

Decay of Θ^+ in a quark model

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We study the decay of Θ^+ in a nonrelativistic quark model. The wave function of Θ^+ is constructed for the two cases $J^P = 1/2^\pm$ as products of color, spin, flavor, and orbital parts respecting total antisymmetrization among the four quarks. We find that for the negative parity Θ^+ the width becomes very large which is of order of several hundreds MeV, while it is about a several tens MeV for the positive parity. By assuming additionally diquark correlations, the width is reduced to be of order 10 MeV. It is also pointed out that a similar calculation for $3/2^-$ results in strong suppression of its width.

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I. INTRODUCTION

One of the distinguished features of the pentaquark particle Θ^+ is its very narrow width [1]. Many experiments so far have reported only upper limits which are less than experimental resolution. The pioneering work of the LEPS group at SPring-8 has indicated $\Gamma \lesssim 25$ MeV [2], while the ITEP group has reported $\Gamma \lesssim 9$ MeV [3]. Recent analysis of the K^+ scattering from the xenon or deuteron implies even smaller value $\Gamma \lesssim 1$ MeV [4–6]. It has been often mentioned that a width of order of 10 MeV or less for baryon resonances is very small as compared with a typical value of around 100 MeV, though such a criterion should be quantified on a better theoretical ground [7]. So far the chiral soliton model has predicted the masses and widths of the pentaquark baryons with less theoretical ambiguity based on the SU(3) flavor algebra [8]. The model indicates the width of Θ^+ around a few tens MeV [9]. Then one might wonder if the chiral soliton model does something exotic in contrast with the conventional knowledge of hadron physics.

The purpose of this paper is to consider the width of Θ^+ in a nonrelativistic quark model. The model has been successful for the description of the conventional baryons made dominantly by three quarks. The detailed study in the quark model must be useful in order to understand the microscopic dynamics of the pentaquarks [10]. Even the result of the chiral soliton model may be interpreted just as for the nucleon in the large- N_c limit [11]. This is, however, beyond our scope in this paper. Another question is related to the intrinsic parity of the pentaquarks. Since we do not know it, we perform the calculation for both cases. As we will see, the decay width depends strongly on the parity of Θ^+ . Therefore, the study of the decay will help to know the parity and hence the internal structure of the pentaquarks.

In order to prepare for the present study, we briefly look at the general aspect for the width of baryons in this

section. Consider a decay of Θ^+ going to the nucleon and kaon. Assuming the spin of the Θ^+ , $J = 1/2$, the interaction Lagrangian takes the form

$$L_{\pm} = g_{KN\Theta} \bar{\psi}_N \gamma_{\pm} \psi_{\Theta} K, \quad (1)$$

where $\gamma_+ = i\gamma_5$ if the parity of Θ^+ is positive, while $\gamma_- = 1$ if the parity of Θ^+ is negative. The formula for the decay width is given by

$$\Gamma_{\pm} = \frac{g_{KN\Theta}^2}{2\pi} \frac{M_N q^3}{E_N(E_N + M_N)M_{\Theta}}, \quad (2)$$

for the positive parity, where M_N and M_{Θ}^+ are the masses of the nucleon and Θ^+ , and $E_N = \sqrt{q^2 + M_N^2}$ with \vec{q} being the momentum of the final state kaon in the kaon-nucleon center-of-mass system, or equivalently in the rest frame of Θ^+ . The width for the negative parity is related to the one of the positive parity by

$$\Gamma_{-} = \frac{(E_N + M_N)^2}{q^2} \Gamma_{+}. \quad (3)$$

The difference arises due to the different coupling nature: p-wave coupling for positive parity Θ^+ and s-wave coupling for negative parity Θ^+ , representing the effect of the centrifugal repulsion in the p-wave. In the kinematical point of the Θ^+ decay, $M_{\Theta} = 1540$ MeV, $M_N = 940$ MeV, and $m_K = 490$ MeV, the factor on the right-hand side of (3) becomes about 50 and brings a significant difference in the widths of the positive and negative parity Θ^+ . If we take $g_{KN\Theta} \sim 10$ as a typical strength for strong interaction coupling constants, we obtain $\Gamma_{+} \sim 100$ MeV, while $\Gamma_{-} \sim 5$ GeV. Both numbers are too large as compared with experimentally observed width. Therefore, the relevant question is whether some particular structure of Θ^+ will suppress the above naive values, or not.

In the quark model, assuming that the meson, nucleon, and pentaquark states are dominated by two, three, and five valence quarks, the decay of the pentaquark occurs through

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the so-called fall apart process, in which the five quarks dissociate into a three-quark cluster, a nucleon, a quark-antiquark cluster, a meson, without pair creation of the quarks [12,13]. This should be contrasted with an ordinary meson-baryon coupling in which a creation of quark-antiquark pair must accompany. If we treat the meson as a fundamental field, as expected to be valid for the Nambu-Goldstone boson, and introduce a meson-quark interaction of the Yukawa type, \mathcal{L}_{int} , the two processes involve matrix elements of the types $\langle 0 | \mathcal{L}_{\text{int}} | q\bar{q} \rangle$ for the fall apart process, while $\langle q | \mathcal{L}_{\text{int}} | q \rangle$ for the ordinary meson-baryon coupling. These are depicted in Figs. 1(b) and 1(a), respectively. In 1(a) the coupling is spacelike, while in 1(b) timelike. To the extent that the meson is regarded as a pointlike, the effect of the form factor is neglected. This is one of the assumptions we adopt in the present work. The quantity we will investigate in this paper is essentially the former matrix element $\langle 0 | \mathcal{L}_{\text{int}} | q\bar{q} \rangle$ for the decay of Θ^+ [8].

