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Wilson loop, Regge trajectory, and hadron masses in a Yang-Mills theory from semiclassical strings

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We compute the one-loop string corrections to the Wilson loop, glueball Regge trajectory and stringy hadron masses in the Witten model of nonsupersymmetric, large-N Yang-Mills theory. The classical string configurations corresponding to the above field theory objects are, respectively: open straight strings, folded closed spinning strings, and strings orbiting in the internal part of the supergravity background. For the rectangular Wilson loop we show that besides the standard Lüscher term, string corrections provide a rescaling of the field theory string tension. The one-loop corrections to the linear glueball Regge trajectories render them nonlinear with a positive intercept, as in the experimental soft Pomeron trajectory. Strings orbiting in the internal space predict a spectrum of hadroniclike states charged under global flavor symmetries which falls in the same universality class of other confining models.

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I. INTRODUCTION

One of the most remarkable developments in the holographic approach to solving gauge theories has been the realization of certain scenarios where the correspondence can be taken beyond the supergravity approximation [1,2]. The main idea behind this development asserts that some classical configurations of the string sigma model in a given supergravity background are dual, in the holographic sense, to states of the gauge theory. In particular, the conserved quantities of the classical configurations are to be identified with the quantum numbers describing gauge theory states.

Being ultimately interested in realistic gauge theories, a natural question in this approach pertains to the properties of states of confining gauge theories. Unlike the case of the standard duality between strings in $AdS_5 \times S^5$ and $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM), for the case of confining strings perturbative field theory calculations are extremely difficult. The only data available to test the predictions of the holographic approach come from lattice simulations of SU(N_c) gauge theories and some low energy phenomenology of QCD. Evidently, holographic calculations are valid for large N_c and a direct comparison with QCD is still beyond our current reach. Nevertheless, it is appropriate to compare the general behavior on both sides. For example, a recent holographic analysis of corrections to the glueball Regge trajectories seems to be compatible with experimental data describing the soft Pomeron trajectory [3]. Other studies point to a possible emergence of the chiral perturbation Lagrangian from a holographic point of view [4].

In the search for a holographic dual to nonsupersymmetric Yang-Mills one is faced with the daunting task of producing a holographic background which ideally retains some of the control provided by supersymmetric theories. Several examples of soft breaking of $\mathcal{N} = 1$ backgrounds have been provided [5,6], however, in this process one usually looses analytic control over the supergravity background if the scale of the breaking is high enough for the study of YM theory. A different route to pure YM was proposed by Witten in [7] and consists of a stack of D4 branes wrapped on a thermal circle. This implies that on the gauge theory side we deal with a 5D gauge theory compactified on a circle along which the fermions have antiperiodic boundary conditions. For the appropriate energy scale this theory is essentially nonsupersymmetric YM in 4D.

In this paper we present a detailed study of the one-loop sigma-model corrections to various classical string solutions, in the asymptotic region of the dual supergravity background conjectured to be relevant for the description of the IR regime of the field theory.

The paper is organized as follows. In Sec. II we present the background and the string theory action needed for the computation of the one-loop corrections.

For supergravity backgrounds dual to confining gauge theories the classical string describing a rectangular Wilson loop with area-law behavior goes into the bulk with a bathtub-shaped configuration. In Sec. III we consider the open string configuration that approximates this Wilson loop sufficiently well: an open straight string of length L lying at the minimal transverse radius of the dual background. We then calculate the leading sigma-model correction to its energy. There are other evaluations in the Witten model of the one-loop quantum correction to this classical configuration [8–10]. A distinctive aspect of our calculation is the systematic treatment of the fermionic sector and that we provide a consistency check via cancellation of anomalies. Our calculation provides a stringy prediction for the leading quantum correction to the linear quark antiquark potential in a theory "close" to pure, large N, YM. We find that the dominant term in this correction is not of Lüscher $(1/L)$ type, but amounts to a kind of "renormalization" of the effective string tension[1]. We find a Lüscher-like term, with negative sign, as a subleading correction.

In Sec. IV we consider a closed string at the minimal radius spinning in the (warped) flat 4D directions of the background. In the dual gauge theory this configuration describes glueball Regge trajectories. Even if we are not able to fully solve the string spectral problem for this configuration, we show that in some interesting regimes the one-loop-corrected glueball Regge trajectory becomes nonlinear with a positive intercept. These features are qualitatively shared (through in a different regime) by the best fit to the experimental soft Pomeron trajectory[2] given by the UA8 Collaboration [12]. We also find, again, a renormalization of the string tension.

In Sec. V we consider closed multispinning string states rotating on the internal S^4 of the IR region of the supergravity background. These are argued to correspond to multicharged hadronic states in the dual field theory, whose constituents are massive adjoint fields[3]. We consider the classical energy in the large spin limit and discuss the quantum corrections for the simplest circular solutions. Their behavior is expected to be qualitatively equivalent to the one of the multispinning string solutions examined in the literature both in the confining and in the nonconfining cases. In particular, in the semiclassical limit of large total spin J, the leading quantum correction to the energy reduces to the one corresponding to collapsed pointlike strings with spin J orbiting along the equator of S^4. The one-loop corrections in this case can be simply deduced by quantizing closed strings on the plane-wave background obtained from the Penrose limit of the Witten model. The corresponding string spectrum is studied in Sec. VA. It shares the universal properties of all the other confining examples examined in literature [6,14–16]. The string results give thus the expected universal predictions on the energy spectrum of multicharged hadronic states, called annulons, in gauge theory.

We conclude in Sec. VI with comments on the stringy predictions for the dual field theory and the nature of their universality.

We include also several appendixes with some technical details. In Appendix A we briefly review the supergravity background paying special attention to the regime of validity of the supergravity description and to aspects of the holographic relations with the dual gauge theory. In Appendix B we discuss the general form of the classical solutions at constant radius. Appendix C shows how the straight open string approximates well the bathtub-shaped configuration describing the actual Wilson loop, in the large L limit, both from the classical and semiclassical point of view. We discuss the cancellation of the Weyl anomaly for the Regge trajectory configuration in Appendix D. Finally, Appendix E provides details on the Penrose limit of the Witten background.

The supergravity background we consider in this paper has recently received renewed attention; many of the classical configurations we analyze are also discussed in [17–19]. Our main contribution is a systematic description of the one-loop corrections to such classical configurations.

II. GENERAL SETUP

In this section we introduce the Witten background, we rewrite the metric in the relevant IR regime [formulas (2.5) and (2.6)] and present the bosonic and fermionic string actions we will use in the following in (2.10) and (2.19). More details of the supergravity background, with some notes on the holographic relations with the field theory, are collected in Appendix A.

The ten-dimensional string frame metric and dilaton of the Witten model are given by

$$\begin{aligned} ds^2 &= \left(\frac{u}{R}\right)^{3/2}\left[\eta_{\mu\nu}dx^\mu dx^\nu + \frac{4R^3}{9u_0}f(u)d\theta^2\right] \\ &\quad + \left(\frac{R}{u}\right)^{3/2}\frac{du^2}{f(u)} + R^{3/2}u^{1/2}d\Omega_4^2, \\ f(u) &= 1 - \frac{u_0^3}{u^3}, \qquad R = (\pi N g_s)^{1/3}\alpha'^{1/2}, \\ e^\Phi &= g_s\frac{u^{3/4}}{R^{3/4}}. \end{aligned} \tag{2.1}$$

The geometry consists of a warped, flat 4D part, a radial direction u, a circle parametrized by θ with radius vanishing at the horizon $u = u_0$, and a four-sphere whose volume is instead everywhere nonzero. It is nonsingular at $u = u_0$. Notice that in the $u \to \infty$ limit the dilaton diverges: this implies that in this limit the completion of the present IIA model has to be found in M theory. The background is completed by a constant four-form field strength

$$F_4 = 3R^3\omega_4, \tag{2.2}$$

where ω_4 is the volume form of the transverse S^4.

[1]We call it a renormalization throughout the paper for brevity. It is really a correction to the dependence of the tension on the UV coupling λ, rather than a dependence on the running coupling.

[2]For an interesting recent study of the glueball Regge trajectories on the lattice and in a "old" string model of QCD, see [11].

[3]For a study of baryonic spectra from strings in AdS space see [13].

The main gauge theory parameter we will use in the following is the KK mass scale $1/R_\theta$, which is given by

$$\frac{1}{R_\theta} = \frac{3}{2} m_0, \quad \text{where } m_0^2 = \frac{u_0}{R^3}. \tag{2.3}$$

As can be read from the metric, m_0 is also the typical glueball mass scale and, as we will show in the following [see formula (3.2)], its square is proportional to the ratio between the confining string tension $T_{\rm QCD}$ and the UV 't Hooft coupling λ. As usual, the supergravity approximation is reliable in the regime opposite to that in which the KK degrees of freedom can decouple from the low energy dynamics. The condition $T_{\rm QCD} \ll m_0^2$ implies in fact $\lambda \ll 1$, which is beyond the supergravity regime of validity.

We will be mainly interested in classical string configurations localized at the horizon $u = u_0$, since this region is dual to the IR regime of the dual field theory. In this case the coordinate u is not suitable because the metric written in this coordinate looks singular at $u = u_0$. Then, as a first step, let us introduce the radial coordinate

$$r^2 = \frac{u - u_0}{u_0}, \tag{2.4}$$

so that the metric expanded to quadratic order around $r = 0$ becomes

$$ds^2 \approx \left(\frac{u_0}{R}\right)^{3/2}\left[1 + \frac{3r^2}{2}\right](\eta_{\mu\nu} dx^\mu dx^\nu) + \frac{4}{3} R^{3/2}\sqrt{u_0}(dr^2 + r^2 d\theta^2) + R^{3/2} u_0^{1/2}\left[1 + \frac{r^2}{2}\right] d\Omega_4^2. \tag{2.5}$$

In order to simplify the study of the classical string configurations and their semiclassical quantization, it is useful to rescale the dimensional coordinates with the KK mass parameter which represents the reference scale of our theory. We do it by defining the dimensionless coordinates $X^\mu = m_0 x^\mu$. Introducing the parameter $\xi = \lambda/3$, the metric in the IR takes the form

$$ds^2 = l_s^2 \xi\left\{\left[1 + \frac{3}{2}(y_1^2 + y_2^2)\right] dX^\mu dX_\mu + \frac{4}{3}(dy_1^2 + dy_2^2) + \left[1 + \frac{1}{2}(y_1^2 + y_2^2)\right] d\Omega_4\right\}, \tag{2.6}$$

where we have used Cartesian coordinates y_a, $a = 1, 2$, such that $r^2 = y_1^2 + y_2^2$, and the metric is expanded up to the second order in y_a^2. From (2.6) one can easily see that the string action contains only ξ as external parameter.

A. Action

As a first step we are going to consider classical solutions sitting at the origin $y^a = 0$ of the y^a's plane for which we can use the following effective Polyakov action in the conformal gauge

$$S = -\frac{\xi}{4\pi}\int d\tau d\sigma [\partial_\alpha X^\mu \partial^\alpha X_\mu + \partial_\alpha \zeta^A \partial^\alpha \zeta^A - \Lambda(\zeta^A \zeta^A - 1)], \tag{2.7}$$

where the Lagrange multiplier Λ constraints ζ^A, $A = 1, \ldots, 5$, to define the S^4, i.e., $\zeta^A\zeta^A = 1$.

