

# Chiral symmetry breaking and stability of strangelets

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We discuss the stability of strangelets by considering dynamical chiral symmetry breaking and confinement. We use a  $U(3)_L \times U(3)_R$  symmetric Nambu-Jona-Lasinio (NJL) model for chiral symmetry breaking supplemented by a boundary condition for confinement. It is shown that strangelets (finite number of strange quarks) with baryon number  $A \lesssim 10^3$  is the lowest quark droplets. For the observables, we obtain the masses and the charge-to-baryon number ratios of the strangelets. These quantities are compared with the observed data of the exotic particles.

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## I. INTRODUCTION

The strange matter, containing the  $u$ ,  $d$  and  $s$  quarks, has been considered to be the ground state of QCD at finite density [1–3], and expected to play an important role in the astrophysical phenomena such as the quark stars and the early universe [4]. It is also interesting that droplets of the strange matter (strangelets) could be a candidate of the dark matter. There is also a possibility to observe strangelets in the relativistic heavy ion collisions.

The physics of the quark matter is interesting in many respects. However, it is difficult to describe its properties directly from QCD. Until now, the stability of the strange matter has been discussed by using effective models of QCD. In the early stage, the MIT bag model was used with an assumption that the strange matter could be treated as a system of free fermi gas in a bag [1–3,5]. There, the strange matter becomes stable than the  $ud$  quark matter due to the large number of degrees of freedom by including the strangeness as the quark density increases. In these works, not only the bulk quark matter, but also the strangelet of finite size has also been studied. It was then shown that the strangelets could be more stable than the normal nuclei.

The MIT bag model has an advantage that it is a simple and useful model for various applications with quark confinement. However, we believe the existence of another important ingredient in QCD, which is the chiral symmetry and its spontaneous breaking [6]. It is known that the Nambu-Jona-Lasinio (NJL) model is one of the useful effective models for chiral symmetry breaking. It has been shown by using the NJL model that the strange matter cannot be absolutely stable [7–9]. The pattern of chiral symmetry restoration in the finite density quark matter is different for the  $ud$  quarks and the  $s$  quarks. The chiral symmetry for the  $ud$  quarks is sufficiently restored at stable densities ( $n_B \approx 2 - 3n_B^0$ ), while that for the  $s$  quarks is still largely broken. Then, the transition from the  $ud$  quark matter to the strange matter by weak process is disfavored because of the large dynamical quark mass in the strange quark sector. This result is qualitatively very different from

the results of the absolute stability obtained in the MIT bag model, in which only current quark masses are used.

Though the strange matter of infinite volume was discussed by taking into account dynamical chiral symmetry breaking [7–9], the strangelet with finite volume is not discussed yet. In this paper, we study the stability of the strangelet by considering dynamical chiral symmetry breaking. In Sec. II, we formulate the Lagrangian to describe dynamical chiral symmetry breaking supplemented by the confinement, which is treated approximately by a boundary condition. In Sec. III, numerical results are shown and we discuss the stability of the strangelet and present several observables. In Sec. IV, we conclude the present study and make discussions.

## II. FORMULATION

We consider a finite size system of a quark droplet, in which quarks are interacting through a four-point interaction of the NJL model. In order to incorporate a finite size system, first, we introduce the MIT bag boundary condition and prepare a set of quark wave functions. Using this basis set, we solve the Hartree equation with the NJL type correlation. Therefore, our model Lagrangian for the strangelet is given by [10,11]

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m^0)\psi + \frac{G}{2} \sum_{a=0}^8 [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] - \bar{\psi}M\psi\theta(r-R), \quad (1)$$

where  $\psi = (u, d, s)^t$  is the quark field and  $m^0 = \text{diag}(m_u^0, m_d^0, m_s^0)$  current quark mass matrix. The second term of Eq. (1) is the four-point quark interaction invariant under  $U(N_f)_L \times U(N_f)_R$  symmetry, in which  $\lambda^a$  ( $a = 0, \dots, 8$ ) are the Gell-Mann matrices normalized by  $\text{tr}\lambda^a\lambda^b = 2\delta^{ab}$ . In our formulation, we do not consider  $U(1)_A$  breaking for simplicity.