In Ref. [14], similar calculation was performed, where the matrix element of the axial-vector current between the Θ^+ and nucleon states were computed. In order to relate the matrix element of the axial-vector current to the axial transition form factor $g_A(\Theta^+ \rightarrow N)$, they have assumed the partially conserved axial-vector current (PCAC) relation, which is, however, not applicable to their quark model approach, since the quark model without the chiral mesons does not generate the meson pole term in the current matrix element and hence does not satisfy the PCAC relation.

Since we do not know the spin and parity of the Θ^+ , we will study the decay width for several spin-parity states. Naively, the negative parity state of $(0s)^5$ configuration appears lower than positive parity states of $(0s)^4 0p$ configuration. Although mechanisms which lower the positive parity states have been discussed [12,15,16], the quantitative prediction for the mass is not yet fully done. However, the mass of the Θ^+ is an important input for the decay width, since it changes the phase space volume and also the q (momentum) dependent transition form factor. In the present study, we use as inputs of masses experimental values in order to exclude the dependence coming from the phase space and the q dependent form factor. In this way, comparison of the results of different states reflects the difference in the structure of, in particular, the internal spin-flavor-color wave functions.

This paper is organized as follows. In section II, we establish the necessary ingredients of the quark model,

especially for the basic meson-quark interaction and how to compute the matrix element of the fall apart process. In section III, we calculate the transition matrix element from the five-quark state of Θ^+ to KN in the nonrelativistic quark model of harmonic oscillator. An advantage of the model is that the separation of the center-of-mass coordinate is completely performed. We compute the matrix elements for both positive and negative parity Θ^+ . In the fall apart process the decay of Θ^+ into KN proceeds first by forming a nucleonlike qqq state and a kaonlike $q\bar{s}$ state in the Θ^+ wave function. It is then necessary to compute the spectroscopic factor for a given quark model wave function for Θ^+ . This is shown in detail in the appendix. In section IV numerical values are presented with some discussions. In the final section the paper is concluded.

II. INGREDIENTS OF THE QUARK MODEL

Our starting point is an interaction Lagrangian of chiral mesons and quarks [17]:

$$\mathcal{L}_{\text{int}} = -ig\bar{\psi}\gamma_5\Phi\psi \sim \frac{g}{2m}\chi^\dagger\vec{\sigma}\cdot\vec{\nabla}\Phi\chi, \quad (4)$$

where $\psi = (\psi_u, \psi_d, \psi_s)$ is a four-component Dirac spinor field, $\chi = (\chi_u, \chi_d, \chi_s)$ the two component spinor field, and

$$\Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix} \quad (5)$$

is a flavor octet meson field. In the second equation of (4), we have shown an expression familiar in the nonrelativistic quark model with a quark mass m . The meson-quark coupling constant g may be determined from the πNN coupling constant $g_{\pi NN} \sim 13$. Taking the matrix element of (4) in the nucleon states of the quark model, we find

$$\langle N(p_2)\pi^a | \mathcal{L}_{\text{int}} | N(p_1) \rangle \sim i\frac{5g}{6m}\vec{\sigma}_N \cdot \vec{q}\tau^a, \quad (6)$$

where $\vec{q} = \vec{p}_2 - \vec{p}_1$ and $\vec{\sigma}_N$ is the spin matrix acting on the two component nucleon spinor. Assuming that quark mass is 1/3 of nucleon mass, $m \sim M_N/3$, and comparing (6) with the πNN interaction $(ig_{\pi NN}/2M_N)\vec{\sigma}_N \cdot \vec{q}$, we find

$$g = \frac{g_{\pi NN}}{5} = 2.6. \quad (7)$$

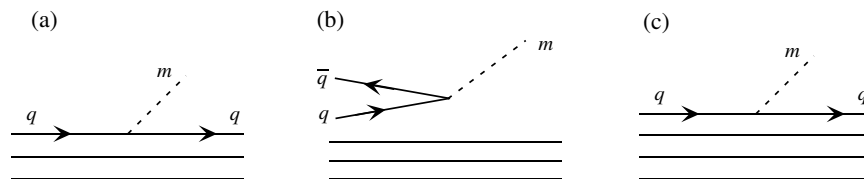


FIG. 1. Meson-baryon couplings involving an $m\bar{q}q$ coupling. (a) Transition of a three-quark baryon to another three-quark baryon. (b) A decay of pentaquark baryon into a three-quark baryon and a meson. (c) A diagram equivalent to (b).

This interaction has been often used in the quark model to compute meson-baryon couplings and transition amplitudes of, for instance, $N^* \rightarrow \pi N$ where N^* is an ordinary nucleon resonance made from three quarks as shown in Fig. 1(a). The same interaction can be used for the decay of a pentaquark baryon if one reverses the outgoing quark line for the incoming antiquark line as shown in Fig. 1(b). Here the quark-antiquark pair has the quantum numbers of the kaon. When we treat the quarks as identical particles, it is convenient to consider the diagram 1(c), where the antiquark line in the initial pentaquark state is once again reversed to an outgoing quark line. This is the ‘‘particle-hole transformation’’ which relates the interaction between

