$b\rightarrow s$ penguin amplitude in charmless $B\rightarrow PP$ decays

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The $b \rightarrow s$ penguin amplitude affects a number of B meson decays to two pseudoscalar (P) mesons in which potential anomalies are being watched carefully, though none has yet reached a statistically compelling level. These include (a) a sum of rates for $B^0 \rightarrow K^0\pi^0$ and $B^+ \rightarrow K^+\pi^0$ enhanced relative to half the sum for $B^0 \rightarrow K^+\pi^-$ and $B^+ \rightarrow K^0\pi^+$, (b) a time-dependent CP asymmetry parameter S for $B^0 \rightarrow K^0\pi^0$ which is low in comparison with the expected value of $\sin2\beta \simeq 0.73$, and (c) a similar deviation in the parameter S for $B^0 \rightarrow \eta'K_S$. These and related phenomena involving vector mesons in the final state are discussed in a unified way in and beyond the standard model. Future experiments which would conclusively indicate the presence of new physics are identified. Several of these involve decays of the strange B meson B_s. In the standard model we prove an approximate sum rule for CP rate differences in $B^0 \rightarrow K^+\pi^-$, $B^+ \rightarrow K^+\pi^0$ and $B^0 \rightarrow K^0\pi^0$, predicting a negative sign for the latter asymmetry.

I. INTRODUCTION

The decays of B mesons to final states consisting of mesons with u, d, and s quarks are rich sources of information on the phases and magnitudes of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and on possible physics beyond the standard model. Many of these decays are dominated by an amplitude in which a b quark undergoes a virtual transition through an intermediate state of a W^- and a quark with charge $2/3$ (u, c, or t) to a strange quark s. This transition does not occur in vacuum, being eliminated by renormalization, but can occur in the presence of the chromoelectric field. Such a transition is known as a $b \rightarrow s$ *penguin amplitude* [1].

The $b \rightarrow s$ penguin amplitude has turned out to be significant in many B meson decays. It is responsible for branching ratios for $B \rightarrow K\pi$ decays of order 1 to 2×10^{-5}, depending on whether the pion is neutral or charged. It also leads to large branching ratios for $B \rightarrow K\eta' \sim 7 \times 10^{-5}$. Several potential anomalies in these modes have attracted attention, including a sum of rates for $B^0 \rightarrow K^0\pi^0$ and $B^+ \rightarrow K^+\pi^0$ slightly enhanced with respect to that expected from the sum for $B^0 \rightarrow K^+\pi^-$ and $B^+ \rightarrow K^0\pi^+$, a time-dependent asymmetry parameter S for $B^0 \rightarrow K^0\pi^0$ decay which is low with respect to the expected value of $\sin2\beta \simeq 0.73$, and similar discrepancies in S for the decays $B^0 \rightarrow \eta'K^0$ and $B^0 \rightarrow \phi K_S$.

In the present paper we discuss a framework for describing new physics in the $b \rightarrow s$ penguin amplitude. We review the evidence for possible discrepancies from the standard picture and indicate ways in which these discrepancies, if they exist, can be sharpened and correlated with other observations. We also attempt to estimate the experimental accuracies which would permit conclusive identification of new physics. We concentrate on $B \rightarrow PP$ decays, where P is a light pseudoscalar meson.

Section II gives conventions for meson states, decay amplitudes, CKM matrix elements, and time-dependent CP asymmetries. Section III is devoted to a parametrization of $B \rightarrow K\pi$ amplitudes, allowing for new physics contributions of the most general form. Section IV reviews the pattern of rates for $B \rightarrow K\pi$ and possible effects of the new parametrization, while Sec. V is devoted to rate asymmetries in these processes. Section VI treats the possible deviation of $S_{K_S\pi^0}$ from its expected value of $\sin2\beta \simeq 0.73$. A similar discussion for $S_{\eta'K_S}$ occupies Sec. VII. The role of B_s decays in sorting out some of these questions is mentioned in Sec. VIII. Section IX lists some related puzzles in $B \rightarrow VP$ and $B \rightarrow VV$ decays, where V is a vector meson. Section X notes some experimental tests for the pattern of deviations from the standard picture of $b \rightarrow s$ penguin-dominated decays. Section XI concludes.

II. CONVENTIONS: STATES, AMPLITUDES, ASYMMETRIES

We use conventions for states defined in Refs. [2,3]. Quark model assignments are as usual (e.g., $B^+ = u\bar{b}$, $B^0 = d\bar{b}$), with the proviso that states with a $\bar{u}$ quark are defined with a minus sign (e.g., $B^- = -b\bar{u}$) for convenience in isospin calculations. For a similar reason, a neutral pion is $\pi^0 = (d\bar{d} - u\bar{u})\sqrt{2}$.

The CKM matrix V is unitary, implying (e.g.) $V_{ub}^*V_{ud} + V_{cb}^*V_{cd} + V_{tb}^*V_{td} = 0$ and a similar relation with $d \rightarrow s$. We shall make use of these relations in defining all amplitudes in terms of two combinations of CKM elements. The unitarity of V can be depicted in terms of a triangle with angles $\alpha = \mathrm{Arg}(-V_{tb}^*V_{td}/V_{ub}^*V_{ud})$, $\beta = \mathrm{Arg}(-V_{cb}^*V_{cd}/V_{tb}^*V_{td})$, and $\gamma = \mathrm{Arg}(-V_{ub}^*V_{ud}/V_{cb}^*V_{cd})$.

We shall define a set of reduced matrix elements known as tree, color-suppressed, and penguin amplitudes, restricting our attention to strangeness-changing ($|\Delta S| = 1$) pro-

cesses. These processes, which were described by primed amplitudes in Ref. [2], will be presented here as unprimed. A tree amplitude, T, and a color-suppressed amplitude, C, involve a CKM factor $V_{ub}^* V_{us}$, while a penguin amplitude, P, contains a factor $V_{tb}^* V_{ts} = -V_{cb}^* V_{cs} - V_{ub}^* V_{us} = -V_{cb}^* V_{cs}(1 + \mathcal{O}(2\%))$. Color-allowed and color-suppressed electroweak penguin amplitudes, P_{EW} and P_{EW}^c, including a CKM factor $V_{tb}^* V_{ts}$, appear with the tree, color-suppressed, and penguin amplitudes in the independent combinations [3],

$$t \equiv T + P_{EW}^c, \qquad c \equiv C + P_{EW}, \qquad p \equiv P - \frac{1}{3}P_{EW}^c. \tag{1}$$

We will neglect exchange and annihilation amplitudes, E and A, which are suppressed relative to the dominant P amplitude by $|V_{ub}^* V_{us}/V_{cb}^* V_{cs}|(\Lambda_{QCD}/m_b) \sim \mathcal{O}(10^{-3})$ [2,4]. A small isosinglet penguin-annihilation amplitude, PA, will be absorbed in the definition of P. Of these three amplitudes only A contributes in $B \to K\pi$, while E and PA occur in $B_s \to K\bar{K}$ and $B_s \to \pi\pi$.