The last term in Eq. (1) has been used in the MIT bag model to impose quark confinement [12–14]. Assuming that the strangelet has a spherical shape, the step function  $\theta(r-R)$  is introduced, where  $R$  is the bag radius. That term represents a quark mass term with  $M$  for the exterior

region and quarks are confined in the region  $r < R$  by taking the limit  $M \rightarrow \infty$ . It is well known that the last term of Eq. (1) breaks chiral symmetry explicitly at the bag surface. In order to recover chiral symmetry there, we need to introduce the chiral field (pion) which is coupled to the quarks at the bag boundary. This leads to the condition of the chiral bag model, in which the pion cloud exists outside of the bag, and the vacuum structure is modified in the bag due to the pion-quark coupling which is known as the chiral Casimir effect [15]. We assume this pion cloud effect is small for large bag system considered in this paper.

The parameters in Eq. (1), such as the coupling constant  $G$ , a three dimensional momentum cut-off  $\Lambda$  (see Eq. (6) below) and the current masses  $m_u^0 = m_d^0$  and  $m_s^0$  are determined so as to reproduce the pion mass  $m_\pi = 0.139$  GeV, the pion decay constant  $f_\pi = 0.093$  GeV and the averaged mass of the nucleon and the delta  $m_{N+\Delta} = 1.134$  GeV. This mass is used to fix the dynamical quark mass  $m_u^*$  in the vacuum by  $m_{N+\Delta} \simeq 3m_u^*$ . We obtain the parameter set  $G\Lambda^2 = 4.7$ ,  $\Lambda = 0.6$  GeV and  $m_u^0 = m_d^0 = 5.9 \times 10^{-3}$  GeV. In this paper, we consider the current mass of the strange quark  $m_s^0$  as a free parameter. We show the results by setting  $m_s^0 = 0.1$  GeV. Other choices of  $m_s^0$  within a reasonable range do not affect our final conclusions.

Now, let us investigate chiral symmetry breaking in a quark bag. We assume that the quark bag contains sufficient number of quarks, where the following approximations will work. As usual, in the NJL interaction term in Eq. (1), we adopt the mean field approximation  $(\bar{q}q)^2 \rightarrow 2\bar{q}q\langle\bar{q}q\rangle - \langle\bar{q}q\rangle^2$ , where  $q = u, d$  and  $s$ , and solve the following gap equation

$$m_q = m_q^0 - 2G\langle\bar{q}q\rangle. \quad (2)$$

Here we need to solve this equation for a finite quark system in a spherical bag. This requires a treatment of quark states in the quark bag with discretized energy levels, which is rather complicated. In order to simplify the numerical calculations, first we take the mean field mass  $m_q$  as a constant value independent of the position of the quarks. Then, we introduce momentum integral with a density of states for the evaluation of  $\langle\bar{q}q\rangle$ , which is approximately obtained by the multiple reflection expansion (MRE) [5,16]. It is expressed by a smoothed function,

$$\rho_{MRE}(p, m, R) = 1 + \frac{6\pi^2}{pR} f_S(p/m) + \frac{12\pi^2}{(pR)^2} f_C(p/m) + \dots, \quad (3)$$

where  $p$  is the momentum,  $m$  the dynamical quark mass and  $R$  the radius of the quark bag. The second and third terms in Eq. (3) are the correction terms by the surface and curvature effects. The functions  $f_S$  and  $f_C$  are given by

$$f_S(x) = -\frac{1}{8\pi} \left(1 - \frac{2}{\pi} \arctan x\right), \quad (4)$$

$$f_C(x) = \frac{1}{12\pi^2} \left[1 - \frac{3x}{2} \left(\frac{\pi}{2} - \arctan x\right)\right].$$