$$\begin{aligned} \langle f | \int d^4x \mathcal{L}_{\text{int}} | i \rangle &= 2\pi \delta(E_f - E_i) \mathcal{M}_{fi}, \\ \mathcal{M}_{\Theta^+ \rightarrow K^+ n} &= -i \langle n_f K^+ (\vec{q}) | \int d^3x g \bar{\psi} \gamma_5 \Phi \psi | \Theta^+ (uudd\bar{s}) \rangle \\ &= -i\sqrt{2} \langle n_f (udd) | \int d^3x g \bar{\psi} \gamma_5 \psi e^{-i\vec{q}\cdot\vec{x}} | \Theta^+ (uudd\bar{s}) \rangle, \end{aligned} \quad (9)$$

where n_f denotes the final state neutron of three quarks. In the valence quark model, in the last line of (9), we have assumed that the final state kaon is expressed by a non-interacting plain wave of momentum \vec{q} . In practical calculations, we treat the quarks as identical particles. By moving \bar{s} in the initial state into s in the final state, the initial and final states may be treated as systems of four identical particles. Then the operator is written as a sum over the four particles $\mathcal{O} = \sum_{i=1,\dots,4} \mathcal{O}(i)$, and the final state may be antisymmetrized as

$$\begin{aligned} |udds\rangle &= \frac{1}{2} [n(123)s(4) - n(124)s(3) - n(143)s(2) \\ &\quad - n(423)s(1)], \end{aligned} \quad (10)$$

where we have assumed that $n(ijk)$ is already antisymmetrized. The four quarks $uudd$ in the initial state Θ^+ are also antisymmetrized having the same structure as (10) under permutation. The Θ^+ wave function is explicitly constructed in the next section. Combining (9) and (10) with correct counting factors, we find

$$\begin{aligned} \mathcal{M}_{\Theta^+ \rightarrow K^+ n} &= 2\sqrt{2}ga \langle 0 | \int d^3x \bar{\psi} \gamma_5 \psi e^{-i\vec{q}\cdot\vec{x}} | (u\bar{s})^{K^+} \rangle \\ &\quad \times \langle n_f | (udd)^n \rangle. \end{aligned} \quad (11)$$

We will compute the matrix element of (11) for both positive and negative parity Θ^+ . Carlson *et al.* [19,20] computed the constant a for several configurations. Here we repeat the calculations briefly using the method of Young diagram.

III. DECAY AMPLITUDE

A. Negative parity

Now we compute the matrix element (11) using a wave function of harmonic oscillator for the nonrelativistic

the two particle states with the one for the particle and hole states [18].

Now the pentaquark Θ^+ wave function $|\Theta^+\rangle$ can be written by four light quarks $uudd$ and an \bar{s} . The state contains a component of the first three quarks having the neutron quantum numbers and the remaining quark and antiquark having kaon quantum numbers,

$$|\Theta^+\rangle = a |(u(1)d(2)d(3))^n (u(4)\bar{s}(5))^{K^+}\rangle + \dots, \quad (8)$$

where a is the spectroscopic factor, probability amplitude of finding the $K^+ n$ state in $|\Theta^+\rangle$. Then transition amplitude for $\Theta^+ \rightarrow K^+ n$ can be written as

quark model [21]. For simplicity, we assume that all quarks are in a potential with the common oscillator parameter, $m\omega \equiv \alpha_0$. Hence the $(0s)^5$ configuration for the negative parity state may be written as

$$|\Theta^+, (0s)^5\rangle = \psi(\vec{x}_1)\psi(\vec{x}_2)\psi(\vec{x}_3)\psi(\vec{x}_4)\psi(\vec{x}_5) |\Theta_{csf}^+\rangle, \quad (12)$$

where the color-spin-flavor wave function is given by (A9) in the appendix with the spectroscopic factor $a = 1/(2\sqrt{2})$. The single particle wave function is given by

$$\psi(\vec{x}_i) = \left(\frac{\alpha_0^2}{\pi}\right)^{3/4} \exp\left(-\frac{\alpha_0^2}{2} |\vec{x}_i|^2\right). \quad (13)$$

By introducing various coordinates as defined in Fig. 2, we can decompose the single particle state (12) into a product of parts of the corresponding coordinates. After the separation of the wave function for the total center-of-mass coordinate \vec{X}_{tot} , and then replacing it by the plane wave of the total momentum $\vec{P}_{\text{tot}} = 0$, we can write

$$\begin{aligned} |\Theta^+\rangle &\sim a |(u(1)d(2)d(3))^n (u(4)\bar{s}(5))^{K^+}\rangle + \dots \\ &= a e^{i\vec{P}_{\text{tot}}\cdot\vec{X}_{\text{tot}}} \phi_{KN}(\vec{x}) \phi_N(\vec{\rho}, \vec{\lambda}) \phi_K(\vec{r}) \\ &\quad \times (\text{color}) \cdot (\text{spin}) \cdot (\text{flavor}) + \dots, \end{aligned} \quad (14)$$

where the color-spin-flavor wave function is presented in the appendix, and the dots in the last line contain all possible states composed of products of color singlet $3q$ and $q\bar{q}$ states which are orthogonal to the $K^+ n$ state in the first term. The wave function $\phi_{KN}(\vec{x})$ is for the relative motion of the nucleon and kaonlike clusters, $\phi_N(\vec{\rho}, \vec{\lambda})$ for the intrinsic state of the nucleonlike part, and $\phi_K(\vec{r})$ for the intrinsic (relative) state of the kaonlike part. For instance,

$$\phi_{KN}(\vec{x}) = \left(\frac{\alpha^2}{\pi}\right)^{3/4} \exp\left(-\frac{\alpha^2}{2} x^2\right), \quad (15)$$

