Color-octet contributions to *P*-wave B_c meson hadroproduction

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The contributions from the color-octet components $|(c\bar{b})_{\mathbf{8}}(^1S_0)g\rangle$ and $|(c\bar{b})_{\mathbf{8}}(^3S_1)g\rangle$ to the h_{B_c} or $\chi^J_{B_c}$ (the *P*-wave B_c meson) hadroproduction are estimated in terms of the complete $\mathcal{O}(\alpha_s^4)$ calculation. As necessary inputs in the estimate, we take the values of the octet matrix elements according to the nonrelativistic quantum chromodynamics scaling rules, and as a result, we have found that the contributions to the *P*-wave production may be the same in order of magnitude as those from the color-singlet ones, $|(c\bar{b})_{\mathbf{1}}(^1P_1)\rangle$ and $|(c\bar{b})_{\mathbf{1}}(^3P_J)\rangle$ ($J = 1, 2, 3$). Especially, our result indicates that the observation of the color-octet contributions to the *P*-wave production in the low transverse momentum region is not very far from the present experimental capability at Tevatron and LHC.

I. INTRODUCTION

B_c meson has been observed experimentally [1,2], and within theoretical uncertainties and experimental errors the observations are consistent with the theoretical predictions. In view of the prospects in B_c physics at the Fermilab Tevatron II and LHC, B_c physics is attracting more and more attention. The hadronic production of the B_c meson (its ground state mainly), has been studied quite well [3–10]. Especially, a generator BCVEGPY [9,10] for hadronic production of B_c meson has been available, which can be easily complimented to the PYTHIA environment [11] and enhances the efficiency to generate the full B_c events greatly in comparison with using PYTHIA itself.

In the framework of effective theory of nonrelativistic quantum chromodynamics (NRQCD) [12,13], a heavy quarkonium is considered as an expansion of various Fock states. The relative importance among those infinite ingredients is evaluated by the velocity scaling rule. A similar idea can be applied to the double heavy meson system too. That is, the physical state of B_c, B_c^*, h_{B_c} and $\chi^J_{B_c}$ mesons can be decomposed into a series of Fock states:

$$
\begin{aligned}
|B_c\rangle &= \mathcal{O}(v^0)|(c\bar{b})_{\mathbf{1}}(^1S_0)\rangle + \mathcal{O}(v^1)|(c\bar{b})_{\mathbf{8}}(^1P_1)g\rangle \\
&\quad + \mathcal{O}(v^1)|(c\bar{b})_{\mathbf{8}}(^3S_1)g\rangle + \cdots \\
|B_c^*\rangle &= \mathcal{O}(v^0)|(c\bar{b})_{\mathbf{1}}(^3S_1)\rangle + \mathcal{O}(v^1)|(c\bar{b})_{\mathbf{8}}(^3P_J)g\rangle \\
&\quad + \mathcal{O}(v^1)|(c\bar{b})_{\mathbf{8}}(^3S_1)g\rangle + \cdots
\end{aligned}
\tag{1}
$$

and

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$$
\begin{aligned}
|h_{B_c}\rangle &= \mathcal{O}(v^0)|(c\bar{b})_{\mathbf{1}}(^1P_1)\rangle + \mathcal{O}(v^1)|(c\bar{b})_{\mathbf{8}}(^1S_0)g\rangle \\
&\quad + \mathcal{O}(v^1)|(c\bar{b})_{\mathbf{8}}(^3P_J)g\rangle + \cdots \\
|\chi^J_{B_c}\rangle &= \mathcal{O}(v^0)|(c\bar{b})_{\mathbf{1}}(^3P_J)\rangle + \mathcal{O}(v^1)|(c\bar{b})_{\mathbf{8}}(^3S_1)g\rangle \\
&\quad + \mathcal{O}(v^1)|(c\bar{b})_{\mathbf{8}}(^1P_1)g\rangle + \cdots,
\end{aligned}
\tag{2}
$$