In the limit  $m \rightarrow 0$ ,  $f_S$  and  $f_C$  become constants;

$$\lim_{m \rightarrow 0} f_S(p/m) = 0, \quad (5)$$

$$\lim_{m \rightarrow 0} f_C(p/m) = -\frac{1}{24\pi^2}.$$

By using the MRE method, the energy density  $\epsilon$  in a strangelet with a radius  $R$  is given as [11],

$$\epsilon = \sum_{q=u,d,s} \left[ \frac{(m_q - m_q^0)^2}{4G} - \nu \int_{p_q^f}^{\Lambda} \sqrt{p^2 + m_q^2} \rho_{MRE}(p, m_q, R) \frac{p^2 dp}{2\pi^2} \right] - \epsilon_0, \quad (6)$$

where the integral is modified by the density of state (3). Note that the dynamical quark mass  $m_q$  in Eq. (6) is determined self-consistently by Eq. (2) as stated below. In Eq. (6),  $\nu = N_{\text{spin}} \times N_{\text{color}} = 6$  is the degrees of degeneracy of spin and color, and  $\Lambda$  in the integral is a three dimensional momentum cutoff. The value  $p_q^f$  is the Fermi momentum which is determined by

$$\nu \int_0^{p_q^f} \rho_{MRE}(p, m_q, R) \frac{p^2 dp}{2\pi^2} = n_q, \quad (7)$$

for a given quark number density  $n_q$  for each flavor  $q = u, d$  and  $s$ . In Eq. (6), the last term  $\epsilon_0$  is the energy density in the chirally broken vacuum of infinite volume

$$\epsilon_0 = \sum_{q=u,d,s} \left[ \frac{(m_q^* - m_q^0)^2}{4G} - \nu \int_0^{\Lambda} \sqrt{p^2 + m_q^{*2}} \frac{p^2 dp}{2\pi^2} \right], \quad (8)$$

where  $m_q^*$  is the dynamical quark mass in the vacuum. Note that the energy density (6) is written as a sum of the kinetic energy of the valence quarks and the effective bag constant just as in the MIT bag model,

$$\epsilon = \sum_{q=u,d,s} \nu \int_0^{p_q^f} \sqrt{p^2 + m_q^2} \rho_{MRE}(p, m_q, R) \frac{p^2 dp}{2\pi^2} + B_{\text{eff}}, \quad (9)$$

where the effective bag constant is defined by

$$B_{\text{eff}} = \sum_{q=u,d,s} \left[ \frac{(m_q - m_q^0)^2}{4G} - \nu \int_0^{\Lambda} \sqrt{p^2 + m_q^2} \rho_{MRE}(p, m_q, R) \frac{p^2 dp}{2\pi^2} \right] - \epsilon_0. \quad (10)$$

Note that the effective bag constant  $B_{\text{eff}}$  depends on the quark density, which is different from the bag constant  $B$  in the MIT bag model. Then, by taking  $\partial\epsilon/\partial m_q = 0$ , the gap Eq. (2) is written as;

$$m_q = m_q^0 + 2G\nu \frac{\partial}{\partial m_q} \int_{p_q^F}^{\Lambda} \sqrt{p^2 + m_q^2} \rho_{MRE}(p, m_q, R) \times \frac{p^2 dp}{2\pi^2}. \quad (11)$$

We mention that the dynamical quark mass depends not only on the Fermi momentum, but also on the quark bag radius  $R$ . It is a characteristic feature of a finite size system.

Now, by the energy density (9), the total energy of the strangelet with a radius  $R$  is given by

$$E = \epsilon V + E_c - \frac{\alpha}{R}, \quad (12)$$

where  $V = (4\pi/3)R^3$  is the volume of the strangelet. By assuming a uniform charge distribution in the strangelet, the Coulomb energy  $E_c$  is given by

$$E_c \approx \frac{3}{5} \frac{e^2 Q^2}{R}, \quad (13)$$

where the total electric charge is given by  $Q = \frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s$  with  $N_q$  being the number of each quark,  $q = u, d$  and  $s$ . The last term of Eq. (12) is a phenomenological zero point energy of the bag. Physically, this expresses the subtraction of the center of mass motion of the finite system. We adopt  $\alpha \approx 2.04$  in the following calculation [17]. This term is negligible for strangelet of our interest with  $A \geq 100$ .