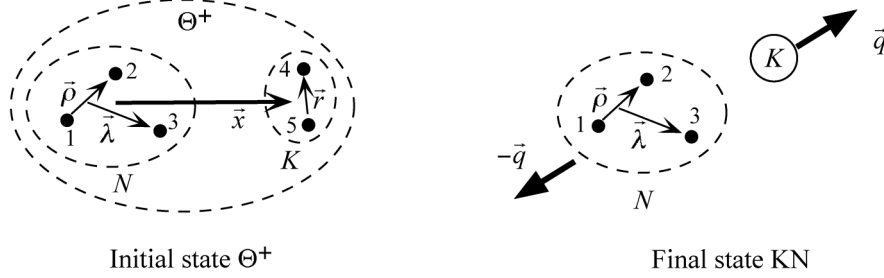


FIG. 2. Definition of various coordinates of the five-quark system.

where the parameter α is for the relative motion of the kaon and nucleonlike clusters and is related to α_0 by

$$\alpha^2 = \frac{6}{5} \alpha_0^2. \quad (16)$$

The final state wave function takes the form

$$e^{i\vec{p}_{\text{tot}} \cdot \vec{x}_{\text{tot}}} e^{i\vec{q} \cdot \vec{x}} \phi_N(\vec{\rho}, \vec{\lambda}), \quad (17)$$

where we have assumed that the intrinsic structure of the final state nucleon is the same as the one of the three-quark cluster in the initial state, and the relative motion of the kaon and the nucleon is described by a plane wave of momentum \vec{q} .

The remaining computation is rather straightforward but it is worth mentioning a few remarks. First, the overlap of the three-quark wave functions in the initial and final states is set one by assuming that the oscillator parameter of the five quarks in the initial state Θ^+ is the same as that of the nucleon in the final state. Therefore, we have $\langle n_f | (udd)^n \rangle = 1$. If spatial structure for the nucleon and pentaquark states are different, this overlap factor will be suppressed from 1. Furthermore, a small repulsive force in the KN scattering channel also reduce the overlap. As discussed in Ref. [14], strong diquark correlation has a significant effect on the suppression. Whether sufficient spatial correlation will be developed or not is, however, a dynamical question [22]. In any case, we expect some suppression in a more realistic study, and therefore the estimation in the present work provides the upper bound in the quark model calculation. Second, the matrix element for the annihilation of the kaonlike cluster in the initial state reduces to

$$\langle 0 | \bar{\psi}_s \gamma_5 \psi_u | (u\bar{s})^{K^+} \rangle \sim \sqrt{2} \left(\frac{\alpha_0^2}{2\pi} \right)^{3/4} \phi_K(\vec{x}). \quad (18)$$

Here, γ_5 is replaced by one in the nonrelativistic approximation, since the lower component of the antiquark wave function is a large component. The factor $\sqrt{2}$ is from the spin part of the matrix element for $S = 0$ pair of u and \bar{s} quarks whose wave function is given by $1/\sqrt{2}(\uparrow\downarrow - \downarrow\uparrow)$, and $(\alpha_0^2/2\pi)^{3/4}$ is the value of the kaonlike wave function $\phi_K(\vec{r})$ at the origin.

The resulting matrix element of (11) is compared with the matrix element of the s -wave coupling,

$$\begin{aligned} \langle n(-\vec{q}) | g_{KN\Theta} \int d^3x \bar{\psi}_n \psi_\Theta e^{-i\vec{q} \cdot \vec{x}} | \Theta^+(\vec{0}) \rangle \\ = (2\pi)^3 \delta^3(0) g_{KN\Theta}, \end{aligned} \quad (19)$$

where we have set the normalization factor $\sqrt{(E+M)/2M} \rightarrow 1$. We find the result

$$g_{KN\Theta} = 4ga \left(\frac{5}{3} \right)^{3/4} F(q), \quad (20)$$

where the transition form factor is defined by

$$F(q) = \left(\frac{\alpha^2}{2\pi} \right)^{3/2} \int d^3x e^{-(1/2)\alpha^2 x^2} e^{-i\vec{q} \cdot \vec{x}} = e^{-(q^2/2\alpha^2)}. \quad (21)$$

B. Positive parity

For a positive parity Θ^+ , one of the four light quarks must be excited into a p-orbit, and hence the five-quark configuration is $(0s)^4 0p$. For this case, four independent configurations are available [12]. In general, the lowest energy configuration may be a linear combination of these states. Here we consider three simple cases; the one minimizing the spin-flavor interaction [19], the one minimizing the spin-color interaction, and the one with strong $S = I = 0$ diquark correlation [23]. For illustration, however, we show detailed computation only for the first one of minimizing the spin-flavor interaction.

In the nonrelativistic quark model of harmonic oscillator, we can write the two terms of the p-states of (A18) in terms of KN relative coordinate, as

$$\begin{aligned} | \Theta^+, (0s)^4 0p \rangle = \sqrt{\frac{5}{96}} \alpha [\vec{x}, \chi]_{J=1/2, m_i} \left(\frac{\alpha}{2\pi} \right)^{3/2} e^{-(1/2)\alpha^2 x^2} \\ \times \left(\frac{5}{3} \right)^{3/4} e^{i\vec{p}_{\text{tot}} \cdot \vec{x}_{\text{tot}}} \phi_N(\vec{\rho}, \vec{\lambda}), \end{aligned} \quad (22)$$

where we have set $\vec{x}_4 \rightarrow \vec{x}_5$ prior to computation of the matrix element. In (22), $[\vec{x}, \chi]_{J=1/2, m_i}$ represents the coupling of the relative coordinate between the kaon and nucleonlike clusters \vec{x} and the spin-1/2 state χ to form

the total spin $J = 1/2$, m_i . Furthermore, we have recovered the spectroscopic factor $a = \sqrt{5/96}$ as derived by Carlson *et al.* [19].

