Generalized parton distribution functions and the nucleon spin sum rules in the chiral quark soliton model

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The theoretical predictions are given for the forward limit of the unpolarized spin-flip isovector generalized parton distribution function $(E^u - E^d)(x, \xi, t)$ within the framework of the chiral quark soliton model, with full inclusion of the polarization of Dirac sea quarks. We observe that $[(H^u - H^d) + (E^u - E^d)](x, 0, 0)$ has a sharp peak around $x = 0$, which we interpret as a signal of the importance of the pionic $q\bar{q}$ excitation with large spatial extension in the transverse direction. Another interesting indication given by the predicted distribution in combination with Ji's angular momentum sum rule is that the $\bar{d}$ quark carries more angular momentum than the $\bar{u}$ quark in the proton, which may have some relation with the physics of the violation of the Gottfried sum rule.

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I. INTRODUCTION

A distinguishable feature of the chiral quark soliton model (CQSM) as compared with many other effective model of baryons, like the naive quark model or the MIT bag model (at least in its most primitive version), is that it is a field theoretical model which takes account not only of three valence quarks but also of infinitely many Dirac sea-quark degrees of freedom [1,2]. As emphasized by Diakonov in his recent review [3], this feature is essential in explaining the so-called "nucleon spin crisis" [4,5] as well as the quite large experimental value of the πN sigma term [6,7], what he calls the two stumbling blocks of the naive quark models. In fact, the nucleon spin sum rule within the CQSM was first derived in [2]. It was shown there that the sizable amount of the nucleon spin comes from the orbital angular momentum carried by Dirac sea quarks. How the CQSM can explain the huge experimental value of πN sigma term is also very interesting. It predicts that only a small portion of the large πN sigma term is due to the main constituents of the nucleon, i.e. the three valence quarks, and the dominant contribution originates from the Dirac sea quarks [8,9]. Moreover, it was recently found that the Dirac sea contribution to the πN sigma term resides in a peculiar delta-function type singularity at $x = 0$ in the chiral-odd twist-3 distribution function $e(x)$ of the nucleon [10–13]. Also clarified there is that this delta-function singularity of $e(x)$ is a rare manifestation of the nontrivial vacuum structure of QCD, characterized by the nonzero quark condensate, in a baryon observable. The superiority of the CQSM manifests even more drastically in high-energy observables. It is almost only one effective model that can give reliable predictions for the quark and antiquark distribution functions of the nucleon satisfying the fundamental field theoretical restrictions like the positivity of the antiquark distribution functions [14–20].

Coming back to the nucleon spin problem, we claim that the CQSM already gives one possible solution to it. The physical reason why this model predicts small quark spin fraction, or large orbital angular momentum is clear. It is connected with the basic nucleon picture of this model, i.e. "rotating hedgehog." Naturally, this unique nucleon picture takes over that of the Skyrme model. Immediately after the EMC measurement, Ellis, Karliner, and Brodsky showed that this unique model predicts $\Delta\Sigma = 0$, i.e. vanishing quark spin fraction [21]. An important difference between these two intimately connected models (the Skyrme model as an effective pion theory and the CQSM as an effective quark model) should not be overlooked, however. In the CQSM, $\Delta\Sigma$ receives small but definitely nonzero contribution from the three valence quarks forming the core of the nucleon. (Another way to get $\Delta\Sigma \neq 0$ is the generalization of the Skyrme model so as to include the short range fields like the vector mesons [22].)

At any rate, an interesting solution to the nucleon spin puzzle, provided by the chiral soliton picture of the nucleon, emphasizes the importance of the orbital angular momentum of quark and antiquarks [2]. On the other hand, there is another completely different scenario giving a possible solution to the nucleon spin puzzle. It claims that the small quark spin fraction is compensated by the large gluon polarization (or the gluon orbital angular momentum.) Which scenario is favored by nature is still an unsolved question, which must be answered by some experiments in the future. An experimental test of the first scenario has been thought to be an extremely difficult task, because the quark orbital angular momentum in the nucleon was not believed to be an experimentally observable quantity. The situation has changed drastically after Ji's proposal [23–26]. He showed that the quark total angular momentum and also the quark orbital angular momentum in the nucleon can in principle be extracted through the measurement of the deeply virtual Compton scattering (DVCS) cross sections in combination with analyses of the standard inclusive reactions. The key quantity here is

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the unpolarized spin-flip generalized parton distribution function $E^q(x, \xi, t)$ appearing in the DVCS cross section formula. Especially interesting in the context of the above argument is its forward limit $E^q(x, 0, 0) \equiv \lim_{t \to 0, \xi \to 0} E^q(x, \xi, t)$. It satisfies the following second moment sum rule:

$$\frac{1}{2} \int_{-1}^{1} x[f_1^q(x) + E^q(x, 0, 0)]\, dx = J^q, \tag{1}$$

which is widely known as Ji's angular momentum sum rule [23–26]. Here $f_1^q(x)$ is the standard unpolarized quark (and antiquark) distribution function of flavor q, while J^q is the total angular momentum of quarks (and antiquark) with flavor q. Since $f_1^q(x)$ is already well known, the new experimental knowledge of $E^q(x, 0, 0)$ would completely determine J^q. Combining it with the available knowledge of the longitudinal quark polarization $\Delta\Sigma$, together with the relation

$$J^q = L^q + \frac{1}{2}\Delta\Sigma, \tag{2}$$

this opens up the possibility to extract the quark orbital angular momentum in the nucleon purely experimentally. Very recently, Ossmann *et al.* reported a very interesting calculation of $E^q(x, 0, 0)$, or more precisely the isoscalar combination $E^u(x, 0, 0) + E^d(x, 0, 0)$, within the framework of the CQSM [27]. They found that the contribution of Dirac sea quarks to $E^u(x, 0, 0) + E^d(x, 0, 0)$ dominates over that of the three valence quarks in the small x region. Especially interesting is their finding that the Dirac sea contribution to $E^u(x, 0, 0) + E^d(x, 0, 0)$ has a $1/x$ singularity around $x = 0$ in the chiral limit. Because of this peculiar feature, it turns out that the Dirac sea term gives negligible contribution to the first moment, i.e. $\int_{-1}^{1}[E^u(x, 0, 0) + E^d(x, 0, 0)]\, dx$, or equivalently to the isoscalar anomalous magnetic moment sum rule, while it gives a sizable contribution to the second moment sum rule, i.e. $\int_{-1}^{1} x[E^u(x, 0, 0) + E^d(x, 0, 0)]\, dx$, or to the quark angular momentum sum rule. They also investigated the second moment sum rule for $E^u(x, 0, 0) + E^d(x, 0, 0)$ within the CQSM, and confirmed that it reduces to the nucleon spin sum rule first derived in [2]. This was an expected result, since, in the CQSM, any physical observables, including the nucleon spin, is saturated by the quark field alone (the quark intrinsic spin and the quark orbital angular momentum in the present case), and since it is already known that the model satisfies the energy momentum sum rule as well [15]. Although only the flavor singlet combination appears in the total nucleon spin sum rule, we also need the independent isovector combination $E^u(x, 0, 0) - E^d(x, 0, 0)$, to make a flavor decomposition of the quark angular momentum. The isovector combination $E^u(x, 0, 0) - E^d(x, 0, 0)$ has already been addressed partially, but in an incomplete way [28–30]. The purpose of the present study is to carry out more complete investigation of this quantity. We shall show the results of exact numerical calculation of this quantity without recourse to the derivative expansion type approximation [28,29]. We also investigate the first and the second moment sum rule of $E^u(x, 0, 0) - E^d(x, 0, 0)$, which is expected to give valuable information on the theoretical consistency of the model.

