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Electroweak absorptive parts in the matching conditions of nonrelativistic QCD

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Electroweak corrections associated with the instability of the top quark to the next-to-next-to-leading logarithmic (NNLL) total top pair threshold cross section in e^+e^- annihilation are determined. Our method is based on absorptive parts in electroweak matching conditions of the operators of nonrelativistic QCD and the optical theorem. The corrections lead to ultraviolet phase space divergences that have to be renormalized and lead to NLL mixing effects. Numerically, the corrections can amount to several percent and are comparable to the known NNLL QCD corrections.

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I. INTRODUCTION

The line-shape scan of the threshold top pair production cross section $\sigma(e^+e^- \to \gamma^*, Z^* \to t\bar{t})$ constitutes a major part of the top quark physics program at the International Linear Collider (ILC) project that is currently being initiated. Because in the Standard Model the top quark width $\Gamma_t \approx 1.5$ GeV is much larger than the typical hadronization energy $\Lambda_{\rm QCD}$, it is expected that the line shape of the total cross section is a smooth function of the c.m. energy, and that nonperturbative effects are strongly suppressed [1,2]. The determination of the top quark mass (in a threshold mass scheme [3,4]) is the most important measurement that can be obtained from the threshold scan since an uncertainty of only around 100 MeV is expected [5,6]. This prospect is quite robust, from the theoretical as well as from the experimental point of view, since it relies mostly on the determination of the c.m. energy where the cross section rises. Because the $t\bar{t}$ pair is produced predominantly in an S-wave state, the rise of the cross section is quite rapid and easily measurable even in the presence of beam effects [5]. In addition it will also be possible to determine the strong coupling α_s, the total top quark width Γ_t and, if the Higgs boson is light, the top Yukawa coupling $g_{\rm tth}$. However, the latter measurements are sensitive to the form and the normalization of the line shape. Since the observable cross section is a convolution of the theory prediction with the partly machine-dependent luminosity spectrum arising from QED effects [5,7], luminosity high demands are imposed on theoretical predictions and experimental analyses to make these measurements possible. In particular, theoretical predictions need to have a precision at the level of only a few percent.

The common theoretical tool to make computations for top threshold observables is nonrelativistic QCD (NRQCD).[1] It provides an economic and systematic treatment for the nonrelativistic expansion and for QCD radiative corrections coming from high and low energies. It has also become evident that it is advantageous to use renormalization group methods [8] to resum logarithms of the top quark velocity v to all orders of QCD perturbation theory in order to avoid large normalization uncertainties of at least 20% that are obtained in fixed-order predictions [6]. Concerning QCD effects at next-to-leading-logarithmic (NLL) order for the total cross section only the $t\bar{t}$ production current has a nontrivial running, which is fully known [8–11]. At next-to-next-to-leading-logarithmic (NNLL) order for the total cross section, the QCD evolution of almost all required couplings is known [9,12–14] except for missing subleading mixing effects in the running of the heavy quark pair production current. Theoretical analyses for the total cross section at NNLL order were given in [15–17]. Currently the normalization uncertainties of the total cross section from QCD effects are estimated to be around 6% [17].

While the major focus in recent work was directed on a better understanding of QCD corrections at NNLL order, a systematic treatment of electroweak effects at the same level has not yet been accomplished. Electroweak corrections are responsible for a variety of different physical effects. At leading order, the three basic electroweak effects are the e^+e^- annihilation process that leads to top pair production by virtual photon and Z exchange, the finite top lifetime, which can be implemented by the replacement rule $E \to E + i\Gamma_t$ [2] ($E = \sqrt{s} - 2m_t$ being the c.m. energy with respect to the top threshold and Γ_t being the on-shell top quark width), and the luminosity spectrum mentioned above. All three effects have so far been treated independently. It has become the convention that only the

[1] We use the term NRQCD to refer to a generic low-energy effective theory which describes nonrelativistic $t\bar{t}$ pairs and bound state effects and not for a theory valid only for scales $m_t > \mu > m_t v$. For the presentation we use the conventions and notations of vNRQCD established in [8,9], but we emphasize that our results are generally true.

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first two effects are included into theoretical predictions while the luminosity spectrum is accounted for in the experimental simulations [5].

At the subleading level a coherent treatment of electroweak effects has not yet been achieved, although previous partial analyses have indicated that they can reach the level of a few percent [18,19]. It is also evident that at the subleading level an independent treatment of various different electroweak effects will, eventually, be impossible. Since a systematic treatment relies on the consistent separation of off-shell (nonresonant) and close-to-mass-shell (resonant) fluctuations, the concept of effective theories appears again to be a highly efficient tool to make progress [20]. For the $t\bar{t}$ threshold such a framework is already provided by the effective theory NRQCD itself, since it automatically achieves an expansion in the off-shellness of the top quark through the nonrelativistic expansion in v. The NRQCD effective theory formalism can be extended to account for electroweak corrections. For example, including the electroweak radiative corrections to the top quark two-point function in the NRQCD matching conditions, one obtains the additional heavy quark bilinear terms[2]

$$\delta\mathcal{L} = \sum_{p}\psi_{p}^{\dagger}\frac{i}{2}\Gamma_{t}\psi_{p} + \sum_{p}\chi_{p}^{\dagger}\frac{i}{2}\Gamma_{t}\chi_{p} \tag{1}$$

in the effective Lagrangian, where $\psi_{\mathbf{p}}$ and $\chi_{\mathbf{p}}$ represent Pauli spinor field operators destroying top and antitop quarks, respectively. For simplicity color indices are suppressed throughout this paper. The terms in Eq. (1) reproduce the replacement rule $E \to E + i\Gamma_t$. They render the effective theory Lagrangian non-Hermitian because they describe an absorptive on-shell process that has been integrated out from the theory. Nevertheless, they allow for a correct determination of the total cross section from the forward scattering amplitude using the optical theorem and the unitarity of the underlying theory. In fact, this issue is analogous to the so-called strong phases in QCD amplitudes that are the basis for the search for Cabibbo-Kobayashi-Maskawa CP violation in a number of B meson decays [21].

In this paper we extend this approach and investigate the role of absorptive parts related to the top quark decay in electroweak loop corrections to the NRQCD matching conditions that contribute to the NNLL total cross section.[3] For simplicity we neglect the bottom quark mass and set $V_{tb} = 1$, and approximate the W boson and bottom quark as stable particles. We demonstrate that these electroweak corrections properly account for the interference of the dominant double-resonant process $e^+e^- \to t\bar{t} \to bW^+\bar{b}W^-$ with the v^2-suppressed single-resonant amplitudes for $e^+e^- \to bW^+\bar{t} \to bW^+\bar{b}W^-$ and $e^+e^- \to t\bar{b}W^- \to bW^+\bar{b}W^-$. We also show that absorptive parts that do not contribute to the $bW^+\bar{b}W^-$ final state, and are therefore also not accounted for in line-shape measurements, can be excluded in a gauge invariant way. This requires to include also fields for the electrons and positrons from the initial state into the effective theory, which act like classic fields for QCD interactions. The new corrections show interesting features. They slightly modify the form of the cross section line shape and lead to UV phase space divergences that are directly related to the fact that the top quarks are unstable. The divergences lead to anomalous dimensions of $(e^+e^-)(e^+e^-)$ operators that also contribute to the absorptive part of the forward scattering amplitude. In the total cross section these mixing effects contribute at NLL order and represent a novel NLL effect that has remained unnoticed in previous work. Numerically, the size of the new corrections ranges up to 5% and partly compensates for the large QCD corrections found recently in [13].

