Remarks on the high-energy behavior of string scattering amplitudes in warped spacetimes. II

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We study the Regge limit of string amplitudes within the model of Polchinski-Strassler for string scattering in warped spacetimes. We also present some numerical estimations of the Regge slopes and intercepts. It is quite remarkable that the real values of those are inside a range of ours.

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I. INTRODUCTION

In recent years, a whole body of knowledge has been developed about duality between gauge and string theories. A first point of duality is a remarkable proposal for string theory whose tension is running [1] and its spectacular implementation in the case of type IIB string theory on $AdS_5 \times S^5$ that turns out to be dual to $N = 4$ supersymmetric Yang-Mills theory [2]. Although the corresponding string sigma models are still out of control that slows further progress, one can get some fascinating insights from simplified models. One is that of Polchinski and Strassler [3]. They proposed to build string amplitudes from old-fashioned amplitudes A_n integrated over the tension with an appropriate weight factor as[1]

$$\hat{A}_n(p_1, \ldots, p_n; \xi_i, \ldots, \xi_n) = \int_{r_0}^{\infty} dr r^{3-\Delta_n} A_n(p_1, \ldots, p_n; \xi_i, \ldots, \xi_n)|_{\alpha' \to \alpha' R^2/r^2}, \tag{1.1}$$

where p_i's and ξ_i's are momenta and wave functions of particles. R is a radius of AdS_5.

In the hard scattering limit such defined amplitudes do fall as powers of momentum as it should be [5]. This also shows that Δ_n is related to a total number of constituents in hadronic states. More recently it was argued in [6] that power law behavior is a feature of string amplitudes in warped spacetimes like AdS_5.

In the Regge limit evaluation of the amplitudes (1.1) is more subtle. The result of [3] based on the approximation of the integral by its dominant saddle point shows that the amplitudes have the desired behavior for special kinematical regions, but otherwise they develop logarithms. However, the use of the semiclassical technique seems questionable as it is not clear what a large parameter is in the problem at hand. The purpose of the present paper is twofold. The first is to propose a possible scheme for studying the Regge limit that is not based on semiclassical approximation. The second is to compare values of the Regge parameters provided by the model (1.1) with those of the real world.

This paper is organized as follows. In Sec. II, we study the Regge limit and find the leading corrections to the Regge behavior. In Sec. III, we discuss the physics behind violation of the Regge behavior and compare it with the known technique of resummation of logarithms in QCD. In Sec. IV, we present our estimations of the Regge parameters. We close the paper with a summary and discussions.

II. REGGE BEHAVIOR

In this section we will discuss the Regge behavior of the amplitudes (1.1). It is also of some interest to evaluate the leading corrections to it because this issue has not been addressed in the literature. In contrast, the leading corrections to the scaling have already been discussed in [4,6], where they turned out to be exponential.

As the first example, we take a tree amplitude of massless vectors in type I theory

$$A_4(\alpha') = (\alpha')^2 K \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1 - \alpha' s - \alpha' t)}, \tag{2.1}$$

with a kinematical factor K as in [7]. Using Eq. (1.1), we get

$$\hat{A}_4 = \alpha' R^2 \frac{K}{t} \int_{r_0}^{\infty} dr r^{1-\Delta} \int_0^1 du u^{-1-\alpha' R^2 s/r^2} (1-u)^{-\alpha' R^2 t/r^2}, \tag{2.2}$$

where $\Delta_4 = \Delta$. Note that one can avoid the poles of the integrand (Γ functions) by increasing r_0 or by deforming the integration contour in their vicinity.

The integrals can be evaluated by first substituting $y = \alpha' R^2/r^2$ and then $x = y/(\hat{\alpha} - y)$ with $\hat{\alpha} = \alpha' R^2/r_0^2$. The result is[2]

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[1]For further developments, see [4].

[2]We omit some irrelevant prefactors, here and below.

$$\hat{A}_4 = \hat{\alpha}\frac{K}{t}\int_0^1 du u^{-1-\hat{\alpha}s}(1-u)^{-\hat{\alpha}t}\int_0^\infty dx x^{-2+\Delta/2}(1+x)^{-\Delta/2} \times \exp\left(\frac{\hat{\alpha}}{(1+x)}[s\ln u + t\ln(1-u)]\right). \quad (2.3)$$

By expanding the exponent we get

$$\hat{A}_4 = \frac{K}{t}\sum_{n=0}^{\infty} c_n \hat{\alpha}^{n+1}\frac{\partial^n}{\partial\hat{\alpha}^n}\mathrm{B}(-\hat{\alpha}s, 1-\hat{\alpha}t), \quad (2.4)$$

where $c_n = (-)^n/(\frac{\Delta}{2}-1)_{n+1}$. $(x)_n$ stands for a Pochhammer polynomial.

