Two photon decay of neutral scalars below 1.5 GeV in a chiral model for *qq* **and** *qqqq* **states**

Simón Rodríguez and Mauro Napsuciale

Instituto de Fı´sica, Universidad de Guanajuato,

Lomas Del Bosque 103, Fraccionamiento Lomas del Campestre, 37150, Leo´n, Guanajuato, Me´xico

(Received 18 November 2004; published 14 April 2005)

We study the two photon decay of neutral scalars below 1.5 GeV in the context of a recently proposed chiral model for $\overline{q}q$ and $\overline{q}qqq$ states. We find good agreement with experimental results for the $a_0(980) \rightarrow$ $\gamma\gamma$ decay. Our calculations for $f_0(980) \to \gamma\gamma$ shows that further work is necessary in order to understand the structure of this meson. The model predicts $\Gamma(a_0(1450) \to \gamma \gamma) = 0.16 \pm 0.10 \text{ KeV}, \Gamma(\sigma \to \gamma \gamma) =$ $(0-1.13)$ KeV, $\Gamma(f_0(1370) \to \gamma \gamma) = (0-0.22)$ KeV, $\Gamma(f_0(1500) \to \gamma \gamma) = (0-1.51)$ KeV.

DOI: 10.1103/PhysRevD.71.074008 PACS numbers: 12.39.Fe

I. INTRODUCTION

The understanding of the lowest lying scalar mesons remains as one of the most challenging problems in low energy QCD. Over the past years, experimental evidence has accumulated for the existence of light scalar mesons [1,2], but there is still an intense debate about the structure of these hadrons. In spite of the fact that a light isoscalar scalar, the so-called sigma meson, was predicted by unitarized models [1] and effective models [2,3], it reentered the Particle Data Group classification of particles until the 2002 edition as $f_0(600)$ [4]. After an intense activity on the experimental side over the past few years, compelling evidence has been accumulated for the existence of this state and its mass has been measured [5]. Nowadays, we can safely say that we have a universal acceptance for the existence of enhancements at low energy in the *s*-wave isoscalar meson-meson scattering, but the interpretation of this phenomena is still controversial. Concerning the isospinor channel, a resonance with a mass around 900 MeV has been predicted by unitarized quark models [3], effective models [6], and unitarized chiral perturbation theory calculations [7]. The $K\pi$ s-wave data has been reanalyzed, finding a relatively strong attraction in the $I = 1/2$ channel [8]. This enhancement in the isoscalar channel of $K\pi$ scattering is identified with an *S*-matrix pole at approximately 900 MeV [κ (900)]. More recently, the existence of this state was claimed to be necessary in order to explain the mass distribution data in $D^+ \rightarrow K^- \pi^+ \pi^+$ [9]. It must be mentioned, however, that there are objections to the existence of this state [10].

There are essentially two proposals for the structure of light scalar mesons: a $q\overline{q}$ structure and a $\overline{q}q\overline{q}q\overline{q}$ one. In the latter case, there are at least three possible dynamical configurations: a meson-meson molecule, a diquarkdiquark state, and a compact genuine $\overline{q\overline{q}qq}$ state. In addition, there exists the possibility that isosinglet scalars contain a certain amount of glueball. Most recent work has been dedicated to understanding the properties of light scalars in light of these pictures, explored in different formalisms.

Since photons couple to charge, electromagnetic decays of mesons have been intensively used in the past to obtain information on their structure. As for scalar mesons, there exist calculations for the $a_0(980)$, $f_0(980) \rightarrow 2\gamma$ decays using a variety of approaches [11–14], in particular, in different versions of the quark model [11,15], with very different results depending on the details of the model. The generally accepted conclusion is that the measured $a_0(980)$, $f_0(980) \rightarrow \gamma \gamma$ decay widths [16] are not consistent with a $q\bar{q}$ structure. In light of these results, other possibilities for the structure of light scalars such as a molecule picture [12] and a $\overline{qq}q\overline{q}$ structure [14] were explored.