We want to classify the solutions by choosing J^{12} and J^{34}, i.e., the generators of the rotations in the 1-2 and 3-4 planes of the coordinates ζ^A, as a basis of the Cartan subalgebra of the isometry group SO(5). A useful parametrization of S^4 is the following:

$$\zeta^5 = \sin\psi, \qquad \zeta^M = \cos\psi Z^M, \qquad M = 1, \ldots, 4, \quad \text{with } Z^M Z^M = 1. \tag{2.8}$$

Then, the S^4 metric takes the form

$$d\Omega_4^2 = d\psi^2 + \cos^2\psi d\Omega_3^2. \tag{2.9}$$

If we consider solutions located at the point $\zeta^5 = 0 \Rightarrow \psi = 0$[4], we can take as effective action

$$S = -\frac{\xi}{4\pi}\int d\tau d\sigma [\partial_\alpha X^\mu \partial^\alpha X_\mu + \partial_\alpha Z^M \partial^\alpha Z^M - \Lambda(Z^M Z^M - 1)]. \tag{2.10}$$

The other ingredient we need is the type IIA GS action expanded up to the second order in the fermions. In his Polyakov form, the quadratic fermionic action is [20]

$$S_F = \frac{i}{4\pi\alpha'}\int d\tau d\sigma \sqrt{-h}\bar{\theta}(1 + \Gamma_{F1})\Gamma^\alpha D_\alpha \theta = \frac{i}{4\pi\alpha'}\int d\tau d\sigma \bar{\theta}[(\sqrt{-h}h^{\alpha\beta} + \epsilon^{\alpha\beta}\Gamma^{\underline{11}})\Gamma_\alpha D_\beta]\theta, \tag{2.11}$$

where $\theta(\tau, \sigma)$ is a ten-dimensional Majorana spinor, Γ_α is the pullback on the string world sheet of $\Gamma_m = e_m^{\underline{a}}\Gamma_{\underline{a}}$ [m($\underline{a}$) are the generic curved (flat) ten-dimensional indexes[5]], D_m is the usual generalized covariant derivative entering the supersymmetry transformations of the gravitino in type IIA supergravity, and

$$\Gamma_{F1} = \frac{\epsilon^{\alpha\beta}}{2\sqrt{-h}}\Gamma_{\alpha\beta}\Gamma^{\underline{11}} \tag{2.12}$$

is the natural chiral world-sheet operator. The kappa symmetry acts on the θ in the following way:

$$\delta_\kappa \theta = (1 - \Gamma_{F1})\kappa, \tag{2.13}$$

where κ is an arbitrary Majorana spinor. In our case where only the Ramond-Ramond (RR) $F_{(4)}$ is turned on, the

[4] In this way, we are looking for classical configurations corresponding to highest weight states of SO(5) with respect to the Cartan subalgebra defined by J^{12} and J^{34}.

[5] In order to avoid ambiguities, we underline flat indexes.

generalized covariant derivative reduces to

$$D_m = \partial_m + \frac{1}{4}\omega_{m\underline{ab}}\Gamma^{\underline{ab}} - \frac{1}{8\cdot 4! g_s} e^{\Phi} F^{(4)}_{\underline{abcd}}\Gamma^{\underline{abcd}}\Gamma_m. \tag{2.14}$$

The configurations we will consider can be grouped into two kinds of solutions both satisfying the static gauge condition $X^0 \equiv t = k\tau$ and either moving in the $\mathbb{R}^{1,3}$ parametrized by the X^μ's or rotating on the S^4. Let us then introduce the following natural vielbein for the IR region of our background (2.6)

$$\begin{aligned}
&e^{\underline{\mu}} = \sqrt{\xi\alpha'}dX^\mu, \qquad \mu = 0, \ldots, 3,\\
&e^{\underline{4}} = \sqrt{\frac{4}{3}\xi\alpha'}dy^1, \qquad e^{\underline{5}} = \sqrt{\frac{4}{3}\xi\alpha'}dy^2,\\
&e^{\underline{6}} = \sqrt{\xi\alpha'}d\psi, \qquad e^{\underline{7}} = \sqrt{\xi\alpha'}\cos\psi d\chi,\\
&e^{\underline{8}} = \sqrt{\xi\alpha'}\cos\psi\sin\chi d\phi_-,\\
&e^{\underline{9}} = \sqrt{\xi\alpha'}\cos\psi\cos\chi d\phi_+.
\end{aligned} \tag{2.15}$$

In this basis the RR field strength is given by

$$F_{\underline{6789}} = \frac{3R^3}{\alpha'^2\xi^2}. \tag{2.16}$$

If we focus on the study of fluctuations around bosonic configurations with a nontrivial shape $X^\mu(\tau, \sigma)$ only in the flat directions (as for the Wilson loop and the Regge trajectory in the following), the pullback on the world sheet of the spin-connection vanishes and the quadratic fermionic action (2.11) in the conformal gauge, and after a rescaling[6] $\theta \to (\alpha'\xi)^{1/4}\theta$, reduces to

$$\begin{aligned}
S_F =& \frac{i\xi}{4\pi}\int d\tau d\sigma[\partial_\alpha X^\mu\bar{\theta}(\eta^{\alpha\beta} + \epsilon^{\alpha\beta}\Gamma^{\underline{11}})\Gamma_{\underline{\mu}}\partial_\beta\theta\\
&- \frac{3}{8}\partial_\alpha X^\mu\partial^\alpha X_\mu\bar{\theta}\tilde{\Gamma}\theta - \frac{3}{8}\epsilon^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu\bar{\theta}\Gamma^{\underline{11}}\Gamma_{\underline{\mu\nu}}\tilde{\Gamma}\theta],
\end{aligned} \tag{2.17}$$

where $\tilde{\Gamma} = \Gamma_{\underline{6789}}$. In order to write the kappa symmetry in the most transparent way, it is useful to go to a reference frame $\hat{e}_{\underline{a}}$ where the first two directions $\hat{e}_{\underline{\alpha}}$ are tangent to the world sheet

$$\hat{e}_{\underline{\alpha}} = \frac{\partial_\alpha X^\mu}{\sqrt{\partial_\sigma X^\nu\partial_\sigma X_\nu}}e_{\underline{\mu}}. \tag{2.18}$$

Now, if we introduce new $\hat{\Gamma}_{\underline{a}}$'s associated to the vielbein $\hat{e}^{\underline{a}}_m$ ($\hat{e}^{\underline{a}}_m\hat{\Gamma}_{\underline{a}} = e^{\underline{a}}_m\Gamma_{\underline{a}}$) and split $\theta = \theta^1 + \theta^2$, where θ^I ($I = 1, 2$) have opposite ten-dimensional chirality $\Gamma^{\underline{11}}\theta^I = (-)^{I+1}$, the kappa symmetry takes the form

$$\delta_\kappa\theta^1 = (1 - \hat{\Gamma}_{\underline{01}})\kappa, \qquad \delta_\kappa\theta^2 = (1 + \hat{\Gamma}_{\underline{01}})\kappa. \tag{2.19}$$

Then, in order to fix the kappa symmetry, it is clear that the most natural gauge is given by the conditions

$$\begin{cases}(1 - \hat{\Gamma}_{\underline{01}})\theta^1 = 0\\ (1 + \hat{\Gamma}_{\underline{01}})\theta^2 = 0\end{cases} \Leftrightarrow \begin{cases}\hat{\Gamma}^-\theta^1 = 0\\ \hat{\Gamma}^+\theta^2 = 0\end{cases}, \tag{2.20}$$

with $\hat{\Gamma}^\pm = \hat{\Gamma}^{\underline{0}} \pm \hat{\Gamma}^{\underline{1}}$. Even if transparent from the geometrical point of view, this kappa fixing depends in general from τ and σ and requires some attention. An explicit example in which we will use a σ dependent kappa fixing will be considered in Sec. IV.

III. CORRECTION TO THE RECTANGULAR WILSON LOOP

In this section we consider the quadratic fluctuations around the classical string configuration corresponding to the gauge theory rectangular Wilson loop. The quantization around the true string solution for the Wilson loop is extremely complicated, and in the literature it is always performed around an approximated configuration, namely, the straight string. This approximation turns out to be quite reliable, the corrections being exponentially suppressed in the regime of interest. We show in Appendix C how it accurately approximates the actual solution for the Wilson loop, at both the classical and semiclassical level.

In the following subsection we will calculate the bosonic quadratic fluctuations for the straight string sitting at the horizon $u = u_0$ [or $y^a = 0$ in the coordinates used in (2.6)]. We then compute the fermionic fluctuations and the quantum correction in (3.2) and (3.3). The resulting quantum corrected energy in formula (3.16) will be discussed in Sec. VI A.

A. Straight string at minimal radius: bosonic fluctuations

The configuration we are going to study is that of a straight open string of length L lying at $y^a = 0$ in our background (2.6)

$$X^0 = \tau, \qquad X^1 = \sigma, \qquad \sigma \in \left[-\frac{L}{2}, \frac{L}{2}\right]. \tag{3.1}$$

The bosonic action (2.10) is very simple in this case and the evaluation of the energy of the configuration is straightforward. Its classical value (per unit time) satisfies the area law[7]

$$E = T_{\mathrm{QCD}}L, \qquad T_{\mathrm{QCD}} = \frac{u_0^{3/2}}{2\pi\alpha' R^{3/2}} = \frac{1}{6\pi}\lambda m_0^2, \tag{3.2}$$

signaling that the dual gauge theory is confining.

[6]This is the natural rescaling in order to see the θ as the fermions associated to the bosons X^μ in the canonical way $X^\mu \sim \bar{\theta}\Gamma^{\underline{\mu}}\theta$.

[7]Throughout the paper we often pass from dimensionless quantities to dimensional ones. The context should help in avoiding confusion.

The bosonic quadratic fluctuations around this solution are trivial for the modes on the four-sphere and for the flat 4D part, as is transparent from the metric (2.6) and from the fact that the world-sheet volume factor is equal to one. The fluctuations of these modes give just six free massless bosons Φ^i, $i = 1, \ldots, 6$. The configuration has to satisfy Dirichlet boundary conditions at $\sigma = \pm L/2$, then

$$\Phi^i = \mathcal{N} \sum_n a_n e^{i\omega_n \tau} \sin\left[\frac{n\pi}{L}\left(\sigma + \frac{L}{2}\right)\right], \tag{3.3}$$

where $\mathcal{N}$ is a normalization and a_n are constants. The associated frequencies are

$$\omega_n = \frac{\pi n}{L}. \tag{3.4}$$

The nontrivial part of the action is the one for the two $y^{1,2}$ modes. From (2.6) it follows that they get a mass term from the coupling with the on-shell $\partial_\alpha X^\mu \partial^\alpha X_\mu$ part. The action for the fluctuation of these modes, expanded in inverse powers of $\sqrt{\xi}$ around the classical value $y^{1,2} = 0$, reads

$$S = -\frac{1}{4\pi} \int d\tau d\sigma \sum_{a=1,2} \left[\frac{4}{3} \partial_\alpha y^a \partial^\alpha y_a + 3 y^a y_a\right]. \tag{3.5}$$

Upon canonically normalizing the kinetic term and considering the solutions for y^a as in (3.3), the two frequencies read (after reintroducing the dependence on R, u_0)

$$\omega_n = \sqrt{\frac{\pi^2 n^2}{L^2} + \frac{9}{4} m_0^2}, \qquad m_0^2 = \frac{u_0}{R^3}. \tag{3.6}$$

In Appendix C it is shown that these frequencies correctly approximate the ones for the actual Wilson loop in the large L limit; if the string does not sit at the horizon, we only have to replace u_0 with u_m, the minimal value of the radius reached by the configuration.

B. Fermionic quadratic fluctuations

In order to study the fermionic contribution to the one-loop correction to the Wilson loop, it is sufficient to consider the fermionic action (2.17) expanded around the minimal radius configuration (3.1).