CP-violating decay asymmetries are defined as

$$A_{CP}(B \to f) \equiv \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)}, \tag{2}$$

while *CP*-averaged decay rates are defined by

$$\bar{\Gamma}(f) \equiv \frac{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})}{2}. \tag{3}$$

For decay to a *CP* eigenstate f, one can measure time-dependent asymmetry parameters A_f and S_f which occur in the expression

$$\frac{\Gamma(\bar{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\bar{B}^0(t) \to f) + \Gamma(B^0(t) \to f)}$$
$$= A_f \cos\Delta mt + S_f \sin\Delta mt. \tag{4}$$

Here $\Delta m \simeq 0.5$ ps^{-1} is the mass difference between neutral B mass eigenstates, while $B^0(t)$ or $\bar{B}^0(t)$ denotes a time-evolved state which has been identified as a B^0 or $\bar{B}^0$ at proper time $t = 0$. One sometimes sees also the notation $C_f = -A_f$. The time-integrated rate asymmetry $A_{CP}(f)$ is equal to A_f.

III. PARAMETRIZATION OF $B \to K\pi$ DECAY AMPLITUDES

The four $B \to K\pi$ decay amplitudes may be written in a standard flavor-SU(3) decomposition [2,3] as

$$A(B^+ \to K^0\pi^+) = p, \tag{5}$$

$$A(B^+ \to K^+\pi^0) = -(p + t + c)/\sqrt{2}, \tag{6}$$

$$A(B^0 \to K^+\pi^-) = -(p + t), \tag{7}$$

$$A(B^0 \to K^0\pi^0) = (p - c)/\sqrt{2}. \tag{8}$$

They satisfy an isospin sum rule [5]

$$A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0)$$
$$= A(B^0 \to K^+\pi^-) + \sqrt{2}A(B^0 \to K^0\pi^0) \tag{9}$$

which is a consequence of there being only three independent amplitudes (two with $I(K\pi) = 1/2$ and one with $I(K\pi) = 3/2$) to describe the four processes. The linear combinations shown are those with $I(K\pi) = 3/2$.

Motivated by early suggestions that the $b \to s$ penguin amplitude was a promising source of effects due to new physics [6–9], many modifications of it have been proposed [10]. We shall consider the case of separate new physics operators for $b \to su\bar{u}$, $b \to sd\bar{d}$, and $b \to ss\bar{s}$ transitions, denoted by ΔP_u, ΔP_d, and ΔP_s, with a superscript $^{(c)}$ to denote those transitions in which members of the light $q\bar{q} = u\bar{u}, d\bar{d}, s\bar{s}$ pair end up in different mesons. These will be seen to resemble electroweak penguin terms, though they could arise from a variety of new-physics sources.

The $B \to K\pi$ decay amplitudes then may be written in the form of Eqs. (5)–(8) with the identifications

$$p = P - \frac{1}{3}P_{EW}^c + \Delta P_d^c. \tag{10}$$

$$t = T + P_{EW}^c + \Delta P_u^c - \Delta P_d^c, \tag{11}$$

$$c = C + P_{EW} + \Delta P_u - \Delta P_d. \tag{12}$$

The nonelectroweak penguin part of p is identified with ΔP_d^c, while the amplitudes t and c acquire new pieces $\Delta P_u^c - \Delta P_d^c$ and $\Delta P_u - \Delta P_d$, respectively. Whereas the standard model amplitude P behaves like an isosinglet and is therefore common (up to a factor $\sqrt{2}$) to all four $B \to K\pi$ decays, this is not necessarily the case for the new physics amplitudes which generally obey $\Delta P_u^{(c)} \neq \Delta P_d^{(c)}$. It is convenient to classify potential anomalies in $B \to K\pi$ in terms of the new physics amplitudes $\Delta P_{u,d}^{(c)}$. For instance, the term ΔP_d^c would show up as a *CP* asymmetry in $B^\pm \to K\pi^\pm$, assuming in general that this term involves strong and weak phases which differ from those of $P - P_{EW}^c/3$. In Secs. IV and VI we will give examples for signatures characterizing the other three terms. Note that the isospin quadrangle relation (9) of course continues to hold as long as one assumes that new physics is given by four-quark $b \to sq\bar{q}$ operators implying the absence of $\Delta I > 1$ transitions.

IV. PATTERN OF $B \to K\pi$ RATES

Current averages for branching ratios for $B \to K\pi$ decays [11] are quoted in Table I. To compare decay rates one also needs the lifetime ratio $\tau_+/\tau_0 \equiv \tau(B^+)/\tau(B^0)$, for which the latest average [11] is 1.081 ± 0.015.

TABLE I. *CP*-averaged branching ratios (in units of 10^{-6}) and *CP* asymmetries A_{CP} (see Sec. V) for $B \to K\pi$ decays.

Decay mode	Branching ratio	A_{CP}
$B^+ \to K^0\pi^+$	24.1 ± 1.3	-0.020 ± 0.034
$B^+ \to K^+\pi^0$	12.1 ± 0.8	0.04 ± 0.04
$B^0 \to K^+\pi^-$	18.2 ± 0.8	-0.109 ± 0.019
$B^0 \to K^0\pi^0$	11.5 ± 1.0	-0.08 ± 0.14

CP-violating asymmetries are also quoted for use in Sec. V.