More recently, some tools to determine the glueball content of mesons from their branching fractions in radiative quarkonium decays and production cross sections in $\gamma\gamma$ collisions were developed in Ref. [17]. Also, the two photon decay of the lightest scalar, $f_0(600)$ or σ , has been studied using different formalisms [18–20] yielding very disparate predictions. The possibility that this meson has a large glueball content was analyzed in Ref. [21] using the two photon decay channel. As shown in this work, the extraction of the two photon coupling of light isoscalars from data on $\gamma \gamma \rightarrow \pi \pi$ is not straightforward and requires a careful analysis in order to get reliable results.

The two photon decay of $a_0(980)$ and $f_0(980)$ mesons was also studied in Ref. [22] in the framework of a chiral theory involving scalars and a linear realization of chiral symmetry, a linear sigma model ($L_{\sigma}M$), where the key interaction is the one violating $U_A(1)$ symmetry [23]. This interaction is assumed to be the manifestation at the hadron level of the effective six-quark interaction (for $N_F = 3$) induced by instantons [24]. In this concern, this interaction identifies the fields entering the model as $q\overline{q}$ states. In this formalism, the two photon decay of neutral scalars is induced at the one-loop level. The scalars decay first into two (real or virtual) charged mesons which annihilate into two photons. As to the information on the internal structure of scalars which can be inferred in this calculation, we must stress that a naive estimation of the distances explored by the photons in this decay is of the order of

FIG. 1. Charged meson loop contributions to $S \to \gamma \gamma$.

 $d \approx 1/k = 2/m_S = 0.4$ fm, which is of the same order as the kaon charge radius; thus, the effective degrees of freedom seen by the photons are actually mesons rather than quarks. In this sense, we can infer that the decaying mesons are $q\overline{q}$ states, but this bare state has large quantum fluctuations into meson-meson states whose exact amount is difficult to quantify.

Recently, we pointed out the existence of a quasidegenerate chiral nonet in the energy region around 1.4 GeV and studied the possibility that mesons below 1.5 GeV arise as admixtures of normal $q\bar{q}$ and $\bar{q}q\bar{q}q\bar{q}$ states, with the latter lying at its natural scale as dictated by the linear rising of meson masses with the number of constituent quarks [25,26]. This model has the nice feature of reducing to the one explored in Ref. [23] in the case when we decouple the $\overline{q\overline{q}}q\overline{q}$ states.

In this work, we explore the predictions of the model presented in Refs. [25,26] for the two photon decay widths of all neutral scalars below 1.5 GeV.

II. MESON LOOP CONTRIBUTIONS TO $S \rightarrow \gamma \gamma$

The most general form for the $S(p) \to \gamma(k, \epsilon) \gamma(q, \eta)$ transition amplitude is dictated by Lorentz covariance and gauge invariance as

$$
\mathcal{M}(S \to \gamma(k, \epsilon)\gamma(q, \eta))
$$

=
$$
\frac{i\alpha}{\pi f_K} V^S(g^{\mu\nu}k \cdot q - k^{\mu}q^{\nu})\eta_{\mu}\epsilon^{\nu},
$$
 (1)

where f_K denotes the kaon weak decay constant and α denotes the electromagnetic fine constant. The factors α , π , f_K in Eq. (1) are introduced for convenience in future manipulations.

In the effective theory formulated in Refs. [25,26], the two photon decays of neutral scalar mesons below 1.5 GeV are induced by loops of charged mesons as depicted in Fig. 1. A straightforward calculation yields

$$
V_M^S = \frac{2f_{K}g_{SMM}}{m_S^2} \bigg[-\frac{1}{2} + \xi_M^S I(\xi_M^S) \bigg],
$$
 (2)

where $\xi_M^S = m_M^2/m_S^2$, g_{SMM} denotes the coupling constant of the decaying scalar *S* to the meson pair (M^+M^-) in the loops, and $I(x)$ denotes the loop integral

$$
I(x) = \begin{cases} 2\left(\arcsin\sqrt{\frac{1}{4x}}\right)^2 & x > \frac{1}{4} \\ 2\left[\frac{\pi}{2} + i\ln\left(\sqrt{\frac{1}{4x}} + \sqrt{\frac{1}{4x}} - 1\right)\right]^2 & x < \frac{1}{4} \end{cases}
$$
(3)