The matrix element (11) can now be computed with the result

$$\mathcal{M} = (2\pi)^3 \delta^3(0) \sqrt{\frac{5}{96}} \frac{4g}{\sqrt{3}\alpha} \langle m_f | \vec{\sigma} \cdot \vec{q} | m_i \rangle F(q) \times \left(\frac{\alpha}{2\pi}\right)^{3/2} \left(\frac{5}{3}\right)^{3/4} \quad (23)$$

which is compared with the p-wave coupling defined by

$$g_{KN\Theta} \langle n(-\vec{q}) | \int d^3x \bar{\psi}_n \gamma_5 \psi_\Theta e^{-i\vec{q}\cdot\vec{x}} | \Theta(\vec{0}) \rangle = (2\pi)^3 \delta^3(0) g_{KN\Theta} \frac{\vec{\sigma} \cdot \vec{q}}{2M_N}. \quad (24)$$

Hence we find

$$g_{KN\Theta} = g \sqrt{\frac{5}{96}} \frac{8M_N}{\sqrt{3}\alpha} \left(\frac{5}{3}\right)^{3/4} F(q), \quad (25)$$

where $\alpha = \sqrt{6/5}\alpha_0$ and $F(q)$ has been defined by (21).

The appearance of the oscillator parameter α in the denominator is worth being pointed out. It indicates that as the size (inversely proportional to α) of Θ^+ decreases, the coupling constant and hence the decay width decreases. This is a feature of the fall apart decay into a p or higher partial wave state. For a small Θ^+ the decay is suppressed, since the overlap with the decaying p-wave final state is suppressed. This feature is very much different from the decay (transition) of an ordinary baryon which is accompanied by the creation of quark-antiquark pair for a meson. Such a decay remains finite in the limit that the size of the baryon is zero.

IV. NUMERICAL VALUES AND DISCUSSIONS

The decay width is given by the square of the coupling constant times the phase space volume. Since the change in the mass affects the phase space volume and the transition form factor, our study here is considered to be for the coupling constant $g_{KN\Theta}$ at the realistic kinematical point by fixing the masses at experimental values: $m_K \sim 490$ MeV, $M_N = 940$ MeV, and $M_\Theta = 1540$ MeV. For numerical estimation, we consider the following two cases corresponding to different sizes of harmonic oscillator potential:

$$\begin{aligned} \langle r^2 \rangle_N^{1/2} = 1.0 \text{ fm} &\rightarrow \alpha_0^2 = 1.5 \text{ fm}^{-2}, \\ \langle r^2 \rangle_N^{1/2} = 0.7 \text{ fm} &\rightarrow \alpha_0^2 = 3.0 \text{ fm}^{-2}. \end{aligned} \quad (26)$$

The resulting coupling constants and the decay widths are summarized in Table I. As discussed before, since we do not consider possible difference in the structures of the spatial wave functions of the nucleons and pentaquark, the values in Table I should be upper bounds.

TABLE I. The $KN\Theta^+$ coupling constant $g_{KN\Theta^+}$ and decay width (in MeV) of Θ^+ for $J^P = 1/2^\pm$. In the columns of SF, SC, and JW presented are the results for the configuration that minimize the spin-flavor interaction, that minimizes the spin-color interaction, and that with the $S = I = 0$ diquark correlation of the Jaffe-Wilzcek type.

		$g_{KN\Theta}$			Γ (MeV)				
		$J^P = 1/2^-$	$1/2^+$		$J^P = 1/2^-$	$1/2^+$			
$\langle r^2 \rangle^{1/2}$	α_0^2	SF	SC	JW	SF	SC	JW		
1/ $\sqrt{2}$ fm	3 fm $^{-2}$	4.1	7.7	5.5	3.2	890	63	32	11
1 fm	1.5 fm $^{-2}$	3.2	8.4	5.9	3.4	520	74	37	12

From the table, we see that the width of the negative parity Θ^+ is too wide for the state to be regarded as a sharp resonance, as consistent with the naive estimate made in section I. In this paper, we have shown this by explicitly calculating the matrix element. However, for the ground state configuration $(0s)^5$, this could have been expected, if we have noticed that this is the unique configuration, unlike the positive parity Θ^+ . Because of this uniqueness, the Θ^+ wave function is completely written as a KN -like state as given in (10). The spectroscopic factor 1/2 (for finding two KN states) is then identical to the normalization factor of (10). Hence, unless there is some attraction and/or coupled channels, $1/2^-$ state cannot accommodate a resonance [7,12]. If one could include a higher excited state such as a $(0s)^4 1s$ configuration, there could be a resonance state, but the energy would become very large.

For the positive parity Θ^+ , the column SF (spin-flavor) shows the results for the Θ^+ configuration minimizing the spin-flavor interaction as we have discussed so far. The column SC (spin-color) is for the result for the configuration minimizing the spin-color interaction. The SC configuration has a spectroscopic factor $\sqrt{5/192}$ which is smaller than that of the SF by the factor $1/\sqrt{2}$. Therefore, the expected decay width becomes half. We have also shown in the column JW the result for the case where the Jaffe-Wilzcek type of diquark correlation is developed [23]. In this case, the spectroscopic factor becomes $\sqrt{5/576}$ [19] instead of $\sqrt{5/96}$, which reduces the decay width by the factor 6 from the result of SF.