The paper is organized as follows. In Sec. II, we briefly summarize some basic properties of the unpolarized generalized parton distribution functions (GPDF) necessary for our later discussion. Section III is devoted to the theoretical analyses of the unpolarized GPDF based on the CQSM. The main concern here is the first and the second moments of the isovector spin-flip unpolarized GPDF $E^{(I=1)}(x, \xi, t)$, which we know has intimate connection with the nucleon isovector magnetic moment and the isovector combination of the quark spin fraction of the nucleon. Next, in Sec. IV, we shall present the CQSM predictions for the forward limit $E^{(I=1)}(x, 0, 0)$ of the isovector spin-flip unpolarized GPDF. We also show the detailed numerical contents of the first and the second moment sum rules of this quantity. Finally, we summarize our findings in Sec. V.

II. GENERAL PROPERTIES OF THE UNPOLARIZED GPDF

Here, we briefly summarize some important features of the unpolarized quark generalized parton distribution functions $H^q(x, \xi, t)$ and $E^q(x, \xi, t)$ with flavor q, which are necessary for later discussion. They are defined by

$$\begin{aligned}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle \boldsymbol{P}', s' | \bar{\psi}_q\left(-\frac{\lambda n}{2}\right) \not{n} \psi_q\left(\frac{\lambda n}{2}\right) | \boldsymbol{P}, s\rangle &= H^q(x, \xi, t) \bar{U}(\boldsymbol{P}', s') \not{n} U(\boldsymbol{P}, s) \\ &\quad + E^q(x, \xi, t) \bar{U}(\boldsymbol{P}', s') \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M_N} U(\boldsymbol{P}, s).\end{aligned} \tag{3}$$

Here (and hereafter) we omit the light-cone gauge link, for brevity. We use the standard notation,

$$\Delta = P' - P, \qquad t = \Delta^2, \qquad \xi = -\frac{1}{2} n \cdot \Delta, \tag{4}$$

with n the lightlike vector satisfying the relations.

$$n^2 = 0, \qquad n \cdot (P' + P) = 2, \tag{5}$$

It is a well known fact that the first moments of $H^q(x, \xi, t)$ and $E^q(x, \xi, t)$ reduce to the Dirac and Pauli form factors, respectively

$$\int_{-1}^{1} H^q(x, \xi, t)\, dx = F_1^q(t), \tag{6}$$

$$\int_{-1}^{1} E^q(x, \xi, t)\, dx = F_2^q(t). \tag{7}$$

For convenience, we introduce the isoscalar and isovector combinations as follows:

$$H^{(I=0)}(x, \xi, t) \equiv H^u(x, \xi, t) + H^d(x, \xi, t), \tag{8}$$

$$H^{(I=1)}(x, \xi, t) = H^u(x, \xi, t) - H^d(x, \xi, t), \tag{9}$$

and similarly for $E^{(I=0)}(x, \xi, t)$ and $E^{(I=1)}(x, \xi, t)$. (In the present paper, we neglect the strange quark degrees of freedom, and confine to the two flavor case.) The forward limit $(\xi \to 0, t \to 0)$ of the first moment sum rule then gives

$$\int_{-1}^{1} H^{(I=0)}(x, 0, 0)\, dx = 3, \tag{10}$$

$$\int_{-1}^{1} H^{(I=1)}(x, 0, 0)\, dx = 1, \tag{11}$$

$$\int_{-1}^{1} E^{(I=0)}(x, 0, 0)\, dx = \kappa^u + \kappa^d = 3(\kappa^p + \kappa^n) = 3\kappa^{(I=0)}, \tag{12}$$

$$\int_{-1}^{1} E^{(I=1)}(x, 0, 0)\, dx = \kappa^u - \kappa^d = \kappa^p - \kappa^n = \kappa^{(I=1)}. \tag{13}$$

Here κ^u and κ^d stand for the anomalous magnetic moments of the u- and d quarks, while κ^p and κ^n are those of the proton and neutron. The first moment sum rule of $E(x, 0, 0)$ has especially interesting physical interpretation. Namely, $E(x, 0, 0)$ gives the distribution of the nucleon anomalous magnetic moments in the Feynman momentum x space not in the ordinary coordinate space. Also noteworthy is the second moment sum rule given as

$$\frac{1}{2}\int_{-1}^{1} x(H^{(I=0)} + E^{(I=0)})(x, 0, 0)\, dx = J^{(I=0)} = J^u + J^d, \tag{14}$$

which is known as Ji's quark angular momentum sum rule [23–26]. Here, $J^u + J^d$ represents the total quark (spin and orbital angular momentum) contribution to the nucleon spin. The forward limit of $H^q(x, \xi, t)$ is known to reduce to the standard unpolarized distribution function, which is rather precisely known by now. On the other hand, the forward limit of $E^q(x, \xi, t)$ is believed to be extracted from the analysis of the so-called deeply virtual Compton scatterings on the nucleon target [23–26]. This means that the total quark angular momentum fraction of the nucleon spin can be determined purely experimentally. Subtracting the known value of the quark intrinsic spin fraction $\Delta\Sigma^{(I=0)}$ of the nucleon. we can thus know the quark orbital angular momentum fraction of the total nucleon spin as well. Furthermore, making a different flavor combination (isovector combination) from (14), one expects another sum rule:

$$\frac{1}{2}\int_{-1}^{1} x(H^{(I=1)} + E^{(I=1)})(x, 0, 0)\, dx = J^{(I=1)} = J^u - J^d. \tag{15}$$

Thus, with combined use of the isoscalar and isovector sum rule, one would make a complete flavor decomposition of the quark total angular momentum.