The program of this paper is as follows. In Sec. II we discuss the power counting for the total cross section with respect to electroweak effects associated with the top quark decay. We introduce the $(e^+e^-)(t\bar{t})$ effective theory operators needed to account for the electroweak absorptive parts that arise in top pair production or annihilation. In particular, we show that up to NNLL order there are no contributions from interference effects originating from (ultrasoft) gluons carrying momenta of order $m_t v^2$. In Sec. III the electroweak absorptive parts of the matching conditions for the $(e^+e^-)(t\bar{t})$ effective theory operators relevant for the $bW^+\bar{b}W^-$ final state are computed. In Sec. IV the resulting NNLL corrections for the total cross section are determined and the renormalization of the $(e^+e^-)(e^+e^-)$ effective theory operators needed to account for the phase space divergences is discussed. In particular, we compute and solve the anomalous dimensions and determine their contribution to the total cross section. Section V contains a brief numerical analysis and in Sec. VI we conclude.

II. POWER COUNTING AND MATCHING CONDITIONS

In this work we are interested in the total cross section and not in any differential information on the top decay final states. We therefore include all effects related to the top quark decay as non-Hermitian matching conditions of effective theory operators that describe the nonrelativistic top and antitop dynamics and their interactions with soft and ultrasoft gluons. We employ gauge invariant operators, and the matching conditions are computed for on-shell external lines. This allows to maintain gauge invariance in a transparent way.

[2]Note that in the effective theory we use positive energy spinors for the antiparticles.

[3]In a complete treatment of all electroweak effects one can integrate out all electroweak effects associated with the massive W, Z and Higgs bosons at the scale m_t. Below the scale m_t gluons, photons and quarks remain as dynamical degrees of freedom.

To illustrate the power counting needed to classify the order at which these electroweak effects can contribute, let us recall the matching conditions for the bilinear quark field operators. They are obtained by matching top or antitop 2-point functions in the effective theory to those in the full electroweak and QCD theory. The result up to NNLL order reads

$$\mathcal{L}_{\rm bilinear}(x) = \sum_{\boldsymbol{p}} \psi_{\boldsymbol{p}}^\dagger(x)\Big\{iD^0 - \frac{(\boldsymbol{p}-i\mathbf{D})^2}{2m_t} + \frac{\boldsymbol{p}^4}{8m_t^3} + \frac{i}{2}\Gamma_t \times \Big(1-\frac{\boldsymbol{p}^2}{2m_t^2}\Big) - \delta m_t\Big\}\psi_{\boldsymbol{p}}(x) + (\psi_{\boldsymbol{p}} \to \chi_{\boldsymbol{p}}), \tag{2}$$

and includes the terms shown in Eq. (1). Here, the fields $\psi_{\boldsymbol{p}}$ and $\chi_{\boldsymbol{p}}$ destroy top and antitop quarks with momentum $\boldsymbol{p}$, $D^\mu = (D^0, -\mathbf{D}) = \partial^\mu + igA^\mu$ is the ultrasoft gauge covariant derivative and Γ_t is the top quark width defined at the top quark pole. At order g^2, g being the SU(2) gauge coupling, the width has the form

$$\Gamma_t = \frac{\alpha |V_{tb}|^2 m_t}{16 s_w^2 x}(1-x)^2(1+2x), \tag{3}$$

where s_w (c_w) is the sine (cosine) of the weak mixing angle, α the fine structure constant and

$$x \equiv \frac{M_W^2}{m_t^2}. \tag{4}$$

We use the usual v-counting $D^0 \sim m_t v^2 \sim \Gamma_t \sim m_t g^2$, which leads to the scaling relation

$$v \sim \alpha_s \sim g \sim g' \tag{5}$$

for the SU(2) and U(1) gauge couplings g and g'. Because the weak mixing is of order one, we apply the same counting to the SU(2) and U(1) gauge couplings. The term δm_t is a residual mass term of order v^2 that arises if a threshold mass scheme [6] is used. The LL order terms in Eq. (2) lead to the top/antitop propagator

$$\frac{i}{p^0 - \boldsymbol{p}^2 - /(2m_t) + i\Gamma_t/2 - \delta m_t}. \tag{6}$$

Although the exchange of timelike ultrasoft A^0 gluons contributes at LL order according to the v-counting, their contribution at LL order can be removed from the particle-antiparticle sector of the theory by a redefinition of the top and antitop fields related to static Wilson lines [22,23].[4] The time dilatation term $\propto \Gamma_t(\mathbf{p}^2/2m_t^2)$ originates from the momentum dependence of the full theory spinors and contributes at NNLL order.

The $t\bar{t}$ pair is produced by an electroweak process. As long as electroweak effects are only treated at leading order in the v-expansion, it is sufficient to describe $t\bar{t}$ production by a bilinear quark-antiquark current. However, as shown below it is necessary to include the initial-state (e^+e^-) fields to ensure electroweak gauge invariance at subleading order in the v-expansion. The dominant operators that have to be used describing $t\bar{t}$ spin-triplet production have the form

$$\mathcal{O}_{V,\boldsymbol{p}} = [\bar{e}\gamma_j e]\mathcal{O}^j_{\boldsymbol{p},1}, \tag{7}$$

$$\mathcal{O}_{A,\boldsymbol{p}} = [\bar{e}\gamma_j \gamma_5 e]\mathcal{O}^j_{\boldsymbol{p},1}, \tag{8}$$

where

$$\mathcal{O}^j_{\boldsymbol{p},1} = [\psi_{\boldsymbol{p}}^\dagger \sigma_j (i\sigma_2)\chi^*_{-\boldsymbol{p}}]. \tag{9}$$

They give the contribution $\Delta\mathcal{L} = \sum_{\boldsymbol{p}}(C_V \mathcal{O}_{V,\boldsymbol{p}} + C_A \mathcal{O}_{A,\boldsymbol{p}}) +$ H.c. to the effective theory Lagrangian where the Hermitian conjugation is referring to the operators only. The index $j = 1, 2, 3$ is summed. The corresponding operators describing $t\bar{t}$ annihilation are obtained by the Hermitian conjugation. Since in this work we only focus on the electroweak effects related to the top quark decay and, in particular, neglect QED radiative corrections (including QED binding and the beam effects mentioned above), the electron and positron fields act like classic fields in the effective theory.[5] In a more complete treatment of electroweak effects, however, their interactions with photons have to be accounted for [20]. The leading order matching conditions of the operators $\mathcal{O}_{V/A,\boldsymbol{p}}$ obtained from the full theory Born diagrams with photon and Z exchange are of order g^2 and read

$$C_V^{\rm born}(\nu = 1) = \frac{\alpha\pi}{m_t^2(4c_w^2 - x)}\Big[Q_e Q_t(4-x) + Q_t - Q_e - \frac{1}{4s_w^2}\Big], \tag{10}$$

$$C_A^{\rm born}(\nu = 1) = -\frac{\alpha\pi}{m_t^2(4c_w^2 - x)}\Big[Q_t - \frac{1}{4s_w^2}\Big], \tag{11}$$

where ν is the vNRQCD renormalization scaling parameter.[6] Except for the QED beam effects, which we will not consider here, the electroweak effects in Eqs. (3), (10), and

[4]In an explicit computation, quark pair production and quark-antiquark scattering diagrams involving the timelike gluons cancel at LL order. In Coulomb gauge all leading order diagrams with timelike gluons are dimensionless and individually zero in dimensional regularization [8].

[5]For a treatment of nonrelativistic QED effects within vNRQED, see Ref. [24].

[6]In vNRQCD the renormalization scales for soft and ultrasoft fluctuations, μ_S and μ_U, are correlated through the heavy quark equation of motion, $\mu_U = \mu_S^2/m_t$. The correlated running from the hard scale down to the soft and ultrasoft scales is described by the dimensionless scaling parameter ν defined by $\mu_S = m_t\nu$ and $\mu_U = m_t\nu^2$. Thus $\nu = 1$ corresponds to the hard matching scale.