Although the results depend significantly on the choice of model parameters, the general tendency is that the decay width for the negative parity Θ^+ is too wide, while that for the positive parity can be of order of 10 MeV when strong correlation in the color-spin-flavor space is developed. As anticipated spatial correlation will further suppress the decay width. Interference between different configurations could be another source of reduction. This, however, is a difficult problem at this point, since it depends very much on the type of interaction.

The present analyses can be extended straightforwardly to the case of spin-3/2. For the negative parity case, the spin-1 state of the four quarks in the Θ^+ may be combined

with the spin of \bar{s} for the total spin-3/2. In this case the final KN state must be in d-wave, and therefore, the spectroscopic factor of finding a d-wave KN state in the initial configuration which is $(0s)^5$ is simply zero. If a tensor interaction induces a small admixture of a d-wave configuration, it can decay into a d-wave KN state. The decay rate, however, would be small. Therefore the decay of $3/2^- \Theta^+$ into KN is expected to be suppressed. There could be a possible decay channel of the nucleon and the vector K^* of $J^P = 1^-$. This decay, however, does not occur since the total mass of the decay channel is larger than the mass of Θ^+ . Hence this $J^P = 3/2^-$ state could be another candidate for the observed narrow state. This state does not have a spin-orbit partner and forms a single resonance peak around its energy.

For the positive parity case, the p-state orbital excitation may be combined with the spin of \bar{s} for the total spin-3/2. In this case, the calculation of the decay width is precisely the same as before except for the last step of Eqs. (A16) and (A18), where the total spin should be 3/2. After taking the average over the angle \vec{q} , however, the coupling yields the same factor as for the case $J = 1/2$. Hence the decay rate of spin-3/2 Θ^+ is the same as that of Θ^+ of spin-1/2 in the present simple treatment.

V. CONCLUSION

We have studied the decay of the pentaquark baryon Θ^+ in the nonrelativistic quark model. The matrix element of the fall apart process has been calculated using the meson-quark coupling of the Yukawa type. The method is a natural extension of the standard quark model calculation for meson-baryon couplings which involves the matrix element of the axial current between quark states to the one of quark-antiquark annihilation. If the quark-meson coupling would be fundamental, such an approach should be reliable.

In the quark model, we can consider various states, in the present study positive and negative parity pentaquark states. This is perhaps an advantage over the chiral soliton model, since the latter negative parity states require an excitation beyond the rigid rotation, which is rather difficult to construct. In the quark model of harmonic oscillator, ground state of $(0s)^5$ is dominated by the KN scattering state and is necessarily lead to a large decay width of the $1/2^-$ pentaquark state. Hence, $J^P = 1/2^-$ pentaquarks dominated by the $(0s)^5$ configuration are hardly regarded as a resonance as observed in experiments.

In contrast, for the positive parity Θ^+ there are more configurations available, which together with the factor due to the centrifugal barrier suppresses the transition probability of $\Theta^+ \rightarrow KN$. The suppression rate depends significantly on the configuration. In the three cases we have studied, one minimizing the spin-flavor interaction, spin-color interaction, and with the $I = S = 0$ diquark correlations, the decay widths turned out to be $70 \sim$

10 MeV. These values, however, were computed only when color-spin-flavor part of the wave functions were properly treated. The inclusion of possible spatial correlations and/or with more realistic configuration mixing would reduce these values. Therefore, the width of the pentaquark baryon around 10 MeV may be explained if the parity would be positive when the spin of Θ^+ is 1/2. If it will be of order 1 MeV, then we will still need further reduction of order 10 for the width, or about 3 for the coupling constant.

Finally, we have pointed out another possibility of a narrow resonance of $J^P = 3/2^-$. In the hadronic language, this state is dominated by the K^*N s-wave bound state. Such a picture may not only explain a narrow width of the pentaquark state but also provide a simple production mechanism as dominated by the K^* -exchange.

As we have seen, the decay width is extremely sensitive to the structure of the baryons. Therefore, in the quark model more accurate treatment of the five-body system with reliable model Hamiltonian should be desired. As several attempts have been reported in a recent workshop [1], more development will be expected to appear to clarify the structure of the pentaquark state.

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APPENDIX: COMPUTATION OF SPECTROSCOPIC FACTOR

1. Negative parity Θ^+

First we start by noting that the color wave function of the q^4 system must be $[211]_c$ to form the color singlet state with \bar{s} of $[11]_c$. This condition should be satisfied not only for the negative parity but also for positive parity Θ^+ . All four light quarks are then assumed to occupy the lowest s-state and therefore the orbital wave function is totally symmetric $[4]_o$. Therefore, the spin-flavor wave function must have the symmetry $[31]_{fs}$ which is combined with the color wave function $[211]_c$ to form the totally antisymmetric state $[1111]_{csfo}$. In the Young diagram,

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} \underset{csfo}{=} \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} \cdot \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} \underset{o}{=} \quad (\text{A-1})$$

The subscripts c , s , f , and o in the diagram denote color (c), spin (s), flavor (f), and orbital (o) parts of the wave function. Furthermore, center dot “ \cdot ” denotes the inner product of wave functions in different functional space. The csf wave function is now decomposed into color and spin-flavor part. In the Young tableaux with particle number assignment, it can be written as

$$\begin{array}{|c|c|c|} \hline 1 \\ \hline 2 \\ \hline 3 \\ \hline 4 \\ \hline \end{array}_{csf} = \frac{1}{\sqrt{3}} \left(\begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array}_c \cdot \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 \\ \hline \end{array}_{sf} - \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}_c \cdot \begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline 3 \\ \hline \end{array}_{sf} + \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}_c \cdot \begin{array}{|c|c|c|} \hline 1 & 3 & 4 \\ \hline 2 \\ \hline \end{array}_{sf} \right) \quad (\text{A-2})$$