Following the general setting of Sec. II A, in this case $\hat{e}^{\underline{a}} = e^{\underline{a}}$ and the (kappa-unfixed) fermionic action takes the form

$$S_F = \frac{i}{2\pi} \int d\tau d\sigma \Big[\bar{\theta}^1 \Gamma^+ \partial_+ \theta^1 + \bar{\theta}^2 \Gamma^- \partial_- \theta^2 - \frac{3}{8}(\bar{\theta}^1 \tilde{\Gamma} \theta^2 + \bar{\theta}^2 \tilde{\Gamma} \theta^1) + \frac{3}{8}(\bar{\theta}^1 \Gamma_{\underline{01}} \tilde{\Gamma} \theta^2 - \bar{\theta}^2 \Gamma_{\underline{01}} \tilde{\Gamma} \theta^1) \Big], \tag{3.7}$$

where $\partial_\pm = (1/2)(\partial_\tau \pm \partial_\sigma)$. The kappa fixing (2.20) can be rendered explicit by introducing the following set of gamma matrices $\Gamma_{\underline{a}}$:

$$\Gamma_{\underline{0}} = i\sigma_2 \otimes \mathbb{I}, \qquad \Gamma_{\underline{1}} = \sigma_1 \otimes \mathbb{I}, \qquad \Gamma_{\underline{A}} = \sigma_3 \otimes \gamma_{\underline{A}} \quad (A = 2, \ldots, 9), \tag{3.8}$$

where $\gamma_{\underline{A}}$ are Euclidean Dirac matrices in eight dimensions and we can split the two θ^I each into two Euclidean 8D Majorana-Weyl fermions of opposite chirality. Then, the kappa-fixing conditions (2.20) become $\sigma_3 \theta^I = (-)^{I+1} \theta^I$ and we can write

$$\theta^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \Theta^1 \\ 0 \end{pmatrix}, \qquad \theta^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \Theta^2 \end{pmatrix}, \tag{3.9}$$

where Θ^I are two Euclidean 8D Majorana-Weyl spinor of the same chirality (with respect to the 8D chirality operator $\gamma^{\underline{11}} \equiv \gamma_{\underline{2}} \cdots \gamma_{\underline{9}}$). Then, the kappa-fixed quadratic fermionic action becomes

$$S_F = -\frac{i}{2\pi} \int d\tau d\sigma \Big[\Theta^1 \partial_+ \Theta^1 + \Theta^2 \partial_- \Theta^2 - \frac{3}{8}(\Theta^1 \tilde{\gamma} \Theta^2 - \Theta^2 \tilde{\gamma} \Theta^1) \Big], \tag{3.10}$$

where $\tilde{\gamma} = \gamma_{\underline{6789}}$. From this action the following (squared) equation of motion follows for Θ^I

$$\left(\partial_\tau^2 - \partial_\sigma^2 + \frac{9}{16}\right) \Theta^I = 0. \tag{3.11}$$

Thus we find eight massive fermionic modes whose frequency reads (we reinsert here the dependence on m_0)

$$\omega_n = \sqrt{\frac{\pi^2 n^2}{L^2} + \frac{9}{16} m_0^2}. \tag{3.12}$$

In the nonminimal radius case we have simply to substitute m_0^2 with u_m/R^3.

C. The one-loop correction to the energy

Let us collect the results found in the previous subsections. The effective one-loop sigma model describing the quadratic fluctuations of the string around the classical solutions is given by the following collection of free modes: six massless and two massive bosonic modes, and eight massive fermions. The corresponding nontrivial frequencies are given in (3.6) and (3.12).

The one-loop correction to the classical energy $E_c = T_{\mathrm{QCD}} L$ thus reads

$$E_1 = \frac{\pi}{2L} \sum_{n \geq 1} \left[6\sqrt{n^2} + 2\sqrt{n^2 + \frac{9 m_0^2 L^2}{4\pi^2}} - 8\sqrt{n^2 + \frac{9 m_0^2 L^2}{16\pi^2}} \right]. \tag{3.13}$$

This is, as usual [16,21,22], a negative function of the effective masses, and hence of L. Let us notice that the straight string configuration gives a finite UV theory (without Weyl anomaly [23])[8] , since

$$\sum_{\text{bosons}} \omega^2_{\text{bosons}} - \sum_{\text{fermions}} \omega^2_{\text{fermions}} = 0. \qquad (3.14)$$

Thus E_1 is not divergent and in the large L limit it reads

$$E_1 \approx -\frac{9m_0^2 L}{8\pi} \log 2 - \frac{\pi}{4L}. \qquad (3.15)$$

The leading term in this expression comes from approximating the series above by integrals. The Lüscher-like term $\pi/4L$ comes from the subleading contribution of the six massless modes. The remaining subleading terms are exponentially suppressed in the large L limit and are related to the massive modes. See [16,22] for details on the evaluation of series like the one above.

In the large L limit we find that the energy of the string configuration corresponding to the Wilson loop is given, up to the one-loop sigma-model correction, by

$$E \approx E_c + E_1 = \frac{m_0^2 \lambda}{6\pi}\left(1 - \frac{27}{4\lambda} \log 2\right)L - \frac{\pi}{4L}$$
$$= T_{\text{QCD}}\left(1 - \frac{27}{4\lambda} \log 2\right)L - \frac{\pi}{4L}. \qquad (3.16)$$

We see that, at the level of approximations we are taking, string theory gives a prediction on the way the YM string tension "renormalizes," $T^{(\text{ren})}_{\text{QCD}}(\lambda) = [1 - (27/4\lambda) \times \log 2] T_{\text{QCD}}$. We will discuss more extensively these results in Sec. VI.

IV. FOLDED CLOSED SPINNING STRING: GLUEBALL REGGE TRAJECTORY

In this section we consider the closed string configuration corresponding to the glueball Regge trajectories. The relevant closed folded spinning string configuration is described, in the coordinates used in (2.6), by the following one parameter family of solutions

$$X^0 = k\tau, \qquad X^1 = k \cos\tau \sin\sigma, \qquad X^2 = k \sin\tau \sin\sigma, \qquad (4.1)$$

and all the other coordinates fixed.

Following standard calculations [3] and reintroducing the dimensional energy, it is easy to find that

$$E = \frac{g_{00}(u_0)}{\alpha'} k = \frac{\lambda m_0}{3} k, \qquad J = \frac{g_{00}(u_0)}{2\alpha'} k^2 = \frac{\lambda}{6} k^2. \qquad (4.2)$$

Thus the relation between the energy E and the angular momentum J of the string (in the 12 plane) has indeed the expected Regge-like form

$$E^2 = \frac{2}{\alpha'}\left(\frac{u_0}{R}\right)^{3/2} J = \frac{2}{3}\lambda m_0^2 J = 4\pi T_{\text{QCD}} J. \qquad (4.3)$$

Note that the tension of the adjoint string T_{adj} (the one relevant for the Regge trajectories of adjoint particles we are considering) is given by the relation $E^2 = J/\alpha'_{\text{adj}} = T_{\text{adj}} J/2\pi$, i.e., $T_{\text{adj}} = 2T_{\text{QCD}}$. This is not surprising in field theory since T_{adj} is expected to be related to the string tension between particles in the fundamental, T_{QCD}, by the relation

$$\frac{T_{\text{adj}}}{T_{\text{QCD}}} = \frac{C_{\text{adj}}}{C_{\text{fund}}} = \frac{2N^2}{N^2 - 1}, \qquad (4.4)$$

which in the large N limit reduces to the relation above.

In order to obtain the one-loop quantum corrections to these classical results, we have to study the quadratic fluctuations around the classical string configuration (4.1). The problem is that in this case the induced metric is the nonconstant conformally flat metric

$$h_{\alpha\beta} = k^2 \cos^2\sigma \, \eta_{\alpha\beta}. \qquad (4.5)$$

We will see that as a result the quadratic action has σ-dependent external terms coupled to the fluctuating fields. The one-loop correction to the classical energy are more complicated than in the other cases considered in this paper. For this reason, after computing the bosonic and fermionic quadratic fluctuations around (4.1), we calculate the correction to the Regge trajectory in the limits $k \ll 1$ in (4.2) and $k \gg 1$ in (4.3); the final results in the two cases, presented in (4.40), (4.56), and (4.57), respectively, will be also discussed in Sec. VI B. We study the finiteness of the theory in Appendix D.

A. Quadratic fluctuations

By expanding the bosonic world-sheet fields around the classical solution (4.1) one finds that all the fluctuating fields are free except two y_a's, which have action

$$S = -\frac{1}{4\pi} \int d\tau d\sigma \sum_{a=1,2} \left[\frac{4}{3}(\partial_\alpha y_a \partial^\alpha y_a) + 3k^2 \cos^2\sigma (y_a)^2\right]. \qquad (4.6)$$

The equations of motion are naturally written in the form

$$\left[\partial_\alpha \partial^\alpha - \frac{9k^2}{4} \cos^2\sigma\right] y_a = 0. \qquad (4.7)$$

Thus the spectrum of the bosonic fluctuations around the classical configuration on a flat world sheet consists of six massless and two massive modes with an effective σ-dependent mass parameter

$$m_B^2 = \frac{9}{4} k^2 \cos^2\sigma \equiv k^2 \ell_B^2 \cos^2\sigma. \qquad (4.8)$$

This is qualitatively similar to what happens for the bo-

[8]The cancellation of this divergence provides us with a consistency check for our calculation of the frequency of the fluctuations. This check is the more crucial due to the various conflicting results appearing in the literature [8–10].

sonic fluctuations around the classical string solution corresponding to the Regge trajectory in other confining backgrounds [3]. The only difference is in the number of massive fluctuations which is three for the Klebanov-Strassler [24] and Maldacena-Núñez [25] backgrounds while in our case we have only two massive bosonic fluctuations.

Let us now consider the fermionic sector. From the general discussion of Sec. II A we know we can start with the action (2.17). As we have explained, the most natural κ fixing is given by

$$(1-\hat{\Gamma}_{\underline{01}})\theta^1=0, \qquad (1+\hat{\Gamma}_{\underline{01}})\theta^2=0. \tag{4.9}$$