III. UNPOLARIZED GPDF IN THE CQSM

The theoretical expressions of $H(x, \xi, t)$ and $E(x, \xi, t)$ in the CQSM were already given in several previous papers [27–30], so that we do not repeat the detailed derivation here. We describe only some main features and differences for the sake of later discussion. Here we closely follow the notation in [27], and introduce the quantities

$$\mathcal{M}^{(I=0)}_{s's} \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle \boldsymbol{P}'.s' | \bar{\psi}\Big(-\frac{\lambda n}{2}\Big) \not{n} \psi\Big(\frac{\lambda n}{2}\Big) | \boldsymbol{P}, s\rangle, \tag{16}$$

$$\mathcal{M}^{(I=1)}_{s's} \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle \boldsymbol{P}', s' | \bar{\psi}\Big(-\frac{\lambda n}{2}\Big) \tau_3 \not{n} \psi\Big(\frac{\lambda n}{2}\Big) | \boldsymbol{P}, s\rangle. \tag{17}$$

The relations between these quantities and the generalized parton distribution functions $H(x, \xi, t)$ and $E(x, \xi, t)$ are obtained most conveniently in the Breit frame. They are given by

$$\mathcal{M}^{(I=0)}_{s's} = 2\delta_{s's} H_E^{(I=0)}(x, \xi, t) - \frac{i\epsilon^{3kl}\Delta^k}{M_N} (\sigma^l)_{s's} E_M^{(I=0)}(x, \xi, t), \tag{18}$$

$$\mathcal{M}^{(I=1)}_{s's} = 2\delta_{s's} H_E^{(I=1)}(x, \xi, t) - \frac{i\epsilon^{3kl}\Delta^k}{M_N} (\sigma^l)_{s's} E_M^{(I=1)}(x, \xi, t), \tag{19}$$

where

$$H_E^{(I=0/1)}(x, \xi, t) \equiv H^{(I=0/1)}(x, \xi, t) + \frac{t}{4M_N^2} E^{(I=0/1)}(x, \xi, t), \tag{20}$$

$$E_M^{(I=0/1)}(x, \xi, t) \equiv H^{(I=0/1)}(x, \xi, t) + E^{(I=0/1)}(x, \xi, t). \tag{21}$$

These two independent combinations of $H(x, \xi, t)$ and $E(x, \xi, t)$ can be extracted through the spin projection of $\mathcal{M}^{(I)}$ as [27,28]

$$H_E^{(I)}(x, \xi, t) = \frac{1}{4}\mathrm{tr}\{\mathcal{M}^{(I)}\}, \tag{22}$$

$$E_M^{(I)}(x, \xi, t) = \frac{iM_N \epsilon^{3bm}\Delta^b}{2\boldsymbol{\Delta}_\perp^2} \mathrm{tr}\{\sigma^m \mathcal{M}^{(I)}\}, \tag{23}$$

where "tr" denotes the trace over spin indices, while $\mathbf{\Delta}_\perp^2 = \mathbf{\Delta}^2 - (\Delta^3)^2 = -t - (-2M_N\xi)^2$. Now, the right-hand side (rhs) of (22) and (23) can be evaluated within the framework of the CQSM. Here, we briefly describe the basic features of the CQSM leading to the theoretical expressions given below. The CQSM is a relativistic mean field theory with hedgehog assumption which breaks the rotation symmetry in addition to the translational symmetry at the mean field level. Two zero-energy modes must be taken into account to recover these symmetries. To recover the translational invariance, we use an approximate method which projects on the nucleon state with given center-of-mass momentum $\boldsymbol{P}$ by integrating out the shift $\boldsymbol{x}$ of the soliton center-mass coordinate [14,15],

$$\langle \boldsymbol{P}'|\cdots|\boldsymbol{P}\rangle = \int d^3\boldsymbol{x} e^{i(\boldsymbol{P}'-\boldsymbol{P})\cdot\boldsymbol{x}}\cdots. \tag{24}$$

Naturally, this procedure is justified only when the soliton is heavy enough and its center-of-mass motion is nonrelativistic. Another zero-energy mode corresponds to the soliton rotational motion. As usual, the velocity of this time-dependent rotation is assumed to be much slower than that of the intrinsic quark motions in the mean field [1,2]. This allows us to evaluate any nucleon observables in a perturbation theory with respect to the soliton rotational velocity Ω. This then leads to the following general structure of the theoretical expressions for nucleon observables in the CQSM. The leading contribution just corresponds to the mean field prediction, which is independent of Ω. The next-to-leading order term takes account of the linear response of the intrinsic quark motion to the rotational motion as an external perturbation, and consequently it is proportional to Ω. Here we confine ourselves to the mean field results [$O(\Omega^0)$ contribution] to the above GPDF. This leading term contributes to the isoscalar combination of $H_E(x,\xi,t)$, while it contributes to the isovector combination of $E_M(x,\xi,t)$:

$$\begin{aligned} H_E^{(I=0)}(x,\xi,t) = M_N N_c \int \frac{dz^0}{2\pi} \sum_{n\le 0} e^{iz^0(xM_N - E_n)} \int d^3\boldsymbol{x}\Phi_n^\dagger(\boldsymbol{x}) \\ \times(1+\gamma^0\gamma^3) e^{-i(z^0/2)\hat{p}_3} e^{i\mathbf{\Delta}\cdot\boldsymbol{x}} e^{-i(z^0/2)\hat{p}_3}\Phi_n(\boldsymbol{x}), \end{aligned} \tag{25}$$

$$\begin{aligned} E_M^{(I=1)}(x,\xi,t) = & \frac{2iM_N^2 N_c}{3(\mathbf{\Delta}^\perp)^2} \\ & \times \int \frac{dz^0}{2\pi} \sum_{n\le 0} e^{i\cdot z^0(xM_N - E_n)} \int d^3\boldsymbol{x}\Phi_n^\dagger(\boldsymbol{x}) \\ & \times (1+\gamma^0\gamma^3)(\boldsymbol{\tau}\times\mathbf{\Delta})^3 \\ & \times e^{-i(z^0/2)\hat{p}_3} e^{i\mathbf{\Delta}\cdot\boldsymbol{x}} e^{-i(z^0/2)\hat{p}_3}\Phi_n(\boldsymbol{x}). \end{aligned} \tag{26}$$

As shown by several previous papers, the first moments of the above GPDF reduce to the following forms [27–30]:

$$\int_{-1}^{1} H_E^{(I=0)}(x,\xi,t)\,dx = \int d^3x e^{i\mathbf{\Delta}\cdot\boldsymbol{x}} N_c \sum_{n\le 0} \Phi_n^\dagger(\boldsymbol{x})\Phi_n(\boldsymbol{x}), \tag{27}$$

$$\begin{aligned} \int_{-1}^{1} E_M^{(I=1)}(x,\xi,t)\,dx = \frac{2iM_N N_c}{3(\mathbf{\Delta}^\perp)^2} \int d^3\boldsymbol{x} \sum_{n\le 0} \Phi_n^\dagger(\boldsymbol{x}) \\ \times(1+\gamma^0\gamma^3)(\boldsymbol{\tau}\times\mathbf{\Delta})^3 e^{i\mathbf{\Delta}\cdot\boldsymbol{x}}\Phi_n(\boldsymbol{x}). \end{aligned} \tag{28}$$

Here the symbol $\sum_{n\le 0}$ denotes the summation over the occupied (the valence plus negative-energy Dirac sea) single quark orbitals in the hedgehog mean field. These expressions are slightly different from the ones given in several previous studies [27–29]. In these studies, on the basis of the large N_c argument, the left-hand side (lhs) of (18) is replaced by $H^{(I=0)}(x,\xi,t)$, while the lhs of (19) is replaced by $E^{(I=1)}(x,\xi,t)$, since the remaining terms are subleasing in N_c. Here, we retain these subleading terms because of the reason explained shortly. The rhs of these equations are the known theoretical expressions of the Sachs form factor and the isovector magnetic form factor within the CQSM, i.e.