(11) are the only ones at leading order. In particular, at this order there are no electroweak effects contributing to the Coulomb potential acting between the $t\bar{t}$ pair,

$$\mathcal{L}_{\rm pot} = -\sum_{\boldsymbol{p},\boldsymbol{p}'} \frac{\mathcal{V}_c^{(s)}(\nu)}{(\mathbf{p}-\mathbf{p}')^2} \psi^\dagger_{\boldsymbol{p}'}\psi_{\boldsymbol{p}}\chi^\dagger_{-\boldsymbol{p}'}\chi_{-\boldsymbol{p}}, \qquad (12)$$

where $\mathcal{V}_c^{(s)}(\nu) = -4\pi C_F \alpha_s(m_t\nu)$ is the LL Coulomb Wilson coefficient for a color singlet heavy quark pair.

For the NLL order approximation it has been frequently stated that there are no new operators that can contribute and that power counting tells that we only need to consider $\mathcal{O}(\alpha_s)$ QCD corrections to the LL matching conditions in Eq. (2) to account for all electroweak effects [25,26]. However, it was also noted in [19] that the mismatch between the $t\bar{t}$ phase space in the full and the effective theory leads to additional NLL matching corrections for unstable top quarks. We come back to the role played by these specific NLL contributions in Sec. IV and ignore them for the following considerations. Thus, concerning NLL effects related to the top decay only the one-loop QCD corrections to the on-shell top decay width have to be accounted for [27]. In particular, there are no QCD interference effects from gluon radiation off the top/antitop quark or its decay products [25,26]. In our approach, which only aims at the total cross section, one can show that such QCD interference effects do not even contribute at NNLL order in the nonrelativistic expansion.

To discuss the QCD interference contributions we have to consider ultrasoft gluons, which carry momenta of order $m_t v^2 \sim \Gamma_t$, because they can interact with a resonant top quark without kicking it off-shell. For the timelike A^0 gluons, we already mentioned that their leading interaction with the quarks can be removed by a field redefinition related to static Wilson lines [22,23]. Moreover, QCD gauge invariance ensures that the dominant electroweak matching corrections to the A^0 interaction vertex vanish because we can set the ultrasoft gluon momentum to zero. Radiative corrections can, however, generate an anomalous interaction in analogy to the $g-2$ in QED. Yet this interaction is suppressed by a factor $1/m_t^2$ and cannot contribute to the cross section matrix elements at NNLL level without even having accounted for additional powers of the coupling constants. It remains to discuss the spacelike ultrasoft $\mathbf{A}$ gluons which couple to the quarks with the $\mathbf{p}.\mathbf{A}/m_t$ coupling. In pure QCD spacelike ultrasoft gluon exchange contributes to the cross section matrix elements at NNLL order and also to the renormalization group running of the $t\bar{t}$ production operators at the NLL level [8] through operator mixing. Accounting for the g^2-suppression from an additional electroweak loop correction to the interaction vertex then also leads to a contribution beyond the NNLL order.

At NNLL order let us first consider whether one needs to account for any operator in addition to those present already at the LL level. Ultrasoft gluon interactions have been discussed above. Soft gluon operators first contribute at the NLL level in pure QCD (for example as corrections to the Coulomb potential [28,29]), so g^2 electroweak corrections to these operators are beyond NNLL order. Such corrections can also not contribute at NNLL order through mixing since the Coulomb potential does not cause UV divergences. It remains to discuss four-quark operators. Because an electroweak loop would require a factor g^4 in addition to the $1/m_t^2$ suppression for dimensional reasons, such an operator could not contribute at NNLL order either [30].

It remains to discuss g^2 corrections to the operators contributing at the LL level. For the bilinear quark operators in the effective theory Lagrangian one obtains the terms shown in Eq. (2). Concerning the instability of the top quark, only the time dilatation correction is obtained, and the $\mathcal{O}(\alpha_s^2)$ and one-loop electroweak corrections to the on-shell top quark width [31,32] have to be accounted for. The $\mathcal{O}(\alpha_s^2)$ correction to the top width is easy to implement together with the Born and one-loop QCD results in the width term Γ_t and will not be discussed further in this work. On the other hand, for the Coulomb potential all dominant g^2 corrections cancel due to SU(3) gauge invariance because in the first approximation one can neglect the gluon momentum flowing into the vertex correction [33,34]. The mechanism is equivalent to the gauge cancellation discussed above for the timelike A^0 gluon. The order g^4 matching conditions of the production operators $\mathcal{O}_{V,\boldsymbol{p}}$ and $\mathcal{O}_{A,\boldsymbol{p}}$, on the other hand, do not cancel and have to be determined from matching to the one-loop Standard Model amplitudes for the process $e^+e^- \to \gamma, Z \to t\bar{t}$. At this order the matching computation can be carried out for an on-shell top-antitop pair at rest. The full set of one-loop electroweak corrections was determined in Ref. [18]. For the examinations in this work we have to account only for the bW^+ and $\bar{b}W^-$ cuts because, as shown in Sec. IV, only these cuts are relevant for the $bW^+\bar{b}W^-$ final state that can interfere with top pair production. Because in Ref. [18] only the sum of all contributions was presented, including the $b\bar{b}$ and W^+W^- cuts, we rederive the results for the bW^+ and $\bar{b}W^-$ cuts in the next section.

III. ABSORPTIVE MATCHING CONDITIONS

The top pair production diagrams in the full theory that need to be considered to determine the absorptive bW^+ and $\bar{b}W^-$ cuts are shown in Fig. 1. The external (on-shell) top quarks can be taken to be at rest. The results for the cuts in the full theory amplitude have the form

$$\begin{aligned}\mathcal{A} &= i[\bar{v}_{e^+}(k')\gamma^\mu(iC_V^{\rm bW,abs} + iC_A^{\rm bW,abs}\gamma_5)u_{e^-}(k)] \\ &\quad \times [\bar{u}_t(p)\gamma_\mu v_{\bar{t}}(p)], \end{aligned} \qquad (13)$$