In the standard model the four $B \to K\pi$ amplitudes are dominated by the amplitude p. Expanding decay rates in $|t/p|$ and $|c/p|$, one observes a simple sum rule for B decay rates, which holds to first order in these ratios [12,13],

$$2\Gamma(B^+ \to K^+\pi^0) + 2\Gamma(B^0 \to K^0\pi^0)$$
$$= \Gamma(B^+ \to K^0\pi^+) + \Gamma(B^0 \to K^+\pi^-). \quad (13)$$

In terms of specific contributions, this reads

$$2|p|^2 + 2\text{Re}(p^*t) + |t|^2 + 2|c|^2 + 2\text{Re}(c^*t)$$
$$= 2|p|^2 + 2\text{Re}(p^*t) + |t|^2. \quad (14)$$

A similar sum rule holds for $\bar{B}$ decay rates and for *CP*-averaged rates. Thus,

$$2\bar{\mathcal{B}}(B^+ \to K^+\pi^0) + 2\frac{\tau_+}{\tau_0}\bar{\mathcal{B}}(B^0 \to K^0\pi^0)$$
$$= \bar{\mathcal{B}}(B^+ \to K^0\pi^+) + \frac{\tau_+}{\tau_0}\bar{\mathcal{B}}(B^0 \to K^+\pi^-), \quad (15)$$

where $\bar{\mathcal{B}}$ denotes a *CP*-averaged branching ratio. Using experimental values for branching ratios and for the lifetime ratio, this sum rule reads in units of 10^{-6}

$$(24.2 \pm 1.6) + (24.9 \pm 2.2) = (24.1 \pm 1.3)$$
$$+ (19.7 \pm 0.9), \quad (16)$$

or

$$49.1 \pm 2.7 = 43.8 \pm 1.6. \quad (17)$$

The two sides differ by 5.3 ± 3.1, or $(12 \pm 7)\%$ of the better known right-hand side. This fraction is given to leading order by second order terms, $\text{Re}\langle c^*(c + t)\rangle/|p|^2$, where an average is taken over B and $\bar{B}$ contributions. Typical estimates of these terms (see, e.g., [14]) in the standard model limit them to no more than a few percent. Fits based on flavor SU(3) [15,16] predict branching ratios satisfying Eq. (15) more accurately, obtaining a slightly smaller value of $\bar{\mathcal{B}}(B^0 \to K^0\pi^0)$ than observed due to destructive interference between the two dominant terms contributing to this process, $(P - \frac{1}{3}P_{EW}^c)/\sqrt{2}$ and $-P_{EW}/\sqrt{2}$.

An equivalent approach to the sum rule can be presented in terms of an equality between two ratios of *CP*-averaged branching ratios (equivalently, of decay rates) defined as [17]

$$R_c \equiv \frac{2\bar{\mathcal{B}}(B^+ \to K^+\pi^0)}{\bar{\mathcal{B}}(B^+ \to K^0\pi^+)}, \qquad R_n \equiv \frac{\bar{\mathcal{B}}(B^0 \to K^+\pi^-)}{2\bar{\mathcal{B}}(B^0 \to K^0\pi^0)}. \quad (18)$$

The experimental values are

$$R_c = 1.00 \pm 0.09, \qquad R_n = 0.79 \pm 0.08,$$
$$R_c - R_n = 0.21 \pm 0.12. \quad (19)$$

Expanding R_c and R_n in ratios t/p, c/p and their charge conjugates, one can show that the difference $R_c - R_n$ is quadratic in these ratios. Attention has been called [18–21] to the fact that if the difference $R_c - R_n$ is maintained with improved statistics this could signal new physics.

In the absence of differences between penguin terms ΔP_u and ΔP_d or ΔP_u^c and ΔP_d^c, one would most naturally ascribe a large term of the form $\text{Re}\langle c^*(c + t)\rangle/|p|^2$ to color-favored electroweak penguin terms [22] of magnitude larger than expected. The point we wish to stress here is that any four-quark operator which contributes to $\Delta P_u - \Delta P_d$ will emulate the color-allowed electroweak penguin P_{EW} in Eq. (12), while any four-quark operator which contributes to $\Delta P_u^c - \Delta P_d^c$ will emulate P_{EW}^c in Eq. (11). Thus, both c and t can receive contributions from new physics aside from enhanced electroweak penguins as long as $b \to sq\bar{q}$ operators produce $u\bar{u}$ pairs differently from $d\bar{d}$ pairs.

It is interesting to note, as has been pointed out [23], that the Fleischer-Mannel ratio [24],

$$R \equiv \frac{\bar{\Gamma}(B^0 \to K^+\pi^-)}{\bar{\Gamma}(B^+ \to K^0\pi^+)}, \quad (20)$$

is currently

$$R = 0.816 \pm 0.058, \quad (21)$$

differing from 1 by 3.2σ. At 95% confidence level $R < 0.911$. Neglecting P_{EW}^c terms, this would lead through the Fleischer-Mannel bound $\sin^2\gamma \leq R$ to an upper limit $\gamma \leq 73°$.

However, as we mention in the next section, P_{EW}^c and C are not much suppressed relative to P_{EW} and T, respectively, as has been customarily assumed. Including the P_{EW}^c term, the Fleischer-Mannel bound becomes [25]

$$\sin^2\gamma \leq \frac{R}{|1 + P_{EW}^c/(P - \frac{1}{3}P_{EW}^c)|^2} \approx \left|1 - \frac{P_{EW}^c}{P}\right|^2 R. \quad (22)$$

The effect on the bound depends on the magnitude of P_{EW}^c/P, which is typically a few percent, and on the phase of this ratio. Using, for instance, Fit III in [15] based on

SU(3), one finds central values $|P^c_{EW}/P| = 0.044$, $\mathrm{Arg}(P^c_{EW}/P) = -69°$, implying $\sin^2\gamma \lesssim 0.97R \leq 0.884$, or $\gamma \lesssim 70°$. This upper bound is consistent with other standard model constraints on γ [26,27]. A potential inconsistency would have been ascribed to ΔP^c_u or ΔP^c_d, which occur in $p + t$ and p, respectively.

V. RATE ASYMMETRIES IN $B \to K\pi$

The penguin dominance of the $B \to K\pi$ decay amplitudes was used in Ref. [12] to derive in the standard model a relation between direct CP-violating rate differences in various $B \to K\pi$ processes. The simplest of these was based on assuming that the only important amplitude interfering with p was t, in which case the relation

$$\Delta(K^+\pi^-) \sim 2\Delta(K^+\pi^0) \qquad (23)$$

was obtained. Here

$$\Delta(K^+\pi^-) \equiv \Gamma(B^0 \to K^+\pi^-) - \Gamma(\bar{B}^0 \to K^-\pi^+), \quad (24)$$

$$\Delta(K^+\pi^0) \equiv \Gamma(B^+ \to K^+\pi^0) - \Gamma(B^- \to K^-\pi^0), \quad (25)$$

with similar definitions for $\Delta(K^0\pi^+)$ and $\Delta(K^0\pi^0)$. These rate asymmetries are related to the CP asymmetries as defined in Sec. II by $\Delta(f) = -2A_{CP}(f)\bar{\Gamma}(f)$, where the CP-averaged rate $\bar{\Gamma}(f)$ was defined in Sec. II.