Having derived the series, we can use it to study the Regge limit. It is clear from (2.4) that the first term simply provides the desired Regge behavior, while all higher terms provide contributions that do not have the Regge form. The dominant contributions contain $\hat{\alpha}t\ln\hat{\alpha}s$ factors. So, by keeping only such dominant contributions one can think of the series as an expansion in $\hat{\alpha}t\ln\hat{\alpha}s$. To illustrate the point, let us consider the amplitude at next-to-leading order. It is given by

$$\hat{A}_4 = A_4(\hat{\alpha})\left(1+\frac{2}{\Delta}\hat{\alpha}[s\psi(-\hat{\alpha}s) + t\psi(1-\hat{\alpha}t) + u\psi(1+\hat{\alpha}u)]\right), \quad (2.5)$$

where $\psi(x) = \Gamma'(x)/\Gamma(x)$. Using the reflection formula together with $\psi(z) = \ln z + O(\frac{1}{z})$ for $z\to\infty$ in $|\arg z| < \pi$, we find

$$\hat{A}_4 \sim \left(1+\frac{2}{\Delta}\hat{\alpha}|t|\ln\hat{\alpha}s\right)(\hat{\alpha}s)^{1+\hat{\alpha}t}, \quad (2.6)$$

which implies that the amplitude has the Regge behavior with a linear trajectory

$$\alpha(t) = 1 + \alpha'_{\mathrm{eff}}t, \qquad \alpha'_{\mathrm{eff}} = \alpha'\left(\frac{R}{r_0}\right)^2 \quad (2.7)$$

for a special kinematical region, where

$$\hat{\alpha} \ll \frac{\Delta}{2|t|\ln\hat{\alpha}s}. \quad (2.8)$$

From (2.6) it follows that the leading correction to the Regge behavior is logarithmic.

It is straightforward to extend the analysis to type II theories. As an example, let us take a tree amplitude for massless scalars rewritten as

$$A_4(\alpha') = (\alpha')^2\frac{K}{u^2}\int d^2z|z|^{-2-(1/2)\alpha' s}|1-z|^{-2-(1/2)\alpha' t}, \quad (2.9)$$

with a kinematical factor K as in [8], and then modify it according to Eq. (1.1):

$$\hat{A}_4 = (\alpha' R^2)^2\frac{K}{u^2}\int_{r_0}^\infty dr r^{-1-\Delta} \times \int d^2z|z|^{-2-(1/2)\alpha' R^2 s/r^2}|1-z|^{-2-(1/2)\alpha' R^2 t/r^2}, \quad (2.10)$$

where $\Delta_4 = \Delta$. Note that the integrand as a ratio of Γ functions has poles at $r = R\sqrt{\alpha' s/4n}$. One can avoid them by increasing r_0 or by deforming the integration contour in their vicinity. The computation proceeds as before. The result is

$$\hat{A}_4 = \frac{K}{u^2}\sum_{n=0}^{\infty} c_n\hat{\alpha}^{n+2}\frac{\partial^n}{\partial\hat{\alpha}^n}\int d^2z|z|^{-2-(1/2)\hat{\alpha}s}|1-z|^{-2-(1/2)\hat{\alpha}t}, \quad (2.11)$$

where $c_n = (-)^n/(\frac{\Delta}{2})_{n+1}$.

Let us now examine more closely this expansion in the Regge limit. If we restrict to leading order, what we will get is an expansion in $\hat{\alpha}t\ln\hat{\alpha}s$ again. At next-to-leading order, the amplitude takes the form

$$\hat{A}_4 = A_4(\hat{\alpha})\left[1+\frac{2}{\Delta+2}f(\hat{\alpha}s/4, \hat{\alpha}t/4, \hat{\alpha}u/4)\right], \quad (2.12)$$

where $f(x,y,z) = x[\psi(-x)+\psi(1+x)] + y[\psi(-y)+\psi(1+y)] + z[\psi(z)+\psi(1-z)]$. Discarding subleading terms, what is left in the limit $s\to\infty$ is

$$\hat{A}_4 \sim \left(1+\frac{1}{\Delta+2}\hat{\alpha}|t|\ln\hat{\alpha}s\right)(\hat{\alpha}s)^{2+(1/2)\hat{\alpha}t}. \quad (2.13)$$