where $k + k' = 2p = (2m_t, 0)$ and

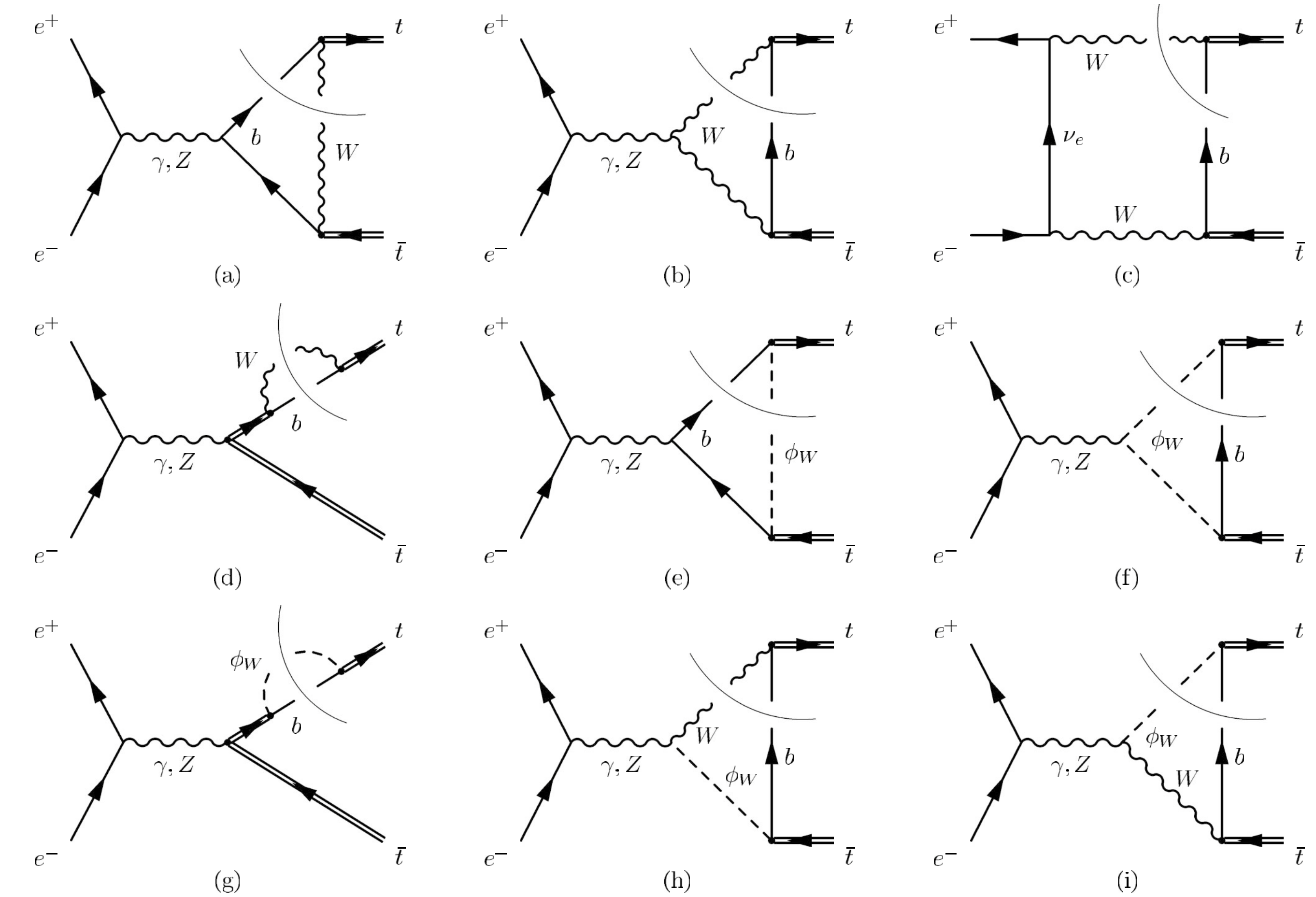


FIG. 1. Full theory diagrams in Feynman gauge that have to be considered to determine the electroweak absorptive parts in the Wilson coefficients C_A and C_V related to the physical bW^+ and $\bar{b}W^-$ intermediate states. Only the bW^+ cut is drawn explicitly.

$$\begin{aligned} iC_V^{\mathrm{bW,abs}} &= -i\frac{\alpha^2\pi|V_{tb}|^2}{12m_t^2 s_w^2 x(4c_w^2 - x)(1+x)}\bigg[\frac{3x(1+x)}{(1-x)}\bigg(1+\frac{x-4}{4s_w^2}\bigg)\ln\bigg(\frac{2-x}{x}\bigg) + Q_e Q_t(1-x)(4-x)(1+2x)(1+x+x^2) \\ &\quad + Q_e(x-1)(1+4x+2x^2+2x^3) + Q_t(1-x)(1+2x)(1+x+x^2) - \frac{1}{2}(1+12x+9x^2+2x^3) \\ &\quad + \frac{1}{8s_w^2}(2+41x+28x^2-x^3+2x^4)\bigg], \\ iC_A^{\mathrm{bW,abs}} &= i\frac{\alpha^2\pi|V_{tb}|^2}{12m_t^2 s_w^2 x(4c_w^2 - x)(1+x)}\bigg[\frac{3x(1+x)}{(1-x)}\bigg(1+\frac{x-4}{4s_w^2}\bigg)\ln\bigg(\frac{2-x}{x}\bigg) + Q_t(1-x)(1+2x)(1+x+x^2) \\ &\quad - \frac{1}{2}(1+12x+9x^2+2x^3) + \frac{1}{8s_w^2}(2+41x+28x^2-x^3+2x^4)\bigg]. \end{aligned} \tag{14}$$

The results for the charge conjugated process describing top pair annihilation, on the other hand, read

$$\begin{aligned} \bar{\mathcal{A}} &= i[\bar{u}_{e^-}(k)\gamma^\mu(iC_V^{\mathrm{bW,abs}} + iC_A^{\mathrm{bW,abs}}\gamma_5)v_{e^+}(k')] \\ &\quad \times[\bar{v}_{\bar{t}}(p)\gamma_\mu u_t(p)]. \end{aligned} \tag{15}$$

We used the cutting equations to obtain the result and checked electroweak gauge invariance by carrying out the computation in unitary and Feynman gauge. In both cases we computed the cut W lines with physical polarizations as well as with unphysical ones including also the charged Goldstone exchange. We note that the contributions that arise from off-shell corrections in the top self-energy graphs are necessary for electroweak gauge invariance. Since the $b\bar{b}$ and W^+W^- cuts lead to different distinct phase space factors, we found that it is possible to identify the results also from the formulae given in [18].

It is an important fact that the sign of the imaginary part of the amplitude does not change in the charge conjugated amplitude. As for the quark field bilinear terms discussed before in Eq. (2) this is related to the unitarity of the underlying theory. It is straightforward to match the amplitudes for the operators $\mathcal{O}_{V/A,\mathbf{p}}$ and $\mathcal{O}^{\dagger}_{V/A,\mathbf{p}}$ to the full theory results in Eqs. (13) and (15). The resulting matching conditions at the hard scale read

$$\begin{aligned} C_V(\nu=1) &= C_V^{\rm born} + iC_V^{\rm bW,abs}, \\ C_A(\nu=1) &= C_A^{\rm born} + iC_A^{\rm bW,abs}, \end{aligned} \tag{16}$$

where we have included also the Born level contributions from Eqs. (10) and (11). We emphasize again that these matching conditions are valid for the operators $\mathcal{O}_{V/A,\mathbf{p}}$ and $\mathcal{O}^{\dagger}_{V/A,\mathbf{p}}$. In a full treatment of electroweak and QCD effects, the coefficients $C_{V/A}$ also include the real parts of the full set of electroweak one-loop diagrams indicated in Fig. 1 and the QCD matching corrections known from previous work [16,35,36]. These corrections lead to an energy-independent multiplicative modification of the cross section normalization which is, however, not a subject of the investigations in this work. The results for the real parts of the full set of electroweak one-loop diagrams were given in [18].

IV. TIME-ORDERED PRODUCT AND RENORMALIZATION

Using the optical theorem the NNLL order corrections to the total cross section that come from the absorptive one-loop electroweak matching conditions for the operators $\mathcal{O}_{V/A,\mathbf{p}}$ and from the time dilatation corrections can be computed from the imaginary part of the $(e^+e^-)(e^+e^-)$ forward scattering amplitude,

$$\sigma_{\rm tot} \sim \frac{1}{s}\,{\rm Im}[(C_V^2(\nu) + C_A^2(\nu))L^{lk}\mathcal{A}_1^{lk}], \tag{17}$$

where [$k+k'=(\sqrt{s},0)$ and $\hat{\mathbf{e}} = \mathbf{k}/|\mathbf{k}|$]

$$\begin{aligned} L^{lk} &= \frac{1}{4}\sum_{e^\pm\ \rm spins}[\bar v_{e^+}(k')\gamma^l(\gamma_5)u_{e^-}(k)][\bar u_{e^-}(k)\gamma^k(\gamma_5)v_{e^+}(k')] \\ &= \frac{1}{2}(k+k')^2(\delta^{lk} - \hat e^l\hat e^k) \end{aligned} \tag{18}$$