Since $\bar{\Gamma}(K^+\pi^-) \approx 2\bar{\Gamma}(K^+\pi^0)$, the relation (23) reduces to the prediction

$$A_{CP}(B^0 \to K^+\pi^-) \sim A_{CP}(B^+ \to K^+\pi^0), \qquad (26)$$

which is rather far from what is observed. According to the averages in Table I, the left-hand side of Eq. (26) is -0.11 ± 0.02, while the right-hand side is 0.04 ± 0.04. The two sides thus differ by more than 3σ. Is this a problem? Does this indicate isospin-violating new physics [28]?

As in Ref. [12], we define $2\vec{P}\,\vec{T}$ to be the interference between P and T contributing to the rate difference $\Delta(K^+\pi^-)$, with similar notations for other interference terms and rate differences. One then finds [12] that

$$\Delta(K^0\pi^+) \simeq 0, \qquad (27)$$

$$\Delta(K^+\pi^0) \simeq \vec{P}\,\vec{T} + \vec{P}\,\vec{C} + \left(\vec{P}_{EW} + \frac{2}{3}\vec{P}^c_{EW}\right)(\vec{T} + \vec{C}), \quad (28)$$

$$\Delta(K^+\pi^-) = 2\vec{P}\,\vec{T} + \frac{4}{3}\vec{P}^c_{EW}\vec{T}, \qquad (29)$$

$$\Delta(K^0\pi^0) = -\vec{P}\,\vec{C} + \vec{P}_{EW}\vec{C} + \frac{1}{3}\vec{P}^c_{EW}\vec{C}. \qquad (30)$$

The only interference terms which contribute to direct CP-violating rate differences are those which have differing weak and strong phases. Thus one sees no interference

between C and T or between electroweak penguin terms and P.

The relation (23) was derived by neglecting all terms in the rate differences except $\vec{P}\,\vec{T}$. An argument was given for the relative smallness of the term $\vec{P}\,\vec{C}$ under the assumption that $|C/T| = \mathcal{O}(1/5)$. Recent fits based on flavor SU(3) [15,29,30] indicate that $|C/T|$ is more like 0.7 to 0.9 (quoting the results of fits in [15] which include processes involving η and η' as well as kaons and pions; $|C/T|$ is even larger in a fit studying only $B \to K\pi$ [30]). Also, arguments based on a soft collinear effective theory [4,31] show that C and T are comparable. In this case an improved relation based on similar reasoning retains the $\vec{P}\,\vec{C}$ term and is

$$\Delta(K^+\pi^-) \approx 2\Delta(K^+\pi^0) + 2\Delta(K^0\pi^0). \qquad (31)$$

This relation ignores terms on the right-hand side which can be written as

$$(\vec{P}_{EW} + \vec{P}^c_{EW})(\vec{T} + \vec{C}) + (\vec{P}_{EW}\vec{C} - \vec{P}^c_{EW}\vec{T}) \approx 0. \quad (32)$$

An argument for the smallness of the first term was given in [12] using a property of the $I(K\pi) = 3/2$ amplitude, $(T + C) + (P_{EW} + P^c_{EW})$, in which the two terms involve approximately a common strong phase [32]. The second term in (32) vanishes approximately due to a relation $P^c_{EW}/P_{EW} \approx C/T$ [33]. Both approximations follow from flavor SU(3) when neglecting electroweak penguin operators with small Wilson coefficients (c_7 and c_8).

Using the approximate relations $\bar{\Gamma}(K^+\pi^-) \simeq 2\bar{\Gamma}(K^+\pi^0) \simeq 2\bar{\Gamma}(K^0\pi^0)$, Eq. (31) may be transcribed as

$$A_{CP}(K^+\pi^-) \approx A_{CP}(K^+\pi^0) + A_{CP}(K^0\pi^0), \qquad (33)$$

which reads, according to Table I, as

$$-0.109 \pm 0.019 \approx (0.04 \pm 0.04) + (-0.08 \pm 0.14). \tag{34}$$

Another way to put this relation is that $A_{CP}(K^0\pi^0)$ is predicted to be -0.15 ± 0.04, i.e., nonzero at a level greater than 3σ. A more precise prediction, using the measured rates of the above three processes, is $A_{CP}(K^0\pi^0) = -0.13 \pm 0.04$. CPT requires that the overall direct CP asymmetry vanishes in eigenstates of the strong S matrix. Our prediction excludes the possibility that the asymmetry in $B^0 \to K^0\pi^0$ alone compensates for the observed asymmetry in $B^0 \to K^+\pi^-$ [34]. The two asymmetries are predicted to have equal signs.

In Ref. [35] we noted a relation between CP rate differences, which holds in the limit of SU(3) when neglecting annihilation-like amplitudes $(E + PA)$ in $B^0 \to \pi^0\pi^0$,

$$\Delta(\pi^0\pi^0) = -\Delta(K^0\pi^0), \qquad (35)$$

or

TABLE II.   Time-dependent CP asymmetry parameters for $B^0 \to K_S \pi^0$.

Parameter	BABAR [37]	Belle [38]	Average
$S_{K\pi}$	$0.35^{+0.30}_{-0.33} \pm 0.04$	$0.32 \pm 0.61 \pm 0.13$	$0.34^{+0.27}_{-0.29}$
$A_{K\pi}$	$-0.06 \pm 0.18 \pm 0.03$	$-0.11 \pm 0.20 \pm 0.09$	-0.08 ± 0.14

$$A_{CP}(\pi^0 \pi^0) = -\frac{\bar{\mathcal{B}}(B^0 \to K^0 \pi^0)}{\bar{\mathcal{B}}(B^0 \to \pi^0 \pi^0)} A_{CP}(K^0 \pi^0). \quad (36)$$

Using our prediction, $A_{CP}(K^0 \pi^0) = -0.13 \pm 0.04$, and the two branching ratios [11], $\bar{\mathcal{B}}(B^0 \to K^0 \pi^0) = (11.5 \pm 1.0) \times 10^{-6}$, $\bar{\mathcal{B}}(B^0 \to \pi^0 \pi^0) = (1.45 \pm 0.29) \times 10^{-6}$ we find

$$A_{CP}(\pi^0 \pi^0) = +1.0 \pm 0.4. \quad (37)$$

This *large and positive* value should be compared with the current world-averaged value [11], $A_{CP}(\pi^0 \pi^0) = +0.28 \pm 0.39$. It would be interesting to watch the decrease of experimental errors in order to learn the effects of SU(3) breaking corrections and annihilation-like amplitudes.