We obtain the energy of strangelet in the following way. First, we give a baryon number  $A$  and a strangeness fraction  $r_s = N_s/(N_u + N_d + N_s)$  with  $N_u = N_d$ . Then, for several radii  $R$ , we solve the gap Eq. (11) and obtain the dynamical quark mass  $m_q$  in the cavity, which is a function of the radius  $R$ . Then we find the minimum of the energy (12) with respect to the radius  $R$ .

We consider that the present mean field approximation approach should be valid for the baryon number  $A$  larger

than a certain value. This is because the MRE is valid when a sufficient number of states are contained. For smaller baryon number, we need to treat the discretized levels explicitly.

### III. NUMERICAL RESULT

#### A. Chiral restoration in a cavity

The confinement term in our model Lagrangian (1) is responsible for the effects of the finite volume of the strangelet. In this subsection, we investigate dynamical chiral symmetry breaking in an empty cavity without valence quarks.

In Fig. 1, we show the energy density (6) as a function of the dynamical quark mass in the empty cavity with several radii for  $ud$  and  $s$  quarks (dashed lines). For comparison, the energy density of the bulk vacuum without the boundary condition is also plotted in the same figure (solid lines). Minimum points of the energy density provide the dynamical quark masses. Concerning the  $ud$  quark sector in the bulk vacuum, we obtain the dynamical quark mass  $m_u = 0.378$  GeV. On the other hand, in the cavity, the dynamical quark masses are  $m_u = 0.322, 0.258$  and  $5.9 \times 10^{-3} (= m_u^0)$  GeV for  $R = 20, 11.5$  and  $8$  fm, respectively. We see that the dynamical quark mass becomes smaller as the radius decreases. This shows that chiral symmetry in the cavity tends to be restored. It is also true for the  $s$  quark sector. In the bulk vacuum, we obtain the dynamical quark mass  $m_s = 0.539$  GeV, while we find  $m_s = 0.361, 0.175$  and  $0.1 (= m_s^0)$  GeV for the radius  $R = 5.0, 3.1$  and  $2.5$  fm, respectively.

In Fig. 2, the dynamical quark mass is shown as a function of the radius of the cavity. The chiral symmetry in the  $ud$  quark sector is restored at  $R = 11.5$  fm, while that in the  $s$  quark sector is restored at  $R = 3.1$  fm. We see that the chiral restoration in the  $s$  quark is suppressed as compared with the  $ud$  quark. This is due to the large current mass of the  $s$  quark as compared with the  $ud$  quark. The difference in the tendency of the chiral symmetry restoration in the  $ud$  quark sector and the  $s$  quark sector

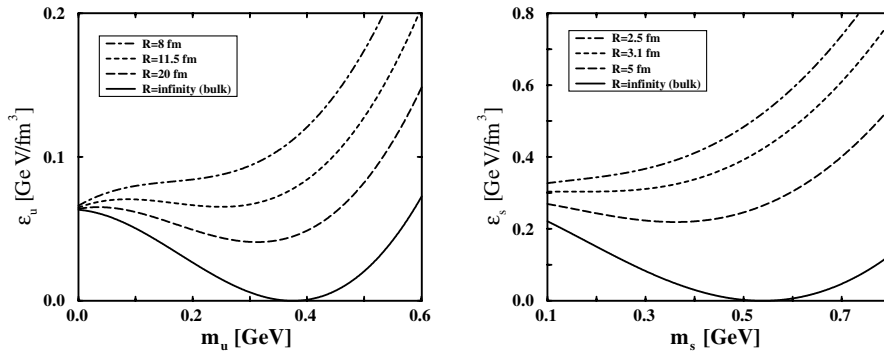


FIG. 1. The energy density  $\epsilon_q$  as a function of the dynamical quark mass  $m_q$  for various radius  $R$ . Left:  $u$  quark, right:  $s$  quark.

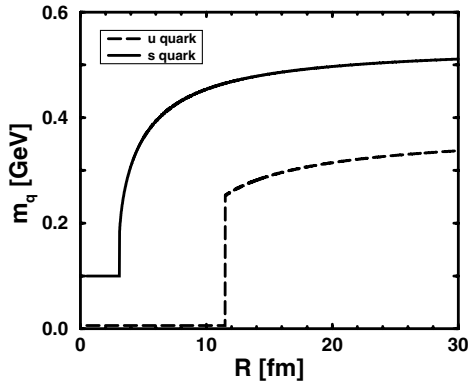


FIG. 2. The dynamical quark mass  $m_q$  of the  $u$  and  $s$  quarks as a function of the radius  $R$ .

causes the difference in the stability of the  $ud$  quark droplets and the strangelets.