is the spin-averaged lepton tensor and [$\hat q \equiv (\sqrt{s}-2m_t, 0)$]

$$\begin{aligned} \mathcal{A}_1^{lk} &= i\sum_{\mathbf{p},\mathbf{p}'}\int d^4x e^{-i\hat q\cdot x}\langle 0|T\mathcal{O}^{l\dagger}_{\mathbf{p},1}(0)\mathcal{O}^{k}_{\mathbf{p}',1}(x)|0\rangle \\ &= 2N_c\delta^{lk}G^0(a,v,m_t,\nu) \end{aligned} \tag{19}$$

is the time-ordered product of the $t\bar t$ production and annihilation operators $\mathcal{O}^j_{\mathbf{p},1}$ and $\mathcal{O}^{j\dagger}_{\mathbf{p},1}$ [16]. In dimensional regularization the result reads

$$\begin{aligned} \Delta\sigma_{\rm tot}^{\Gamma,1} = 2N_c\,{\rm Im}\{&2i[C_V^{\rm born}C_V^{\rm bW,abs} \\ &+ C_A^{\rm born}C_A^{\rm bW,abs}]G^0(a,v,m_t,\nu) + [(C_V^{\rm born})^2 \\ &+ (C_A^{\rm born})^2]\delta G^0_\Gamma(a,v,m_t,\nu)\}, \end{aligned} \tag{20}$$

where $a \equiv -\mathcal{V}_c^{(s)}(\nu)/4\pi = C_F\alpha_s(m_t\nu)$. The term G^0 is the zero- S-wave Green function of the nonrelativistic Schrödinger equation which is obtained from the LL order terms in the Lagrangian shown in Eqs. (2) and (12). In dimensional regularization it has the form [16]

$$\begin{aligned} G^0(a,v,m_t,\nu) = \frac{m_t^2}{4\pi}\Big\{&iv - a\Big[\ln\Big(\frac{-iv}{\nu}\Big) - \frac{1}{2} + \ln 2 + \gamma_E \\ &+ \psi\Big(1 - \frac{ia}{2v}\Big)\Big]\Big\} + \frac{m_t^2 a}{4\pi}\,\frac{1}{4\epsilon}, \end{aligned} \tag{21}$$

where

$$v = \sqrt{\frac{\sqrt{s} - 2(m_t + \delta m_t) + i\Gamma_t}{m_t}}, \tag{22}$$

$\sqrt{s}$ being the c.m. energy. The term δG^0_Γ represents the corrections originating from the time dilatation correction in Eq. (2) and reads

$$\delta G^0_\Gamma(a,v,m_t,\nu) = -i\frac{\Gamma_t}{2m_t}\Big[1 + \frac{v}{2}\frac{\partial}{\partial v} + a\frac{\partial}{\partial a}\Big]G^0(a,v,m_t,\nu). \tag{23}$$

Note that the Wilson coefficients $C_{V/A}$ do not run at LL order, so only the matching conditions at $\nu = 1$ appear in Eq. (20).

It is straightforward to check that the terms proportional to $C_{V/A}^{\rm bW,abs}$ in Eq. (20) are in agreement with the full theory matrix elements from the interference between the double-resonant amplitudes for the process $e^+e^- \to t\bar t \to bW^+\bar bW^-$ [Fig. 2(a)] and the single-resonant amplitudes describing the processes $e^+e^- \to t + \bar bW^- \to bW^+\bar bW^-$ and $e^+e^- \to bW^+\bar t \to bW^+\bar bW^-$ [Figs. 2(b)–2(i)] in the $t\bar t$ threshold limit for $m_t \to \infty$. Note that diagram (a) dominates in the nonrelativistic limit due to two resonant top/antitop lines, while diagrams (b)–(i) are v^2-suppressed having only one resonant top/antitop line. Diagram (a) also contains a subleading v^2-suppressed contribution that has to be accounted for. Diagrams with no top/antitop line are suppressed by v^4 and do not need to be considered. This also means that pure background diagrams containing no intermediate top quark can be neglected at this order. We also note that to find literal agreement between full and effective theory matrix elements one has to replace the $i\epsilon$ terms in the resonant full theory top propagators by the Breit-Wigner term $im_t\Gamma_t/2$. The circle shown in Fig. 2(a) represents the QCD form factors for the $t\bar t$ vector and axial-vector currents. In the nonrelativistic limit they reduce to the insertions of Coulomb potentials described by the higher order terms in Eq. (21). Because of the cancellation

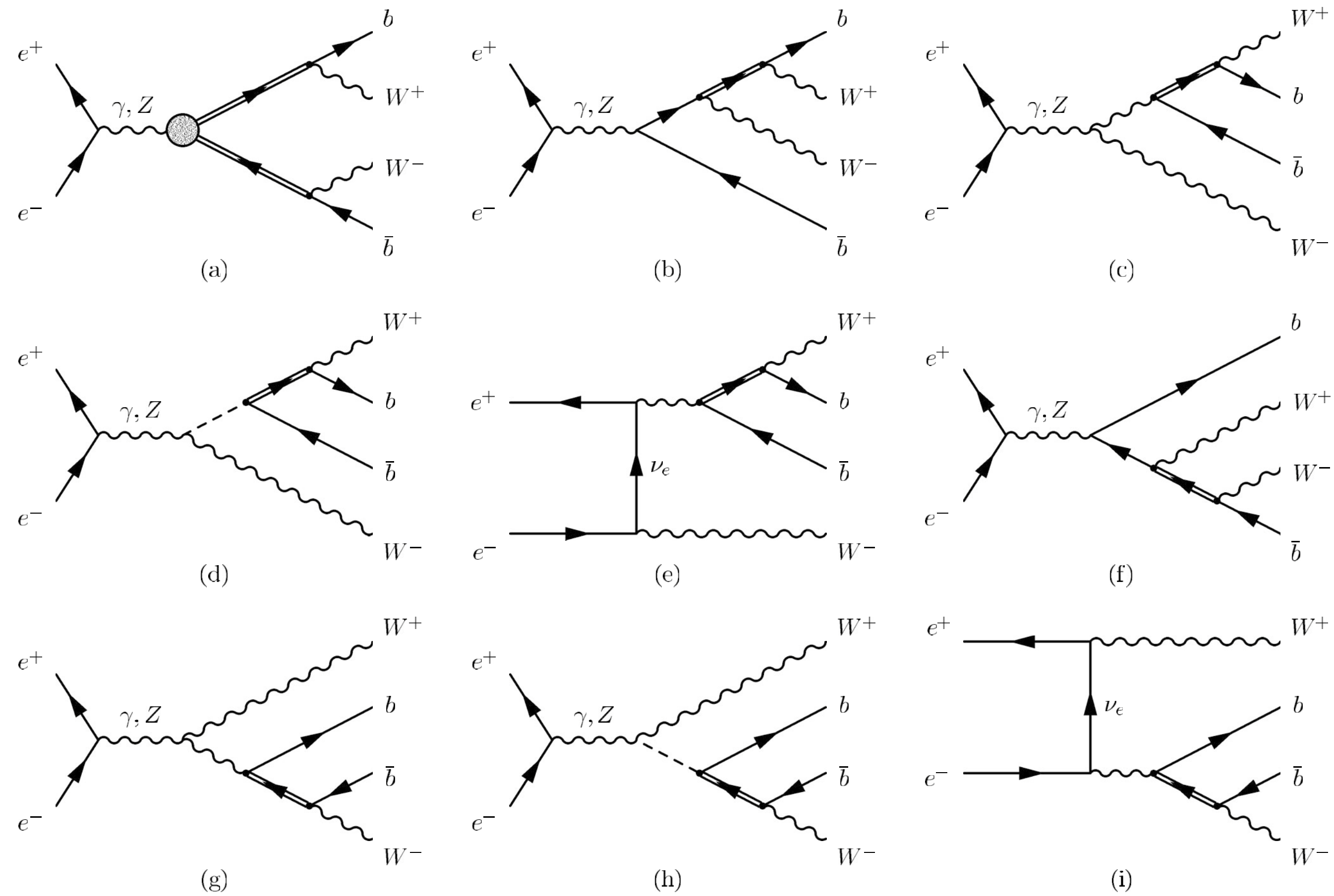


FIG. 2. Full theory Feynman diagrams describing the process $e^+e^- \to bW^+\bar{b}W^-$ with one or two intermediate top or antitop quark propagators. The circle in diagram (a) represents the QCD form factors for the $t\bar{t}$ vector and axial-vector currents.

of the QCD interference effects caused by gluons with ultrasoft momenta, there are no further QCD corrections in the nonrelativistic limit.