VI. DEVIATIONS OF $S_{K\pi}$ FROM ITS NOMINAL VALUE

The dominance of the $b \to s$ penguin amplitude in $B^0 \to K^0 \pi^0$ implies that the parameter $S_{K\pi} \equiv S_{K_S \pi^0}$ should be very close to the value $\sin(2\beta)$ expected from interference between B^0–$\bar{B}^0$ mixing and B^0 decay alone. One has

$$S_{K\pi} = \frac{2\mathrm{Im}\lambda_{K\pi}}{|\lambda_{K\pi}|^2 + 1}, \qquad A_{K\pi} = \frac{|\lambda_{K\pi}|^2 - 1}{|\lambda_{K\pi}|^2 + 1}, \quad (38)$$

where

$$\lambda_{K\pi} \equiv -e^{-2i\beta} \frac{A(\bar{B}^0 \to \bar{K}^0 \pi^0)}{A(B^0 \to K^0 \pi^0)}. \quad (39)$$

Rewriting Eq. (8) for $A(B^0 \to K^0 \pi^0)$ in terms of two contributions A_P and A_C involving CKM factors $V_{cb}^* V_{cs}$ and $V_{ub}^* V_{us}$, respectively, and a relative strong phase δ,

$$A(B^0 \to K^0 \pi^0) = A_P + A_C = |A_P|e^{i\delta} + |A_C|e^{i\gamma}, \quad (40)$$

where

$$A_P \equiv \left(P - P_{EW} - \frac{1}{3} P_{EW}^c\right) \Big/ \sqrt{2}, \qquad A_C \equiv -C/\sqrt{2}, \quad (41)$$

one obtains to first order in $|A_C/A_P|$ [36]

$$\Delta S_{K\pi} \equiv S_{K\pi} - \sin 2\beta \approx 2|A_C/A_P| \cos 2\beta \cos \delta \sin \gamma,$$

$$A_{K\pi} \simeq -2|A_C/A_P| \sin \delta \sin \gamma. \quad (42)$$

With the help of information on the $B^0 \to \pi^0 \pi^0$ decay rate and an upper limit on $\bar{\mathcal{B}}(B^0 \to K^+ K^-)$, it was found [using flavor SU(3)] that [35]

$$-0.11 \le \Delta S_{K\pi} \le 0.12, \qquad |A_{K\pi}| \le 0.17, \quad (43)$$

under the assumption that $A(B^0 \to K^+ K^-)$ could be neglected, or

$$-0.18 \le \Delta S_{K\pi} \le 0.16, \qquad |A_{K\pi}| \le 0.26, \quad (44)$$

when taking into account a possible nonzero amplitude for $B^0 \to K^+ K^-$. [These constraints are modified slightly by recent updates of $\bar{\mathcal{B}}(B^0 \to K^0 \pi^0)$ and $\bar{\mathcal{B}}(B^0 \to \pi^0 \pi^0)$ [11].] Under the first, more restrictive, assumption one could actually *exclude* a small elliptical region in the $S_{K\pi}, A_{K\pi}$ plane with center at $(0.76, 0)$ and semiaxes $(0.06, 0.08)$. Our prediction (33) of a negative direct asymmetry is consistent with these bounds and implies $\sin \delta > 0$.

The current experimental situation for measurements of $S_{K\pi}$ and $A_{K\pi}$ is summarized in Table II. The observed $\Delta S_{K\pi} = -0.39^{+0.27}_{-0.29}$ differs from zero by 1.4σ. If one were to ascribe this difference to nonstandard behavior of the $b \to s q \bar{q}$ penguin amplitude, one would have to blame the amplitude ΔP_d^c or $\Delta P_u - \Delta P_d$, modifying, respectively, the p or c amplitude. At this point, however, it is clearly premature to speculate on such modifications.

A flavor SU(3) fit to $B \to PP$ amplitudes [15] predicts a *positive* $\Delta S_{K\pi} \simeq 0.1 \pm 0.01$ as well as a negative $A_{K\pi} \simeq -0.12 \pm 0.03$. The latter prediction is in accord with the discussion of the previous section. The sign of the former may be understood from the following qualitative argument. In SU(3) fits the two terms, $p = P - \frac{1}{3}P_{EW}^c$ and P_{EW}, are found to involve a relative strong phase smaller than π and thus interfere destructively in A_P. To account for the somewhat large measured CP-averaged rate of $B^0 \to K^0 \pi^0$, which is equal to half the rate of $B^+ \to K^0 \pi^+$ given by p alone, this requires constructive interference between A_P and A_C in the CP-averaged rate for $B^0 \to K^0 \pi^0$. This implies $\cos \delta > 0$ and consequently $\Delta S_{K\pi} > 0$.

VII. DEVIATIONS OF $S_{\eta' K_S}$ FROM ITS NOMINAL VALUE

The experimental situation with regard to the time-dependent parameter $S_{\eta' K}$ for $B^0 \to \eta' K^0$ is not clear. The BABAR Collaboration sees a significant deviation from the standard picture prediction of $\sin 2\beta \simeq 0.73$, while Belle's value is consistent with the standard picture. The values of $S_{\eta' K}$ and $A_{\eta' K}$ and their averages are summarized in Table III. Here, in view of the discrepancy between Belle and BABAR values, we have multiplied the error (as quoted

TABLE III. Time-dependent CP asymmetry parameters for $B^0 \to \eta' K_s$. Errors on averages include scale factor $S = \sqrt{\chi^2}$.

Parameter	BABAR [39]	Belle [38]	S	Average
$S_{\eta' K}$	$0.30 \pm 0.14 \pm 0.02$	$0.65 \pm 0.18 \pm 0.04$	1.51	0.43 ± 0.17
$A_{\eta' K}$	$0.21 \pm 0.10 \pm 0.02$	$-0.19 \pm 0.11 \pm 0.05$	2.53	0.04 ± 0.20

in Ref. [11]) by a scale factor $S = \sqrt{\chi^2}$, where χ^2 is the value for the best fit to the BABAR and Belle values.