Thus the amplitude has the desired Regge behavior with a linear trajectory

$$\alpha(t) = 2 + \alpha'_{\mathrm{eff}}t, \qquad \alpha'_{\mathrm{eff}} = \frac{1}{2}\alpha'\left(\frac{R}{r_0}\right)^2 \quad (2.14)$$

for a special kinematical region, where

$$\hat{\alpha} \ll \frac{\Delta+2}{|t|\ln\hat{\alpha}s}. \quad (2.15)$$

From Eq. (2.13) it follows that the leading correction to the Regge behavior is logarithmic.

III. SUMMING CORRECTIONS

If s grows, the logarithmic corrections become more and more relevant. So, it is necessary to resum contributions to all orders in Eqs. (2.4) and (2.11). To see how it works, consider, for instance, the logarithmic terms in (2.4). Summing gives[3]

[3]In fact, one can obtain this expression from (2.3) by noting that in the Regge limit the integral over u is dominated by $u = 1 - t/s$.

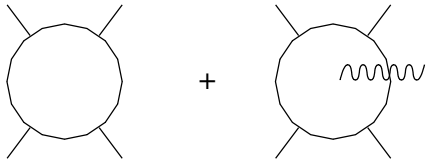

FIG. 1. Contributions to the amplitude (3.3). The wavy line corresponds to the soft dilaton.

$$\hat{A}_4 \sim (\hat{\alpha}s)^{1+\hat{\alpha}t} \int_0^\infty dx x^{(\Delta-5)/2}(1+x)^{(1-\Delta)/2} \times \exp\left(\frac{\hat{\alpha}|t|}{1+x}\ln\hat{\alpha}s\right). \tag{3.1}$$

We focus on s dependence, so s-independent contributions have been dropped. The integral is dominated by $x = (\Delta - 5)/2\hat{\alpha}|t|\ln\hat{\alpha}s$, and so it is proportional to $(\ln\hat{\alpha}s)^{(3-\Delta)/2} \times (\hat{\alpha}s)^{\hat{\alpha}|t|}$. The last factor is of great importance as it is responsible for cancellation ($t < 0$) of the prefactor in (3.1). As a result, the Regge form is lost. We end up with

$$\hat{A}_4 \sim s(\ln\hat{\alpha}s)^{(3-\Delta)/2}. \tag{3.2}$$

We will not attempt a similar derivation in detail for (2.11). However, we claim that the answer is that of [3].

To see the physical interpretation, take the amplitude at next-to-leading order [Eq. (2.5)] and rewrite it as

$$\hat{A}_4 = c\left(A_4(\hat{\alpha}) - \frac{2}{\Delta+2}\delta A_4^{(1)}(\hat{\alpha})\right), \tag{3.3}$$

where $c = 1 + 2/\Delta$. One line of thought is to think of the right-hand side as a string theory with the fixed tension defined by $\hat{\alpha}$. If so, then the first term is just the tree amplitude. As to the second, it is nothing else but the one-loop planar amplitude of four massless vectors integrated over a corner of moduli space near $q = 0$ [7]. We define such a corner as a range between $\varepsilon e^{-(1/e^2N)}$ and ε, where ε is an arbitrary small parameter. Then, to leading order the integral over the modular parameter gives $1/e^2N$ that allows us to consider both the terms on equal footing. In fact, this is the case for higher corrections also.

To make the connection to QCD, we first note that $\delta A_4^{(1)}(\hat{\alpha})$ can be expressed as a tree level diagram by virtue of the soft dilaton theorem.[4] The amplitude (3.3) then takes the form shown schematically in Fig. 1.

Second, as noted above, the leading corrections are terms $(\ln\hat{\alpha}s)^n$. One of the approaches to resum the leading logarithms in QCD is that of [9]. At next-to-leading order, gluons can be radiated into the final state. This effect is shown schematically in Fig. 2. We see that on the string theory side the soft dilatons play a similar role of that of the radiated soft gluons. However, the final results are quite different. In QCD the resummation leads to the Regge behavior, while in the string theory case it destroys such a behavior.

FIG. 2. Meson scattering via two-gluon exchange. The wavy lines correspond to gluons.