This gauge fixing is clearly world-sheet coordinate dependent. In order to simplify this situation, one could proceed in two ways. One could choose a rotating vielbein (in order to have $\hat{\Gamma}_{\underline{a}}$ as constant matrices) or, more conveniently, one can rotate directly the spinors. In particular, if the rotation $M(\tau,\sigma)_{\underline{a}}{}^{\underline{b}}$ connecting the two vielbeins [i.e., $\hat{e}_{\underline{a}}=M(\tau,\sigma)_{\underline{a}}{}^{\underline{b}}e_{\underline{b}}$] is implemented on the spinors by $\Lambda(\tau,\sigma)$ (i.e., $\Lambda\Gamma_{\underline{a}}\Lambda^{-1}=M_{\underline{a}}{}^{\underline{b}}\Gamma_{\underline{b}}=\hat{\Gamma}_{\underline{a}}$), we can introduce the rotated spinors $\tilde{\theta}^I=\Lambda^{-1}\theta^I$ and the κ fixing becomes

$$(1-\Gamma_{\underline{01}})\tilde{\theta}^1=0, \qquad (1+\Gamma_{\underline{01}})\tilde{\theta}^2=0, \tag{4.10}$$

which can be solved following the Wilson loop example. Since, after an obvious constant rescaling of the metric, on the world sheet the first two one-forms of the adapted (co)vielbein are given by

$$\hat{e}_{\underline{\alpha}}=\frac{\partial_\alpha X^\mu}{\sqrt{\partial_\sigma X^\mu\partial_\sigma X_\mu}}\partial_\mu, \tag{4.11}$$

we can choose the following rotation generator

$$\Lambda=e^{(\pi/4)[1-\mathrm{sign}(\cos\sigma)]\Gamma_{\underline{12}}}e^{-(1/2)\tau\Gamma_{\underline{12}}}e^{(1/2)\chi(\sigma)\Gamma_{\underline{02}}}, \tag{4.12}$$

where[9] $\chi(\sigma)=\cosh^{-1}\frac{1}{|\cos\sigma|}=\sinh^{-1}\frac{\sin\sigma}{\cos\sigma}$. The action (2.17) can be written as

$$S_F=\frac{i}{4\pi}\int d\tau d\sigma\Big[k|\cos\sigma|\bar{\theta}(\eta^{\alpha\beta}+\epsilon^{\alpha\beta}\Gamma^{\underline{11}})\hat{\Gamma}_{\underline{\alpha}}\partial_\beta\theta - \frac{3}{4}k^2\cos^2\sigma\bar{\theta}\,\tilde{\Gamma}\,\theta-\frac{3}{4}k^2\cos^2\sigma\bar{\theta}\Gamma^{\underline{11}}\hat{\Gamma}_{\underline{01}}\tilde{\Gamma}\theta\Big], \tag{4.13}$$

where $\tilde{\Gamma}=\hat{\Gamma}_{\underline{6789}}=\Gamma_{\underline{6789}}$ and $\theta=\theta^1+\theta^2$. Then, in terms of the rotated spinors it becomes

$$S_F=\frac{i}{4\pi}\int d\tau d\sigma\Big[k|\cos\sigma|\bar{\tilde{\theta}}(\eta^{\alpha\beta}+\epsilon^{\alpha\beta}\Gamma^{\underline{11}})\Gamma_{\underline{\alpha}}\partial_\beta\tilde{\theta} - \frac{3}{4}k^2\cos^2\sigma\bar{\tilde{\theta}}\,\tilde{\Gamma}\,\tilde{\theta}-\frac{3}{4}k^2\cos^2\sigma\bar{\tilde{\theta}}\Gamma^{\underline{11}}\Gamma_{\underline{01}}\tilde{\Gamma}\,\tilde{\theta} + k|\cos\sigma|\bar{\tilde{\theta}}(\eta^{\alpha\beta}+\epsilon^{\alpha\beta}\Gamma^{\underline{11}})\Gamma_{\underline{\alpha}}(\Lambda^{-1}\partial_\beta\Lambda)\tilde{\theta}\Big]. \tag{4.14}$$

Using the fact that the Majorana-Weyl spinors $\tilde{\theta}^1$ and $\tilde{\theta}^2$ have opposite space-time chirality and satisfy the κ-fixing conditions (4.10), which can be read as a condition of opposite "world-sheet chirality" for the two spinors, the last term of the action reduces to

$$\begin{aligned}
&k|\cos\sigma|\bar{\tilde{\theta}}(\eta^{\alpha\beta}+\epsilon^{\alpha\beta}\Gamma^{\underline{11}})\Gamma_{\underline{\alpha}}(\Lambda^{-1}\partial_\beta\Lambda)\tilde{\theta}\\
&=-\frac{k}{2}\mathrm{sign}(\cos\sigma)\sin\sigma\bar{\tilde{\theta}}(\Gamma_{\underline{1}}+\Gamma^{\underline{11}}\Gamma_{\underline{0}})\tilde{\theta}\\
&\quad-\frac{k\pi}{4}\sin\sigma\mathrm{sign}(\cos\sigma)\partial_\sigma[\mathrm{sign}(\cos\sigma)]\bar{\tilde{\theta}}(\Gamma_{\underline{0}}+\Gamma^{\underline{11}}\Gamma_{\underline{1}})\tilde{\theta}\\
&=k\mathrm{sign}(\cos\sigma)\sin\sigma[\bar{\tilde{\theta}}^1\Gamma_{\underline{1}}\tilde{\theta}^1+\bar{\tilde{\theta}}^2\Gamma_{\underline{1}}\tilde{\theta}^2].
\end{aligned} \tag{4.15}$$

Indeed, any $\Gamma_{\underline{a}}$ changes the space-time chirality while only the two $\Gamma_{\underline{\alpha}}$'s ($\alpha=0,1$) change the world-sheet chirality. Then the above result follows from the fact that $\Lambda^{-1}\partial_\tau\Lambda=-\frac{1}{2}(\cosh\chi\Gamma_{\underline{12}}+\sinh\chi\Gamma_{\underline{01}})$ and $\Lambda^{-1}\partial_\sigma\Lambda=-\frac{\pi}{4}\partial_\sigma[\mathrm{sign}(\cos\sigma)](\cosh\chi\Gamma_{\underline{12}}+\sinh\chi\Gamma_{\underline{01}})$, and any term containing an even number of $\Gamma_{\underline{a}}$'s and an odd number of $\Gamma_{\underline{\alpha}}$ vanishes.

We can now choose the same gamma matrices used in the Wilson loop case, Eq. (3.8), and the κ fixing can be implemented as in (3.9):

$$\tilde{\theta}^1=\frac{1}{\sqrt{2}}\begin{pmatrix}\Theta^1\\0\end{pmatrix}, \qquad \tilde{\theta}^2=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\\Theta^2\end{pmatrix}, \tag{4.16}$$

where Θ^I are again two Euclidean 8D Majorana-Weyl spinor of the same chirality. Then, the kappa-fixed quadratic fermionic action becomes

$$S_F=-\frac{i}{2\pi}\int d\tau d\sigma\Big[k|\cos\sigma|(\Theta^{1T}\partial_+\Theta^1+\Theta^{2T}\partial_-\Theta^2) - \frac{k}{4}\mathrm{sign}(\cos\sigma)\sin\sigma(\Theta^{1T}\Theta^1-\Theta^{2T}\Theta^2) - \frac{3}{8}k^2\cos^2\sigma(\Theta^{1T}\tilde{\gamma}\Theta^2-\Theta^{2T}\tilde{\gamma}\Theta^1)\Big], \tag{4.17}$$

where $\tilde{\gamma}=\gamma_{\underline{6789}}$.

Performing the Weyl rescaling of the fermions $\Theta^I\to\Theta^I/\sqrt{k|\cos\sigma|}$, we obtain

$$S_F=-\frac{i}{2\pi}\int d\tau d\sigma\Big[(\Theta^{1T}\partial_+\Theta^1+\Theta^{2T}\partial_-\Theta^2) - \frac{3}{8}k|\cos\sigma|(\Theta^{1T}\tilde{\gamma}\Theta^2-\Theta^{2T}\tilde{\gamma}\Theta^1)\Big].$$

Thus we find that the eight fermionic fluctuations have the

[9]Notice that this transformation is degenerate at the ends of the folded string $\sigma=\pi/2, 3\pi/2$.

same σ-dependent effective mass

$$m_F = \frac{3}{4} k \cos\sigma \equiv k\ell_F \cos\sigma. \tag{4.18}$$

The equations of motion become

$$\begin{aligned} \partial_+ \Theta^1 - \frac{3k}{8} |\cos\sigma| \tilde{\gamma} \Theta^2 = 0, \\ \partial_- \Theta^2 + \frac{3k}{8} |\cos\sigma| \tilde{\gamma} \Theta^1 = 0, \end{aligned} \tag{4.19}$$

which can be squared to yield the following equations on a flat world sheet

$$\begin{aligned} \left[\partial^\alpha \partial_\alpha + \tan\sigma(\partial_\tau + \partial_\sigma) - \frac{9k^2}{16}\cos^2\sigma\right]\Theta^1 = 0, \\ \left[\partial^\alpha \partial_\alpha - \tan\sigma(\partial_\tau - \partial_\sigma) - \frac{9k^2}{16}\cos^2\sigma\right]\Theta^2 = 0. \end{aligned} \tag{4.20}$$

Note that these squared equations are not valid at the degenerate points $\sigma = \pm \frac{\pi}{2}$, since the equations have been derived by writing one of the Θ^I in terms of the other from Eqs. (4.19), and this operation is clearly degenerate at the turning points. Nevertheless, this equation must be valid for $\sigma \neq \pm \frac{\pi}{2}$ and shows how the parameter k enters the diagonalized equations with its square k^2, as in the bosonic case. This will be useful in the following sections.

Let us write the squared equations in a nondegenerate nondiagonalized form by adopting the 2D Majorana spinor formalism. The equations of motion in the flat world-sheet gauge become

$$[\tau^\alpha \partial_\alpha - k\ell_F \cos\sigma \tilde{\gamma}]\psi = 0, \tag{4.21}$$

where the index i of ψ^i is understood. These equations can be squared into

$$[\partial_\alpha \partial^\alpha - k\ell_F \partial_\sigma |\cos\sigma| \tilde{\gamma} \tau_{\underline{1}} - k^2 \ell_F^2 \cos^2\sigma]\psi = 0. \tag{4.22}$$

Unfortunately, the nontrivial σ dependence of the bosonic and fermionic masses does not allow a simple solution of the spectral problem. In the other cases considered in this paper it is possible to compute the characteristic frequencies and to show explicitly how the one-loop energy is finite and then the theory is consistent. In this case we will be able to attack the spectral problem by considering the limits of large and small bosonic and fermionic masses: $k\ell_{B,F} \ll 1$ and $k\ell_{B,F} \gg 1$. We will do so by implementing perturbative techniques usual in quantum mechanics. The problem of the finiteness of the theory is instead analyzed in Appendix D.

As usual, we will solve the spectral problem by looking for eigenfunctions with a τ dependence of the form $\sim e^{i\omega\tau}$ and solving the resulting spectral equation for the characteristic frequency ω. For the two massive bosons, we obtain the spectral equation

$$\left[-\frac{d^2}{d\sigma^2} + k^2 \ell_B^2 \cos^2\sigma\right] y = \omega_B^2 y, \tag{4.23}$$

while for the eight fermions the spectral equations read

$$\left[-\frac{d^2}{d\sigma^2} + k\ell_F \partial_\sigma |\cos\sigma| \tilde{\gamma} \tau_{\underline{1}} + k^2 \ell_F^2 \cos^2\sigma\right]\psi = \omega_F^2 \psi. \tag{4.24}$$

Since in the diagonalized equations (4.20), $k\ell_F$ enters the equations of motion with its square, we can expect that ω admits an expansion in powers of $(k\ell_{B,F})^2$ or $1/(k\ell_{B,F})^2$ in the $k\ell_{B,F} \ll 1$ and $k\ell_{B,F} \gg 1$ limits, respectively.