### B. Stability of strangelet

Now we discuss the stability of the strangelet. For this purpose, we add  $3A$  valence quarks in the cavity, and calculate the energy per baryon number  $E/A$  for several baryon numbers  $A$  and the strangeness fractions  $r_s$ . In order to simplify the discussions, first, we fix the strangeness fraction  $r_s = 0$  for the  $ud$  quark droplets and  $r_s = 1/3$  for the strangelets, respectively. In the case of  $r_s = 1/3$ , the Coulomb energy of Eq. (13) vanishes. In this subsection, we turn off the Coulomb term.

The cavity radius  $R$  of a strangelet is determined by the variation of  $E/A$ . In Fig. 3(a), we show the energy per baryon number  $E/A$  as a function of the cavity radius  $R$  for the baryon numbers  $A = 10^2, 10^3$  and  $10^4$ . The minimum of  $E/A$  gives the energy and the radius of the strangelet. The existence of the minimum is understood in the following way. In the total energy (12) with Eq. (9), there are two terms: the kinetic energy from the valence quarks and the volume energy  $B_{\text{eff}}V$  from the effective bag constant. The

kinetic energy decreases, and the volume energy increases with the bag radius. Hence, we find the equilibrium radius at a certain radius, being balanced by the kinetic energy and the volume energy.

For example, for the baryon number  $A = 10^4$ , the energy per baryon number in the  $ud$  quark droplet and the strangelet are  $(E/A)_{ud} = 1.27$  GeV and  $(E/A)_{uds} = 1.33$  GeV, respectively. For the baryon number  $A = 10^2$ , the energy per baryon number in the  $ud$  quark droplet and the strangelet are  $(E/A)_{ud} = 1.60$  GeV and  $(E/A)_{uds} = 1.48$  GeV, respectively. In our results, the strangelets are more stable than the  $ud$  quark droplets for smaller baryon numbers  $A \lesssim 2 \times 10^3$ . The stability of the strangelets with small baryon numbers is very much different from the result for the bulk quark matter, where the strange matter with infinite volume is not absolutely stable [7–9]. This is because of the effect of the confinement leading to the restoration of chiral symmetry in the cavity. In order to show the restoration of chiral symmetry in the strangelets in Fig. 3(b), the dynamical quark masses  $m_u$  and  $m_s$  of the  $ud$  and  $s$  quarks are shown as functions of the cavity radius  $R$ . We see that chiral symmetry of the  $ud$  quark and  $s$  quark in the strangelet has a tendency to be restored for small radii as seen in the empty cavity.

In Fig. 4(a), we show explicitly the energy per baryon number  $E/A$  as a function of the baryon number  $A$  for  $ud$  quark droplets and strangelets. We show the results of the strangelets with the baryon number more than ten, which would be treated by the mean field approximation. In Fig. 4(a), it is shown that the strangelets are more stable than the  $ud$  quark droplets for the baryon number  $A \lesssim 2 \times 10^3$ . It is generally expected that the strange matter can be more stable than the  $ud$  quark matter, when the dynamical quark mass of  $s$  quark  $m_s$  is smaller than the Fermi energy  $\epsilon_{F,u}$  of the  $ud$  quark. When this relation is satisfied, the weak transition from  $ud$  quarks to  $s$  quarks can occur by the weak processes  $u \rightarrow d + e^+ + \nu_e$  and  $u + d \rightarrow u + s$ . In Fig. 4(b), we compare  $m_s$  and  $\epsilon_{F,u}$  in the  $ud$  quark droplets. It is shown that  $m_s < \epsilon_{F,u}$  is satisfied in the quark droplets with  $A \lesssim 2 \times 10^3$ .