here v is the relative velocity. Note that throughout this paper we use the symbols h_{B_c} and $\chi^J_{B_c}$ to denote the four physical *P*-wave states [h_{B_c} denotes the *P*-wave state with the dominant color-singlet state $(c\bar{b})_{\mathbf{1}}(^1P_1)$, while $\chi^J_{B_c}$ denotes the *P*-wave states with the dominant color-singlet states $(c\bar{b})_{\mathbf{1}}(^3P_J)$ as described in Eq. (2)] instead of $B^*_{cJ,L=1}$ that is used in Ref. [14]. Here the thickened subscripts of the $(c\bar{b})$ denote for color indices, **1** for color singlet and **8** for color octet; the relevant angular momentum quantum numbers are shown in the parentheses accordingly. According to the velocity scaling rule, the probability of each Fock state in the expansion is proportional to the scales in a definite power of v, squared the indicated one as the above. Generally, the leading Fock states of the B_c and B_c^* states are $|(c\bar{b})_{\mathbf{1}}(^1S_0)\rangle$ and $|(c\bar{b})_{\mathbf{1}}(^3S_1)\rangle$, respectively, whose probability in respect of the relative velocity are set to be the order of $O(v^0)$. Also for the production of the B_c and B_c^* states, there is no problem at all, one can be sure that the contributions from leading Fock state(s) are dominant.

As implied in Eqs. (2), at leading and next leading order, the production of *P*-wave B_c mesons may involve in the Fock states: $|(c\bar{b})_{\mathbf{1}}(^1P_1)\rangle$, $|(c\bar{b})_{\mathbf{8}}(^1S_0)g\rangle$, $|(c\bar{b})_{\mathbf{8}}(^3P_J)g\rangle \cdots$; $|(c\bar{b})_{\mathbf{1}}(^3P_J)\rangle$, $|(c\bar{b})_{\mathbf{8}}(^3S_1)g\rangle$, $|(c\bar{b})_{\mathbf{8}}(^1P_1)g\rangle \cdots$. For color-singlet components (*P*-wave), the long distance nonperturbative matrix elements can be directly connected with the first derivative of the wave functions at the origin, which can be computed via potential models [15] and/or potential NRQCD (pNRQCD) [16] and/or lattice QCD etc. at the accuracy for the present purposes, whereas, there is no

reliable way to offer such accurate values for the color-octet nonperturbative matrix elements as those for color-singlet ones even for lattice QCD [13]. So far, we may only have rough values of these matrix elements relating to the components $|(c\bar{b})_\mathbf{8}(^1S_0)g\rangle$, $|(c\bar{b})_\mathbf{8}(^3S_1)g\rangle \cdots$ by means of the velocity power counting rule of the NRQCD effective theory [12]. In order to make a reasonable estimate on the contributions from the color-octet components to the hadronic production so as to see if the color-octet components are visible experimentally or not, we take a similar way as done in Ref. [17], i.e., the color-octet matrix elements are taken to be smaller by certain order in v^2 than those obtained by relating them to the color-singlet S-wave wave functions at the origin $|\psi(0)|^2$. Whereas, the first derivative of the wave functions (P wave) at the origin for the color-singlet components with proper factor of m (the reduce mass of the bound state), i.e. $\psi'(0)/m$, is in a higher order in v than the wave functions at the origin for the S-wave functions, $\psi(0)$, therefore the contributions from the color-octet components $|(c\bar{b})_\mathbf{8}(^1S_0)g\rangle$, $|(c\bar{b})_\mathbf{8} \times (^3S_1)g\rangle \cdots$ (S-wave) to the production of the P-wave excited states h_{B_c} and $\chi^J_{B_c}$ may be comparable with those from the color-singlet components $|(c\bar{b})_\mathbf{1}(^1P_1)\rangle$ and $|(c\bar{b})_\mathbf{1}(^3P_J)\rangle$ (P wave).

The color-singlet contributions to the P-wave B_c meson hadronic production have been studied in Refs. [10,14,18,19]. In Ref. [18], the hadronic productions of h_{B_c} and $\chi^J_{B_c}$ were estimated by applying the fragmentation approach. While in Refs. [10,14,19] the authors adopted the lowest order (here α_s^4) complete approach but only color-singlet components $|(c\bar{b})_\mathbf{1}(^1P_1)\rangle$ and $|(c\bar{b})_\mathbf{1}(^3P_J)\rangle$ were taken into account, and found that the total cross section is much larger than what predicted by the fragmentation approach [18], and the fragmentation results tend to be comparable with those from the α_s^4 complete calculation only in the cases when the transverse momentum p_t of the meson B_c is so high as $p_t \gtrsim 30$ GeV (higher than S-wave production case) [14,19]. For the α_s^4 complete estimation, to be complete and to see the characteristics, we compute the contributions from the color-octet components to the P-wave B_c-meson production precisely in the paper.