B. Small bosonic and fermionic masses

Let us first consider the bosons. Equation (4.23) could be solved in terms of Mathieu functions, determining the characteristic frequencies in an expansion in k^2, as done in [3]. Since we are interested in the first order correction, it is more direct to consider the term

$$V_B = k^2 \ell_B^2 \cos^2\sigma \tag{4.25}$$

as a perturbation to the equation and to use the standard techniques of perturbation theory of nonrelativistic quantum mechanics. The most useful choice for the eigenfunctions of the unperturbed equation is given by the following complex basis

$$\langle \sigma | 0 \rangle = \frac{1}{\sqrt{2\pi}}, \qquad \langle \sigma | n, \pm \rangle = \frac{1}{\sqrt{2\pi}} e^{\pm i n \sigma}. \tag{4.26}$$

It follows immediately that the squared frequencies are given by

$$\begin{aligned} \omega_{B(n,\pm)}^2 &= n^2 + \langle n, \pm | V_B | n, \pm \rangle + \mathcal{O}(k^4) \\ &= n^2 + \frac{k^2 \ell_B^2}{2} + \mathcal{O}(k^4), \end{aligned} \tag{4.27}$$

and then

$$\omega_{B(n,\pm)} = \sqrt{n^2 + \frac{k^2 \ell_B^2}{2} + \mathcal{O}(k^4)}. \tag{4.28}$$

Turning now to the fermionic case, the perturbation term in (4.24) is the sum of two terms, $V_F = V_{F1} + V_{F2}$, where

$$V_{F1} = k\ell_F \partial_\sigma |\cos\sigma| \tilde{\gamma} \tau_{\underline{1}}, \qquad V_{F2} = k^2 \ell_F^2 \cos^2\sigma. \tag{4.29}$$

Since we are interested in the corrections of order $(k\ell_F)^2$, we must compute the first and second order corrections given by V_{F1} and the first order corrections given by V_{F2}. Analogously to the bosonic case, V_{F2} gives the following correction of order $(k\ell_F)^2$

$$\delta_2 \omega_{F(n,\pm)}^2 = \frac{k^2 \ell_F^2}{2}. \tag{4.30}$$

On the other hand, it is easy to see that V_{F1} does not give any correction at order k, since $\langle n, \pm | V_{F1} | n, \pm \rangle = 0$, while it gives a nontrivial correction of order k^2. By using the result

$$\left| \int_0^{2\pi} \frac{d\sigma}{2\pi} \partial_\sigma \left| \cos\sigma \right| e^{ir\sigma} \right|^2 = \begin{cases} 0 & \text{if } r = 2q+1, \\ \frac{16}{\pi^2} \frac{q^2}{(4q^2-1)^2} & \text{if } r = 2q, \end{cases} \tag{4.31}$$

where r and q are integer numbers, it is straightforward to obtain the following corrections by using standard perturbation theory at second order:

$$\delta_1 \omega^2_{F(0)} = -\frac{8k^2\ell_F^2}{\pi^2} \sum_{q\geq 1} \frac{1}{(4q^2-1)^2} \equiv -ck^2\ell_F^2,$$
$$\delta_1 \omega^2_{F(n,\pm)} = \frac{-4k^2\ell_F^2}{\pi^2} \sum_{q\neq -n} \frac{q}{(n+q)(4q^2-1)^2} \equiv -c_n k^2\ell_F^2. \tag{4.32}$$

The series appearing above are convergent and thus we can give the explicit expression for the constants previously introduced

$$c = \frac{(\pi^2-8)}{2\pi^2} \approx 0.095,$$
$$c_n = \frac{8+96n^2-(4n^2-1)^2\pi^2}{2\pi^2(4n^2-1)^3}. \tag{4.33}$$

Then the zero mode frequency is given by

$$\omega_{F(0)} = \frac{k\ell_F}{\sqrt{2}}\sqrt{1-2c}. \tag{4.34}$$

On the other hand the nonzero modes get a correction of the form

$$\omega_{F(n,\pm)} = \sqrt{n^2 + \frac{k^2\ell_F^2}{2} - c_n k^2\ell_F^2 + \mathcal{O}(k^4)}, \tag{4.35}$$

and then the one-loop correction to the space-time energy, after reinserting the dimensional factor, is given by

$$E_1 = \frac{m_0}{2k}\Bigg\{6\times 2\sum_{n\geq 1} n + 2 \times \left[\frac{k\ell_B}{\sqrt{2}} + 2\sum_{n\geq 1}\sqrt{n^2+\frac{k^2\ell_B^2}{2}+\mathcal{O}(k^4)}\right] - 8\times\left[\frac{k\ell_F\sqrt{1-2c}}{\sqrt{2}} + 2\sum_{n\geq 1}\sqrt{n^2+\frac{k^2\ell_F^2}{2}-c_n k^2\ell_F^2+\mathcal{O}(k^4)}\right]\Bigg\}. \tag{4.36}$$

By expanding in powers of $(k\ell_B)^2$ and $(k\ell_F)^2$, using the mass matching condition $2\ell_B^2 = 8\ell_F^2$ and the fact that $c_n \sim 1/n^2$ for large n, it is easy to see that the sum is indeed finite and that the zero modes contribute to a correction at order 1 while the other modes at order k, leading to

$$E_1 = z_0 + kz_1 + \mathcal{O}(k^2), \tag{4.37}$$

where z_0 and z_1 are given by

$$z_0 = \frac{m_0}{\sqrt{2}}[\ell_B - 4\ell_F\sqrt{1-2c}] = \frac{3m_0}{\sqrt{2}}\left[\frac{1}{2} - \sqrt{1-2c}\right] \approx -0.85m_0, \tag{4.38}$$

and

$$z_1 = \frac{4m_0}{k^2}\sum_{n\geq 1}\frac{c_n k^2\ell_F^2}{n} = \frac{2\ell_F^2 m_0}{\pi^2}[-24+\pi^2+16\log 2-\pi^2\log 4+14\zeta(3)] \approx 0.012m_0. \tag{4.39}$$

The limit of small m_B and m_F requires that $\lambda \gg J$. In particular, by considering J finite (and then E_0 of order $\sqrt{\lambda}$) and neglecting all terms of order $1/\lambda^2$, the corrected Regge trajectory takes the following form:

$$J = \alpha'_{\text{adj}}\left(1 - \frac{6z_1}{\lambda m_0}\right)[E^2 - 2Ez_0 + z_0^2], \tag{4.40}$$

where $\alpha'_{\text{adj}} = 3/2\lambda m_0^2$. Thus we see that the effect of the z_1 term is to shift the effective slope of the Regge trajectory, while z_0^2 gives a positive intercept. Notice that the renormalization effect on the effective slope occurs in the form $1 - a^2/\lambda$: thus the effective "adjoint tension" is rescaled as $T^{(\text{ren})}_{\text{adj}} \sim (1/2\pi\alpha'_{\text{adj}})(1 + a^2/\lambda)$, i.e., with an opposite sign with respect to the other cases examined in the paper. Finally notice that we are still restricting ourselves to a large J regime in order to thrust the semiclassical approximation.

C. Large bosonic and fermionic masses

In order to present a unified treatment of the bosonic and fermionic contributions we find it convenient to think about the large-mass limit as a large-k limit. Note that equivalently we have a large J/λ limit for $k \gg 1$. To solve the spectral problem we first define $\omega \equiv k\alpha$.

Let us start with the bosonic case. The spectral Eq. (4.23) takes the form

$$\left[-\frac{1}{k^2}\frac{d^2}{d\sigma^2} + \ell_B^2\cos^2\sigma\right]y = \alpha_B^2 y. \tag{4.41}$$

This can be seen as a one-dimensional stationary Schrödinger equation, where $1/k$ plays the role of $\hbar$, α_B^2 is the energy and $V_B(\sigma) \equiv \ell_B^2\cos^2\sigma$ the potential. Then the $k \gg 1$ regime corresponds to the quasiclassical regime $\hbar \ll 1$, where the WKB approach is reliable. It consists of making the formal substitution $y(\sigma) = e^{ik\chi(\sigma)}$, to obtain the equation

$$(\chi')^2 - \frac{i}{k}\chi'' = \alpha^2 - \ell_B^2\cos^2\sigma. \tag{4.42}$$

Since we are interested only in the leading order result in the $k \gg 1$ limit, we can simply drop the $1/k$ term. Then the above equation can be easily integrated

$$\chi(\sigma) = \pm \int_0^\sigma d\tilde{\sigma} \sqrt{\alpha^2 - \ell_B^2 \cos^2\tilde{\sigma}}. \tag{4.43}$$

We have then an oscillating wave function in the region where $\alpha^2 > \ell_B^2\cos^2\sigma$ while a rapidly decreasing exponential wave function where $\alpha^2 < \ell_B^2\cos^2\sigma$. In particular, at the turning points $\bar{\sigma}$ where $\ell_B^2\cos^2\bar{\sigma} = \alpha^2$ the WKB approximation looses its reliability.

If $\alpha^2 > \ell_B^2$ we can use the WKB approximation for any point $\sigma \in [0, 2\pi]$. For a given α^2 two independent symmetric and antisymmetric eigenfunctions are then given by

$$\begin{aligned} y &\sim \cos\left\{k \int_0^\sigma d\tilde{\sigma} \sqrt{\alpha^2 - \ell_B^2\cos^2\tilde{\sigma}}\right\}, \\ y &\sim \sin\left\{k \int_0^\sigma d\tilde{\sigma} \sqrt{\alpha^2 - \ell_B^2\cos^2\tilde{\sigma}}\right\}. \end{aligned} \tag{4.44}$$

By imposing the periodicity condition

$$k \int_0^{2\pi} d\tilde{\sigma} \sqrt{\alpha^2 - \ell_B^2\cos^2\tilde{\sigma}} = 2\pi n, \tag{4.45}$$

we get

$$E(\zeta_B^2) = \frac{\pi}{2\ell_B} \zeta_B x, \tag{4.46}$$

where we have introduced $\zeta_B \equiv \ell_B/\alpha \in (0, 1]$, $x = n/k$ and $E(\zeta^2)$ is the complete elliptic integral of the second kind

$$E(\zeta^2) = \int_0^{\pi/2} d\sigma \sqrt{1 - \zeta^2\sin^2\sigma}, \qquad \zeta^2 < 1. \tag{4.47}$$

When $\zeta_B \geq 1$, in the $k \to \infty$ limit the corresponding quantum mechanical particle lives in one of the two potential wells, with subleading tunnel effect. Choosing the domain $\sigma \in [-\pi, \pi]$, we can then consider an eigenfunction $y_0(\sigma)$ concentrated in the right well and construct the two eigenfunctions (symmetric and antisymmetric)

$$y_\pm(\sigma) \sim y_0(\sigma) \pm y_0(-\sigma). \tag{4.48}$$

These correspond to the same eigenvalue, up to exponentially suppressed terms. If $\sigma_{\mathrm{min,max}} \in [0, \pi]$ are the turning points ($\sigma_{\max} = \pi - \sigma_{\min}$), the Bohr-Sommerfeld quantization condition reads

$$k \int_{\sigma_{\min}}^{\sigma_{\max}} d\sigma \sqrt{\alpha^2 - \ell_B^2\cos^2\sigma} = \pi\left(n + \frac{1}{2}\right). \tag{4.49}$$

In the large n, k limit with $x = n/k$ fixed, we can rewrite this relation as

$$F(\zeta_B^2) = \frac{\pi}{2\ell_B} \zeta_B x, \qquad 2F(\zeta^2) \equiv \int_{\sigma_{\min}}^{\sigma_{\max}} d\sigma \sqrt{1 - \zeta^2\cos^2\sigma}, \qquad \zeta^2 \geq 1. \tag{4.50}$$

This expression is continuously connected with (4.46), both approaching the same expression when $\zeta_B \to 1$. In the following we will recall E the function covering the whole interval $\zeta \in (0, \infty)$. Let us notice that, from (4.50) it can be deduced that when $\zeta \to \infty$, then $x \to 0$.

Let us now turn to the fermionic case. After the substitution $\omega_F = k\alpha_F$ Eq. (4.24) becomes

$$\left[-\frac{1}{k^2}\frac{d^2}{d\sigma^2} + \frac{1}{k}\ell_F \partial_\sigma |\cos\sigma| \tilde{\gamma}\tau_{\underline{1}} + \ell_F^2\cos^2\sigma\right]\psi = \alpha_B^2\psi. \tag{4.51}$$

It is now easy to see that by making the formal substitution $\psi = e^{ik\chi}$ and keeping again only the leading order terms, we obtain the very same spectral equation of the bosonic case,

$$(\chi')^2 = \alpha^2 - \ell_F^2\cos^2\sigma, \tag{4.52}$$

with ℓ_F instead of ℓ_B. We can then use all the results obtained for the bosonic case by simply substituting ℓ_B and ζ_B with ℓ_F and ζ_F, respectively.