The decay width is given by

$$
\Gamma(S \to \gamma \gamma) = \frac{\alpha^2}{64\pi^3} \frac{m_S^3}{f_K^2} |V^S|^2. \tag{4}
$$

The experimental data for $a_0(980)$, $f_0(980)$ decay into two photons is [16]

$$
\Gamma(a_0(980) \to \gamma \gamma) \times BR(a_0(980) \to \pi^0 \eta) = 0.24^{+0.08}_{-0.07} \text{ KeV},
$$

$$
\Gamma(f_0(980) \to \gamma \gamma) = 0.39^{+0.10}_{-0.13} \text{ KeV},
$$

and, assuming that the decay $a_0(980) \rightarrow \pi \eta$ is the dominant mode $\left[BR(a_0(980) \to \pi^0 \eta \right) \simeq 1$, we obtain the form factors $|V^{a_0(980)}|$ and $|V^{f_0(980)}|$ at $p^2 = m_S^2$ as

$$
|V_{Exp}^{a_0(980)}| = 0.34 \pm 0.05, \qquad |V_{Exp}^{f_0(980)}| = 0.44 \pm 0.07.
$$

There exists no confident experimental information for the two photon decays of $a_0(1450)$, $f_0(600)$, $f_0(1370)$, and $f_0(1500)$ mesons [16].

III. TWO PHOTON DECAY OF $a_0(980)$ **AND** $a_0(1450)$

For the $a_0(980) \rightarrow \gamma \gamma$ and $a_0(1450) \rightarrow \gamma \gamma$ decays, the model in Refs. [25,26] yields contributions coming from *K*, κ (900) and their respective heavy "partners" *K*(1469) and $K_0^*(1430)$ in the loops. For the sake of simplicity, we denote these mesons as K , κ , \hat{K} , and $\hat{\kappa}$ respectively. The couplings entering in the loops were calculated in Ref. [26] and are listed in Table I; we refer the reader to Ref. [26] for details of the notation.

It is worth noticing that the coupling of light scalars to light pseudoscalars quoted above reduce to the ones obtained in the linear sigma model [23] when we decouple the heavy fields. The two photon decays are, of course, dominated by light meson in the loops, heavy meson contributions being suppressed by powers of the corresponding inverse mass. The values extracted in Ref. [25] for the mixing angles entering these expressions are

TABLE I. Predictions of the chiral model for $\overline{q}q$ and $\overline{q}qqq$ states for the SP^+P^- and $SS^+S^$ couplings entering the loops in $a_0 \rightarrow \gamma \gamma$ decays.