Finally, the point about the string models with running tension which may sound surprising is that tree amplitudes might be built in terms of multiloop amplitudes of the theories with fixed tension. We have provided some evidence supporting this idea. We believe that the issue is worthy of future study.

IV. PHENOMENOLOGICAL PROSPECTS

A. Estimates of slopes

From the early days of old-fashioned string theories (dual resonance models), it was of great interest to use them for the demands of experiment. All those models have free parameters which are not predicted by the theory but determined by fitting the experimental data (see, e.g., [10,11]). On the other hand, it became clear later that the string theories are more appropriate for the description of gravity, where a fundamental length is given by the Planck length. If so, a typical slope of trajectories is of order 10^{-38} GeV^{-2}. Neither of these seems acceptable.

Having derived the explicit formulas for the slopes, there is too great a temptation to check whether the slopes are able to meet the challenge of the experimental data. We begin with Eq. (2.14) which corresponds to the Pomeron.[5] As in [3], let us define r_0 in terms of a typical strong-interaction scale Λ as $r_0 = \Lambda R^2$. Thus, the slope takes the form $\alpha'_{\text{eff}} = \alpha'/2R^2\Lambda^2$. The ratio α'/R^2 can be traded for the 't Hooft coupling constant λ. The precise relation is not known, in general, for an arbitrary value of λ. We are led therefore to investigate the behavior of the slope by using the original Maldacena's relation [2]

$$\frac{R^2}{\alpha'} = \sqrt{4\pi e^2 N_c} \tag{4.1}$$

valid for large λ as well as the modified relation[6]

[4] See, e.g., [7] and references therein.

[5] Strictly speaking, the closed string amplitudes of Sec. III quite likely describe elastic scattering of glueballs. In the Regge limit, however, scattering is dominated by the exchange of the Pomeron, so it is possible to get the Pomeron intercept from such amplitudes. We believe that its form is independent of the nature of the scattering particles.

[6] It is known from several contexts that it provides similar results to perturbative QCD. See, e.g., [12].

TABLE I. Estimates of the Pomeron slope at high scales set by $\sqrt{s}$.

$\alpha_{\rm eff}^{\rm (I)}$ (GeV^{-2})	1.81	1.59	1.53	1.13	1.02
$\alpha_{\rm eff}^{\rm (II)}$ (GeV^{-2})	0.26	0.20	0.19	0.10	0.08
α_s (Q^2)	0.10	0.13	0.14	0.26	0.32
Q (GeV)	91	58	35	7	1.7

$$\frac{R^2}{\alpha'} = 4\pi e^2 N_c \tag{4.2}$$

valid for small λ.

The use of these relations, respectively, gives

$$\alpha_{\rm eff}^{\rm (I)} = \frac{1}{8\pi\sqrt{\alpha_s N_c}\Lambda^2} \tag{4.3}$$

and

$$\alpha_{\rm eff}^{\rm (II)} = \frac{1}{32\pi^2\alpha_s N_c \Lambda^2}, \tag{4.4}$$

where $\alpha_s = e^2/4\pi$.

For $N_c = 3$, $\Lambda = 200$ MeV, and some values of α_s, our estimates are presented in Table I.

Here the smallest value of α_s corresponds to the scale Q set by the neutral weak boson with $M_z = 91$ GeV, while the largest one corresponds to the scale set by the τ lepton with $m_\tau = 1.7$ GeV. All values are taken from [13]. The value of the soft-Pomeron slope (0.25 GeV^{-2}) is verified at $\sqrt{s} = 53$ GeV for elastic pp scattering, while for the hard Pomeron (0.1 GeV^{-2}) it is extracted from the processes with $\sqrt{s}$ between 6 and 94 GeV [14]. This time we assume that the effective coupling is defined by the scale $\sqrt{s}$, so we use the values given above.

Since there is more than one scale in the problem at hand, it seems natural to repeat the above analysis for the effective coupling defined by the scale $\sqrt{|t|}$. To do so, we need values of α_s at scales between 0.05 and 1 GeV^2 [14]. Unfortunately, no reliable values is possible. The problem is well known: QCD becomes strongly coupled at low scales. Leaving aside the problem of the effective QCD coupling at low scales,[7] we give a few estimates without referring to scales. Our results for the slopes are present in Table II.