The average value of $A_{\eta' K}$ is consistent with zero, while $S_{\eta' K}$ differs from $\sin 2\beta = 0.726 \pm 0.037$ by $\Delta S_{\eta' K} = -0.30 \pm 0.17$, or 1.76σ. In contrast to the case of $B^0 \to K^0 \pi^0$, there are a wide range of possible contributors to new physics in $b \to s q \bar{q}$ amplitudes. In the flavor-SU(3) decomposition of Ref. [15] the amplitude for $B^0 \to \eta' K^0$ is

$$A(B^0 \to \eta' K^0) = (3p + 4s + c)/\sqrt{6}, \quad (45)$$

where s denotes a singlet-penguin amplitude contributing mainly to η' production. It is expressed in terms of a genuine singlet-penguin term S and an electroweak penguin correction P_{EW} as $s = S - (1/3)P_{EW}$. New-physics contributions for $b \to s u \bar{u}$ or $b \to s s \bar{s}$ can enter into the s and c amplitudes, while those for $b \to s d \bar{d}$ can enter into all three amplitudes. Thus, it becomes particularly hard to identify the source of new physics if the only deviation from the standard prediction for S is that seen in $B^0 \to \eta' K^0$.

A question arises as to the accuracy with which the standard picture can predict $\Delta S_{\eta' K}$ and $A_{\eta' K}$. We have addressed this in two ways in previous work. (1) In Ref. [40] we used flavor SU(3) (or only its U-spin subgroup [41,42]) to bound the effects of nonpenguin amplitudes which could give rise to nonzero $\Delta S_{\eta' K}$ and $A_{\eta' K}$. (2) In Ref. [15] we performed a fit to a wide variety of $B \to PP$ processes based on flavor SU(3), obtaining predictions for these quantities

$$\Delta S_{\eta' K} \approx 0.02 \pm 0.01, \qquad A_{\eta' K} \approx 0.06 \pm 0.02. \quad (46)$$

Other explicit calculations (see, e.g., [43,44]) also obtain such very small values in the standard picture.

While it is difficult to estimate the deviations from Eq. (46) that might cause us to question the standard picture, one can at least give a range of such deviations that would *not* be a cause for immediate concern. Proceeding in the same manner as for $B^0 \to K^0 \pi^0$ in the previous section, we decompose the amplitude for $B^0 \to \eta' K^0$ into two terms A_P and A_C (dropping primes in comparison with Ref. [40]) involving intrinsic CKM factors $V_{cb}^* V_{cs}$ and $V_{ub}^* V_{us}$, and strong and weak phases δ and γ, respectively:

$$A(B^0 \to \eta' K^0) = A_P + A_C = |A_P| e^{i\delta} + |A_C| e^{i\gamma}. \quad (47)$$

First order expressions for $\Delta S_{\eta' K}$ and $A_{\eta' K}$ are the same as

in $B^0 \to K^0 \pi^0$:

$$\Delta S_{\eta' K} \equiv S_{\eta' K} - \sin 2\beta \approx 2|A_C/A_P| \cos 2\beta \cos \delta \sin \gamma,$$
$$A_{\eta' K} \approx -2|A_C/A_P| \sin \delta \sin \gamma. \quad (48)$$

The predictions of Ref. [15] that $\Delta S_{\eta' K} \gtrsim 0$, $A_{\eta' K} \gtrsim 0$ do not seem to have a simple interpretation, in contrast to that for the sign of $\Delta S_{K\pi}$ in the previous section, since strong phases of several small amplitudes are involved. Nonetheless, all the terms in Eq. (48) with the exception of δ may be considered to be fairly stable in the SU(3) fit, so that a crude estimate of possible deviations would be to let δ range through all possible values, thereby tracing an ellipse (shown as the dotted curve in Fig. 1) passing through the point (46). Indeed, we would regard any measurement lying *within* this ellipse as providing little challenge to the standard picture, given the rudimentary nature of our understanding of strong phases.

A more conservative estimate of eventual limits of the standard picture for $\Delta S_{\eta' K}$ and $A_{\eta' K}$ may be obtained by
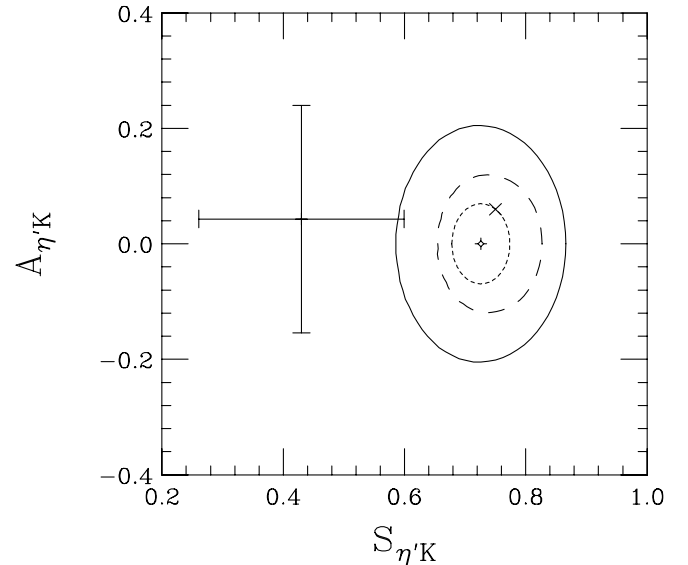

FIG. 1. Regions in the $S_{\eta' K}, A_{\eta' K}$ plane satisfying updated limits based on the last line in Table IV. Solid curve: limits based on flavor SU(3) without neglect of spectator amplitudes; dashed curve: limits with spectator amplitudes neglected. Plotted open point: $(S_{\eta' K}, A_{\eta' K}) = (0.726, 0)$. Point labeled $\times$: central value of a prediction in Ref. [15]. The dotted ellipse passing through this point denotes the range of values of $S_{\eta' K}, A_{\eta' K}$ in which only the strong phase δ varies.

TABLE IV. Comparison of current and anticipated 90% C.L. upper limits on branching ratios (in units of 10^{-6}) for B^0 decays to final states conisisting of pairs of neutral particles which may be used to place correlated bounds on $\Delta S_{\eta'K}$ and $A_{\eta'K}$.