\blacksquare \blacksquare ັ	
$g_{a_0(980)K^+K^-}$	$-\frac{1}{\sqrt{2}(a+b)}(m_K^2+m_K^2-m_a^2-m_A^2+\hat{\mu}_1^2-\hat{\mu}_{1/2}^2)\cos\phi_1\cos^2\theta_{1/2}$
$g_{a_0(980)\kappa^+\kappa^-}$	$\frac{1}{\sqrt{2(b-a)}}(m_{\kappa}^2 + m_{\kappa}^2 - m_a^2 - m_A^2 + \hat{\mu}_1^2 - \hat{\mu}_{1/2}^2)\cos\phi_1\cos^2\phi_{1/2}$
$g_{a_0(1450)K^+K^-}$	$-\frac{1}{\sqrt{2}(a+b)}(m_K^2 + m_{\hat{K}}^2 - m_a^2 - m_A^2 + \hat{\mu}_1^2 - \hat{\mu}_{1/2}^2)\sin\phi_1\cos^2\theta_{1/2}$
$g_{a_0(1450)\kappa^+\kappa^-}$	$\frac{1}{\sqrt{2(b-a)}}(m_{\kappa}^2 + m_{\kappa}^2 - m_a^2 - m_A^2 + \hat{\mu}_1^2 - \hat{\mu}_{1/2}^2) \sin \phi_1 \cos^2 \phi_{1/2}$
$g_{a_0(980)\hat{K}^+\hat{K}^-}$	$-\frac{1}{\sqrt{2}(a+b)}(m_K^2 + m_{\hat{K}}^2 - m_a^2 - m_A^2 + \hat{\mu}_1^2 - \hat{\mu}_{1/2}^2)\cos\phi_1\sin^2\theta_{1/2}$
$g_{a_0(980)\hat{\kappa}^+\hat{\kappa}^-}$	$\frac{1}{\sqrt{2}(b-a)}(m_{\kappa}^2 + m_{\kappa}^2 - m_a^2 - m_A^2 + \hat{\mu}_1^2 - \hat{\mu}_{1/2}^2)\cos\phi_1\sin^2\phi_{1/2}$
$g_{a_0(1450)\hat{K}^+\hat{K}^-}$	$-\frac{1}{\sqrt{2}(a+b)}(m_K^2 + m_{\hat{K}}^2 - m_a^2 - m_A^2 + \hat{\mu}_1^2 - \hat{\mu}_{1/2}^2) \sin \phi_1 \sin^2 \theta_{1/2}$
$g_{a_0(1450)\hat{\kappa}^+\hat{\kappa}^-}$	$\frac{1}{\sqrt{2}(b-a)}(m_{\kappa}^2 + m_{\kappa}^2 - m_a^2 - m_A^2 + \hat{\mu}_1^2 - \hat{\mu}_{1/2}^2)\sin\phi_1\sin^2\phi_{1/2}$

Contr.		К. к	K, κ, \hat{K}	$K, \kappa, \hat{K}, \hat{\kappa}$
$ V^{a_0(980)} $	0.33 ± 0.08	0.32 ± 0.08	0.32 ± 0.08	0.32 ± 0.08
$ V^{a_0(1450)} $	0.15 ± 0.05	0.15 ± 0.05	0.15 ± 0.05	0.15 ± 0.05

TABLE III. Comparison of the results of the present model with those of Ref. [22] and experimental results.

Form factor	LσM	This model	Exp.
$ V^{a_0(980)} $	0.35 ± 0.04	0.32 ± 0.08	0.34 ± 0.05
$ V^{a_0(1450)} $	\cdots	0.15 ± 0.05	\cdots

TABLE IV. Predictions of the chiral model for $\overline{q}q$ and $\overline{q}qqq$ states for the SP^+P^- and $SS^+S^$ couplings entering the loops in $f_0 \rightarrow \gamma \gamma$ decays. \overline{a}

TABLE V. Predictions of the L σM [22] for the form factor describing $f_0(980) \rightarrow \gamma \gamma$ revisited.

Contr.		K, π	K, π, κ
$ V^{f_0(980)} _{\rm I}$ $L\sigma M$	0.45 ± 0.12	0.78 ± 0.20	0.90 ± 0.14

TABLE VI. Loop contributions to the form factors describing $f_0 \rightarrow \gamma \gamma$ decays.

The vacuum expectation values *a; b* are related to the weak decay constants by

$$
a = \frac{f_{\pi}}{\sqrt{2}\cos(\theta_1)}, \qquad a + b = \frac{\sqrt{2}f_K}{\cos(\theta_{1/2})}.
$$
 (5)

Using these values and the masses quoted in Ref. [16], we obtain the results listed in Table II for the form factors. We include the contributions of different mesons in the loops step by step in order to clearly exhibit the corresponding effects.

As expected, the main contribution comes from kaon loops due to its relatively light mass. These results are in good agreement with the world average in the case of the two photon decay of $a_0(980)$. The modifications introduced by the mixing between $\overline{q}q$ and $\overline{q}qqq$ fields to the picture in the conventional (updated) linear sigma model [22] are also exhibited in Table III.

