Erratum: Quantum corrections to the Schwarzschild and Kerr metrics [Phys. Rev. D 68, 084005 (2003)]

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In Eq. (35) a factor of $\log\vec{q}^2$ should be added to the second line from the end.
In Eq. (B19) the coefficient of the antisymmetric tensor should be $i/16$ rather than $i/8$.
In Eq. (A6) in the expression for $R^{(2)}$ the factor of $\frac{1}{2}$ in term $\frac{1}{2}h^{(1)\lambda\alpha}(\ldots)$ should be replaced by unity.
In Eq. (A8) the piece $+\eta_{\mu\nu}\partial^\alpha(\partial^\beta h^{(1)}_{\alpha\beta} - \frac{1}{2}\partial_\alpha h^{(1)})$ should be appended to the left-hand side of the equation.
In Eq. (A13) the piece $+\eta_{\mu\nu}\partial^\alpha(\partial^\beta h^{(2)}_{\alpha\beta} - \frac{1}{2}\partial_\alpha h^{(2)})$ should be appended to the left-hand side of the equation.
In Eq. (A14) the terms

$$\frac{1}{2}\left(\partial_\beta h^{(1)\alpha\beta} - \frac{1}{2}\partial^\alpha h^{(1)}\right)\left[\partial_\mu h^{(1)}_{\nu\alpha} + \partial_\nu h^{(1)}_{\mu\alpha} - \partial_\alpha h^{(1)}_{\mu\nu} - \eta_{\mu\nu}\left(\partial^\lambda h^{(1)}_{\lambda\alpha} - \frac{1}{2}\partial_\alpha h^{(1)}\right)\right] + \frac{1}{2}\partial_\alpha\left(\partial^\beta h^{(1)}_{\lambda\beta} - \frac{1}{2}\partial_\lambda h^{(1)}\right)[\eta^{\alpha\lambda}h^{(1)}_{\mu\nu} - 2\eta_{\mu\nu}h^{(1)\lambda\alpha}]$$

should be appended to the right-hand side of the equation.
In Eqs. (C2) and (C4) the $q_i q_j$ Fourier transforms should read

$$\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} q_i q_j \log\vec{q}^2 = \frac{\delta_{ij}}{\pi r^5} - \frac{5}{2\pi r^5}\left(\delta_{ij} - 3\frac{r_i r_j}{r^2}\right)$$

$$\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{q_i q_j}{\vec{q}^2} \log\frac{\vec{q}^2}{\mu^2} = \frac{-r_i r_j}{2\pi r^5} + \frac{1 - \log\mu r}{2\pi r^3}\left(\delta_{ij} - 3\frac{r_i r_j}{r^2}\right),$$

respectively. Correspondingly, the form of T_{ij} in Eqs. (23) and (41) should read

$$T_{ij}(r) = -\frac{7Gm^2}{4\pi r^4}\left(\frac{r_i r_j}{r^2} - \frac{1}{2}\delta_{ij}\right) + \frac{2Gm\hbar}{\pi^2 r^5}\delta_{ij} + \frac{5Gm\hbar}{2\pi^2 r^5}\left(\delta_{ij} - 3\frac{r_i r_j}{r^2}\right),$$

while the form of h_{ij} in Eqs. (35) and (44) becomes

$$h_{ij}(r) = -\delta_{ij}\frac{2Gm}{r} - \frac{G^2m^2}{r^2}\left(\delta_{ij} + \frac{r_i r_j}{r^2}\right) - \frac{G^2m\hbar}{\pi r^3}\left(\delta_{ij} + 8\frac{r_i r_j}{r^2}\right) + \frac{8G^2m\hbar}{\pi r^3}\left(\delta_{ij} - 3\frac{r_i r_j}{r^2}\right)(1 - \log\mu r).$$

Finally, the vacuum polarization contribution to the metric—Eq. (34)—is modified to become

$$\delta h^{(2)\mathrm{vacpol}}_{ij} = \frac{G^2m\hbar}{15\pi r^3}\left(\delta_{ij} + 44\frac{r_i r_j}{r^2}\right) - \frac{44G^2m\hbar}{15\pi r^3}\left(\delta_{ij} - 3\frac{r_i r_j}{r^2}\right)(1 - \log\mu r).$$

Correspondingly, the forms given in Eqs. (3) and (4) should read

$$g_{ij} = -\delta_{ij}\left[1 + 2\frac{Gm}{r} + \frac{G^2m^2}{r^2} + \frac{14G^2m\hbar}{15\pi r^3} - \frac{76}{15}\frac{G^2m\hbar}{\pi r^3}(1 - \log\mu r)\right] - \frac{r_i r_j}{r^2}\left[\frac{G^2m^2}{r^2} + \frac{76G^2m\hbar}{15\pi r^3} + \frac{76}{5}\frac{G^2m\hbar}{\pi r^3}(1 - \log\mu r)\right]$$

for the Schwarzschild and Kerr metrics. The dependence on $\log\mu$ in the metric can be removed by a coordinate redefinition. Note that with these changes the conditions $\partial^\mu T_{\mu\nu} = 0$ and $\partial^\mu h_{\mu\nu} = \frac{1}{2}\partial_\nu h$ are satisfied in coordinate space, as expected.

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