The paper is organized as follows: In Sec. II we will present the basic formulas for the lowest order (α_s^4) which are used in the complete calculation of the contributions to the P-wave hadroproduction from the color-octet components $|(c\bar{b})_\mathbf{8}(^1S_0)g\rangle$ and $|(c\bar{b})_\mathbf{8}(^3S_1)g\rangle$. In Sec. III, we will explain the options about the input parameters, which are necessary for the calculations and will present the numerical results properly. In the last section, a brief summary and some discussions will be given.

II. FORMULATION AND TECHNIQUE

In hadron-hadron collisions at high energy, the gluon-gluon fusion mechanism is the dominant one over the others. Hence, to study the hadronic production of the P wave B_c mesons, as argued in the preceding section, we need to consider two types of processes, i.e., $gg \to (c\bar{b})_\mathbf{1} + b + \bar{c}$ where $(c\bar{b})_\mathbf{1}$ is in color-singlet $^1P_1$ and ^3P_J states; and $gg \to (c\bar{b})_\mathbf{8} + b + \bar{c}$ where $(c\bar{b})_\mathbf{8}$ is in color-octet $^1S_0$ and $^3S_1$ states. In Ref. [14], the color-singlet contributions to the hadronic production of h_{B_c} and $\chi^J_{B_c}$ have been computed, and the useful formulas and techniques have been explained there. For simplicity, we will not repeat it here. In the present work, our main concern is to complete the calculations on the hadronic production of the P-wave B_c physical states, especially to pay attention to the contributions from the color octets $|(c\bar{b})_\mathbf{8}(^1S_0)g\rangle$ and $|(c\bar{b})_\mathbf{8}(^3S_1)g\rangle$.

The amplitude for the $gg \to (c\bar{b})_\mathbf{8} + b + \bar{c}$ can be analytically obtained by applying the helicity method [9], and $(c\bar{b})_\mathbf{8}$ will be "hadronized" to the state $|(c\bar{b})_\mathbf{8}(^1S_0)g\rangle$ or $|(c\bar{b})_\mathbf{8}(^3S_1)g\rangle$ finally. The details of the helicity technique can be found in Ref. [9] and references therein. The difference of the present calculations is only in the color configuration from those for the leading order $B_c(B_c^*)$ meson hadronic production.

In the case of the hadronic production $gg \to (c\bar{b})_\mathbf{1} + b + \bar{c}$ where $(c\bar{b})_\mathbf{1}$ is in color singlet, $|(c\bar{b})_\mathbf{1}(^1S_0)\rangle$ or $|(c\bar{b})_\mathbf{1}(^3S_1)\rangle$, there are only three independent color factors (i.e., three independent color flows [14,20]). While for the case of the hadronic production of the color-octet states, $gg \to (c\bar{b})_\mathbf{8} + b + \bar{c}$ where $(c\bar{b})_\mathbf{8}$ is in color octet and will be hadronized to $|(c\bar{b})_\mathbf{8}(^1S_0)g\rangle$ or $|(c\bar{b})_\mathbf{8}(^3S_1)g\rangle$ finally, there are totally ten independent color factors, C_{kij} ($k = 1, 2, \cdots, 10$), where $i, j = 1, 2, 3$ are color indices of the quarks $\bar{c}$ and b respectively. They are

$$C_{1ij} = \frac{1}{\sqrt{2}N_c}(T^bT^aT^d)_{ij}, \quad C_{2ij} = \frac{1}{\sqrt{2}N_c}(T^aT^bT^d)_{ij},$$

$$C_{3ij} = \frac{1}{\sqrt{2}}Tr[T^aT^d]T^b_{ij}, \quad C_{4ij} = \frac{1}{\sqrt{2}}Tr[T^bT^d]T^a_{ij},$$

$$C_{5ij} = \frac{1}{\sqrt{2}}Tr[T^bT^aT^d]\delta_{ij}, \quad C_{6ij} = \frac{1}{\sqrt{2}}Tr[T^aT^bT^d]\delta_{ij},$$

$$C_{7ij} = \frac{1}{\sqrt{2}N_c}(T^dT^bT^a)_{ij}, \quad C_{8ij} = \frac{1}{\sqrt{2}N_c}(T^dT^aT^b)_{ij},$$

$$C_{9ij} = \frac{1}{\sqrt{2}N_c}(T^aT^dT^b)_{ij}, \quad C_{10ij} = \frac{1}{\sqrt{2}N_c}(T^bT^dT^a)_{ij},$$