An interesting new conceptual aspect of the corrections shown in Eq. (20) is that they have UV $1/\epsilon$-divergences that arise from a logarithmic high energy behavior of the top-antitop effective theory phase space integration for matrix elements containing a single insertion of the Coulomb potential. In the forward scattering amplitude these UV divergences arise because the imaginary parts of the matching conditions of Eqs. (16) lead to a dependence on the real part of G^0 [see Eq. (21)]. In the full theory this logarithmic behavior is regularized by the top quark mass. While phase space logarithms are known in the literature and can be resummed with renormalization group techniques [37], the divergences here are specific since they would not exist if the top quark were approximated as being stable. In particular, the UV divergences from the time dilatation corrections arise from the Breit-Wigner–type high energy behavior of the effective theory top propagator in Eq. (6) which differs from the one for a stable particle. Likewise, the interference effects described by the absorptive electroweak matching conditions for the operators $\mathcal{O}_{V,\mathbf{p}}$ and $\mathcal{O}_{A,\mathbf{p}}$ would not have to be taken into account if the top quarks were stable particles. UV divergences of the same kind have already been observed and described before in the NNLL relativistic corrections to the S-wave zero- Green function if the unstable propagator in Eq. (6) is used [16,19,38]. For the P-wave zero-Green function, which is generated by $t\bar{t}$ production through an axial-vector current from the Z exchange and that contributes only at NNLL order, a similar UV divergence arises already at leading order in the nonrelativistic expansion. Like for the case of the time dilatation corrections, these divergences originate from the modified high energy behavior of the unstable top propagator. We believe it is evident that these divergences do not represent a deficiency of the effective theory, because the concept of separating resonant and nonresonant fluctuations appears to be the only practical way to make systematic predictions involving unstable particles. Thus these UV divergences should be handled with the renormalization techniques known from effective theories for stable particles. The only difference is that the renormalization procedure will involve operators having non-Hermitian Wilson coefficients.

The operators that are renormalized by the UV divergences displayed in Eq. (20) are the two $(e^+e^-)(e^+e^-)$ operators

$$\tilde{\mathcal{O}}_V = -[\bar{e}\gamma^\mu e][\bar{e}\gamma_\mu e], \tag{24}$$

$$\tilde{\mathcal{O}}_A = -[\bar{e}\gamma^\mu\gamma_5 e][\bar{e}\gamma_\mu\gamma_5 e], \tag{25}$$

which give the additional contribution $\tilde{\Delta}\mathcal{L} = \tilde{C}_V\tilde{\mathcal{O}}_V + \tilde{C}_A\tilde{\mathcal{O}}_A$ to the effective theory Lagrangian, $\tilde{C}_{V/A}$ being the Wilson coefficients. Because in this work we neglect QED effects, the electron and positron act as classic fields and therefore $\tilde{C}_V$ and $\tilde{C}_A$ run only through mixing due to UV divergences such as in Eq. (20). Since only the imaginary parts of the coefficients $\tilde{C}_{V/A}$ can contribute to the total cross section through the optical theorem we neglect the real contributions in the following. Using the standard $\overline{\rm MS}$ subtraction procedure the (non-Hermitian) counterterms of the renormalized $\tilde{\mathcal{O}}_{V/A}$ operators read

$$\begin{aligned}
\delta\tilde{C}_V &= i\frac{N_c m_t^2}{32\pi^2\epsilon}\bigg[(C_V^{\rm born})^2\frac{\Gamma_t}{m_t} + 2C_V^{\rm born}C_V^{\rm bW,abs}\bigg]\mathcal{V}_c^{(s)}(\nu) + i\frac{N_c m_t^2}{32\pi^2\epsilon}(C_V^{\rm born})^2\frac{\Gamma_t}{m_t}[(2c_2(\nu)-1)\mathcal{V}_c^{(s)}(\nu) + \mathcal{V}_r^{(s)}(\nu)]\\
&\quad + i\frac{N_c m_t^2}{48\pi^2\epsilon}(C_V^{\rm ax})^2\frac{\Gamma_t}{m_t}\mathcal{V}_c^{(s)}(\nu),\\
\delta\tilde{C}_A &= i\frac{N_c m_t^2}{32\pi^2\epsilon}\bigg[(C_A^{\rm born})^2\frac{\Gamma_t}{m_t} + 2C_A^{\rm born}C_A^{\rm bW,abs}\bigg]\mathcal{V}_c^{(s)}(\nu) + i\frac{N_c m_t^2}{32\pi^2\epsilon}(C_A^{\rm born})^2\frac{\Gamma_t}{m_t}[(2c_2(\nu)-1)\mathcal{V}_c^{(s)}(\nu) + \mathcal{V}_r^{(s)}(\nu)]\\
&\quad + i\frac{N_c m_t^2}{48\pi^2\epsilon}(C_A^{\rm ax})^2\frac{\Gamma_t}{m_t}\mathcal{V}_c^{(s)}(\nu),
\end{aligned}\tag{26}$$

where the respective first term on the right-hand sides subtract the $1/\epsilon$ divergences shown in Eq. (20) and the other terms account for the UV divergences in the P-wave Green function and the NNLL corrections of the S-wave Green function computed in Ref. [16]. Here, $C_{V/A}^{\rm ax}$ are Born level Wilson coefficients of operators describing top pair production in a P-wave (originating from pure Z exchange), $\mathcal{V}_r^{(s)}$ is the color singlet coefficient of the potential $(\mathbf{p}^2+\mathbf{p}'^2)/(2m_t(\mathbf{p}-\mathbf{p}')^2)$ [9] and c_2 the coefficient of the $\mathbf{p}^2$-suppressed S-wave production current [16]. The explicit formulas read

$$\begin{aligned}
&C_V^{\rm ax} = \frac{\alpha\pi}{m_t^2(4c_w^2-x)}\bigg[Q_e + \frac{1}{4s_w^2}\bigg], \qquad C_A^{\rm ax} = -\frac{\alpha\pi}{4s_w^2m_t^2(4c_w^2-x)}, \qquad \mathcal{V}_r^{(s)}(\nu) = -4\pi C_F\alpha_s(m_t)z\bigg[1+\frac{8C_A}{3\beta_0}\ln(2-z)\bigg],\\
&c_2(\nu) = -\frac{1}{6} - \frac{8C_F}{3\beta_0}\ln\bigg(\frac{z}{2-z}\bigg), \qquad z\equiv\frac{\alpha_s(m_t\nu)}{\alpha_s(m_t)}.
\end{aligned}\tag{27}$$