Mode	$\pi^0 \pi^0$	$\pi^0 \eta$	$\pi^0 \eta'$	$\eta \eta$	$\eta' \eta'$	$\eta \eta'$
Current	1.45 ± 0.29	2.5	3.7	2.0	10	4.6
Anticipated	1.8	1.5	1.6	2.0	1.2	3.3

improving the bounds set in Ref. [40] using *anticipated* rather than current upper bounds on strangeness-conserving B^0 decays to various final states consisting of neutral particles. In Table IV we compare current bounds [11] (mostly used in Ref. [40]) with those that could be set if the data respected 90% C.L. upper limits of the predictions in the flavor SU(3) fits of Ref. [15].

Using the last line in Table IV and the current central value [11,39] for the branching ratio $\mathcal{B}(B^0 \to \eta' K^0) = 68.6 \times 10^{-6}$, we find a modest improvement in the bounds of Ref. [40]. The resulting constraints are illustrated in Fig. 1. The dashed curve denotes SU(3) bounds in which annihilation-like amplitudes were neglected [2,4] as in the discussion of $B \to K\pi$. In that case the previously excluded ellipse was centered at $(0.74, 0)$ with semiaxes $(0.12, 0.18)$. With the new inputs the semiaxes shrink to $(0.09, 0.12)$. Values of $S_{\eta'K}$ less than 0.65 would cause us first of all to question the neglect of annihilation-like amplitudes involving the spectator quark. The absence of a detectable rate for $B^0 \to K^+ K^-$ [11], indicating a low level of rescattering from other states [45], and a theoretical argument presented in [4] are the best justifications for their omission.

To be very conservative, we also present bounds without neglecting annihilation-like amplitudes. For this case, we found in [40] values of $S_{\eta'K}, A_{\eta'K}$ confined roughly to an ellipse with center at $(0.71, 0)$ and semiaxes $(0.22, 0.33)$. With the new inputs we now find this ellipse (solid curve in Fig. 1) to be centered at $(0.73, 0)$ with semiaxes $(0.14, 0.20)$. The lower bound on $S_{\eta'K}$ thus becomes 0.59. If the central value of the present average remains at 0.43 and the error is reduced to ± 0.05, the standard picture will be in trouble. This situation is probably some distance in the future.

VIII. THE ROLE OF B_s DECAYS

A number of decays of strange B's ($B_s \equiv \bar{b}s$) can be related to those of nonstrange B's using flavor SU(3). In particular, its U-spin subgroup involving the interchange $d \leftrightarrow s$ relates CP rate differences in strangeness-conserving and strangeness-changing decays of B^0 and B_s mesons [46–49]. A few examples are

$$\Delta(B_s \to K^+ K^-) = -\Delta(B^0 \to \pi^+ \pi^-), \quad (49)$$

$$\Delta(B_s \to K^- \pi^+) = -\Delta(B^0 \to \pi^- K^+), \quad (50)$$

$$\Delta(B_s \to \bar{K}^0 \pi^0) = -\Delta(B^0 \to K^0 \pi^0). \quad (51)$$

Gross violation of these relations, beyond corrections anticipated from SU(3) breaking, would indicate new physics in $b \to s$ transitions. As pointed out in [6], new physics contributions in these transitions are often accompanied by anomalous contributions to $B_s - \bar{B}_s$ mixing, thereby affecting the B_s mass-difference and time-dependent decays of B_s meson. A well-known example is the time-dependent CP asymmetry in $B_s \to K^+ K^-$ which is related in the standard model to the phase γ [46,50]. These measurements would therefore provide complementary information about potential new-physics operators.

Considering only strangeness-changing decays, any anomalous behavior in $b \to s$ penguin amplitudes should show up in B_s decays as well as in nonstrange B decays. Two of the simplest SU(3) relations, neglecting phase space and form factor differences and small amplitudes involving the spectator quark, are [48]

$$\Gamma(B_s \to K^+ K^-) = |p + t|^2 = \Gamma(B^0 \to K^+ \pi^-), \quad (52)$$

$$\Gamma(B_s \to K^0 \bar{K}^0) = |p|^2 = \Gamma(B^+ \to K^0 \pi^+). \quad (53)$$

These predictions are also obtained in the flavor-SU(3) description of a wide variety of B^+, B^0, and B_s decays in Ref. [16]. The first relation becomes a prediction for an approximate equality of branching ratios under the assumption of equal lifetimes for B_s and B^0, which is consistent with present data [11]. It is not particularly well obeyed since [51]

$$\mathcal{B}(B_s \to K^+ K^-) = (34.3 \pm 5.5 \pm 5.1) \times 10^{-6}, \quad (54)$$

to be compared with the world average in Table I:

$$\mathcal{B}(B^0 \to K^+ \pi^-) = (18.2 \pm 0.8) \times 10^{-6}. \quad (55)$$

If penguin amplitudes factorize, one may parametrize SU(3) breaking in terms of a ratio $F^{B_s K}/F^{B\pi}$ of form factors. The B_s decay rate could be enhanced to the value (54) if this ratio were about 1.4. Such a large value was obtained in a calculation based on QCD sum rules [52]. The result (54) is still preliminary, and is based on a fit involving several contributions. It will be interesting to see if the value (54) persists with improved statistics and better particle identification capabilities. No results have been presented yet for $B_s \to K^0 \bar{K}^0$, which is difficult to detect in a hadronic production environment.

The flavor-SU(3) fit of Ref. [16] predicts branching ratios below 10^{-7}, for $B_s \to \pi^0(\eta, \eta')$, and large branching ratios of around 56×10^{-6} and 23×10^{-6} for $B_s \to \eta' \eta'$ and $B_s \to \eta \eta'$, respectively, partly as a result of a large singlet-penguin contribution. The rates for $B_s \to \pi^0(\eta, \eta')$ could be affected substantially by new physics masquerading as an electroweak penguin. Measurement of

the $B_s \to \eta'\eta'$ and $B_s \to \eta\eta'$ branching ratios could help resolve the question of whether the enhanced rate for $B \to K\eta'$ decays is due in part to a singlet-penguin contribution [53] or whether conventional penguin contributions suffice [43,54].

IX. RELATED PUZZLES INVOLVING VECTOR MESONS

A. The parameter $S_{\phi K_S}$

The time-dependent asymmetry parameter $S_{\phi K_S}$ in $B^0 \to \phi K_S$ differs from the standard prediction of $\simeq 0.73$ by about 1.8σ, as shown in Table V. This decay mode was one which was deemed promising for manifestation of new physics in $b \to s$ penguin amplitudes well before any measurements were made [7].