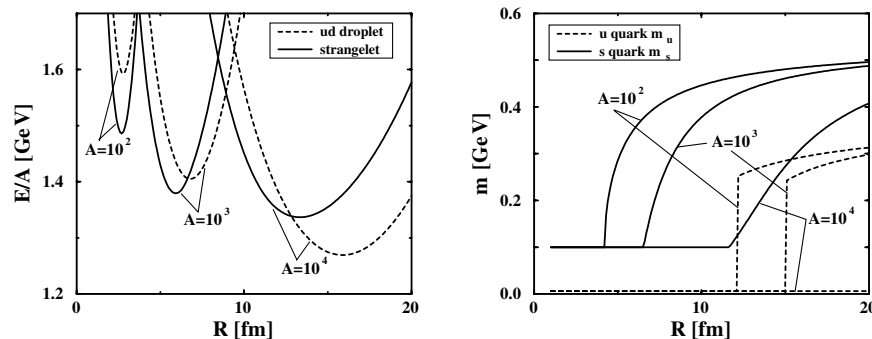


FIG. 3. Left: (a) The energy per baryon number  $E/A$  as a function of the radius  $R$  for the  $ud$  quark droplet ( $r_s = 0$ ) and the strangelet ( $r_s = 1/3$ ). Right: (b) The dynamical quark mass  $m_q$  of the  $ud$  and  $s$  quarks in the strangelets with  $r_s = 1/3$ .

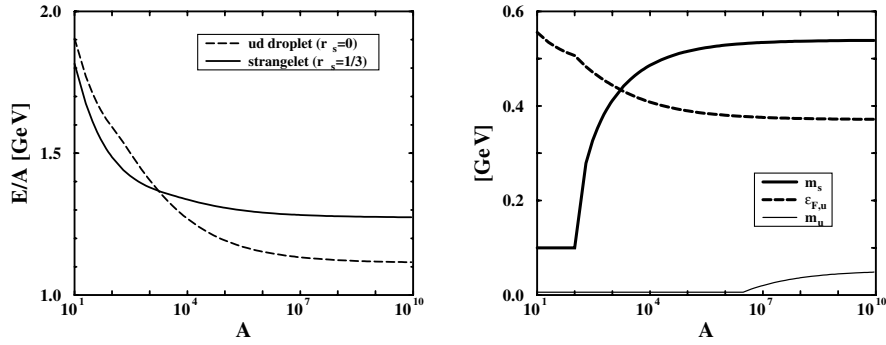


FIG. 4. Left: (a) The energy per baryon number  $E/A$  as a function of the baryon number  $A$ . The solid line for the strangelets ( $r_s = 1/3$ ) and the dashed line for the  $ud$  quark droplets ( $r_s = 0$ ). Right: (b) The dynamical quark mass  $m_u$  and  $m_s$  of the  $u$  and  $s$  quarks, and the Fermi energy  $\epsilon_{F,u}$  of the  $u$  quark in the  $ud$  quark droplets with  $r_s = 0$ .

**C. Observables**

When we consider that the strangelets are formed in the QCD phase transition in the early universe and/or in the explosions of the strange stars, the remaining strangelets could be observed in the cosmic rays. They would be observed as exotic particles with a large mass and a small electric charge. In order to identify such heavy particles, the charge-to-baryon number ratio is an important quantity. In this subsection, we discuss the observables of the strangelets. In order to give a realistic discussion, we switch on the Coulomb term (13).

In the previous subsection, we have fixed the ratio of  $s$  quarks to be  $r_s = 0$  and  $1/3$ . In the following, we consider a variation of the energy (12) with respect to  $r_s$ . When the Coulomb term is switched on, we obtain a fraction number of each flavor due to competition of the Fermi energy and the Coulomb energy. The resulting energy per baryon number  $E/A$  is plotted in Fig. 5. We also show the result without the Coulomb term, which has been discussed in the previous subsection. Here, we see that the strangelets with  $A \lesssim 2 \times 10^3$  are not affected by the Coulomb energy, since the strangelets are almost neutral objects. On the other

hand, the  $ud$  quark droplets with  $A \gtrsim 2 \times 10^3$  obtain some energy by the Coulomb repulsion.