$$\int_{-1}^{1} H_E^{(I=0)}(x,\xi,t)\,dx = G_E^u(t) + G_E^d(t) = 3G_E^{(I=0)}(t), \tag{29}$$

$$\int_{-1}^{1} E_M^{(I=1)}(x,\xi,t)\,dx = G_M^u(t) - G_M^d(t) = G_M^{(I=1)}(t). \tag{30}$$

The reason why we did not drop the subleading terms N_c in the lhs of (18) and (19) is as follows. If we did so, we would have obtained the relations

$$\int_{-1}^{1} H^{(I=0)}(x,\xi,t)\,dx = 3G_E^{(I=0)}(t), \tag{31}$$

$$\int_{-1}^{1} E^{(I=1)}(x,\xi,t)\,dx = G_M^{(I=1)}(t), \tag{32}$$

which contradicts the first moment sum rule expected on the general ground, i.e.

$$\int_{-1}^{1} H^{(I=0)}(x,\xi,t)\,dx = 3F_1^{(I=0)}(t), \tag{33}$$

$$\int_{-1}^{1} E^{(I=1)}(x,\xi,t)\,dx = F_2^{(I=1)}(t). \tag{34}$$

The first case would make little difference, because the difference between $F_1^{(I=0)}(t)$ and $G_E^{(I=0)}(t)$ is small under the circumstance in which the soliton center-of-mass motion is nonrelativistic. This is not the case with the difference between $F_2^{(I=1)}(t)$ and $G_M^{(I=1)}(t)$, as seen from the experimentally known relations:

$$F_2^{(I=1)}(t=0) \simeq \kappa^p - \kappa^n \simeq 3.7, \tag{35}$$

$$G_M^{(I=1)}(t=0) \simeq 1 + (\kappa^p - \kappa^n) \simeq 4.7, \tag{36}$$

although the anomalous magnetic moment term dominates over the Dirac moment term both from the viewpoint of the N_c counting as well as numerically. Next, we consider the forward limit of (25) and (26) . The forward limit of (25) gives

$$\begin{aligned} H_E^{(I=0)}(x,0,0) &= M_N N_c \int \frac{dz^0}{2\pi} \sum_{n\le 0} e^{iz^0(xM_N - E_n)} \\ &\quad \times \int d^3\boldsymbol{x} \Phi_n^\dagger(\boldsymbol{x})(1+\gamma^0\gamma^3) e^{-iz_0\hat{p}_3}\Phi_n(\boldsymbol{x}) \\ &= M_N N_c \\ &\quad \times \sum_{n\le 0} \langle n|(1+\gamma^0\gamma^3)\delta(xM_N - E_n - \hat{p}_3)|n\rangle. \end{aligned} \tag{37}$$

The rhs precisely coincides with the expression of the isoscalar unpolarized quark distribution in the CQSM [28], i.e. we confirm that

$$H_E^{(I=0)}(x,0,0) = f_1^{(I=0)}(x). \tag{38}$$

It is already known that the model expression for $f_1^{(I=0)}(x)$ satisfies the quark number and the energy momentum sum rules [14,15]:

$$\int_{-1}^{1} f_1^{(I=0)}(x)\,dx = 3, \tag{39}$$

$$\int_{-1}^{1} x f_1^{(I=0)}(x)\,dx = 1. \tag{40}$$

The forward limit of (26) reduces to

$$\begin{aligned} E_M^{(I=1)}(x,0,0) &= \frac{1}{3} M_N^2 N_c \int \frac{dz^0}{2\pi} \sum_{n\le 0} e^{iz_0(xM_N - E_n)} \int d^3\boldsymbol{x} \\ &\quad \times \Phi_n^\dagger(\boldsymbol{x})(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3(1+\gamma^0\gamma^3)e^{-iz_0\hat{p}_3}\Phi_n(\boldsymbol{x}) \\ &= \frac{1}{3} M_N^2 \cdot N_c \sum_{n\le 0} \langle n|(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3(1+\gamma^0\gamma^3) \\ &\quad \times \delta(xM_N - E_n - \hat{p}_3)|n\rangle. \end{aligned} \tag{41}$$

The first moment of this quantity gives

$$\int_{-1}^{1} E_M^{(I=1)}(x,0,0)\,dx = -\frac{M_N}{9} N_c \sum_{n\le 0} \langle n|(\hat{\boldsymbol{x}}\times\boldsymbol{\alpha})\cdot\boldsymbol{\tau}|n\rangle. \tag{42}$$

The rhs precisely gives the theoretical expression for the isovector magnetic moment in the CQSM. (It is not the anomalous magnetic moment part.) This is basically a known fact [28], but what we emphasize here is that $E_M^{(I=1)}(x,0,0)$ is interpreted to give the distribution of (isovector) nucleon magnetic moment in the Feynman x space not in the ordinary coordinate space.

Now, we turn to our main concern in this paper, i.e. the second moment of this quantity, which we expect is related to the isovector part of the quark angular momentum fraction of the nucleon. Performing a weighted x integral, we find that

$$\begin{aligned} \int_{-1}^{1} x E_M^{(I=1)}(x,0,0)\,dx &= \frac{1}{3} N_c \sum_{n\le 0} \langle n|(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3(1+\alpha_3) \\ &\quad \times (E_n + \hat{p}_3)|n\rangle. \end{aligned} \tag{43}$$

One notices that the second moment of $E_M^{(I=1)}(x,0,0)$ can be decomposed into four parts as

$$\begin{aligned} \int_{-1}^{1} x E_M^{(I=1)}(x,0,0)\,dx &= \frac{1}{3} N_c \sum_{n\le 0} \langle n|(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3 E_n|n\rangle \\ &\quad + \frac{1}{3} N_c \sum_{n\le 0} \langle n|(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3 \alpha_3 E_n|n\rangle \\ &\quad + \frac{1}{3} N_c \sum_{n\le 0} \langle n|(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3 \hat{p}_3|n\rangle \\ &\quad + \frac{1}{3} N_c \sum_{n\le 0} \langle n|(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3 \alpha_3 \hat{p}_3|n\rangle \\ &\equiv M_1 + M_2 + M_3 + M_4. \end{aligned} \tag{44}$$

Using the Dirac equation (here $\hat{\boldsymbol{r}} \equiv \hat{\boldsymbol{x}}/|\hat{\boldsymbol{x}}|$)

$$H|n\rangle = E_n|n\rangle, \tag{45}$$

with

$$H = \boldsymbol{\alpha}\cdot\hat{\boldsymbol{p}} + M\beta(\cos F(r) + i\gamma_5\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\sin F(r)), \tag{46}$$

the first term can be rewritten as

$$M_1 = \frac{N_c}{3}\sum_{n\le 0}\frac{1}{2}\langle n|\{(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3, H\}|n\rangle = \frac{N_c}{3}\sum_{n\le 0}\frac{1}{2}\langle n| - i(\boldsymbol{\alpha}\times\boldsymbol{\tau})_3 - 2(\boldsymbol{\tau}\times\hat{\boldsymbol{x}})_3\boldsymbol{\alpha}\cdot\hat{\boldsymbol{p}}|n\rangle. \tag{47}$$