We now turn to Eq. (2.7) which describes the Reggeon trajectories. It is straightforward to extend the above analysis. As a result, we get

$$\alpha_{\rm eff}^{\rm (I)} = \frac{1}{4\pi\sqrt{\alpha_s N_c}\Lambda^2} \tag{4.5}$$

[7]See, e.g., [15] and references therein.

and

$$\alpha_{\rm eff}^{\rm (II)} = \frac{1}{16\pi^2\alpha_s N_c \Lambda^2}. \tag{4.6}$$

The experimental data indicate that the slopes are around 0.9 ± 0.1 GeV^{-2}. For example, in the case of pion charge-exchange scattering, the values are 0.93 GeV^{-2} for $\pi^- p \to \pi^0 n$ and 0.79 GeV^{-2} for $\pi^- p \to \eta n$ [16].

For $N_c = 3$, $\Lambda = 200$ MeV, some estimates are presented in Table III. Here we assume that the effective coupling is defined by high scales. In the case of pion charge-exchange scattering, the energy range is typically between 20 and 200 GeV. All values of α_s are taken from [13].

On the other hand, assuming now that the coupling is defined by the scale $\sqrt{|t|}$, we need values of α_s at low scales. For example, $\sqrt{|t|}$ must be below 0.55 GeV for the pion's scattering, otherwise the trajectories are nonlinear [16]. As noted, no reliable values are possible. So, we give a few estimates without referring to scales in Table IV.

Although the values for the slopes we found may in fact differ from the real values by up to 1 order of magnitude, it is still remarkable that this simple model is in principle able to meet the experimental data.

We conclude by making a few comments:

(i) From Tables I, II, III, and IV, we note that $\alpha_{\rm eff}^{\rm (I)}$ provides more acceptable results at low scales, while $\alpha_{\rm eff}^{\rm (II)}$ provides results at high scales. This is in accord with a common belief that going from the relation (4.1) to (4.2) does lead to a more weakly coupled theory. It is now clear that the precise relation $R^2/\alpha' = f(4\pi e^2 N)$ is worthy of further investigation.

TABLE II. Estimates of the Pomeron slope at low scales.

$\alpha_{\rm eff}^{\rm (I)}$ (GeV^{-2})	0.57	0.41	0.26	0.18	0.13
$\alpha_{\rm eff}^{\rm (II)}$ (GeV^{-2})	0.026	0.013	0.005	0.003	0.001
α_s (Q^2)	1	2	5	10	20

TABLE III. Estimates of the Reggeon slope at high scales.

$\alpha_{\rm eff}^{\rm (I)}$ (GeV^{-2})	3.54	3.48	3.46	3.07	2.97
$\alpha_{\rm eff}^{\rm (II)}$ (GeV^{-2})	0.50	0.48	0.48	0.38	0.35
α_s (Q^2)	0.105	0.109	0.11	0.14	0.15
Q (GeV)	200	150	91	35	20

TABLE IV. Estimates of the Reggeon slope at low scales.

$\alpha_{\rm eff}^{\rm (I)}$ (GeV^{-2})	1.15	0.81	0.51	0.36	0.26
$\alpha_{\rm eff}^{\rm (II)}$ (GeV^{-2})	0.050	0.026	0.011	0.005	0.003
α_s (Q^2)	1	2	5	10	20

(ii) It follows from the results of Sec. IV that the relative factor between the Reggeon and Pomeron slopes is 2. The experimental data point out that it is at least 2 times larger. The point is the use of the simplified ansatz (1.1) that inherits this factor from the standard string amplitudes. However, in more realistic models this factor might be close to 4 [17].

(iii) Although we use some ideas inspired by the AdS/CFT correspondence, we do not strictly follow this conjecture. So, we set the number of colors to be 3. Although $1/N_c = 1/3$ is not very small, we cannot say whether this approximation is good or bad. To do so, we must be able to find all terms in the $1/N_c$ expansion that even for the leading ones remains to be done.[8]

(iv) As noted in Sec. IV, the amplitudes exhibit the Regge behavior for special kinematic regions. So it is worth checking that this is consistent with the experimental data. First, let us check this for elastic pp scattering with $\sqrt{s} = 53$ GeV used to extract the value of the soft-Pomeron slope in [14]. For $|t|$ between 0.05 and 0.2 GeV^2, we may replace $\ln\hat{\alpha}s$ in the denominator of (2.15) by $\ln s/|t|$. So, we get