The one-loop correction to the energy, up to terms of order $1/k$, can be expressed as

$$E_1 = \frac{km_0}{2} \lim_{\Lambda\to\infty}\left\{6 \times 2 \int_0^\Lambda dx x + 2 \times 2 \int_0^\Lambda dx \alpha_B(x) - 8 \times 2 \int_0^\Lambda dx \alpha_F(x)\right\}. \tag{4.53}$$

Using (4.46) and the corresponding fermionic equation it is possible to change the variable of integration from x to $\zeta_{B,F}$, obtaining

$$\begin{aligned} E_1 &= \frac{km_0}{2} \lim_{\Lambda\to\infty}\left\{6\Lambda^2 + 2 \times \frac{4\ell_B^2}{\pi}\left(\left[\frac{E(\zeta^2)}{\zeta^2}\right]_{+\infty}^{\ell_B/\Lambda} + \int_{+\infty}^{\ell_B/\Lambda} d\zeta \frac{E(\zeta^2)}{\zeta^3}\right) - 8 \times \frac{4\ell_F^2}{\pi}\left(\left[\frac{E(\zeta^2)}{\zeta^2}\right]_{+\infty}^{\ell_F/\Lambda} + \int_{+\infty}^{\ell_F/\Lambda} d\zeta \frac{E(\zeta^2)}{\zeta^3}\right)\right\} \\ &= \frac{km_0}{2} \lim_{\Lambda\to\infty}\left\{6\Lambda^2 + 2 \times \frac{4\ell_B^2}{\pi}\left(\left[\frac{E(\zeta^2)}{\zeta^2}\right]_{\ell_F/\Lambda}^{\ell_B/\Lambda} + \int_{\ell_F/\Lambda}^{\ell_B/\Lambda} d\zeta \frac{E(\zeta^2)}{\zeta^3}\right)\right\}, \end{aligned} \tag{4.54}$$

where in the last step the mass matching condition $2\ell_B^2 = 8\ell_F^2$ has been used[10].

By expanding $E(\zeta^2)$ for small ζ^2, it is possible to show that the quadratic and logarithmic divergences drop out and we are left with the following one-loop finite correction to the energy

[10]Notice that, due to the mass matching condition, the region $\zeta \in [1, \infty)$ does not contribute to E_1 and so the details of the function F defined as in (4.50) are not necessary (we only need to know its behavior for $\zeta \to \infty$).

$$E_1 = -\frac{km_0}{4}\left[2\times\frac{\ell_B^2}{2}\log\frac{\ell_B^2}{2} - 8\times\frac{\ell_F^2}{2}\log\frac{\ell_F^2}{2}\right] = -m_0\left(\frac{9}{8}\log 2\right)k. \tag{4.55}$$

Let us observe that this correction is negative as in the other cases considered in this paper and that it has the same universal form, with the σ dependent squared masses $m^2(\sigma) = k^2\ell^2\cos^2\sigma$ substituted by their mean value $m^2 = k^2\ell^2/2$. The effect of the one-loop correction in Regge trajectory translates in a renormalization (actually a rescaling) of the effective string tension

$$E^2 = 4\pi T_{\rm QCD}^{\rm (ren)}(\lambda)J, \tag{4.56}$$

with

$$T_{\rm QCD}^{\rm (ren)}(\lambda) = (1 - \frac{27\log 2}{8\lambda})^2 T_{\rm QCD}. \tag{4.57}$$

This is equal to the renormalized effective string tension (3.16) obtained from the one-loop correction to the Wilson loop energy. The relation $T_{\rm adj} = 2T_{\rm QCD}$ is then preserved by quantum corrections in this regime of parameters.

Notice that in the above calculations we have suppressed the subleading orders in the large k limit. Among these there is a (negative) $1/k$ term coming from the six massless modes, shifting the value of E_1. In particular, this term would produce, as in the $k \ll 1$ regime, a nonlinearity of the Regge trajectory and a positive intercept, which are thus fairly general outcomes of the sigma-model calculations. We did not write this term explicitly since it is possible that we are neglecting similar terms while using the WKB approximation.

V. STRINGS SPINNING IN INTERNAL DIRECTIONS: STRINGY HADRONS

In this section we concentrate on a class of string solutions related to sectors of the gauge theory which do not have a direct contact with pure YM. In fact, what we are going to consider are string states with large spins along the transverse S^4 of our background. These correspond to multicharged gauge theory hadrons whose constituents are the adjoint massive fields [6,14–16].

The bosonic action (2.10), with $X^i = \text{const}$, $i = 1, \ldots, 3$, reduces to a case widely studied in literature, which can be connected to the $n = 4$ Neumann integrable system, as already done in [26] following the general approach adopted for the search of string solutions on $AdS_5 \times S^5$ (see [27], and references therein). At the classical level, in fact, this is exactly analogous to the case considered in [27–29] and reviewed in [26]. In the latter paper some solutions were studied, corresponding to circular and folded strings rotating on the internal S^3 of the IR region of the (softly broken) Maldacena-Nùñez (MN) background. These solitonic stringy states were interpreted as corresponding to multicharged hadronic states in the dual gauge theory. We can readily adapt the results of [26] to the present discussion.

First of all, it is useful to introduce the following parametrization of the $S^3 \subset S^4$:

$$Z^1 = \sin\chi\cos\phi_-, \qquad Z^2 = \sin\chi\sin\phi_-, \qquad Z^3 = \cos\chi\cos\phi_+, \qquad Z^4 = \cos\chi\sin\phi_+, \tag{5.1}$$

in which the S^4 metric becomes

$$d\Omega_4^2 = d\psi^2 + \cos^2\psi(d\chi^2 + \sin^2\chi d\phi_-^2 + \cos^2\chi d\phi_+^2). \tag{5.2}$$

In the case of the folded strings, the solution reads (ψ is always kept constant at zero value)

$$\theta_2(\sigma) = \theta_2(\sigma + 2\pi), \qquad X^0 = k\tau, \qquad \phi_+ = \nu\tau, \qquad \phi_- = \omega\tau, \tag{5.3}$$

where $\theta_2 = 2\chi$. The Virasoro constraint implies that

$$\theta_2'^2 - 2(\omega^2 - \nu^2)\cos\theta_2 = 4k^2 - 2(\omega^2 + \nu^2). \tag{5.4}$$

The conserved charges are

$$E = \xi k, \qquad J_+ = \frac{\xi\nu}{4\pi}\int_0^{2\pi} d\sigma[1 + \cos\theta_2(\sigma)], \qquad J_- = \frac{\xi\omega}{4\pi}\int_0^{2\pi} d\sigma[1 - \cos\theta_2(\sigma)], \tag{5.5}$$

from which it follows that

$$E = \frac{k}{\nu}J_+ + \frac{k}{\omega}J_-. \tag{5.6}$$

There are various special cases. For $\omega^2 = \nu^2$ the solution describes circular strings and is given by $\theta_2 = \pm 2\sigma\sqrt{k^2 - \omega^2} + \text{const}$. This is in general an extended solution, except for $k^2 = \omega^2$ where it describes a pointlike string. The periodicity condition implies the energy/charge relation $E = \sqrt{m^2\xi^2 + J^2}$, where m is the number of windings and $J = J_+ + J_-$ is the sum of the two charges.

In the general case $\omega^2 \neq \nu^2$, we can obtain the energy/charge relation in the short and long string limits. For short strings

$$E \sim J + \frac{\xi^2 J_-}{2J^2}, \tag{5.7}$$

which is analogous to the BMN expansion. In the long string limit, the relation is

$$E \sim J + \frac{2\xi^2}{\pi^2 J_+}. \tag{5.8}$$

The homogeneous circular string solutions have instead the form

$$Z^1 + iZ^2 = a_1 e^{iw_1\tau + im_1\sigma}, \qquad Z^3 + iZ^4 = a_2 e^{iw_2\tau + im_2\sigma},$$
$$X^0 = k\tau, \tag{5.9}$$

with $w_i, m_i, i = 1, 2$, satisfying the following relations:

$$w_i^2 = m_i^2 - \Lambda, \qquad a_1^2 + a_2^2 = 1,$$
$$k^2 = 2(a_1^2 w_1^2 + a_2^2 w_2^2) + \Lambda, \qquad a_1^2 w_1 m_1 + a_2^2 w_2 m_2 = 0. \tag{5.10}$$

In the coordinates (5.1), the solutions (5.9) take the form

$$X^0 = k\tau, \qquad \sin\chi = a_1,$$
$$\phi_- = w_1\tau + m_1\sigma, \qquad \phi_+ = w_2\tau + m_2\sigma. \tag{5.11}$$

The charges are $J_- = \xi a_1^2 w_1$, $J_+ = \xi a_2^2 w_2$. In the large J limit one observes that

$$E = J\left[1 + \frac{\xi^2}{2J^2}\left(m_1^2 \frac{J_-}{J} + m_2^2 \frac{J_+}{J}\right) + \ldots\right]. \tag{5.12}$$

All we have said up to now concerns the classical solutions. One can go further and try to calculate the leading one-loop sigma-model corrections to the relations above. This amounts to a calculation of the vacuum energy of the string theory expanded to the first nontrivial order around the classical solution. While for the folded strings there are some subtleties in the quantization procedure, due to the nontrivial world-sheet metric (see, for example, [30]), for the circular solutions the calculation is straightforward. Since the coordinates ϕ_- and ϕ_+ never appear in the components of the background fields, they enter the string action only through the pullback of background fields. This means that, in the expansion of the string action around the solution (5.11), all the coefficients are constants depending only from k, w_i, and m_i. The same holds for all the conserved charges.

Even in the simple example of the circular solutions on the fixed-radius S^5 in $AdS_5 \times S^5$, the calculation of the vacuum energy is very difficult, and only its leading contribution can be computed explicitly, in the limit of very large k [31,32]. The same holds for the more involved MN solution [26], and we expect the very same behavior in our case. But as far as the large k limit (for fixed m_i) is concerned, we can make the following observation. From (5.10) and (5.11) it follows that in this limit we can neglect at the leading order the contributions coming from the m_i. As such, the one-loop expansion is equivalent, in the limit, to the expansion around a collapsed, *pointlike* string rotating on S^4 with an angular velocity $\nu = \sqrt{a_1^2 w_1^2 + a_2^2 w_2^2}$. This in turn is the same as studying the exact (in α') string theory on the Penrose limit of the original background. Then, in the large k limit the leading one-loop quantum correction to the energy of the circular configurations is identical to that obtained in the Penrose limit, i.e., on the corresponding pp wave. This has been explicitly verified in $AdS_5 \times S^5$ [31], where the vacuum energy is vanishing at this order because of the linearly realized supersymmetries in the Penrose limit, and for the MN solution in [26]. We are then going to study the Penrose limit theory in detail in the following section. But let us just quote here for completeness the result for the energy-charge relation of the circular strings (we reinsert the dimensional factor and use $\xi = \lambda/3$)

$$E = m_0 J\left[1 + \frac{\lambda^2}{18J^2}\left(m_1^2 \frac{J_-}{J} + m_2^2 \frac{J_+}{J}\right) + \ldots\right] + m_0 J\left[\frac{45}{8\lambda}\log\frac{3}{4} + \ldots\right]. \tag{5.13}$$

The sigma-model result provides a strong λ coupling renormalization of $m_0 J$. While being a subleading term for large λ, it forbids to extrapolate the classical result (5.12) to the small coupling regime as for the spinning strings in $AdS_5 \times S^5$, and so to compare this result with a field theory calculation. We will comment further about this result and about the field theory duals to the spinning strings in Sec. VI C.