It is easy to see that this term vanishes identically due to the hedgehog symmetry. Alternatively, we can use the parity symmetry to show $M_1 = 0$. The parity also enforces the fourth term M_4 to vanish. In fact, under the parity operation $\mathcal{P}$, we have

$$\mathcal{P}|n\rangle = (-1)^{P_n}|n\rangle, \tag{48}$$

$$\mathcal{P}(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\tau}})_3\mathcal{P}^{-1} = -(\boldsymbol{x}\times\boldsymbol{\tau})_3, \tag{49}$$

$$\mathcal{P}\alpha_3\mathcal{P}^{-1} = -\alpha_3, \tag{50}$$

$$\mathcal{P}\hat{p}_3\mathcal{P}^{-1} = -\hat{p}_3 \tag{51}$$

with $(-1)^{P_n}$ being the parity of the eigenstate $|n\rangle$, so that we conclude that

$$M_4 = \frac{N_c}{3}\sum_{n\le 0}\langle n|\mathcal{P}^{-1}\mathcal{P}(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3 \times \mathcal{P}^{-1}\mathcal{P}\alpha_3\mathcal{P}^{-1}\mathcal{P}\hat{p}_3\mathcal{P}^{-1}\mathcal{P}|n\rangle = \{(-1)^{P_n}\}^2(-1)^3 M_4 = -M_4 = 0. \tag{52}$$

The third term M_3 does not vanish but it can be simplified in the following way because of the hedgehog symmetry (generalized spherical symmetry) as

$$M_3 = \frac{N_c}{3}\sum_{n\le 0}\langle n|(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3\hat{p}_3|n\rangle = \frac{N_c}{3}\sum_{n\le 0}\frac{1}{3}\langle n|(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})\cdot\hat{\boldsymbol{p}}|n\rangle = \frac{N_c}{3}\sum_{n\le 0}\left(-\frac{1}{3}\right)\langle n|\boldsymbol{\tau}\cdot(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{p}})|n\rangle = -\frac{N_c}{3}\sum_{n\le 0}\langle n|\tau_3 L_3|n\rangle. \tag{53}$$

Finally, the second term M_2 can be rewritten in the following manner by using the Dirac equation, commutation relations of γ matrices and isospin matrices, and also the hedgehog symmetry

$$M_2 = \frac{N_c}{3}\sum_{n\le 0}\frac{1}{2}\langle n|\{(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3\alpha_3, H\}|n\rangle = \frac{N_c}{3}\sum_{n\le 0}\frac{1}{2}\langle n|\{(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3\alpha_3, \boldsymbol{\alpha}\cdot\hat{\boldsymbol{p}}\}|n\rangle + \frac{N_c}{3}\sum_{n\le 0}\frac{1}{2}\langle n|\{(\hat{\boldsymbol{x}}\times\boldsymbol{\tau})_3\alpha_3, M\beta(\cos F(r) + \gamma_5\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\sin F(r))\}|n\rangle = -\frac{N_c}{3}\sum_{n\le 0}\langle n|\tau_3 L_3 + \tau_3\Sigma_3|n\rangle - M\cdot\frac{N_c}{9}\sum_{n\le 0}\langle n|r\sin F(r)\gamma^0[\boldsymbol{\Sigma}\cdot\hat{\boldsymbol{r}}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}} - \boldsymbol{\Sigma}\cdot\boldsymbol{\tau}]|n\rangle. \tag{54}$$

Collecting all the four terms, we finally obtain the second moment sum rule of the form

$$\int_{-1}^{1} xE_M^{(I=1)}(x,0,0)\,dx = 2\left(-\frac{N_c}{3}\right)\sum_{n\le 0}\langle n|\tau_3\left(L_3 + \frac{1}{2}\Sigma_3\right)|n\rangle - M\cdot\frac{N_c}{9}\sum_{n\le 0}\langle n|r\sin F(r) \times \gamma^0[\boldsymbol{\Sigma}\cdot\hat{\boldsymbol{r}}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}} - \boldsymbol{\Sigma}\cdot\boldsymbol{\tau}]|n\rangle. \tag{55}$$

The first term of the rhs just coincides with the proton matrix element of the free field quark angular momentum operator, or more precisely its isovector part, given by

$$J_f^{(I=1)} = \langle p\uparrow|\hat{J}_f^{(I=1)}|p\uparrow\rangle, \tag{56}$$

with

$$\hat{J}_f^{(I=1)} \equiv \int \psi^\dagger(\hat{\boldsymbol{x}})\tau_3\left[(\boldsymbol{x}\times\hat{\boldsymbol{p}})_3 + \frac{1}{2}\Sigma_3\right]\psi(\boldsymbol{x})d^3x = \hat{L}_f^{(I=1)} + \frac{1}{2}\hat{\Sigma}^{(I=1)}. \tag{57}$$

In view of Ji's angular momentum sum rule, we would have naively expected to get

$$\frac{1}{2}\int_{-1}^{1} xE_M^{(I=1)}(x,0,0)\,dx = J_f^{(I=1)}. \tag{58}$$

Somewhat unexpectedly, however, we find that

$$\frac{1}{2}\int_{-1}^{1} xE_M^{(I=1)}(x,0,0)\,dx = J_f^{(I=1)} + \delta J^{(I=1)}, \tag{59}$$

with

$$\delta J^{(I=1)} = -M\frac{N_c}{18}\sum_{n\le 0}\langle n|r\sin F(r) \times \gamma^0[\boldsymbol{\Sigma}\cdot\hat{\boldsymbol{r}}\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}} - \boldsymbol{\Sigma}\cdot\boldsymbol{\tau}]|n\rangle. \tag{60}$$

This second moment sum rule should be contrasted with the corresponding sum rule in the isoscalar channel, which was recently proved in [27] :

$$\frac{1}{2}\int_{-1}^{1} xE_M^{(I=0)}(x,0,0)\,dx = L_f^{(I=0)} + \frac{1}{2}\Delta\Sigma^{(I=0)} = J_f^{(I=0)} = 1/2, \tag{61}$$

which appears to be compatible with Ji's general sum rule, considering that the nucleon spin sum rule should be saturated by the quark field alone in the CQSM [2]. How should we interpret the above unexpected result for the isovector case? One may argue that the second moment need not necessarily reduce to the nucleon matrix element of the free field angular momentum operator since the CQSM is anyhow an interacting theory of quark fields. In fact, the above derivation of the sum rule (59) indicates that the cause of the $\delta J^{(I=1)}$ term can be traced back to the dynamical generation of the effective quark mass and the formation of the symmetry breaking mean field containing the scalar product of $\boldsymbol{\tau}$ and $\hat{\boldsymbol{r}}$. At least, one can say that, since the $\delta J^{(I=1)}$ term is proportional to the dynamically generated quark mass M, it vanishes in the perturbative vacuum, although it is meaningless to consider the chiral soliton if the QCD vacuum is perturbative. We recall that a similar breakdown of the second moment sum rule, expected on the general ground of QCD, occurs also in the case of chiral-odd twist-3 distribution functions $e(x)$ of the nucleon. From the general QCD analysis, one expect that the second moment of the isoscalar part of $e(x)$ satisfies the following sum rule in the chiral limit [10],

$$\int_{-1}^{1} xe^{(I=0)}(x)\,dx = 0. \tag{62}$$

In the CQSM, however, we obtain [12,13],

$$\int_{-1}^{1} xe^{(I=0)}(x)\,dx = \frac{M}{M_N}N_c\sum_{n\le 0}\langle n|\frac{1}{2}(U+U^\dagger)|n\rangle, \tag{63}$$

with $U = e^{i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)}$. It is indicative that the violation of the QCD sum rule is again proportional to the dynamical quark mass M, which would vanish in the perturbative vacuum [13].