$$(3)$$

where the indices a and b are color indices for gluon-1 and gluon-2, respectively, and $\sqrt{2}T^d$ stands for the color of the color-octet state $(c\bar{b})_\mathbf{8}$ in the production. $N_c = 3$, for QCD. All the color factors in the subprocess can be expressed by the linear combination of the above ten independent color factors, i.e., in the amplitude all the color factors may be written in terms of these ten explicitly. Thus the total helicity amplitude can be generically expressed as

$$M^{(\lambda_1, \lambda_4, \lambda_5, \lambda_6)}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_1, k_2)$$

$$= \sum_{m=1}^{10} C_{mij} M_m^{(\lambda_1, \lambda_4, \lambda_5, \lambda_6)}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_1, k_2), \quad (4)$$

where λ_1, λ_4, λ_5, and λ_6 denote the helicities of the outgoing b quark, outgoing $\bar{c}$ quark, gluon-1 and gluon-2, respectively. k_1 and k_2 are the momenta of the gluons; q_{c1}, q_{b1} are the momenta of c and b quarks, and q_{c2}, q_{b2} are the momenta of $\bar{c}$ and $\bar{b}$ antiquarks, respectively. The helicity amplitude $M_m^{(\lambda_1, \lambda_4, \lambda_5, \lambda_6)}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_1, k_2)$ can be directly read from the Eqs. (4)–(8) in Ref. [9], with only the color-singlet matrix elements (or the wave function at the origin) being replaced by the color-octet matrix elements $\langle 0|\chi_b^\dagger T^d \psi_c (a_H^\dagger a_H) \psi_c^\dagger T^d \chi_b|0\rangle$ and $\langle 0|\chi_b^\dagger \sigma^i T^d \psi_c (a_H^\dagger a_H) \psi_c^\dagger \sigma^i T^d \chi_b|0\rangle$. To get the matrix element squared, one needs first to deal with the square of the above ten independent color factors, i.e. $(C_{mij} \times C_{nij}^*)$ with $m, n = 1, 2, \cdots 10$. For reference use, whose values are listed in Table I.

According to pQCD factorization theorem, the inclusive B_c meson hadroproduction can be formulated as

$$d\sigma = \sum_{ij} \int dx_1 \int dx_2 F_{H_1}^i(x_1, \mu_F^2)$$

$$\times F_{H_2}^j(x_2, \mu_F^2) d\hat{\sigma}_{ij \to (c\bar{b})X}(x_1, x_2, \mu_F^2, \mu^2, Q^2), \quad (5)$$

where $F_{H_2}^j(x, \mu_F^2)$, $F_{H_2}^j(x, \mu_F^2)$ are parton distribution functions (PDFs) of partons i and j in hadrons H_1, H_2, respectively. μ^2 is the "energy scale squared" where renormalization for the subprocess is made; Q^2 is the "characteristic energy scale of the subprocess squared"; and μ_F^2 is the "energy scale squared" where the factorization of the PDFs and the hard subprocess is made. Usually, in the LO calculation, to avoid the large logarithms the scales of typical scale of hard interaction, factorization, and "renormalization" are set to be the same, i.e. $\mu^2 = \mu_F^2 = Q^2$.

III. NUMERICAL RESULTS

For convenience and as a reference, in the numerical calculation we take the values of the radial wave function at the origin and the first derivative of the radial wave function at the origin as those given by Refs. [15,21], i.e., $|R(0)|^2 = 1.54$ GeV^3 and $|R'(0)|^2 = 0.201$ GeV^5 (namely the two values roughly mean $v^2 \sim 0.1$, if the reduced mass $m \sim 1.2$ GeV), which relate to the nonperturbative matrix elements of the color-singlet production definitely. Whereas there is no reliable way to compute the production color-octet matrix elements $\langle 0|\chi_b^\dagger T^d \psi_c (a_H^\dagger a_H) \psi_c^\dagger T^d \chi_b|0\rangle$ and $\langle 0|\chi_b^\dagger \sigma^i T^d \psi_c (a_H^\dagger a_H) \times \psi_c^\dagger \sigma^i T^d \chi_b|0\rangle$. Although we do not know the exact values of the above two octet matrix elements, according to NRQCD scale rule we know that they are at the same order of the *P*-wave color-singlet matrix elements, which are one order in v^2 higher than the *S*-wave color-singlet matrix elements accordingly. Moreover according to the heavy-quark spin symmetry [12], the values of the above two production matrix elements have the approximate relation:

