to the number of quark (antiquark) pairs in the system. Thus the attraction in the pentaquark systems is (10/3) times as large as that in the three-quark baryons. The attraction is stronger for the  $H$  dibaryon because the number of pairs is 15 in the six-quark system. It is, however, the 3-body III that strongly repulsive for  $H$ . The repulsion is weaker in  $\Theta^+$  (and in  $\Theta_{S=3/2}^+$ ) than in  $H$ , because the number of  $u$ - $d$ - $s$  combinations inside the pentaquark is just a half of that in  $H$ .

The 2-body III is weaker in the  $\Xi^{--}$  system because the  $H^{(2)}$  is proportional to the inverse of the effective quark masses and it is therefore weaker for the strange quark. We have also found that the spin-spin splitting between the  $\Theta^+$  and  $\Theta_{S=3/2}^+$  is larger than the splitting of  $\Theta^+$  and  $\Xi^{--}$ . This is one of the distinctive features of the negative-parity pentaquarks in comparison to the positive-parity states, where the spin-orbit force is responsible for the  $J = 1/2^+ - 3/2^+$  splitting and it is generally weaker than the spin-spin force. Moreover, the contribution of OgE for the  $\Theta_{S=3/2}^+$  is very small. Therefore the splitting of  $\Theta^+$  and  $\Theta_{S=3/2}^+$ , if it is found, should give us important information on the structure of the pentaquarks.

In all, we have shown that the instanton induced interaction is an important ingredient of the pentaquark spectrum and should be taken into account in evaluating their masses. The present results are not completely satisfactory in detail, however. For instance, it can not reproduce the observed  $\Theta^+$  mass. It also underestimates the mass of  $H$ , which has not been found below the  $\Lambda\Lambda$  threshold, 2231 MeV. Possible resolutions are corrections from expected two-body (diquark type) correlations, pionic effects, which may be included in chiral bag models, and also couplings to background  $NK$  scattering states. If (one or some of) these effects are important, the pentaquark spectrum may be well modified. The instanton induced interaction will contribute to the two-body correlation significantly, as has been already suggested in relation to the color superconductivity of the QCD vacuum at finite baryon density. It seems also important to include the effect of the  $NK$  continuum, which should make the width of the negative-parity pentaquark state large.

Despite these defects, the current study is worthwhile because using the simplest possible picture of the hadron,

we demonstrate how large and important are the effects of instantons on the spectrum of pentaquarks and the  $H$  dibaryon. Further analysis including the above-mentioned corrections are to be performed as the next step.

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## APPENDIX A: ONE GLUON EXCHANGE

The wave function of the lowest energy  $S_{1/2}$  state in the MIT bag is given by

$$\psi = \begin{pmatrix} f(m, r) \\ -i(\vec{n} \cdot \vec{\sigma})g(m, r) \end{pmatrix} \chi_s e^{-i\omega_0 t}, \quad (\text{A1})$$

where

$$f(m, r) = N \sqrt{\frac{\omega_0 + m}{2\omega_0}} j_0\left(\frac{x}{R} r\right), \quad (\text{A2})$$

$$g(m, r) = -N \sqrt{\frac{\omega_0 - m}{2\omega_0}} j_1\left(\frac{x}{R} r\right), \quad (\text{A3})$$

where  $N$  denotes the normalization constant,  $R$  is the bag radius,  $j_i$  is the spherical Bessel function, and  $\chi_s$  denotes a two component Pauli spinor. For the antiparticle, we get the negative frequency mode operated by charge conjugation,

$$\psi^c = C\bar{\psi}^t = i\gamma^2\gamma^0\bar{\psi}^t. \quad (\text{A4})$$

Matrix elements of the color-magnetic part of the one-gluon exchange (OgE) between quarks are shown here.  $M_{ij}$  denotes the matrix element for the quark pair of  $i$  and  $j$ . For the ground state, the Hamiltonian is given by