From the practical viewpoint, only the lhs of (59) is observable, and neither of $J_f^{(I=1)}$ nor $\delta J^{(I=1)}$ is observable. We therefore take the following viewpoint, although it is not absolutely mandatory. The second moment of $E_M^{(I=1)}(x,0,0)$ gives the isovector quark angular momentum fraction of the interacting theory (it is the CQSM in the present context):

$$\frac{1}{2}\int_{-1}^{1} xE_M^{(I=1)}(x,0,0)\,dx \equiv J^{(I=1)} \equiv J^u - J^d. \tag{64}$$

Although somewhat arbitrary, this sum rule combined with (61) allows us to carry out the flavor decomposition of the quark angular momentum fraction in the nucleon. In any case, the most important quantity here is $E_M^{(I=1)}(x,0,0)$, since it is the quantity which can be directly measured through the DVCS experiments. In the next section, we perform a numerical calculation of $E_M^{(I=1)}(x,0,0)$ without recourse to the derivative expansion type approximation. We also evaluate $J_f^{(I=1)}$ and $\delta J^{(I=1)}$ in the rhs of (59) directly without any notion of distribution functions. This allows us to check the validity of the sum rule (59) within the model, thereby providing us with a nontrivial check for our numerical result for $E_M^{(I=1)}(x,0,0)$.

IV. NUMERICAL RESULTS AND DISCUSSION

The most important parameter of the CQSM is the dynamical quark mass M, which plays the role of the quark-pion coupling constant, thereby controlling basic soliton properties [1,2]. Here we use the value $M = 375$ MeV, which is favored from the analysis of nucleon low energy observables. The model is an effective theory which is defined with an physical cutoff. We use here the double-subtraction Pauli-Villars regularization scheme proposed in [31]. (Naturally, this is not the only way of introducing regularization. More sophisticated regularization scheme is proposed in [32].) In this scheme, the most nucleon observables are regularized through the subtraction.

$$\langle O\rangle^{\rm reg} \equiv \langle O\rangle^M - \sum_{i=1}^{2} c_i\langle O\rangle^{\Lambda_i}. \tag{65}$$

Here $\langle O\rangle^M$ denotes the nucleon matrix element of an operator O evaluated with the original action of the CQSM having the mass parameter M, while $\langle O\rangle^{\Lambda_i}$ stands for the corresponding matrix element obtained from $\langle O\rangle^M$ by replacing the parameter M with the Pauli-Villars cutoff mass Λ_i. To remove all the ultraviolet divergences of the theory, the parameters c_i and Λ_i must satisfy the conditions

$$M^2 - \sum_{i=1}^{2} c_i\Lambda_i^2 = 0, \tag{66}$$

$$M^4 - \sum_{i=1}^{2} c_i\Lambda_i^4 = 0. \tag{67}$$

We further impose two additional physical conditions, which amounts to requiring that the model reproduces the empirically known value of the vacuum quark condensate as well as the correct normalization of the pion kinetic term in the corresponding bosonized action. This gives

$$\langle \bar{\psi}\psi \rangle_{\text{vac}} = \frac{N_c M^3}{2\pi^2} \sum_{i=1}^{2} c_i \left(\frac{\Lambda_i}{M}\right)^4 \ln\left(\frac{\Lambda_i}{M}\right)^2, \tag{68}$$

$$f_\pi^2 = \frac{N_c M^2}{4\pi^2} \sum_{i=1}^{2} c_i \left(\frac{\Lambda_i}{M}\right)^2 \ln\left(\frac{\Lambda_i}{M}\right)^2. \tag{69}$$

These four conditions (66)–(69) are enough to fix the four parameters c_1, c_2, Λ_1, and Λ_2. Fixing M and f_π to be 375 and 93 MeV, respectively, we find

$$c_1 = 0.399\,37, \qquad c_2 = -0.006\,61, \tag{70}$$

$$\Lambda_1 = 627.653 \text{ MeV}, \qquad \Lambda_2 = 1589.45 \text{ MeV}. \tag{71}$$

As usual, the numerical calculations in the CQSM are carried out by using Kahana-Ripka's discretized momentum basis [33,34]. Our main task here is to evaluate $E_M^{(I=1)}(x,0,0)$ given by (41) making use of the Kahana-Ripka basis, which turns out not so easy. The cause of difficulty lies in the following fact. First, the appearance of the momentum operator $\hat{p}_3$ in the delta function enforces us to work in the momentum representation. On the other hand, the matrix element in (41) contains the coordinate operator $\hat{\boldsymbol{x}}$. This coordinate operator becomes a differential operator in the momentum representation, which is, however, incompatible with the whole calculation scheme of the CQSM making use of the discretized momentum basis. To circumvent this difficulty, we insert a complete set of states $|m_0\rangle$ as follows:

$$E_M^{(I=1)}(x,0,0) = \frac{1}{3} M_N^2 \cdot N_c \sum_{n\le 0} \sum_{m_0=\text{all}} \langle n | (\hat{\boldsymbol{x}} \times \boldsymbol{\tau})_3 \times (1+\gamma^0\gamma^3) | m_0 \rangle \langle m_0 | \delta(x M_N - E_n - \hat{p}_3) | n \rangle. \tag{72}$$

This complete set can in principle be chosen at will. It can be the eigenstates of the full Dirac Hamiltonian H as the states $|n\rangle$ are so, or it can be the eigenstates of the free Dirac Hamiltonian H_0 given by

$$H_0 = \boldsymbol{\alpha} \cdot \hat{\boldsymbol{p}} + \beta M. \tag{73}$$

We choose here the latter. The grand spin K' of the states $|m_0\rangle$ need not be the same as the grand spin K of the states $|n\rangle$, but the finite rank nature of the operator $(\hat{\boldsymbol{x}} \times \boldsymbol{\tau})_3 \times (1+\gamma^0\gamma^3)$ restricts the value of K' to be K, $K \pm 1$, or $K \pm 2$. The advantage of the expression (72) is that the first and the second matrix elements can, respectively, be evaluated in the coordinate representation and the momentum representation.