$$\langle 0|\chi_b^\dagger \sigma^i T^d \psi_c (a_H^\dagger a_H) \psi_c^\dagger \sigma^i T^d \chi_b|0\rangle$$

$$= (2J + 1) \cdot \langle 0|\chi_b^\dagger T^d \psi_c (a_H^\dagger a_H) \psi_c^\dagger T^d \chi_b|0\rangle[1 + \mathcal{O}(v^2)], \quad (6)$$

where J is the total angular momentum of the hadron state. In our numerical calculations, as done in Ref. [17], the values of the color-octet matrix elements are set to be smaller by certain order v^2 than those obtained by relating to the *S*-wave wave functions at the origin $|\psi(0)|^2$ for the color singlet. More specifically, based on the velocity scale rule, we can estimate

$$\langle 0|\chi_b^\dagger T^d \psi_c (a_H^\dagger a_H) \psi_c^\dagger T^d \chi_b|0\rangle$$

$$\simeq \Delta_S(v)^2 \cdot \langle 0|\chi_b^\dagger \psi_c (a_H^\dagger a_H) \psi_c^\dagger \chi_b|0\rangle$$

$$\simeq \Delta_S(v)^2 \cdot |\langle 0|\chi_b^\dagger \psi_c |B_c(^1S_0)\rangle|^2 . [1 + \mathcal{O}(v^4)], \quad (7)$$

TABLE I. The square of the ten independent color factors (including the cross terms), $(C_{mij} \times C_{nij}^*)$ with $m, n = 1, 2, \cdots, 10$ respectively.

	C_{1ij}^*	C_{2ij}^*	C_{3ij}^*	C_{4ij}^*	C_{5ij}^*	C_{6ij}^*	C_{7ij}^*	C_{8ij}^*	C_{9ij}^*	C_{10ij}^*
C_{1ij}	32/81	−4/81	4/9	−1/18	7/18	−1/9	1/162	10/162	1/162	−4/81
C_{2ij}	−4/81	32/81	−1/18	4/9	−1/9	7/18	10/162	1/162	−4/81	1/162
C_{3ij}	4/9	−1/18	4	1/2	0	0	−1/18	4/9	4/9	4/9
C_{4ij}	−1/18	4/9	1/2	4	0	0	4/9	−1/18	4/9	4/9
C_{5ij}	7/18	−1/9	0	0	7/2	−1	7/18	−1/9	7/18	−1/9
C_{6ij}	−1/9	7/18	0	0	−1	7/2	−1/9	7/18	−1/9	7/18
C_{7ij}	1/162	10/162	−1/18	4/9	7/18	−1/9	32/81	−4/81	1/162	−4/81
C_{8ij}	10/162	1/162	4/9	−1/18	−1/9	7/18	−4/81	32/81	−4/81	1/162
C_{9ij}	1/162	−4/81	4/9	4/9	7/18	−1/9	1/162	−4/81	32/81	10/162
C_{10ij}	−4/81	1/162	4/9	4/9	−1/9	7/18	−4/81	1/162	10/162	32/81

TABLE II. Total cross section (in unit of nb) for the hadronic production of the $(c\bar{b})$ meson at LHC (14.0 TeV) and TEVATRON (1.96 TeV), where for short the $|(^1S_0)_1\rangle$ denotes $(c\bar{b})$ state in color-singlet $(^1S_0)$ configuration, and so forth. Here $m_b = 4.90$ GeV, $m_c = 1.50$ GeV, and $M = 6.40$ GeV. For the color-octet matrix elements, we take $\Delta_S(v) \in (0.10, 0.30)$.

	$|(^1S_0)_1\rangle$	$|(^3S_1)_1\rangle$	$|(^1S_0)_8 g\rangle$	$|(^3S_1)_8 g\rangle$	$|(^1P_1)_1\rangle$	$|(^3P_0)_1\rangle$	$|(^3P_1)_1\rangle$	$|(^3P_2)_1\rangle$
LHC	71.1	177	(0.357, 3.21)	(1.58, 14.2)	9.12	3.29	7.38	20.4
TEVATRON	5.50	13.4	(0.0284, 0.256)	(0.129, 1.16)	0.655	0.256	0.560	1.35

where the second equation comes from the vacuum-saturation approximation. $\Delta_S(v)$ is of the order v^2 or so, and we take it to be within the region of 0.10 to 0.30, which is inconsistent with the identification: $\Delta_S(v) \sim \alpha_s(Mv)$ and has covered the possible variation due to the different ways to obtain the wave functions at the origin (S-wave) and the first derivative of the wave functions at the origin (P-wave) etc.