The form factors predicted by the chiral model for $\overline{q}q$ and $\overline{q\overline{q}q}q$ states listed in Table II give the following decay widths:

$$
\Gamma(a_0(980) \to \gamma \gamma) = 0.25 \pm 0.12 \text{ KeV}, \tag{6}
$$

$$
\Gamma(a_0(1450) \to \gamma \gamma) = 0.16 \pm 0.10 \text{ KeV.} \tag{7}
$$

IV. TWO PHOTON DECAY OF ISOSINGLET SCALARS

Next we work out the predictions of the model for $f_0(600)$ (or σ), $f_0(980)$, $f_0(1370)$, and $f_0(1500)$ decay into two photons. In this case, we expect the main contribution to come from π , K, and κ meson loops. In Table IV we list the couplings involved in these processes as arising from the chiral model for $\overline{q}q$ and $\overline{q}qqq$ states.

Again, couplings for light scalars to light pseudoscalars listed in Table IV reduce to the linear sigma model ones when we decouple heavy fields. In this respect, we would

TABLE VII. Predictions of the chiral model for $\overline{q}q$ and $\overline{q}qq$ states for the decay widths of $S \to \gamma\gamma$ and comparison with other predictions in the literature.

Process	Width (KeV) this work	Width (KeV) other work	Comments	Exp. (KeV)
$\sigma \rightarrow \gamma \gamma$	$(0-1.13)$	0.024 ± 0.023 [18]	VMD1	
		0.38 ± 0.09 [18]	VMD ₂	
		123 [20]	$m_{\sigma} = 1$ GeV	
		3.8 ± 1.5 [21]		
$a_0(980) \rightarrow \gamma \gamma$	0.25 ± 0.12	6 [15]	$q\overline{q}$	$0.24^{+0.08}_{-0.07}$
		$(2-5) \times 10^{-4}$ [28]	\overline{q} \overline{q} qq	
		0.19 [29]	$q\overline{q}$	
$f_0(980) \rightarrow \gamma \gamma$	0.92 ± 0.40	0.34 ± 0.21 [19]	φ _s = 18° \pm 2°	$0.39_{-0.13}^{+0.10}$
		0.51 ± 0.23 [19]	φ _c = -18° \pm 2°	
		$0.28_{-0.13}^{+0.09}$ [21]		
		0.36 [29]	$q\overline{q}$	
$f_0(1370) \rightarrow \gamma \gamma$	$(0-0.22)$	3.21 ± 1.26 [19]	$\varphi'_{s} = 18^{\circ} \pm 2^{\circ}$	
		7.71 ± 2.70 [19]	$\varphi'_{s} = -18^{\circ} \pm 2^{\circ}$	
$a_0(1450) \rightarrow \gamma \gamma$	0.16 ± 0.10			
$f_0(1500) \rightarrow \gamma \gamma$	$(0-1.51)$			

like to note that the naive mixing factor used in Ref. [22] for $g_{f_0(980)\kappa^+\kappa^-}$ contains an incorrect minus sign [27]. Even though contributions coming from charged κ loops are suppressed, they are crucial in the understanding of the two photon decay of $f_0(980)$. Indeed, the sign assumed in Ref. [22] drives the result arising from kaon and pion loops in the right direction to match experiment. We recalculated this decay in the $L_{\sigma}M$ corrected for this sign, finding a κ contribution in the opposite direction. The different contributions to this form factor as obtained in the $L_{\sigma}M$ are quoted in Table V.

Using the couplings listed in Table IV, we obtain the predictions of the chiral model for $\overline{q}q$ and $\overline{q}qqq$ states for the different contributions to the form factors describing the two photon decay of $f_0(600)$, $f_0(980)$, $f_0(1370)$, and $f_0(1500)$ shown in Table VI.

These form factors yield the decay widths quoted in Table VII, where we collect our results and compare with other results in the literature.