The theoretical expression for $E_M^{(I=1)}(x,0,0)$ in (41) is given as the summation over the occupied single quark orbitals. An alternative but equivalent expression is obtained for it, which is given as the summation over the nonoccupied quark levels as

$$E_M^{(I=1)}(x,0,0) = -\frac{1}{3} M_N^2 \cdot N_c \sum_{n>0} \langle n | (\hat{\boldsymbol{x}} \times \boldsymbol{\tau})_3 \times (1+\gamma^0\gamma^3) \delta(x M_N - E_n - \hat{p}_3) | n \rangle. \tag{74}$$

The equivalence of these two representations is based on a quite general principle of field theory, i.e. the anticommuting property of two quark field operator with spacelike separation. It is also known that the Pauli-Villars regularization preserves this equivalence. For numerical calculation of $E_M^{(I=1)}(x,0,0)$, it is convenient to use the occupied form (41) for $x > 0$, and the nonoccupied form (74) for $x < 0$. (We recall that the distribution with $x < 0$ is related to the antiquark distribution with $x > 0$.)

Now, we show our numerical result for $E_M^{(I=1)}(x,0,0)$. The long-dashed and the dash-dotted curves in Fig. 1 stand for the contributions of the three valence quarks and of the Dirac sea quarks, respectively. The sum of these two contributions is represented by the solid curve. A remarkable feature here is that the contribution of the Dirac sea quarks has a sharp peak around $x = 0$. This confirms the qualitative result first given in [29]. Here we recall the fact that the x integral or the first moment of $E_M^{(I=0)}(x,0,0)$ gives the isovector magnetic moment $\mu^{(I=1)} = \mu_p - \mu_n$ of the nucleon. This denotes that $E_M^{(I=1)}(x,0,0)$ gives the distribution of the nucleon isovector magnetic moment in the Feynman x space not in the ordinary coordinate space. The sharp peak around $x = 0$ therefore means that the quark and antiquark with small x carry a sizable amount of isovector magnetic moment of the nucleon. We interpret this fact as an indication of the importance of the quark motion in the transverse direction by the following reason. First, the isovector nucleon magnetic moment mainly comes from the quark and antiquark orbital motion in the nucleon, since it is known that the anomalous part domi-

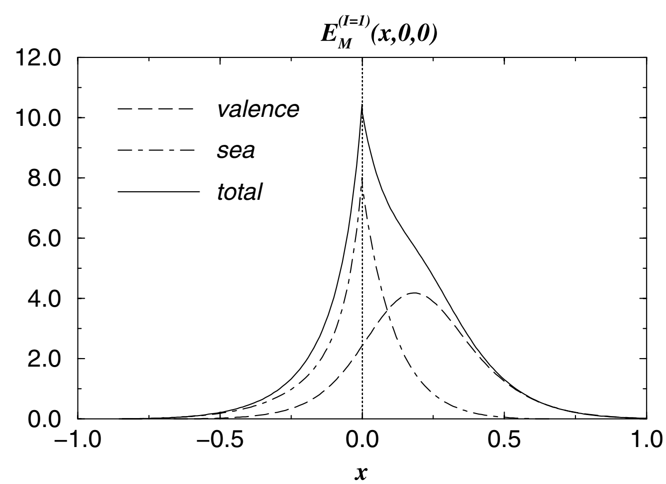

FIG. 1. The CQSM prediction for $E_M^{(I=1)}(x,0,0)$. The long-dashed and dash-dotted curves here stand for the contribution of the valence quarks and of the Dirac sea quarks, while their sum is represented by the solid curve.

nate over the Dirac moment part. Second, the quark and antiquark with $x = 0$ has zero velocity in the longitudinal direction. Accordingly, the large magnetic moment density concentrated in the small x region must come from the motion of quark and antiquarks in the plane perpendicular to the proton spin direction.

The prominent peak of $E_M^{(I=1)}(x, 0, 0)$ around $x = 0$ can also be interpreted as the effect of pionic $q\bar{q}$ excitation with large spatial extension. In fact, it has long been known that the pion cloud around the "bare" nucleon gives a significant contribution to the isovector nucleon magnetic moment. This then indicates that the dominant contribution to $E_M^{(I=1)}(x, 0, 0)$ in the small x domain originates from the motion of correlated quarks and antiquarks, the spatial distribution of which have a long range tail in the transverse direction. The validity of the proposed interpretation may be tested more definitely, if one could evaluate the so-called impact-parameter dependent distribution function defined by [35–39]

$$\varepsilon_M^{(I=1)}(x, \boldsymbol{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} E_M^{(I=1)}(x, 0, -\boldsymbol{\Delta}_\perp^2). \tag{75}$$

The rest of this section will be devoted to the numerical check of the first and second moment sum rule of $E_M^{(I=1)}(x, 0, 0)$ within the framework of the CQSM. This is important not only to confirm the precision of our numerical result for $E_M^{(I=1)}(x, 0, 0)$ but also to verify the internal consistency of the whole theoretical framework. We first discuss the first moment sum rule given by (42). The point is that both sides of this equation can be evaluated totally independently. The lhs can be is calculated numerically integrating the already given $E_M^{(I=0)}(x, 0, 0)$ over x. On the other hand, the evaluation of the rhs, i.e. the isovector magnetic moment of the nucleon, has no trouble, since it is just a nucleon expectation value of a local operator. Numerically, we have got

$$\int_{-1}^{1} E_M^{(I=1)}(x, 0, 0)\, dx \simeq 2.07 + 1.79 \simeq 3.86, \tag{76}$$

while

$$\begin{aligned} -\frac{M_N}{9} N_c \sum_{n \le 0} \langle n | (\hat{\boldsymbol{x}} \times \boldsymbol{\alpha}) \cdot \boldsymbol{\tau} | n \rangle &= \mu_V^{(I=1)} \\ &\simeq 2.05 + 1.87 \\ &\simeq 3.92, \end{aligned} \tag{77}$$

which coincides with the precision of about 1%. One may notice that the theoretical prediction for $\mu_V^{(I=1)}$ is smaller than the experimental value $\mu_V^{(I=1)}(\exp) \simeq 4.7$. We however know that, within the framework of the CQSM, there is a rotational correction to $\mu_V^{(I=1)}$ proportional to the collective angular velocity Ω, which is known to fill this gap [40,41]. (We should however recall some controversy related to this first order rotational correction to some isovector nucleon observables [42–45].) This first order rotational correction is naturally expected to contribute also to $E_M^{(I=1)}(x, 0, 0)$, thereby to both side of the first moment sum rule. The calculation of such a higher order contribution to $E_M^{(I=1)}(x, 0, 0)$ is beyond the scope of the present paper. Next we turn to the second moment sum rule of $E_M^{(I=1)}(x, 0, 0)$ given by (59). We first evaluate the first term of the rhs of (59), i.e. the nucleon matrix element of the free field angular momentum operator in the isovector combination. This term consists of the two terms as

TABLE I. The separate contributions of the valence and the Dirac sea quarks to the quantities $L_f^{(I=1)}$, $\Delta\Sigma^{(I=1)}$, and $J_f^{(I=1)} \equiv L_f^{(I=1)} + \frac{1}{2}\Sigma^{(I=1)}$ defined in the text.