A. The plane-wave theory

The semiclassical quantization around the pointlike string spinning at the speed of light around the large equator of the four-sphere is equivalent to the study of the string theory on the plane-wave background, obtained as a Penrose limit of the original metric (2.1). We present the Penrose limit in Appendix E and just quote the result:

$$ds^2 = -4dx^+dx^- - m_0^2\left[v_3^2 + v\bar{v} + \frac{3}{4}u\bar{u}\right]dx^+dx^+ + dx^i dx^i + du d\bar{u} + dv d\bar{v} + dv_3^2,$$
$$e^{\Phi_0}F_4 = \frac{3i}{2}m_0 dx^+ \wedge dv_3 \wedge dv \wedge d\bar{v}. \tag{5.14}$$

The dilaton becomes a constant $e^{\Phi_0} \approx g_s N^{1/2}$. In the plane-wave metric above, the three coordinates x^i come from the three flat special directions, the three v modes from the four-sphere (the fourth mode of the sphere combines with the time variable into the light-come coordinates x^+, x^-), and the u modes from the radius and circle direction of the original background. The x^i, v fields parameterize the so-called universal sector. The IIA Hamiltonian can be read as the one describing the nonrelativistic motion in flat 3D space (x^i) of gauge theory hadrons made of adjoint massive constituents, along the lines of [14]. The conserved light-cone Hamiltonian H and momentum P^+ read as usual

$$H = i\partial_+ = E - m_0 J, \qquad P^+ = \frac{i}{2}\partial_- = \frac{m_0}{\lambda}J. \tag{5.15}$$

Let us now study the light-cone gauge IIA GS string on the above background. There are three massless bosonic fields corresponding to the three spatial directions x^i, $i = 1, 2, 3$. The remaining five massive fields have frequencies

$$\omega_n^{v,\bar{v}} = \sqrt{n^2 + m^2}, \qquad \omega_n^{v_3} = \sqrt{n^2 + m^2}, \qquad \omega_n^{u,\bar{u}} = \sqrt{n^2 + \frac{3}{4}m^2}, \tag{5.16}$$

where $m = m_0\alpha' p^+$.

Concerning the fermionic fields, their equations of motion read

$$(\partial_\tau + \partial_\sigma)\theta^1 = -\frac{\alpha' p^+}{(4)3!} e^\Phi F_{+ijk}\Gamma^{ijk}\theta^2, \qquad (\partial_\tau - \partial_\sigma)\theta^2 = -\frac{\alpha' p^+}{(4)3!} e^\Phi F_{+ijk}\Gamma^{ijk}\theta^1, \tag{5.17}$$

from which we get that the fermionic fields are all massive, and all have frequency

$$\omega_n^I = \sqrt{n^2 + \frac{9}{16}m^2}, \qquad I = 1, \ldots, 8. \tag{5.18}$$

The light-cone Hamiltonian can be obtained straightforwardly and can be expressed as a sum of a contribution from the momentum and the stringy excitations in the three flat dimensions and a contribution from the massive "zero" modes and excitations of the internal directions and the fermionic fields (see [14]).

Since the sum of the squares of the fermionic frequencies exactly matches with the analogous bosonic sum, our model is finite with nontrivial zero-point energy

$$E_0(m) = \frac{m_0}{2m}\sum_{n=-\infty}^{\infty}\left[3n + 3\sqrt{n^2+m^2} + 2\sqrt{n^2 + \frac{3}{4}m^2} - 8\sqrt{n^2 + \frac{9}{16}m^2}\right]. \tag{5.19}$$

Following the general arguments in [16,21,22] we see that this is negative and increases in absolute value as m in the large m limit. The latter is also the large k limit of the circular string configuration, so it is interesting to exhibit its explicit leading order form

$$E_0(m) \approx \frac{15 m_0 m}{8}\log\frac{3}{4}. \tag{5.20}$$

In the small m limit, instead, we have

$$E_0(m) \to -\frac{(3-\sqrt{3})}{2} m_0. \tag{5.21}$$

We see that the energy/charge relation for the pointlike strings in the large m regime reads[11]

$$E = m_0 J\left(1 + \frac{45}{8\lambda}\log\frac{3}{4} + \ldots\right). \tag{5.22}$$

[11]From (5.15) and (A17) it follows that $m = 3J/\lambda$.

In Sec. VI C we will comment about the field theory duals of our plane-wave string states.

VI. CONCLUDING REMARKS ON THE DUAL FIELD THEORY

Let us summarize the implications of our string theory considerations for the dual field theory. A very generic property of the leading order one-loop corrections to the energies of the various configurations we have analyzed is that they are always negative. Their precise meaning can be sketched case by case.

A. Wilson loop

Let us first discuss the string configuration corresponding to the Wilson loop. Our main result is the fact that for world-sheet supersymmetric strings on the confining Witten background, apart from the Lüscher $1/L$ term, the leading order one-loop correction to the quark-antiquark potential, in the large L limit, gives a renormalization of the YM string tension[12]. As Gross and Ooguri discussed in [34], this fact is somewhat expected. The gauge theory under consideration has a UV cutoff given by $m_0 \sim 1/R_\theta$ and the coupling g_{YM} should be considered as the bare coupling at scales of order m_0. As we have recalled, the classical string tension in the strong 't Hooft coupling limit is given by

$$T_{\rm QCD} = \frac{1}{6\pi}\lambda m_0^2 \quad \text{for } \lambda = g_{YM}^2 N \gg 1. \tag{6.1}$$

To construct the pure QCD we should consider the limit of very large UV cutoff m_0 and the 't Hooft coupling constant should be given by the usual perturbative expression

$$\lambda = \frac{b}{\log(\frac{m_0}{\Lambda_{\rm QCD}})}, \tag{6.2}$$

where b is given by the first coefficient of the beta function. Gross and Ooguri suggested that a reasonable behavior of the tension for small λ should be

$$T_{\rm QCD} = a e^{-2b/\lambda} m_0^2 = a\Lambda_{\rm QCD}^2 \quad \text{for } \lambda \ll 1, \tag{6.3}$$

where a is some numerical constant. Then, there should be an interpolation function $f(\lambda)$ such that

$$T_{\rm QCD} = f(\lambda) m_0^2, \tag{6.4}$$

with the previous asymptotic behaviors (6.1) and (6.3). In our calculation we have computed $f(\lambda)$ to the second order in the large λ limit.

The fact that in the large L limit we have a renormalization of the string tension besides a Lüscher term depends crucially of three facts: (1) our string is

[12]Interestingly, this renormalization of the string tension (without Lüscher term) takes place, at the classical level, in M-theoretic quantum chromodynamics [33].

supersymmetric; (2) we consider fluctuations around the Witten background dual to YM as opposed to flat space; (3) we evaluate the one-loop correction to the energy (E_1) without separately regularizing the bosonic and fermionic contributions, but simply by calculating the whole fermion + boson contribution [22].

Interesting, ignoring any one of these three conditions results in the absence of this correction to the tension. For example, if we reject (3) but not (1) and (2) and evaluate the E_1 with the Δ renormalizations (i.e., the ζ renormalization generalized to allow for world-sheet masses), we find that in the large L limit the only nonvanishing contribution comes from the six massless bosonic fields, i.e., we are left with only the standard Lüscher term. If we reject point (1) and consider only the bosonic contributions we are forced to renormalize and again only the Lüscher term is present.

One way to interpret our result is that it yields falsifiable predictions for the holographic approach to nonsupersymmetric gauge theories. In particular, there is a clear distinction about the nature of the "YM string." If the YM string is an intrinsically bosonic object, then we expect only a Lüscher term correction. On the other hand, if the string is intrinsically fermionic[13] then the most likely scenario for the leading order correction is a renormalization of the string tension. Future studies of the $q\bar{q}$ potential should settle this question.

B. Regge trajectory

One of the most distinctive features of the strong interactions is Regge theory which encompasses the particle spectrum, the forces between particles and the high energy behavior of scattering amplitudes. Regge trajectories are one of the most ubiquitous properties shared by most strongly interacting states.

The UA8 Collaboration [12] has presented a description of the nonlinearities of the Regge trajectory corresponding to the soft Pomeron. Based on its quantum numbers, this trajectory is believed to be composed of glueball states, although no direct evidence is available. The best experimental fit is of the form:

$$J = \alpha(t) = 1.10 + 0.25t + \alpha'' t^2, \tag{6.5}$$

where $\alpha'' = 0.079 \pm 0.012\ \mathrm{GeV}^{-4}$ and t is the mass squared of the particles.

The nonlinearity and the positive intercept that we find as the result of one-loop corrections in the framework of IIA string theory are two important qualitative properties shared with this trajectory. It is worth remarking that this behavior is exhibited by the loop-corrected trajectories of supersymmetric supergravity backgrounds discussed in [3] in the framework of IIB string theory. The constancy of this result encourages us to think that this is indeed a very universal property of confining theories admitting a holographic description in IIA, IIB, supersymmetric or nonsupersymmetric cases.

The string theory nonlinearity is in a term proportional to $\sqrt{t}$ instead of t^2. This different behavior could be due to the different limits of the λ coupling and spin J regimes where the two results are relevant. There are successful phenomenological models [36] interpolating between a squared root and quadratic behavior in t. It is also possible that the t^2 nonlinearity could be due to mixing with fundamental matter and decay effects present in the finite N theory and should disappear in the planar limit. Having its origin in the massive modes of the world-sheet fluctuations, we expect the $\sqrt{t}$ nonlinearity to be due to mixing with the adjoint matter always present in the supergravity regime. These massive terms are intrinsically quantum mechanical and vanish in the string classical result giving the linear trajectory.

Our evaluation of this nonlinear effect is valid for large J and even much larger λ. Then, it is hard to believe that the details of this result can be extrapolated to the decoupled, pure YM regime $\lambda \ll 1$. On the other hand in Sec. IV C we have considered also the limit $J \gg \lambda$, which seems to be more reliable for a comparison with the decoupled theory, and in this case we have obtained that the leading quantum corrections give a renormalization of the effective string tension completely consistent with the result obtained from the Wilson loop analysis. We would like to stress that, in this regime, subleading corrections to the result we found could produce again nonlinearities in the trajectory, including t^2 ones. It would be very interesting to calculate these corrections, in order to see if this picture is actually realized.

Another qualitative agreement with the experimental data comes from the comparison with the slopes of the Regge trajectories for mesons and for the soft Pomeron. The phenomenological data indicate that the former is approximately 3.6 times bigger than the latter. The classical string computation gives instead only a factor of 2, to be traced back to the difference between open and closed strings, consistently with the infinite N limit of the Casimir ratio in formula (4.4). According to the latter, the inclusion in the string calculation of finite N contributions should produce for $N = 3$ just a little improvement to a factor of 2.25. But the first α' stringy quantum correction seems to point in the right direction to solve the discrepancy. In fact, comparing the renormalized fundamental and adjoint string tensions $T^{(\mathrm{ren})}_{\mathrm{QCD}}$ from (3.16) and $T^{(\mathrm{ren})}_{\mathrm{adj}}$ from (4.51), we read

$$\frac{T^{(\mathrm{ren})}_{\mathrm{adj}}}{T^{(\mathrm{ren})}_{\mathrm{QCD}}} \sim 2\left(1 + \frac{4.8}{\lambda}\right). \tag{6.6}$$

[13]It has been suggested that the string dual to YM theory has to include world-sheet supersymmetry in order to get rid of the open string tachyon [35].