V. CONCLUSIONS

In this paper, we work out the predictions of the chiral model for $\overline{q}q$ and $\overline{q}qqq$ states [25,26] for the two photon decay of all neutral scalars below 1.5 GeV. Except for $a_0(980)$ and $f_0(980)$, there is no experimental information on these decays. The predictions of the model for the

 $a_0(980) \rightarrow \gamma \gamma$ decay are in good agreement with experiment. As for $f_0(980) \rightarrow \gamma \gamma$, we recalculate and update the linear sigma model predictions for this decay, finding a discrepancy with the experimental results. The situation is improved by the mixing inherent to the chiral model for $\overline{q}q$ and $\overline{q\overline{q}q}q$ states [25,26]. Nevertheless, on the one side, the extraction of the width from experimental data is not an easy task as shown in Ref. [21]; thus, the world average quoted in Ref. [16] should be taken with some care; on the other side, we expect $f_0(980)$ to arise actually as a mixing of $\overline{q}q$, $\overline{q}qqq$, and glueball. The latter has not been included in the model under consideration, and, according to recent analysis based on chiral symmetry, $f_0(980)$ can acquire some component along the glueball direction [30,31]; thus, we expect modifications to the present picture upon the inclusion of glueball degrees of freedom. Finally, the model gives definite predictions for the two photon decays of $f_0(600)$, $f_0(1370)$, and $f_0(1500)$. The small decay width of the latter two mesons into two photons, even if they are composed of quarks in this model, is particularly interesting. The small width of these mesons has been usually argued as the signal for a large glueball component.

ACKNOWLEDGMENTS

This work was supported by Conacyt- México under Project No. 37234-E.

- [1] N. A. Tornqvist, Phys. Rev. Lett. **49**, 624 (1982); Z. Phys. C **68**, 647 (1995); N. A. Tornqvist and M. Roos, Phys. Rev. Lett. **76**, 1575 (1996).
- [2] M. D. Scadron, Phys. Rev. D **26**, 239 (1982); F. Sannino and J. Schechter, Phys. Rev. D **52**, 96 (1995); M. Harada, F. Sannino, and J. Schechter, Phys. Rev. D **54**, 1991 (1996); D. Black, A. H. Fariborz, F. Sannino, and J. Schechter, Phys. Rev. D **58**, 054012 (1998); J. A. Oller and E. Oset, Nucl. Phys. **A620**, 438 (1997); **A652**, 407(E) (1999); L. Lesniak, hep-ph/9807539.
- [3] E. Van Beveren, T. A. Rijken, K. Metzger, C. Dullemond, G. Rupp, and J. E. Ribeiro, Z. Phys. C **30**, 615 (1986).
- [4] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
- [5] E791 Collaboration, E. M. Aitala *et al.*, Phys. Rev. Lett. **86**, 770 (2001); **86**, 765 (2001).
- [6] S. Ishida, M. Ishida, H. Takahashi, T. Ishida, K. Takamatsu, and T. Tsuru, Prog. Theor. Phys. **95**, 745 (1996); M. Napsuciale, hep-ph/9803396.
- [7] J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. Lett. **80**, 3452 (1998); Phys. Rev. D **59**, 074001 (1999); **60**, 099906(E) (1999); J. A. Oller and E. Oset, Phys. Rev. D **60**, 074023 (1999).
- [8] S. Ishida, M. Ishida, T. Ishida, K. Takamatsu, and T. Tsuru, Prog. Theor. Phys. **98**, 621 (1997); D. Black, A. H.

Fariborz, F. Sannino, and J. Schechter, Phys. Rev. D **58**, 054012 (1998).