As argued in Ref. [14], the equations: $P = q_{c1} + q_{b2}$, $q_{c1}^2 = m_c^2$, $q_{b2}^2 = m_b^2$, and $P^2 = M^2$ must be satisfied simultaneously, and furthermore, only when these equations are satisfied can the gauge invariance of the amplitude be guaranteed. Hence, in this paper we proceed the numerical calculation by making the choice: the masses of c and b quarks are $m_c = 1.50$ GeV and $m_b = 4.90$ GeV, respectively; the mass of the bound states $|(c\bar{b})_1(^1P_1)\rangle$, $|(c\bar{b})_1 \times (^3P_J)\rangle$, $|(c\bar{b})_8(^1S_0)g\rangle$ and $|(c\bar{b})_8(^3S_1)g\rangle$ are therefore to be $M = 6.40$ GeV, the sum of the two heavy-quark masses.

There are a couple of uncertainty sources in the theoretical estimates on the B_c meson hadronic production [22], such as those from the strong coupling constant, the α_s; from the PDFs of different sets of fittings, etc. In the present calculations, the factorization energy scale is set to be the transverse mass squared of the $(c\bar{b})$ bound states, i.e., $Q^2 = M_t^2 \equiv (M^2 + p_t^2)$ where p_t is the transverse momentum of the bound state; the PDF of version

CTEQ6L [23] and the leading order α_s running above $\Lambda_{QCD}^{(n_f=4)} = 0.326$ GeV are adopted.

In Table II, we show the total cross sections for hadronic production of the $(c\bar{b})$ meson at LHC and TEVATRON, respectively, where the $(c\bar{b})$ may mean in $|(c\bar{b})_1(^1P_1)\rangle$, $|(c\bar{b})_1(^3P_J)\rangle$, $|(c\bar{b})_8(^1S_0)g\rangle$ and $|(c\bar{b})_8(^3S_1)g\rangle$ configurations, respectively, and the results of the $|(c\bar{b})_1(^1S_0)\rangle$ (the dominant component for B_c) and $|(c\bar{b})_1(^3S_1)\rangle$ (the dominant component for B_c^*) production are included for comparison. In the table, we take $\Delta_S(v) \in (0.10, 0.30)$. One may observe that the cross sections of the color-octet S-wave states are comparable to those of the color-singlet P-wave states, and they are in the same order in v^2 expansion as expected by NRQCD.

To see this point more clearly, we calculate the distributions of the corresponding bound states on the transverse momentum p_t and the rapidity y respectively, and plot the results in Figs. 1 and 2. From Figs. 1 and 2, one may observe that the contributions to the total cross section from the color-octet component are in comparable with those from the color-singlet component in the P-wave B_c meson hadronic production. However, in the p_t distribution the color-octet contributions drop more rapidly than those from the color-singlet parts, which exhibits more clearly through looking at the ratio, $R(p_{t\text{cut}}) = (\sigma_{\text{octet}}/\sigma_{\text{singlet}})$, where σ_{octet} stands for the total contribution from the color
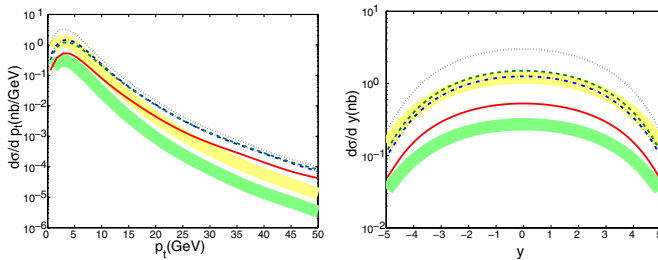