$$H_{OgE} = \sum_{i < j} (\vec{\lambda}_i \cdot \vec{\lambda}_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) M_{ij}, \quad (\text{A5})$$

where

$$M_{ij} = -3\alpha_c \frac{\mu(m_i, R)\mu(m_j, R)}{R^3} I(m_i, m_j, R), \quad (\text{A6})$$

$$I(m_i, m_j, R) = 1 + 2 \int_0^R \frac{dr}{r^4} \mu(m_i, r)\mu(m_j, r), \quad (\text{A7})$$

$$\mu(m, r) = \int_0^r dr' \mu'(m, r'), \quad (\text{A8})$$

$$\mu'(m, r) \equiv \frac{-2}{3} r^3 f(m, r) g(m, r). \quad (\text{A9})$$

For antiquark, we replace  $\lambda$  by  $-\lambda^*$ .

For the octet baryons, OgE is attractive, while, for the decuplet baryons, it has the same strength but the sign is opposite.

$$\langle \phi_N | H_{OgE} | \phi_N \rangle = 8M_{uu}, \quad (\text{A10})$$

$$\langle \phi_\Delta | H_{OgE} | \phi_\Delta \rangle = -8M_{uu}. \quad (\text{A11})$$

For the  $H$  dibaryon, it is strongly attractive, as was first shown by Jaffe [45],

$$\langle \phi_H | H_{OgE} | \phi_H \rangle = 5M_{uu} + 22M_{us} - 3M_{ss}. \quad (\text{A12})$$

For the negative parity  $\Theta^+$ , the isospin decomposition is given by

$$\begin{aligned} \langle \phi_{\Theta^+} | H_{OgE} | \phi_{\Theta^+} \rangle &= -\frac{26}{9} M_{uu} - \frac{26}{9} M_{dd} + \frac{100}{9} M_{ud} \\ &+ \frac{20}{3} M_{us} + \frac{20}{3} M_{ds}. \end{aligned} \quad (\text{A13})$$

Then the matrix element of  $\Theta^+$  is obtained by using  $M_{uu} = M_{dd} = M_{ud}$  and  $M_{us} = M_{ds}$ . For the  $\Xi^{--}$ , we obtain a similar formula by  $u \rightarrow s$  and  $\bar{s} \rightarrow \bar{u}$ :

$$\langle \phi_{\Theta^+} | H_{OgE} | \phi_{\Theta^+} \rangle = \frac{16}{3} M_{uu} + \frac{40}{3} M_{us}, \quad (\text{A14})$$

$$\langle \phi_{\Xi^{--}} | H_{OgE} | \phi_{\Xi^{--}} \rangle = \frac{34}{9} M_{uu} + \frac{160}{9} M_{us} - \frac{26}{9} M_{ss}. \quad (\text{A15})$$

For  $\Theta_{S=3/2}^+$ , we obtain

$$\langle \phi_{\Theta_{S=3/2}^+} | H_{OgE} | \phi_{\Theta_{S=3/2}^+} \rangle = \frac{16}{3} M_{uu} - \frac{20}{3} M_{us}. \quad (\text{A16})$$

## APPENDIX B: INSTANTON INDUCED INTERACTION

We show the matrix elements of the instanton induced interaction (III). For the octet baryon, III is attractive, while III does not affect the decuplet baryon:

$$\begin{aligned} \langle \phi_N | H^{(2)} | \phi_N \rangle &= \frac{G_{ud}^{(2)}}{4\pi} \int_0^R r^2 dr \frac{24}{5} (f_u(r)^4 + 2f_u(r)^2 g_u(r)^2 \\ &+ g_u(r)^4), \end{aligned} \quad (\text{B1})$$

$$\langle \phi_\Delta | H^{(2)} | \phi_\Delta \rangle = 0. \quad (\text{B2})$$

For the  $\Theta^+$ , we obtain

$$\begin{aligned} \langle \phi_{\Theta^+} | H_{ud}^{(2)} | \phi_{\Theta^+} \rangle &= \frac{G_{ud}^{(2)}}{4\pi} \int_0^R r^2 dr \frac{4}{5} (9f_u(r)^4 \\ &+ 2f_u(r)^2 g_u(r)^2 + 9g_u(r)^4), \end{aligned} \quad (\text{B3})$$