The Young tableaux is convenient when projecting out such a term as containing the quarks 123 forming the nucleonic component and of $4\bar{5}$ kaonic one. The first term is the one of such, where the color wave function of 123 is totally antisymmetric $[111]_c$ and spin-flavor part is totally symmetric $[3]_{sf}$. Assuming that the Θ^+ has isospin 0, the flavor wave function is expressed by $[22]_f$ and so the only possible spin wave function is $[31]_s$. Therefore, in the Young tableaux, the spin-flavor wave function can be expressed as

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 \\ \hline \end{array}_{sf} = \frac{1}{\sqrt{2}} \left(\begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline 3 \\ \hline \end{array}_s \cdot \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}_f + \begin{array}{|c|c|c|} \hline 1 & 3 & 4 \\ \hline 2 \\ \hline \end{array}_s \cdot \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}_f \right). \quad (\text{A-3})$$

Finally the \bar{s} wave function is multiplied to the above q^4 wave function. The color, spin-flavor wave function of \bar{s} quark is expressed by

$$\bar{s} \sim \begin{array}{|c|} \hline \\ \hline \end{array}_c \cdot \begin{array}{|c|} \hline \\ \hline \end{array}_f \cdot \begin{array}{|c|} \hline \\ \hline \end{array}_s \quad (\text{A-4})$$

This is combined with the q^4 wave function to yield the Θ^+ wave function

$$\begin{aligned} |\Theta^+\rangle &= \frac{1}{\sqrt{3}} \left(\begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array}_c \cdot \frac{1}{\sqrt{2}} \left(\begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline 3 \\ \hline \end{array}_s \cdot \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}_f + \begin{array}{|c|c|c|} \hline 1 & 3 & 4 \\ \hline 2 \\ \hline \end{array}_s \cdot \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}_f \right) + \dots \right) \\ &\otimes \left(\begin{array}{|c|} \hline \\ \hline \end{array}_c \cdot \begin{array}{|c|} \hline \\ \hline \end{array}_f \cdot \begin{array}{|c|} \hline \\ \hline \end{array}_s \right) \\ &= \frac{1}{\sqrt{3}} \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array}_c \cdot \frac{1}{\sqrt{2}} \left(\begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline 3 \\ \hline \end{array}_s \cdot \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}_f + \begin{array}{|c|c|c|} \hline 1 & 3 & 4 \\ \hline 2 \\ \hline \end{array}_s \cdot \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}_f \right) \\ &+ \dots \quad (\text{A-5}) \end{aligned}$$

In the first term of this equation, the fourth quark and \bar{s} form the desired color (singlet) and flavor (isosinglet) quantum numbers. The spin part needs one more step. For instance, the spin wave function $\begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}_s$ has the coupling structure with $S_{1234} = 1$ as

$$[[S_{123}, S_4]^{S_{1234}}, S_5]^{S_{\text{tot}}} = [[1/2, 1/2]^1, 1/2]^{1/2}, \quad (\text{A6})$$

which may be recoupled for the kaon with spin $S_{45} = 0$:

$$\begin{aligned} [[1/2, 1/2]^1, 1/2]^{1/2} &= \sum_j c_j [1/2[1/2, 1/2]^j]^{1/2}, \\ c_0 &= \frac{\sqrt{3}}{2}, \quad c_1 = \frac{1}{2}. \end{aligned} \quad (\text{A7})$$

Here the coefficients c_0 and c_1 are the amplitude for the spin $S_{45} = 0$ and 1 components. $S_{45} = 1$ corresponds to the K^* vector meson of spin one. Therefore, the coupling strength of K^* to the Θ^+ is $1/\sqrt{3}$ of that of K for the negative parity Θ^+ .

Using the results of Eqs. (A5) and (A7), one finds the amplitude of finding the neutronlike udd and kaon(K^+)-like $u\bar{s}$ is

$$a = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{2} = \frac{1}{2\sqrt{2}}. \quad (\text{A8})$$

Note that the first factor $1/\sqrt{2}$ is needed when extracting the K^+n component from the isospin zero combination of K^+n and K^0p in the flavor wave function of (A5). In other words, we have

$$\begin{aligned} |\Theta_{csf}^+\rangle &= \frac{1}{2\sqrt{2}} |(udd)^n (u\bar{s})^{K^+}\rangle - \frac{1}{2\sqrt{2}} |(uud)^p (d\bar{s})^{K^0}\rangle \\ &+ \dots \end{aligned} \quad (\text{A9})$$

2. Positive parity Θ^+

The wave function for the positive parity Θ^+ contains excitation of one unit of orbital angular momentum and allows four independent states with $J^P = 1/2^+$ and flavor antidecuplet. Assuming that one of the $uudd$ quarks is excited into the $l = 1$ p-orbit, the orbital part of the q^4 wave function takes the symmetry structure $[31]_o$. The totally symmetric state $[4]_o$ represents a center-of-mass motion of the $uudd$ system. As in the negative parity case, the spin-flavor-orbital wave function has the symmetry $[31]_{sfo}$, and therefore, the spin-flavor part can take $[4]_{sf}$ or $[22]_{sf}$. The spin-flavor decomposition of these states with the flavor symmetry $[22]_f$ for antidecuplet is

$$\begin{aligned} [4]_{sf} &= [22]_s \cdot [22]_f, \\ [22]_{sf} &= ([22]_s \text{ or } [31]_s \text{ or } [4]_s) \cdot [22]_f. \end{aligned} \quad (\text{A10})$$