We discuss quark number fraction in the quark droplets. The number fraction  $r_u = N_u/3A$ ,  $r_d = N_d/3A$  and  $r_s$  for  $u$ ,  $d$  and  $s$  quarks are plotted as a function of the baryon number  $A$  in Fig. 6. For the baryon number  $A \lesssim 2 \times 10^3$ , we see that each quark number fraction is close to the number  $1/3$ . There, the Coulomb energy is a dominant term. Indeed, we see that the  $ud$  quark droplets with  $A \gtrsim 10^3$  have number fraction of  $r_u \approx 1/3$  and  $r_d \approx 2/3$  to minimize the Coulomb energy.

Once the number fraction  $r_q$  ( $q = u, d$  and  $s$ ) is obtained, the number of quarks  $N_q$  are also obtained. Then, the electric charge  $Q$  of the strangelets are calculated from  $Q = \frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s$ . In Fig. 6, we show also the charge-to-baryon number ratio  $Q/A$  as a function of the baryon number  $A$ . The electric charge of the strangelet is a few percents of that of the normal nuclei. Such strangelets can have large baryon number, since the Coulomb energy is negligible. Thus, our results show that the strangelets would be exotic particles with small charge-to-baryon number ratio as compared with the normal nuclei.

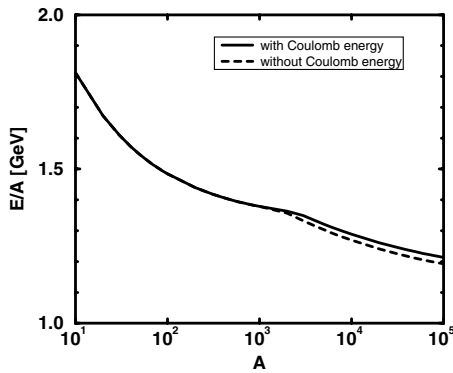


FIG. 5. The energy per baryon number  $E/A$  of quark droplets with the Coulomb energy. The result of no Coulomb energy is also plotted for comparison.

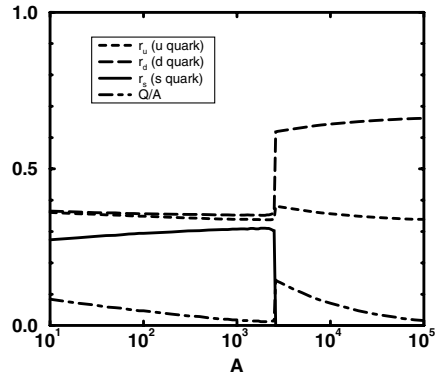


FIG. 6. The quark number fraction  $r_q$  ( $q = u, d$  and  $s$ ) and the electric charge  $Q/A$  of the quark droplets as a function of the baryon number  $A$ .

TABLE I. The baryon number  $A$  and the charge-to-baryon number  $Q/A$  from cosmic ray experiments. See also [4].

Baryon number $A$	Charge-to-baryon number $Q/A$	
$A \sim 350 - 450$	$0.03 - 0.04$	[18]
$A \sim 460$	$0.043$	[19]
$A > 1000$	$0.046$	[20]
$A \sim 370$	$0.038$	[21,22]

Let us compare our theoretical results and the existing data which was reported in the observation of exotic particles in the cosmic rays. We show the baryon number and the charge-to-baryon number ratio of the observed particles in Table I. The baryon numbers  $A$  of these exotic particles are from the order of hundreds to thousands. The charge-to-baryon number  $Q/A$  is around 0.04. It is interesting that these observed values are very close to our theoretical results. These exotic particles are candidates of strangelets.

In Fig. 7, we show the radius  $R$  of the strangelet and the  $ud$  quark droplet as a function of the baryon number  $A$ . The relation between  $R$  and  $A$  is expressed approximately by

$$R = r_0 A^{1/3}, \quad (14)$$

with some constant  $r_0$ . We obtain  $r_0 \simeq 0.57$  fm and 0.76 fm for the strangelets and the  $ud$  quark droplets, respectively. The baryon number density is estimated as  $n_B = A/(4\pi R^3/3) = 7.6n_B^0$  and  $3.2n_B^0$  for the strangelets and the  $ud$  quark droplets, respectively. Here, we use the value of the normal nuclear matter density  $n_B^0 = 0.17$  fm $^{-3}$ . The strangelets and the  $ud$  quark droplets are extremely compact objects.