The penguin amplitude contributing to $S_{\phi K}$ is exclusively a $b \to ss\bar{s}$ term. Both color-suppressed and color-favored matrix elements of this operator can contribute. Thus, this process becomes particularly worth while for identifying a specific four-quark operator in which new physics is appearing. Nonetheless, since the discrepancy with the standard picture is less than 2σ, speculation again seems premature.

A model-independent approach to studying an anomaly in $B^0 \to \phi K_S$ was presented in [56], using flavor SU(3) to normalize the amplitude of this process by the penguin amplitude dominating $B^+ \to K^{*0}\pi^+$. Explicit models of the space-time structure of new four-quark operators for $b \to sq\bar{q}$ [10] will in general treat $B \to VP$ decays (such as $B^0 \to \phi K_S$) differently from the $B \to PP$ decays which have occupied the bulk of our discussion. This should be borne in mind when discussing possible deviations from the standard model in processes dominated by $b \to s$ penguin amplitudes. This is in addition to any differences associated with the flavor q in $b \to sq\bar{q}$ amplitudes. Thus, it is dangerous to quote average values of S_f when discussing different final states f.

B. Helicity structure in $B^0 \to \phi K^{*0}$

The $b \to s$ penguin amplitude (again, with Lorentz structure possibly different from that in $B \to PP$ or $B \to VP$ decays) is expected to dominate the process $B^0 \to \phi K^{*0}$. In contrast, several other processes with large branching ratios such as $B^0 \to \rho^+\rho^-$ and $B^\pm \to \rho^\pm\rho^0$ are expected to be dominated by the tree amplitude. In

these, the vector mesons appear to be almost totally longitudinally polarized [57], while the longitudinal fraction in $B^0 \to \phi K^{*0}$ appears to be more like $1/2$ [58]. Some authors (see, e.g., Ref. [59]) have cited this circumstance as further evidence for the anomalous behavior of the penguin amplitude.

We see no reason why the penguin amplitude should have the same space-time structure as the tree amplitude. If, for example, it is an effective operator driven partly by rescattering from charm-anticharm states, as suggested in Refs. [60,61], its space-time properties may be governed largely by long-distance physics, and not amenable to the usual arguments based on Fierz rearrangement of a $V - A$ current.

X. FUTURE EXPERIMENTAL TESTS

It has sometimes been noted (see, e.g., Refs. [11,23,62]) that processes dominated by $b \to s$ penguin amplitudes give an effective average S_f value of about 0.4 ± 0.1, to be contrasted with the standard model expectation of $\sin 2\beta \simeq 0.73$. We regard this viewpoint as dangerous for three reasons. (1) It does not take account of the discrepancies between the *BABAR* and Belle determinations of S_f for several cases, including that of $\eta'K_S$ mentioned above as well as $K_S f_0(980)$, where $f_0(980) \to \pi\pi$ [38,63], and $K_S K_S K_S$ [64]. When these discrepancies are taken into account and the errors on experimental averages are multiplied by an appropriate scale factor, the significance of the deviation becomes less. (2) The several processes dominated by a $b \to s$ penguin amplitude involve small but different terms proportional to $V_{ub}^* V_{us}$ [35,40–42,65]. This implies *ab initio* different nonzero values for ΔS_f and A_f for different final states f. (3) As we have pointed out, the penguin operators for $b \to su\bar{u}$, $b \to sd\bar{d}$, and $b \to ss\bar{s}$ may differ from one another, both in their intrinsic strengths and in their matrix elements between states of different spins. How would one sort out this situation?

Our first suggestion is to concentrate on processes for which the interpretation is as clean as possible. Thus, $B^0 \to K^0\pi^0$ appears considerably simpler to interpret in terms of specific contributions than $B^0 \to \eta'K_S$, for which even the interpretation of the decay rate itself has been the subject of controversy. (See, e.g., a discussion in Ref. [15]). Pinning down the value of $S_{K\pi}$ is a first priority. Lowering the experimental upper limit on $\mathcal{B}(B^0 \to K^+K^-)$ may soon imply $S_{K\pi} \neq \sin 2\beta$, consistent with our prediction (33) of

TABLE V. Time-dependent CP asymmetry parameters for $B^0 \to \phi K_S$. Errors on averages include scale factor $S = \sqrt{\chi^2}$.

Parameter	*BABAR* [55]	Belle [38]	S	Average
$S_{\phi K}$	$0.50 \pm 0.25 ^{+0.07}_{-0.04}$	$0.08 \pm 0.33 \pm 0.09$	1.03	0.35 ± 0.21
$A_{\phi K}$	$0.00 \pm 0.23 \pm 0.05$	$0.08 \pm 0.22 \pm 0.09$	<1	0.04 ± 0.17

a nonzero (negative) direct asymmetry $A_{K\pi}$. Testing the sum rule (15), or determining whether R_n differs from R_c by a significant amount, obviously has high priority.

Our second suggestion regards the measurement of decay modes listed in Table IV. Improvement on the upper bounds listed there to the level of bounds anticipated from SU(3) fits will not only help sharpen bounds on $S_{\eta'K}$, but may uncover additional unanticipated contributions to amplitudes or shortcomings of the flavor SU(3) fits.

A third suggestion regards confirmation of the patterns of tree-penguin interference seen in nonstrange B decays using B_s decays. There are several B_s decays related via U-spin to B^0 decays, as noted in Sec. VIII. Study of B_s decays will also be helpful in identifying the source of the enhanced rate for $B \to \eta'K$.

A fourth suggestion is to continue the study of $B \to VV$ modes which has begun so auspiciously with the study of such decays as $B \to \rho\rho$, $B \to \phi K^*$, $B \to \rho K^*$, and even $B_s \to \phi\phi$. Information on these modes is approaching the stage that will permit analyses based on flavor SU(3) analogous to those performed for $B \to PP$ [15] and $B \to VP$ [66] decays. Relations among amplitudes have to be analyzed separately for each helicity state, so it does not suffice to have rate information alone.

XI. SUMMARY AND CONCLUSIONS

We have discussed several B meson decay processes governed by the $b \to s$ penguin amplitude, concentrating on processes with two light pesudoscalar mesons P in the final state. Although several indications appear for anomalous behavior, including the rate and time-dependent asymmetry parameter S for $B^0 \to K^0\pi^0$ and the corresponding parameter S for $B^0 \to \eta'K^0$, no deviations of more than 2σ from the standard model expectations have been identified yet. We have indicated several ways in which experimental searches for this anomalous behavior can be sharpened.

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