$$\begin{aligned} \langle \phi_{\Theta^+} | H_{us}^{(2)} | \phi_{\Theta^+} \rangle &= \langle \phi_{\Theta^+} | H_{ds}^{(2)} | \phi_{\Theta^+} \rangle \\ &= \frac{G_{us}^{(2)}}{4\pi} \int_0^R r^2 dr \frac{2}{15} (33f_u(r)^2 f_s(r)^2 + 33g_u(r)^2 g_s(r)^2 - 13f_u(r)^2 g_s(r)^2 - 13g_u(r)^2 f_s(r)^2 \\ &+ 52f_u(r)g_u(r)f_s(r)g_s(r)), \end{aligned} \quad (\text{B4})$$

$$\begin{aligned} \langle \phi_{\Theta^+} | H^{(3)} | \phi_{\Theta^+} \rangle &= \frac{G^{(3)}}{(4\pi)^2} \int_0^R r^2 dr \frac{60}{7} (3f_u(r)^4 f_s(r)^2 - f_u(r)^4 g_s(r)^2 + 8f_u(r)^3 g_u(r) f_s(r) g_s(r) - 4f_u(r)^2 g_u(r)^2 f_s(r)^2 \\ &+ 4f_u(r)^2 g_u(r)^2 g_s(r)^2 - 8f_u(r)g_u(r)^3 f_s(r)g_s(r) + g_u(r)^4 f_s(r)^2 - 3g_u(r)^4 g_s(r)^2). \end{aligned} \quad (\text{B5})$$

For  $\Theta_{S=3/2}^+$ , the matrix element of  $H_{ud}^{(2)}$  is identical to that of  $\Theta^+$ :

$$\langle \phi_{\Theta_{S=3/2}^+} | H_{ud}^{(2)} | \phi_{\Theta_{S=3/2}^+} \rangle = \frac{G_{ud}^{(2)}}{4\pi} \int_0^R r^2 dr \frac{4}{5} (9f_u(r)^4 + 2f_u(r)^2 g_u(r)^2 + 9g_u(r)^4), \quad (\text{B6})$$

$$\begin{aligned} \langle \phi_{\Theta_{S=3/2}^+} | H_{us}^{(2)} | \phi_{\Theta_{S=3/2}^+} \rangle &= \langle \phi_{\Theta_{S=3/2}^+} | H_{ds}^{(2)} | \phi_{\Theta_{S=3/2}^+} \rangle \\ &= \frac{G_{us}^{(2)}}{4\pi} \int_0^R r^2 dr \left( \frac{7}{5} f_u(r)^2 f_s(r)^2 + \frac{7}{5} g_u(r)^2 g_s(r)^2 - \frac{41}{15} f_u(r)^2 g_s(r)^2 - \frac{41}{15} g_u(r)^2 f_s(r)^2 \right. \\ &\left. + \frac{4}{3} f_u(r)g_u(r)f_s(r)g_s(r) \right), \end{aligned} \quad (\text{B7})$$

$$\begin{aligned}
\langle \phi_{\Theta_{S=3/2}^+} | H^{(3)} | \phi_{\Theta_{S=3/2}^+} \rangle &= \frac{G^{(3)}}{(4\pi)^2} \int_0^R r^2 dr \frac{15}{7} (3f_u(r)^4 f_s(r)^2 - 7f_u(r)^4 g_s(r)^2 + 8f_u(r)^3 g_u(r) f_s(r) g_s(r) \\
&\quad + 2f_u(r)^2 g_u(r)^2 f_s(r)^2 - 2f_u(r)^2 g_u(r)^2 g_s(r)^2 - 8f_u(r) g_u(r)^3 f_s(r) g_s(r) + 7g_u(r)^4 f_s(r)^2 \\
&\quad - 3g_u(r)^4 g_s(r)^2).
\end{aligned} \tag{B8}$$