FIG. 1 (color online). Distributions in p_t and y of the hadronic production of $(c\bar{b})$ meson at LHC. The dashed line, solid line, dash-dotted line, dotted line represent the color-singlet $^1P_1$, $^3P_0$, $^3P_1$, and $^3P_2$, respectively. The lower and upper shaded bands stand for the color-octet $^1S_0$ and $^3S_1$ states, respectively, whose upper limit corresponds to $\Delta_S(v) = 0.3$ and lower limit corresponds to $\Delta_S(v) = 0.1$.
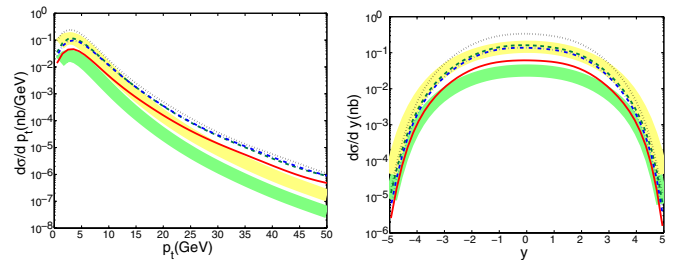

FIG. 2 (color online). Distributions in p_t and y of the hadronic production of $(c\bar{b})$ meson at TEVATRON. The dashed line, solid line, dash-dotted line, and dotted line represent the color-singlet $^1P_1$, $^3P_0$, $^3P_1$, and $^3P_2$, respectively. The lower and upper shaded bands stand for color-octet $^1S_0$ and $^3S_1$ states, respectively, whose upper limit corresponds to $\Delta_S(v) = 0.3$ and lower limit corresponds to $\Delta_S(v) = 0.1$.

TABLE III.   The correlation of $R(p_{tcut}) = (\sigma_{octet}/\sigma_{singlet})$ with the p_{tcut}. For color-octet matrix elements, we take $\Delta_S^2(v) = 0.05$, which is in the middle of the range $(0.01, 0.09)$.

	$p_{tcut} = 0$ GeV	$p_{tcut} = 5$ GeV	$p_{tcut} = 20$ GeV	$p_{tcut} = 35$ GeV	$p_{tcut} = 50$ GeV
LHC	0.22	0.19	0.095	0.058	0.043
TEVATRON	0.25	0.20	0.098	0.060	0.044

octet and $\sigma_{singlet}$ for that from the color singlet. The variation of the dependence of $R(p_{tcut})$ on p_{tcut} is presented in Table III. Here, for simplicity, we take a middle value of $\Delta_S^2(v)$, i.e. $\Delta_S^2(v) = 0.05$ (or $\Delta_S(v) = 0.224$), for the color-octet matrix elements.

Since there is no detector in high energy hadronic collisions which can directly detect all the events, especially those with a small p_t and/or a large rapidity y, so for experimental studies and for practical purpose, only the events with proper kinematic cuts on p_t and/or y are taken into account. Now let us compute the observables with cuts on p_t and/or y, so as to see the cut effects in h_{B_c} and $\chi_{B_c}^J$ production precisely. Namely we have investigated the
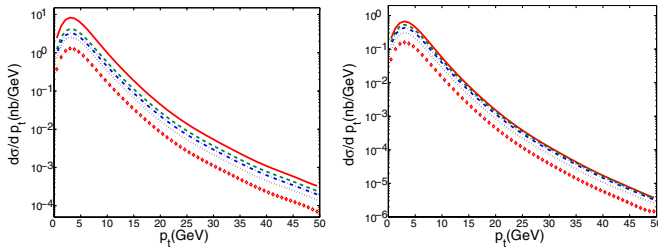

FIG. 3 (color online). p_t distributions of various contributions in h_{B_c} and $\chi_{B_c}^J$ production, with several different cuts in rapidity (y_{cut}) at LHC (left) and at TEVATRON (right) energies. Dashed line (next to top) with $y_{cut} = 2.0$; dash-dotted line (middle) with $y_{cut} = 1.5$; dotted line (next bottom) with $y_{cut} = 1.0$; diamond line (bottom) with $y_{cut} = 0.5$ and solid line (top) without y_{cut}. The color-octet matrix elements are set at $\Delta_S^2(v) = 0.05$.
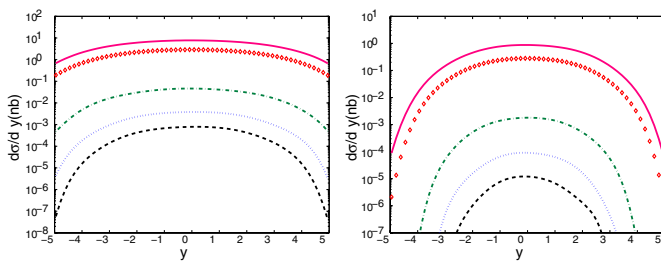