$$\alpha'_{\rm eff} \ll \frac{1}{2}\frac{\Delta+2}{|t|\ln s/|t|}. \tag{4.7}$$

The right-hand side takes its lowest value at $|t| = 0.2$ GeV^2, where

$$\alpha'_{\rm eff} \ll 3.6\ {\rm GeV}^{-2}. \tag{4.8}$$

Here we simply set $\Delta = 12$ as a total number of constituents. It does not make a big difference to our estimate.[9] Note that the bound of [3] coincides with ours up to a shift: $\Delta \to \Delta - 6$. It yields

$$\alpha'_{\rm eff} \ll 2.1\ {\rm GeV}^{-2}. \tag{4.9}$$

Let us now check the consistency condition for the pion's scattering. The right-hand side of (2.8) takes its lowest value at the largest possible $|t|$ and s. For these values we may replace $\ln\hat{\alpha}s$ in the denominator by $\ln s/|t|$. So, we get

$$\alpha'_{\rm eff} \ll \frac{\Delta}{2|t|\ln s/|t|}. \tag{4.10}$$

At $|t| = 0.3$ GeV^2 and $s = 400$ GeV^2 it provides

$$\alpha'_{\rm eff} \ll 2.3\ {\rm GeV}^{-2}. \tag{4.11}$$

Here we set for our estimate $\Delta = 10$. Certainly, the above value is not much larger than the real value of the slope, so the corrections might violate the Regge behavior. Interestingly, the experimental data also indicate a violation of the linear Regge trajectory near this value of $|t|$.

(v) Interestingly enough, our expressions for the slopes (4.4) and (4.6) look like instanton contributions. Indeed, one can rewrite the amplitude as $A \sim \exp(-\frac{c}{\alpha_s}|t|\ln s)$. This might be a hint on a nonperturbative nature of the high-energy scattering in the Regge limit. Other indications are reviewed in [19].

[8]For a more detailed discussion of this issue in QCD, see [18].

[9]The bounds (2.8) and (2.15) are rather crude. We derived them by keeping only leading logarithms. In general, the bounds might include some factors due to subleading terms.

TABLE V. Estimates of the Pomeron intercept.

α_0	0.97	1.10	1.13	1.36	1.42	1.67	1.77	1.85	1.90
α_s (Q^2)	0.10	0.13	0.14	0.26	0.32	1	2	5	10
Q (GeV)	91	58	35	7	1.7	...	...	...	...

B. Estimates of intercepts

According to Sec. II, the results for the Regge intercepts are the same as in the dual resonance models. From this point of view, the model (1.1) does not solve the problem of getting the right values. On the other hand, these values are relatively close to the real ones, so a good idea is to take them as the leading contributions. Recently, a next-to-leading order correction to the Pomeron intercept has been reported by Polchinski [20]. In our notations it is given by $-1/\sqrt{\pi\alpha_s N_c}$. Thus, the intercept is

$$\alpha_0 = 2 - \frac{1}{\sqrt{\pi\alpha_s N_c}}. \tag{4.12}$$

There is too great a temptation to check whether this improved expression is able to meet the challenge of the experimental data. For $N_c = 3$ and some values of α_s, our estimates are presented in Table V.

We do not refer to scales where no reliable values of α_s are possible.

The values of the Pomeron intercepts are known [14]:

$$\begin{aligned} &\alpha_0 = 1.08 \quad \text{for the soft Pomeron,} \\ &\alpha_0 = 1.4 \quad\ \ \text{for the hard Pomeron.} \end{aligned} \tag{4.13}$$

The same as the slopes, both values of the Pomeron intercepts are also inside a range of our estimates. Unfortunately, the above simple estimates do not clarify the issue of the existence of two Pomerons.

V. SUMMARY AND DISCUSSION

In this paper, we studied the model (1.1) in the Regge limit. We found that the Regge behavior holds for the special kinematical regions; otherwise it is violated by logarithms. We revealed the physics behind violation and its counterpart in QCD. We presented the numerical esti-

mates of the Regge parameters. It is quite remarkable that the real values of those are inside a range of ours.

There is a large number of open problems associated with the circle of ideas explored in this paper. Let us mention a couple.

As noted earlier, the absence of control over the string sigma models describing warped spacetime geometries further slows progress in our understanding of gauge/string duality. It is therefore highly desirable to develop a new technique that will allow us to consider more realistic models and apply them to the real world. Our estimations show that even the simplified model of strings in warped spacetimes yields rather fascinating results. This provides some further evidence that such a direction is worthy of future study.

To the contrary, the string sigma models may not be the last word, and new ideas are required to meet the challenge of the experimental data. It still remains to be suggested what these ideas are.

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