For  $\Xi^{--}$ , we obtain

$$\langle \phi_{\Xi^{--}} | H_{ud}^{(2)} | \phi_{\Xi^{--}} \rangle = \frac{G_{ud}^{(2)}}{4\pi} \int_0^R r^2 dr \left( \frac{22}{5} f_u(r)^4 + \frac{22}{5} g_u(r)^4 + \frac{52}{15} f_u(r)^2 g_u(r)^2 \right), \tag{B9}$$

$$\begin{aligned}
\langle \phi_{\Xi^{--}} | (H_{us+ds}^{(2)}) | \phi_{\Xi^{--}} \rangle &= \frac{G_{us}^{(2)}}{4\pi} \int_0^R r^2 dr \left( \frac{58}{5} f_u(r)^2 f_s(r)^2 + \frac{58}{5} g_u(r)^2 g_s(r)^2 - \frac{202}{45} f_u(r)^2 g_s(r)^2 - \frac{202}{45} g_u(r)^2 f_s(r)^2 \right. \\
&\quad \left. + \frac{632}{45} f_u(r) g_u(r) f_s(r) g_s(r) \right),
\end{aligned} \tag{B10}$$

$$\begin{aligned}
\langle \phi_{\Xi^{--}} | H^{(3)} | \phi_{\Xi^{--}} \rangle &= \frac{G^{(3)}}{(4\pi)^2} \int_0^R r^2 dr \frac{20}{21} (27f_u(r)^4 f_s(r)^2 - 14f_u(r)^4 g_s(r)^2 + 28f_u(r)^3 g_u(r) f_s(r) g_s(r) \\
&\quad + 13f_u(r)^2 g_u(r)^2 f_s(r)^2 - 13f_u(r)^2 g_u(r)^2 g_s(r)^2 - 28f_u(r) g_u(r)^3 f_s(r) g_s(r) + 14g_u(r)^4 f_s(r)^2 \\
&\quad - 27g_u(r)^4 g_s(r)^2).
\end{aligned} \tag{B11}$$

For the  $H$  dibaryon, we obtain

$$\langle \phi_H | H_{ud}^{(2)} | \phi_H \rangle = \frac{G_{ud}^{(2)}}{4\pi} \int_0^R r^2 dr \frac{4}{5} (9f_u(r)^4 + 2f_u(r)^2 g_u(r)^2 + 9g_u(r)^4), \tag{B12}$$

$$\begin{aligned}
\langle \phi_H | H_{us}^{(2)} | \phi_H \rangle &= \langle \phi_H | H_{ds}^{(2)} | \phi_H \rangle \\
&= \frac{G_{us}^{(2)}}{4\pi} \int_0^R r^2 dr \frac{2}{5} (18f_u(r)^2 f_s(r)^2 + 18g_u(r)^2 g_s(r)^2 - 7f_u(r)^2 g_s(r)^2 - 7g_u(r)^2 f_s(r)^2 \\
&\quad + 18f_u(r) g_u(r) f_s(r) g_s(r)),
\end{aligned} \tag{B13}$$

$$\begin{aligned}
\langle \phi_H | H^{(3)} | \phi_H \rangle &= \frac{G^{(3)}}{(4\pi)^2} \int_0^R r^2 dr \frac{30}{7} (12f_u(r)^4 f_s(r)^2 - 12g_u(r)^4 g_s(r)^2 - 3f_u(r)^4 g_s(r)^2 + 3g_u(r)^4 f_s(r)^2 \\
&\quad + f_u(r)^2 g_u(r)^2 f_s(r)^2 - f_u(r)^2 g_u(r)^2 g_s(r)^2 + 14f_u(r)^3 g_u(r) f_s(r) g_s(r) - 14f_u(r) g_u(r)^3 f_s(r) g_s(r)).
\end{aligned} \tag{B14}$$

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