So far, we have not included electrons in our discussion. We show that the electrons play only minor role in the strangelets. From Fig. 6, the electric charge of the strangelets are at most  $Q \simeq 52$  for  $A = 2 \times 10^3$ , due to small  $Q/A \simeq 0.026$ . Assuming an electron around the strangelets with such electric charge, the de Broglie wave length of the electron is estimated by the energy of the electron  $E_e = \sqrt{p^2 + m_e^2} - e^2 Q/r$ , where  $p$  is the momentum of the

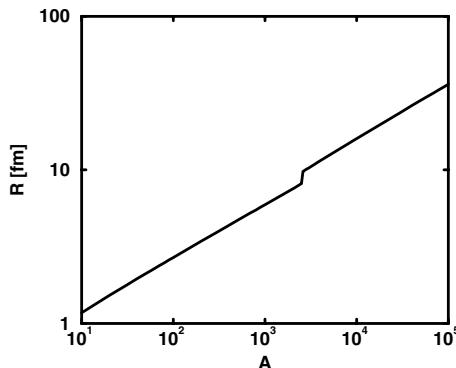


FIG. 7. The radius  $R$  of the strangelet as a function of the baryon number  $A$ .

electron,  $m_e = 0.51$  MeV the mass of the electron and  $r$  the distance from the strangelet. Then, we find that the de Broglie wave length of the electron is  $\lambda \simeq 940$  fm for  $A = 2 \times 10^3$ . That is much larger than the radius of the strangelet, which is shown in Fig. 7. Therefore, the electrons exist far outside of the strangelet. If the size of the strangelets could be larger than the electron de Broglie wave length, we need to consider the electrons in the strangelets.

#### IV. CONCLUSION

We have discussed the structure of the strangelet by considering dynamical chiral symmetry breaking. We have used the NJL interaction for chiral symmetry breaking. In addition, we have incorporated the spherical cavity for the confinement of quarks by the MIT bag boundary condition. In the mean field approximation in the finite volume system, we have obtained the gap equation for the dynamical generation of the quark mass. Then, we have obtained the energy of the strangelets.

As a result, it is shown that the chiral symmetry is restored in the cavity at small radii. The dynamical quark mass becomes small as compared with that in the vacuum of infinite volume, and the strange quark mass can be smaller than the Fermi energy of the  $ud$  quarks in the droplet. We have investigated the stability of the strangelet for several baryon number  $A$  and the strangeness fraction  $r_s = 0$  and  $1/3$ . It is shown that the strangelets are more stable than the  $ud$  quark droplets for the baryon number  $A \lesssim 2 \times 10^3$  for the  $s$  quark current mass  $m_s^0 = 0.1$  GeV. Our result does not change qualitatively for the case of  $m_s^0 = 0.18$  GeV, in which we obtain the stable strangelets for  $A \lesssim 0.5 \times 10^3$ . We obtain the charge-to-baryon number of the strangelets, which is consistent with the experimental data, reported to be observed in the cosmic rays.

In the present analysis, we obtain  $E/A$  larger than that of ordinary nuclei for all baryon number  $A$ . This might cause a transition of the strangelet to ordinary nuclei and/or hyper nuclei. The transition to nonstrange quark matter and ordinary nuclei, however, will be suppressed, because the process should be accompanied by a weak process with a large number of strangeness change. On the other hand, the decay of a strangelet to hadrons with strangeness is possible in the present results on the masses of quark droplets. We do believe at this moment, however, we cannot compare our results of the quark droplets with actual hadrons and nuclei, since we ought to include several correlations in quark droplets to make a careful study of hadrons and mesons in the present model. For instance, the color magnetic interaction reduces the energy of the strangelet. Furthermore, the color pairing correlation and even color flavor locking effects in the quark droplets may affect the energy of the strangelet [23–25]. It is also interesting to discuss the effect of the meson clouds in the strangelets with small baryon number which are expected to be observed in the experiments of heavy ion collisions.

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