As discussed previously [12], we take the most likely configuration for the spin-flavor wave function with the symmetry $[4]_{sf}$ in which the attraction due to the meson-exchange interactions is maximized. Such a spin-flavor state may be expressed by the following Young tableaux

$$\begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 4 \\ \hline \end{array}_{sf} = \frac{1}{\sqrt{2}} \left(\begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}_s \cdot \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}_f + \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}_s \cdot \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}_f \right). \quad (\text{A-11})$$

Accordingly, the color-orbital wave function is totally

antisymmetric:

$$\begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline 3 \\ \hline 4 \\ \hline \end{array}_o = \frac{1}{\sqrt{3}} \left(\begin{array}{|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array}_c \cdot \begin{array}{|c|} \hline 1 & 2 & 3 \\ \hline 4 \\ \hline \end{array}_o - \begin{array}{|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}_c \cdot \begin{array}{|c|} \hline 1 & 2 & 4 \\ \hline 3 \\ \hline \end{array}_o + \begin{array}{|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}_c \cdot \begin{array}{|c|} \hline 1 & 3 & 4 \\ \hline 2 \\ \hline \end{array}_o \right). \quad (\text{A-12})$$

As in the case of negative parity, we need to pick up the term where the 123 quarks form neutron quantum numbers with color singlet and totally symmetric in spin-flavor wave function. We then need one more decomposition for the orbital part. Denoting s ($l = 0$) and p ($l = 1$) states simply by s and p , the orbital wave function in the relevant term is

$$\begin{array}{|c|} \hline 1 & 2 & 3 \\ \hline 4 \\ \hline \end{array}_o = \frac{\sqrt{3}}{2} sssp - \frac{1}{2} \frac{1}{\sqrt{3}} (pss + sps + ssp)s. \quad (\text{A-13})$$

The first term $sssp$ is combined with \bar{s} state, representing a state where the nucleonlike (123) is in the s -state and the kaon either moving in p -state or excited intrinsically:

$$ps = \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} (ps + sp) + \frac{1}{\sqrt{2}} (ps - sp) \right). \quad (\text{A14})$$

For the decay $\Theta^+ \rightarrow K^+ n$, we pick up the first term of (A14). The combination of the first term of (A13) and the first term of (A14) is referred to as the case (1). The second term of (A13) represents that the nucleonlike udd is moving in the p -state while the kaonlike $u\bar{s}$ is in the s -state. We will refer to this term as the case (2). The first and second cases both contribute to Θ^+ decay and are added coherently.

Finally, we need to evaluate the spin rearrangement for $qqq\bar{s}$,

$$\begin{array}{|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \otimes \square. \quad (\text{A-15})$$

Using the notation of (A6), it can be done as

$$[[1/2, 1/2]^0, 1/2]^{1/2} = \sum_J c'_J [1/2, [1/2, 1/2]^J]^{1/2}, \quad (\text{A16})$$

$$c'_0 = \frac{1}{2}, \quad c'_1 = \frac{\sqrt{3}}{2}.$$

Here $c'_1/c'_0 = \sqrt{3}$ represents the ratio of the K^* coupling to K coupling to the Θ^+ , which is the result presented by Close and Dudek [13]. The probabilities of finding the KN state for the cases (1) and (2) in the Θ^+ wave function are

$$P(1) = \left| \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{2} \frac{1}{\sqrt{2}} \frac{1}{2} \right|^2 = \frac{1}{32},$$

$$P(2) = \left| \frac{1}{\sqrt{3}} \frac{1}{2} \frac{1}{2} \right|^2 = \frac{1}{48}, \quad P(1) + P(2) = \frac{5}{96}. \quad (\text{A17})$$

These results agree with that derived in Ref. [19]. Note that these are probabilities of finding $KN \sim (K^+ n + K^0 p)/\sqrt{2}$. Therefore, the probabilities of finding $K^+ n$ are half of them. To complete the decomposition of the wave function, we write

$$|\Theta^+\rangle = \frac{a_1}{\sqrt{2}} |[[udd]_{i=0, S=1/2}^n]_{j=1/2}, (\bar{s}u)_{i=1, S=0}^{K^+}]_{J=1/2}\rangle$$

$$+ \frac{a_2}{\sqrt{2}} |[[udd]_{i=1, S=1/2}^n]_{j=1/2}, (\bar{s}u)_{i=0, S=0}^{K^+}]_{J=1/2}\rangle$$

$$+ ((K^+ n) \rightarrow (K^0 p)) + \dots$$

$$= \sqrt{\frac{1}{32}} |1\rangle - \sqrt{\frac{1}{48}} |2\rangle + \dots$$

$$= \sqrt{\frac{5}{96}} \left(\sqrt{\frac{3}{5}} |1\rangle - \sqrt{\frac{2}{5}} |2\rangle \right) + \dots \quad (\text{A18})$$

Here the states $|1\rangle$ and $|2\rangle$ are for the states of a_1 and a_2 terms, respectively. In these equations, we have shown that the Θ^+ contains a component of relative p -wave motion between the two-quark cluster (mesonlike) and three-quark cluster (nucleonlike) states. This reduces to (22).

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