The resulting renormalization group equations for the Wilson coefficients $\tilde{C}_{V/A}$ have the form

$$\begin{aligned}
\frac{d\tilde{C}_V(\nu)}{d\ln\nu} &= i\frac{N_c m_t^2}{8\pi^2}\bigg\{(C_V^{\rm born})^2\frac{\Gamma_t}{m}(2c_2(\nu)\mathcal{V}_c^{(s)}(\nu) + \mathcal{V}_r^{(s)}(\nu)) + 2C_V^{\rm born}C_V^{\rm bW,abs}\mathcal{V}_c^{(s)}(\nu)\bigg\} + i\frac{N_c m_t^2}{12\pi^2}\bigg\{(C_V^{\rm ax})^2\frac{\Gamma_t}{m_t}\mathcal{V}_c^{(s)}(\nu)\bigg\},\\
\frac{d\tilde{C}_A(\nu)}{d\ln\nu} &= i\frac{N_c m_t^2}{8\pi^2}\bigg\{(C_A^{\rm born})^2\frac{\Gamma_t}{m}(2c_2(\nu)\mathcal{V}_c^{(s)}(\nu) + \mathcal{V}_r^{(s)}(\nu)) + 2C_A^{\rm born}C_A^{\rm bW,abs}\mathcal{V}_c^{(s)}(\nu)\bigg\} + i\frac{N_c m_t^2}{12\pi^2}\bigg\{(C_A^{\rm ax})^2\frac{\Gamma_t}{m_t}\mathcal{V}_c^{(s)}(\nu)\bigg\},
\end{aligned}\tag{28}$$

and the solutions for scales below m_t ($\nu < 1$) read

$$\begin{aligned}
\tilde{C}_V(\nu) &= \tilde{C}_V(1) + i\frac{2N_c m_t^2 C_F}{3\beta_0}\bigg\{\bigg[((C_V^{\rm born})^2 + (C_V^{\rm ax})^2)\frac{\Gamma_t}{m_t} + 3C_V^{\rm born}C_V^{\rm bW,abs}\bigg]\ln(z)\\
&\quad - \frac{4C_F}{\beta_0}\frac{\Gamma_t}{m_t}(C_V^{\rm born})^2\ln^2(z) + \frac{4(C_A+2C_F)}{\beta_0}\frac{\Gamma_t}{m_t}(C_V^{\rm born})^2\rho(z)\bigg\},\\
\tilde{C}_A(\nu) &= \tilde{C}_A(1) + i\frac{2N_c m_t^2 C_F}{3\beta_0}\bigg\{\bigg[((C_A^{\rm born})^2 + (C_A^{\rm ax})^2)\frac{\Gamma_t}{m_t} + 3C_A^{\rm born}C_A^{\rm bW,abs}\bigg]\ln(z)\\
&\quad - \frac{4C_F}{\beta_0}\frac{\Gamma_t}{m_t}(C_A^{\rm born})^2\ln^2(z) + \frac{4(C_A+2C_F)}{\beta_0}\frac{\Gamma_t}{m_t}(C_A^{\rm born})^2\rho(z)\bigg\},
\end{aligned}\tag{29}$$

where

$$\rho(z) = \frac{\pi^2}{12} - \frac{1}{2}\ln^2 2 + \ln 2\ln(z) - {\rm Li}_2\bigg(\frac{z}{2}\bigg), \tag{30}$$

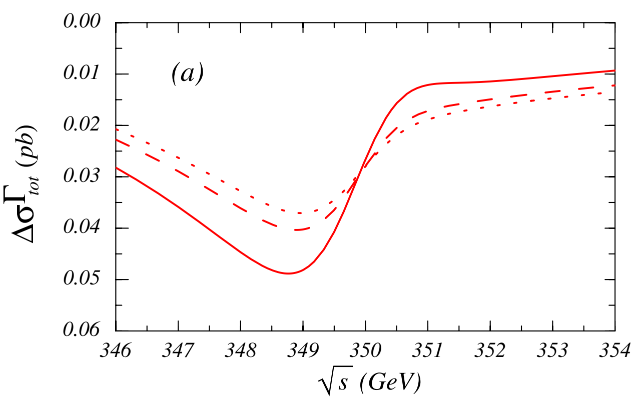
and the $\tilde{C}_{V/A}(1)$ are the hard matching conditions, which are presently unknown. Finally, the contribution of the operators $\tilde{\mathcal{O}}_{V/A}$ to the total cross section reads

$$\Delta\sigma_{\rm tot}^{\Gamma,2} = {\rm Im}[\tilde{C}_V + \tilde{C}_A]. \tag{31}$$

Parametrically $\Delta\sigma_{\rm tot}^{\Gamma,2}$ is of order g^6. Comparing this with the LL cross section which counts as $g^4 v \sim g^5$, we see that it constitutes a NLL contribution. This is also evident from the fact that the corresponding UV divergences were generated in NNLL order effective theory matrix elements. The correction $\Delta\sigma_{\rm tot}^{\Gamma,2}$ is energy independent, but it is scale dependent and compensates the logarithmic scale dependence in the NNLL order matrix elements. As mentioned before, the matching conditions $\tilde{C}_{V/A}(\nu = 1)$ are presently unknown and we therefore set them to zero in the numerical analysis presented below. We note, however, that it was shown in [19] that the difference between the full theory phase space (which is cut off by the large, but finite m_t) and the effective theory phase space (which is infinite in the computation of the forward scattering amplitude) contributes to $\tilde{C}_{V/A}(\nu = 1)$ and also represents a NLL effect.

V. NUMERICAL ANALYSIS

In Fig. 3 we have plotted $\Delta\sigma_{\rm tot}^{\Gamma,1}$ and $\Delta\sigma_{\rm tot}^{\Gamma,2}$ in picobarn in the 1S mass scheme [19,39] for $M_{1S} = 175$ GeV, $\alpha = 1/125.7$, $s_w^2 = 0.23120$, $V_{tb} = 1$ and $M_W = 80.425$ GeV with the renormalization scaling parameter $\nu = 0.1$ (solid curves), 0.2 (dashed curves) and 0.3 (dotted curves). The divergences in $\Delta\sigma_{\rm tot}^{\Gamma,1}$ are subtracted minimally. For the QCD coupling we used $\alpha_s(M_Z) = 0.118$ as an input and employed 4-loop renormalization group running. Note that in the 1S scheme $\delta m_t = M_{1S}(\mathcal{V}_c^{(s)}(\nu)/4\pi)^2/8$. In Fig. 3(a) the sum of $\Delta\sigma_{\rm tot}^{\Gamma,1}$ and $\Delta\sigma_{\rm tot}^{\Gamma,2}$ is shown while in Fig. 3(b) both contributions are presented separately. For the top quark width we adopted the value $\Gamma_t = 1.43$ GeV.[7] We find that the sum of the corrections is negative and shows a moderate ν dependence. Compared to the most recent NNLL QCD predictions for the total cross section [17] the corrections are around -10% for energies below the peak, between -2% and -4% close to the peak and about -2% above the peak. Their magnitude is comparable to the NNLL QCD corrections. Interestingly, they partly compensate the sizable positive QCD corrections found in [13,17]. The peculiar energy dependence of the corrections, caused by the dependence on the real part of the Green function G^0, also leads to a slight displacement of the peak position. Relative to the peak position of the LL cross section, one obtains a shift of (30, 35, 47) MeV for $\nu = (0.1, 0.2, 0.3)$. This shift is comparable to the expected experimental uncertainties of the top mass measurements from the threshold scan [5].