	$L_f^{(I=1)}$	$\Sigma^{(I=1)}$	$J_f^{(I=1)}$
Valence	0.147	0.705	0.5000
Sea	−0.265	0.357	−0.087
Total	−0.115	1.057	0.413

$$J_f^{(I=1)} = L_f^{(I=1)} + \frac{1}{2}\Delta\Sigma^{(I=1)}. \tag{78}$$

Here, $L_f^{(I=1)}$ is the nucleon matrix element of the free field isovector angular momentum operator, while $\Delta\Sigma^{(I=1)}$ is the isovector part of the longitudinal quark polarization.

We show in Table I the separate contribution of the valence quarks and the Dirac sea quarks to $L_f^{(I=1)}$, $\Delta\Sigma^{(I=1)}$, and $J_f^{(I=1)}$. One sees that the valence quark contribution to $J_f^{(I=1)}$ is precisely 1/2 while the Dirac sea contribution to it is slightly negative.

Next, shown in Table II are the contributions of the valence quarks and the Dirac sea quarks to the quantity $\delta J^{(I=1)}$ in the second moment sum rule as well as the sum of $J_f^{(I=1)}$ and $\delta J^{(I=1)}$. (The numerical values of $J_f^{(I=1)}$ already given in Table I are also shown for convenience.) Also shown in this table is a half of the second moment of $E_M^{(I=1)}(x, 0, 0)$ obtained numerically from the weighted x integral of it. One confirms that the second moment sum rule (59) is satisfied with good precision, which in turn

TABLE II. The separate contributions of the valence and the Dirac sea quarks to the quantities $J_f^{(I=1)}$, $\delta J^{(I=1)}$, and their sum. Also shown are the corresponding numbers for $\frac{1}{2} \times \int x E_M^{(I=1)}(x, 0, 0)\, dx$.

	$J_f^{(I=1)}$	$\delta J^{(I=1)}$	$J_f^{(I=1)} + \delta J^{(I=1)}$	$\frac{1}{2}\int x E_M^{(I=1)}(x, 0, 0) dx$
Valence	0.500	−0.289	0.211	0.210
Sea	−0.087	0.077	−0.010	−0.008
Total	0.413	−0.212	0.201	0.202

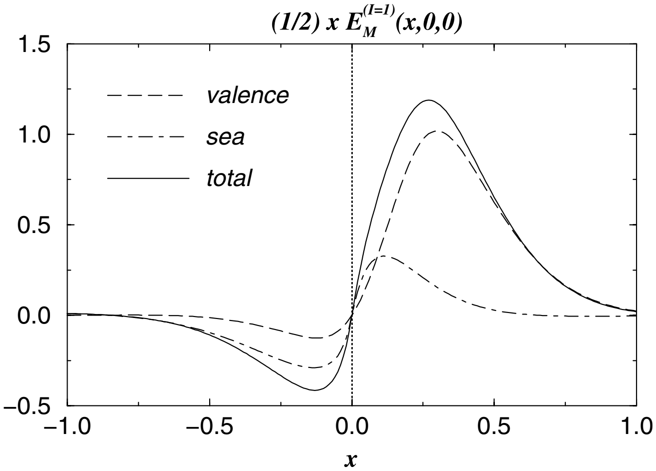

FIG. 2. The CQSM prediction for $\frac{1}{2}xE_M^{(I=1)}(x,0,0)$. The meaning of the curves is the same as in Fig. 1.

assures the numerical accuracy of our numerical calculation of $E_M^{(I=1)}(x,0,0)$. Knowing that the second moment of $E_M^{(I=1)}(x,0,0)$ does not coincide with the nucleon matrix element of the free field quark angular momentum operator in the isovector case, we would rather take a viewpoint that its second moment gives the quark angular momentum in the interacting theory. This amounts to regarding the sum of $J_f^{(I=1)}$ and $\delta J^{(I=1)}$ as the isovector part of the quark angular momentum fraction $J^{(I=1)}$ of the interacting theory, which gives

$$J^{(I=1)} = J^u - J^d \simeq 0.202. \tag{79}$$

The corresponding quark orbital angular momentum fraction can be estimated by subtracting the corresponding quark spin part $\frac{1}{2}\Delta\Sigma^{(I=1)} \simeq 0.529$, which gives

$$L^{(I=1)} = L^u - L^d \simeq -0.327. \tag{80}$$

Worthy of mention here is the x-distribution of the isovector quark angular momentum. Pushing forward with the above interpretation, let us identify $\frac{1}{2}xE_M^{(I=0)}(x,0,0)$ with the isovector quark angular momentum distribution $J^{(I=1)}(x) \equiv J^u(x) - J^d(x)$.

Shown in Fig. 2 is the CQSM prediction for this quantity $\frac{1}{2}xE_M^{(I=0)}(x,0,0)$, or $J^u(x) - J^d(x)$ in the above interpretation. Here, the distribution in the negative region should be interpreted as that of antiquarks, i.e. $J^u(-x) - J^d(-x) = J^{\bar{u}}(x) - J^{\bar{d}}(x)$ with $x > 0$. Then, we observe from this figure that

$$J^u(x) - J^d(x) > 0 \qquad (\text{for } x > 0), \tag{81}$$

while

$$J^{\bar{u}}(x) - J^{\bar{d}}(x) < 0 \qquad (\text{for } x > 0). \tag{82}$$

The first inequality is nothing surprising, since the proton contains two u quarks and one d quark as valence particles. More interesting here is the second inequality, which indicates that the $\bar{d}$ quark carries more angular momentum than the $\bar{u}$ quark in the proton. This reminds us of the violation of the Gottfried sum rule, which has been accepted by now as a clear evidence of the dominance of the $\bar{d}$ quark over the $\bar{u}$ quark in the unpolarized parton distribution functions of the proton. Undoubtedly, the two physics cannot be completely unrelated.

V. CONCLUSION

To conclude, we have given a theoretical prediction for the forward limit of the isovector, spin-flip generalized parton distribution function $E^{(I=1)}(x,\xi,t)$ of the nucleon on the basis of the CQSM. It has been shown that the distribution function $E_M^{(I=1)}(x,0,0) \equiv H^{(I=1)}(x,0,0) + E^{(I=1)}(x,0,0)$ has a sharp peak around $x = 0$ generated by the vacuum polarization of the Dirac sea quark. In view of the fact that the function $E_M^{(I=1)}(x,0,0)$ gives the distribution of the nucleon isovector magnetic moment in the Feynman momentum x space, we interpret this sharp peak around $x = 0$ as an indication of the importance of the pionic $q\bar{q}$ excitation with large spatial extension in the transverse direction. Somewhat unexpectedly, we found that the second moment of $E_M^{(I=1)}(x,0,0)$ does not reduce to the proton matrix element of the free quark angular momentum operator, but receive a peculiar correction term. The cause of this correction term seems to be traced back to the nonperturbative formation of isospin dependent hedgehog mean field. Still, we advocate a viewpoint that the second moment of $E_M^{(I=1)}(x,0,0)$ gives the isovector quark angular momentum fraction of an interacting theory, which leads us to an interpretation of $\frac{1}{2}xE_M^{(I=1)}(x,0,0)$ as the isovector quark (and antiquark) angular momentum distribution. This then indicates that the $\bar{d}$ quark carries more angular momentum than the $\bar{u}$ quark inside the proton.

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