- [9] E791 Collaboration, E. M. Aitala *et al.*, Phys. Rev. Lett. **89**, 121801 (2002).
- [10] S. N. Cherry and M. R. Pennington, Nucl. Phys. **A688**, 823 (2001).
- [11] A. Bramon and M. Greco, Lett. Nuovo Cimento Soc. Ital. Fis. **2**, 522 (1971); S. B. Berger and B. T. Feld, Phys. Rev. D **8**, 3875 (1973); S. Eliezer, J. Phys. G **1**, 701 (1975); J. Babcoc and J. L. Rosner, Phys. Rev. D **14**, 1286 (1976); M. Budnev and A. E. Kaloshin, Phys. Lett. **86B**, 351 (1979); N. N. Achasov, S. A. Denayin, and G. N. Shestakov, Z. Phys. C **16**, 55 (1982); E. P. Shabalin, JETP Lett. **42**, 135 (1985); Z. P. Li, F. E. Close, and T. Barnes, Phys. Rev. D **43**, 2161 (1991).
- [12] J. D. Weinstein and N. Isgur, Phys. Rev. D **27**, 588 (1983).
- [13] T. Barnes, Phys. Lett. **165B**, 434 (1985).
- [14] N.N. Achasov, S.A. Devyanin, and G.N. Shestakov, Phys. Lett. B **108**, 134 (1982); **108**, 435(E) (1982); E. P. Shabalin, Yad. Fiz. **46**, 852 (1987) [Sov. J. Nucl. Phys. **46**, 486 (1987)]; N. N. Achasov and G. N. Shestakov, Z. Phys. C **41**, 309 (1988); N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. **B315**, 465 (1989).
- [15] C. A. Dominguez and N. Paver, Z. Phys. C **39**, 39 (1988).
- [16] Particle Data Group, S. Eidelman *et al.*, Phys. Lett. B **592**,

1 (2004).

- [17] F. E. Close, G. R. Farrar, and Z.-p. Li, Phys. Rev. D **55**, 5749 (1997).
- [18] D. Black, M. Harada, and J. Schechter, Phys. Rev. Lett. **88**, 181603 (2002).
- [19] F. Kleefeld, E. van Beveren, G. Rupp, and M. D. Scadron, Phys. Rev. D **66**, 034007 (2002).
- [20] M.P. Rekalo and E. Tomasi-Gustafsson, Nucl. Phys. **A725**, 116 (2003).
- [21] M. Boglione and M. R. Pennington, Eur. Phys. J. C **9**, 11 (1999).
- [22] J. L. Lucio Martinez and M. Napsuciale, Phys. Lett. B **454**, 365 (1999).
- [23] M. Napsuciale, hep-ph/9803396; M. Napsuciale and S. Rodriguez, Int. J. Mod. Phys. A **16**, 3011 (2001); N. A. Tornqvist, Eur. Phys. J. C **11**, 359 (1999); G. 't Hooft, hepth/9903189.
- [24] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976); **18**, 2199(E)

(1978); Phys. Rep. **142**, 357 (1986).

- [25] M. Napsuciale and S. Rodriguez, Phys. Lett. B **603**, 195 (2004).
- [26] M. Napsuciale and S. Rodriguez, Phys. Rev. D **70**, 094043 (2004).
- [27] In Ref. [22] the mixing factor was assumed to be $\sin\varphi_s$ + In Ker. [22] the mixing ractor was assumed to be $\sin\varphi_s + \sqrt{2}\cos\varphi_s$, where φ_s denotes the scalar mixing angle in the strange-nonstrange basis. A complete calculation yields strange-nonstrange basis. A complete calculation yields
actually $\sin\varphi_s - \sqrt{2}\cos\varphi_s$, which coincides with the present calculation when we decouple heavy scalars. The angle φ_s coincides with the angle γ in the present work.
- [28] S. Narison, Phys. Lett. B **175**, 88 (1986).
- [29] J. R. Ellis and J. Lanik, Phys. Lett. B **175**, 83 (1986).
- [30] A. H. Fariborz, Int. J. Mod. Phys. A **19**, 2095 (2004).
- [31] T. Teshima, I. Kitamura, and N. Morisita, J. Phys. G **28**, 1391 (2002); T. Teshima and S. Oneda, Phys. Rev. D **33**, 1974 (1986).