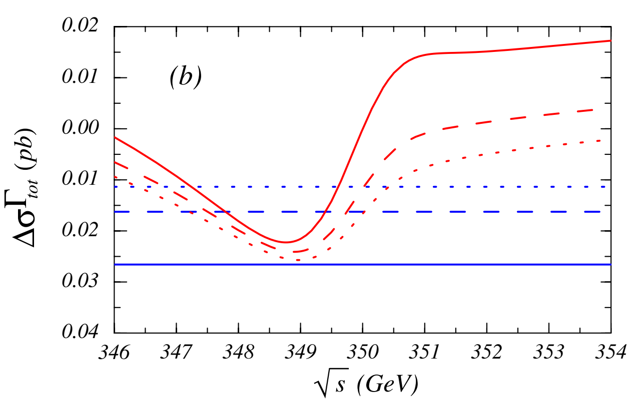


FIG. 3 (color online). The corrections $\Delta\sigma_{\rm tot}^{\Gamma,1}$ and $\Delta\sigma_{\rm tot}^{\Gamma,2}$ in pb for $M_{1S} = 175$ GeV, $\alpha = 1/125.7$, $s_w^2 = 0.23120$, $V_{tb} = 1$, $M_W = 80.425$ GeV, $\Gamma_t = 1.43$ GeV and $\nu = 0.1$ (solid curves), 0.2 (dashed curves) and 0.3 (dotted curves) in the energy range 346 GeV $< \sqrt{s} <$ 354 GeV. Panel (a) shows the sum of both corrections and panel (b) the individual size of $\Delta\sigma_{\rm tot}^{\Gamma,1}$ (energy-dependent lines) and $\Delta\sigma_{\rm tot}^{\Gamma,2}$ (straight lines).

VI. CONCLUSION

We have determined electroweak corrections to the NNLL top pair threshold cross section in e^+e^- annihilation related to the instability of the top quark. Our approach is closely related to the treatment of absorptive processes in the optical theory. It includes the effects of the top instability by accounting for the absorptive parts in electroweak corrections to the effective theory matching conditions that are related to the observable $bW^+\bar{b}W^-$ final state. The matching conditions render the NRQCD Lagrangian non-Hermitian, but they allow for the determination of the total cross section using the $e^+e^- \to e^+e^-$ forward scattering matrix element and the optical theorem. We have shown that the absorptive parts of the electroweak matching conditions for the $t\bar{t}$ production and annihilation operators describe the interference of the double-resonant amplitude for $e^+e^- \to t\bar{t} \to bW^+\bar{b}W^-$ with the single-resonant (and v^2-suppressed) amplitudes for $e^+e^- \to bW^+\bar{t} \to bW^+\bar{b}W^-$ and $e^+e^- \to t\bar{b}W^- \to bW^+\bar{b}W^-$. We have also shown that at NNLL order there are no further interference effects caused by the exchange and radiation of ultrasoft gluons.

The novel feature of the NNLL corrections is that they lead to new UV divergences. These divergences originate

[7]In a complete analysis of electroweak effects, the top quark width depends on the input parameters given above and is not an independent parameter. For the purpose of the numerical analysis in this work, however, our treatment is sufficient.

from the logarithmic high energy behavior of the $t\bar{t}$ phase space in the effective theory forward scattering amplitudes, which is caused by the interferences and by the modified propagators of an unstable top quark. The divergences renormalize $(e^+e^-)(e^+e^-)$ operators that contribute to the forward scattering amplitude already at NLL order. The corrections determined in this work slightly modify the cross section shape and are comparable to the known NNLL QCD corrections. The size of the corrections shows that a complete treatment of all NNLL electroweak effects is desirable.

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[1] I. I. Bigi, Y. L. Dokshitzer, V. Khoze, J. Kühn, and P. Zerwas, Phys. Lett. B **181**, 157 (1986).
[2] V. S. Fadin and V. A. Khoze, JETP Lett. **46**, 525 (1987).
[3] A. H. Hoang, hep-ph/0204299.
[4] M. Battaglia *et al.*, hep-ph/0304132.
[5] M. Martinez and R. Miquel, Eur. Phys. J. C **27**, 49 (2003).
[6] A. H. Hoang *et al.*, Eur. Phys. J. direct C **2**, 1 (2000).
[7] D. Cinabro, hep-ex/0005015.
[8] M. Luke, A. Manohar, and I. Rothstein, Phys. Rev. D **61**, 074025 (2000).
[9] A. H. Hoang and I. W. Stewart, Phys. Rev. D **67**, 114020 (2003).
[10] A. V. Manohar and I. W. Stewart, Phys. Rev. D **63**, 054004 (2001).
[11] A. Pineda, Phys. Rev. D **66**, 054022 (2002).
[12] A. Pineda, Phys. Rev. D **65**, 074007 (2002).
[13] A. H. Hoang, Phys. Rev. D **69**, 034009 (2004).
[14] A. A. Penin, A. Pineda, V. A. Smirnov, and M. Steinhauser, Nucl. Phys. **B699**, 183 (2004).
[15] A. H. Hoang, A. V. Manohar, I. W. Stewart, and T. Teubner, Phys. Rev. Lett. **86**, 1951 (2001).
[16] A. H. Hoang, A. V. Manohar, I. W. Stewart, and T. Teubner, Phys. Rev. D **65**, 014014 (2002).
[17] A. H. Hoang, Acta Phys. Pol. B **34**, 4491 (2003).
[18] R. J. Guth and J. H. Kühn, Nucl. Phys. **B368**, 38 (1992).
[19] A. H. Hoang and T. Teubner, Phys. Rev. D **60**, 114027 (1999).
[20] M. Beneke, A. P. Chapovsky, A. Signer, and G. Zanderighi, Phys. Rev. Lett. **93**, 011602 (2004); M. Beneke, A. P. Chapovsky, A. Signer, and G. Zanderighi, Nucl. Phys. **B686**, 205 (2004).
[21] BABAR Collaboration, P. F. Harrison and H. R. Quinn, SLAC-R-0504.
[22] G. P. Korchemsky and A. V. Radyushkin, Phys. Lett. B **279**, 359 (1992).
[23] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002).
[24] A. V. Manohar and I. W. Stewart, Phys. Rev. Lett. **85**, 2248 (2000).
[25] V. S. Fadin, V. A. Khoze, and A. D. Martin, Phys. Rev. D **49**, 2247 (1994).
[26] K. Melnikov and O. I. Yakovlev, Phys. Lett. B **324**, 217 (1994).
[27] M. Jezabek and J. H. Kühn, Nucl. Phys. **B314**, 1 (1989).
[28] A. V. Manohar and I. W. Stewart, Phys. Rev. D **62**, 014033 (2000).
[29] A. V. Manohar and I. W. Stewart, Phys. Rev. D **62**, 074015 (2000).
[30] M. Beneke and G. Buchalla, Phys. Rev. D **53**, 4991 (1996).
[31] I. Blokland, A. Czarnecki, M. Slusarczyk, and F. Tkachov, Phys. Rev. Lett. **93**, 062001 (2004).
[32] A. Denner and T. Sack, Nucl. Phys. **B358**, 46 (1991).
[33] W. Mödritsch and W. Kummer, Nucl. Phys. **B430**, 3 (1994).
[34] S. Su and M. B. Wise, Phys. Lett. B **510**, 205 (2001).
[35] A. Czarnecki and K. Melnikov, Phys. Rev. Lett. **80**, 2531 (1998); M. Beneke, A. Signer, and V. A. Smirnov, Phys. Rev. Lett. **80**, 2535 (1998).
[36] A. H. Hoang, Phys. Rev. D **56**, 7276 (1997).
[37] C. W. Bauer, A. F. Falk, and M. E. Luke, Phys. Rev. D **54**, 2097 (1996).
[38] A. H. Hoang and T. Teubner, Phys. Rev. D **58**, 114023 (1998).
[39] A. H. Hoang, Z. Ligeti, and A. V. Manohar, Phys. Rev. Lett. **82**, 277 (1999); Phys. Rev. D **59**, 074017 (1999).