FIG. 4 (color online). y distributions of all the considered contributions, i.e., from both color-octet S-wave states and color-singlet P-wave states, with various transverse momentum cuts (y_{cut}) at LHC (left) and at TEVATRON (right) energies. Here, the diamond line represents what with $p_{tcut} = 5$ GeV; dash-dotted line with $p_{tcut} = 20$ GeV; dotted line with $p_{tcut} = 35$ GeV; dashed line with $p_{tcut} = 50$ GeV, and the solid line with $p_{tcut} = 0$. Of the color-octet matrix elements, we take $\Delta_S^2(v) = 0.05$.

effects in production rate and various differential cross sections and for the color-octet production and the color-singlet production of S-wave (B_c and B_c^*) and P-wave (h_{B_c} and $\chi_{B_c}^J$), respectively. Explicitly, we take $\Delta_S^2(v) = 0.05$ for the color-octet matrix elements in the investigation. Considering the abilities on measuring p_t and y rapidity of B_c for CDF, D0, BTeV at TEVATRON and for ATLAS, CMS, and LHC-B at the LHC, we compute the p_t distributions with the rapidity cuts $y_{cut} = 1.5$, and the y distributions with the transverse momentum cut $p_{tcut} = 5$ GeV accordingly. The results with four rapidity cuts, $y_{cut} = (0.5, 1.0, 1.5, 2.0)$, are put together in Fig. 3 and the results with four transverse momentum cuts, $p_{tcut} = 5, 20, 35, 50$ GeV, are put in Fig. 4.

IV. DISCUSSIONS AND SUMMARY

In the paper, according to the NRQCD expectation about the importance for the components, we have precisely investigated the contributions of the S-wave color-octet components to the hadronic production of the P-wave B_c states. Our final results show that contributions from the color-octet ones are comparable to those from the color-singlet ones in the production of the P-wave B_c excited states if the scale rule of NRQCD works well and the value of the relative velocity squared v^2 is really in the possible region $0.1 \sim 0.3$, as indicated by potential models for the $(c\bar{b})$ binding system for instance. Therefore, to make a sound prediction to the hadronic production of the P-wave $(c\bar{b})$ meson at leading order in v^2, one needs to take both contributions from the color-octet S-wave Fock states and those from the color-singlet P-wave Fock states into account. With $\Delta_S^2(v) = 0.05$ and $p_{tcut} = 0.0$ GeV, the total contributions from the color-octet S-wave Fock states to the total production cross section is about $\sim 22\%$ from the color-singlet P-wave states at LHC and about $\sim 25\%$ at TEVATRON, respectively, and such contributions decrease with the increment in p_{tcut}, e.g. at $p_{tcut} = 50$ GeV, it reduces to $\sim 4\%$ at both energies of LHC and TEVATRON. This character may used for people to distinguish the color-singlet contributions from those from color-octet ones in future when there are enough h_{B_c} and $\chi_{B_c}^J$ data.

Note that for shortening the paper and not changing the main feature, here we have not taken the possible mixing between h_{B_c} and $\chi_{B_c}^{J=1}$ into account at all. According to NRQCD scale rule, except the components $|(c\bar{b})_8(^1S_0)g\rangle$

and $|(c\bar{b})_8(^3S_1)g\rangle)$, there is no enhancement factor, such as that comes from the wave function at the origin for the *S*-wave components virus the derivative of the wave function at the origin for the leading *P*-wave color-singlet ones, so we consider the contributions from the other high order Fock components in Eq. (2) small, and ignore them at all.

In summary, the total cross section of the h_{B_c} and $\chi^J_{B_c}$ production, including both singlet and octet contributions, at the lowest order in α_s and v^2 expansions, may be much bigger than a half of the direct production rate of the ground state $B_c(^1S_0)$ (see Table II, roughly speaking: ~ 0.8 for LHC and ~ 0.7 for TEVATRON). Considering the fact that almost all of the low-laying excited states decay to the ground state B_c, it also means that the full low-laying *P*-wave state production "promptly" contrib-

utes the $B_c(^1S_0)$ production by a bigger factor than 0.5 of the direct one. Especially, if $\Delta^2_s(v) \sim 0.05$, the contributions from color-octet components themselves may contribute promptly a factor ~ 0.1 of the direct hadronic production of B_c meson. Furthermore, our complete calculations also indicate that the observation of the *P*-wave B_c states is not very far from the present experimental capability in principle.

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