The plus sign in the correction suggests that reducing the value of the coupling toward the realistic regime the ratio of the tensions correctly increases. Thus, even if the very large J and λ regime where our computation is reliable is not the phenomenological one, we take formula (6.6) as an indication that the inclusion of higher stringy corrections points toward a better agreement of the string models with the real world[14]. As a final caveat, we should stress that in (6.6) we used the mesonic string tension calculated via the Wilson loop. It would be very interesting to check if this value, as expected, is exactly the same one that appears in the open string calculation of the mesonic Regge trajectories.

C. Stringy hadrons

In this section we comment on the field theory states dual to the rotating strings of Sec. V. The general matching between (some of) the string massless excitations and the field theory operators, first analyzed in [14], is discussed in [16]. Let us review the basic features.

The field theory object dual to our string ground state must be made out of some massive fields charged under the global SO(5) flavor group. As such, the gauge degrees of freedom are ruled out. The natural building blocks for describing the field theory states are the five adjoint scalars Φ_i, $i = 1, \ldots, 5$, living on the D4 world volume before wrapping the supersymmetry breaking cycle. After wrapping, they get [the same, since SO(5) is preserved] mass due to loop corrections. The string prediction is that this mass, in the strong coupling regime and in the hadron bound state, gets renormalized to m_0. The scalars transform in the vector representation of SO(5), so, with respect to the two U(1)'s subgroups, corresponding to the J_+ and J_- charges of the spinning strings (in Sec. VA J_+ was called simply J), we can arrange a complex linear combination, call it Z, with charges (1,0), respectively, a second complex combination W with charges (0,1), and we are left with an uncharged field Φ.

From these data we can immediately conclude that the ground state of the string on the plane wave of Sec. VA is dual to a hadron created by acting on the field theory vacuum as $\mathrm{Tr}[Z^{J_+}]|0\rangle_{FT}$. More precisely, in the absence of state/operator correspondence we can simply identify the hadron as the lowest energy state created by the action of the given operator on the gauge theory vacuum. The latter has in fact the right quantum numbers: its mass is J_+ times m_0 and its charge is precisely J_+, so, according to the energy formula (5.15), its dual string state has light-cone Hamiltonian $H = E - m_0 J_+ = 0$ and is, therefore, the perfect candidate for the string ground state (5.22).

Let us propose an identification of some of the string theory zero modes with gauge theory states. If we look at the modes coming from the S^4 of the original Witten background, we see that the $v, \bar{v}$ coordinates of the plane wave (5.14) are the ones charged under J_-, while v_3 is not charged under it, but the three of them are not charged under J_+. Their zero modes have mass m_0, so their action on the string theoretic ground state gives a state with energy $H = m_0$. In field theory we can easily identify them with the fields $W, \bar{W}$ and Φ, respectively: all of them have mass m_0 and zero J_+ charge, so their insertion in the hadron $\mathrm{Tr}[Z^{J_+}]|0\rangle_{FT}$ gives a total contribution to the Hamiltonian as $H = m_0(J_+ + 1) - m_0 J_+ = m_0$.

It was also argued in [14] that the three massless string modes x_i should describe the nonrelativistic motion (and excitations) of the hadron in the three special direction in field theory. We refer the interested reader to that paper for a discussion of this point. For what concerns the $u, \bar{u}$ modes in (5.14), their identification has always been very difficult [6,14–16], and we do not have any definite proposal.

Let us come to the multicharged string solitons of Sec. V. The ingredients in field theory are the same as before, the only difference is that the string ground state should be identified with a hadron carrying two very large charges J_+, J_-. Unfortunately, this is the only identification we will make given the absence of the spectrum of excitation. Identifying the ground state is an easy task since a state created by J_+ fields Z and J_- fields W, such as $\mathrm{Tr}[Z^{J_+} W^{J_-}]|0\rangle_{FT}$ will contain all the needed quantum numbers. There is, however, an ambiguity in the ordering structure that should account for the m_1, m_2 integers in formula (5.17).

Next, we discuss the problem of the sigma-model corrections. Up to now, we have ignored them, pretending the ground state energy of our strings to be the classical value $E = m_0 J$. How do we interpret the corrections in (5.13) and (5.22)? A natural suggestion is a renormalization of the mass of the constituents. Basically one can think of it as a strong coupling renormalization, that is, the value m_0 is the "bare" one in the strict $\lambda = \infty$ limit, while for finite (but large) λ this is corrected as in (5.13) and (5.22). Coincidentally, we learn from Sec. III that the string tension *is* renormalized for finite λ. These are the same kind of corrections that are detected in the hadronic bound states. The ratio between the renormalized scalar masses $m(\lambda)$ and the renormalized string tension is

$$\frac{m^2(\lambda)}{T_{\mathrm{QCD}}^{(\mathrm{ren})}} = \frac{6\pi}{\lambda}\left(1 + \frac{c}{\lambda}\right), \tag{6.7}$$

with $c > 0$. Then the quantum correction increases the attitude of these fields to decouple as we lower λ. Nevertheless, the dimensionless scalar masses $\hat{m}(\lambda) = m(\lambda)/m_0$ decrease with λ, consistently with the fact that

[14]Note that in this argument we used the $k \ll 1$ (i.e., $J \ll \lambda$) result. In the opposite limit of $k \gg 1$ we find no λ correction to the ratio of the tensions. This suggests that for generic values of k the correction should be present, although with a smaller magnitude with respect to the one in (6.6). The main result is thus the same.

the expected one-loop mass square at weak coupling $m^2_{\rm scalars} \sim \lambda m_0^2$ is much smaller than m_0^2.

In summary, we can adapt to our case all the solitonic solutions already discussed in the case of $AdS_5 \times S^5$ (e.g., see the review article [27]) which correspond to strings at rest in AdS_5 and spinning on a $S^3 \subset S^5$. In particular, we have considered the explicit case of multispinning folded and circular strings which are particular examples of the solutions called "regular" in [27]. These solutions admit a regular expansion in the 't Hooft coupling of the dual $\mathcal{N} = 4$ SYM theory and it is believed that their quantum corrections to the classical energy are subleading in the infinite J limit. In confining backgrounds we have that one-loop corrections are not in general subleading but our results suggest that at one-loop the general energy-spin relation for these regular solutions (including the string excitations on the pp wave) admit an expansion in $\lambda/J \ll 1$ of the form

$$E = m(\lambda)J\left[1 + a_1 \frac{\lambda^2}{J^2} + a_2\left(\frac{\lambda^2}{J^2}\right)^2 + \ldots\right], \qquad (6.8)$$

where $a_1, a_2, \ldots$ are given by the classical energy-spin relation while the one-loop quantum effects can be reabsorbed in a redefinition of the mass of the single hadron constituent $m(\lambda)$. Let us observe that, differently from what happens in $AdS_5 \times S^5$, in our case the regular solutions admit an expansion in even powers of λ/J [up to a nontrivial dependence of $m(\lambda)$]. It would be interesting to understand better this structure from the dual gauge theory point of view and if it is preserved by world-sheet higher order corrections.

Last, we recall that in [26] it was speculated that, while the sigma-model correction could be connected with the renormalization of the mass m_0 of the single constituents (but also accounting for the mean field of the other constituents), the classical string dependence in (5.13) could be related to the correction in the binding energy in the chain of constituents of the hadron, due to some mixing between hadrons with different internal structure. This would explain why in the hadron built up by only one type of scalar, and so with only one possible structure, those corrections are not present.

D. Universality of the results

Let us make some simple observations about the generality of the results presented in this paper. The universality of the properties of the hadrons studied in Secs. V and VI C, first advocated in [14], was extensively discussed in [16], and we refer the interested reader to those papers. We concentrate instead in the results for the Wilson loop and the Regge trajectory. Apart from numerical coefficients, these calculations are completely determined by the confining nature (and smoothness) of the background. Namely, the spectrum of string fluctuations is dictated by the fact that the metric in the IR is of the type $R^{1,p} \times S^q \times R^{9-p-q}$, with a nonvanishing g_{00} component. Crucially, all the known confining backgrounds are of this form in the IR. The actual value of g_{00} (up to quadratic order in the radius) and the values of p, q only affect the final results via the numerical coefficients in formulas like (3.16) and (4.40). As such, those formulas are expected to be the same, modulo the coefficients, in all the confining duals. We then state that the energy of the rectangular Wilson loop and of the Regge trajectory for glueballs, up to the first sigma-model correction, are given by the universal formulas

$$E_{\rm Wilson} = T_{\rm QCD}\left(1 - \frac{w_1}{\lambda}\right)L - \frac{w_2}{L}, \qquad (6.9)$$

$$J = \alpha'_{\rm adj}\left(1 - \frac{r_1}{\lambda}\right)[E_{\rm Regge} - r_0]^2, \qquad (6.10)$$

with coefficients w_1, w_2 (both positive), r_1, r_0 and bare value of $T_{\rm QCD}$ (and so $\alpha'_{\rm adj}$) depending on the specific background. The first formula is reliable, to this order, up to exponentially suppressed (in L) terms, while the second one includes only the first order terms in a series expansion.

The Wilson loop gives a renormalization of the string tension and a Lüscher term. The nonlinear glueball Regge trajectory generically has a renormalization of the string tension and positive intercept. The relations (6.9) and (6.10) can be thought of as the strong coupling expansions for the physical quantities $E_{\rm Wilson}$, $E_{\rm Regge}$.

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APPENDIX A: REVIEW OF WITTEN'S MODEL

Let us briefly review the model proposed in [7] as a possible IIA string dual of large N Yang-Mills theory in four dimensions. The background is generated by N D4 branes wrapped on a circle where antiperiodic boundary conditions for the fermions are assumed. This breaks supersymmetry explicitly and the low energy gauge theory on the D4 world volume turns out to contain a nonsupersymmetric YM theory in 4D. As usual, in the supergravity approximation we cannot really decouple the pure gauge theory from the other modes, so in this limit only a confining theory sitting, presumably, in the same universality class as pure YM can be really examined.

It is useful to present the 11D origin of the solution since this will help us understand it in a wider range of parameters. The starting point is the (rescaled, large-mass) Schwarzschild-$AdS_7 \times S^4$ solution

$$ds_{11}^2 = h(\rho)d\tau^2 + h^{-1}(\rho)d\rho^2 + \rho^2 \sum_{i=1}^{5} dx_i dx_i + \frac{b^2}{4} d\Omega_4^2,$$
$$h(\rho) = \frac{\rho^2}{b^2}\left(1 - \frac{b^6}{\rho^6}\right), \qquad b = 2\alpha'^{1/2}(\pi N)^{1/3}, \quad \text{(A1)}$$

where $N \gg 1$ is the number of $M5$ branes, wrapped along the circle parametrized by τ, generating the solution. The background also includes a four-form field strength which we will write in the following. The coordinate τ has periodicity $\delta\tau = 2\pi b/3$: this ensures regularity in the $\rho \to b$ limit. It is possible to use a normal angular variable θ of period 2π instead of τ, by changing coordinates as $\tau = (b/3)\theta$. The fermionic boundary conditions in going around the θ circle are taken to be antiperiodic, this ensures a breaking of supersymmetry on the world volume. The coordinates ρ and τ have dimension of a length, while the x_i are dimensionless.

Now let us wrap the branes on a second cycle orthogonal to the first one and parametrized by the angular variable $\beta = 3Nx_5/\lambda$, where λ measures the ratio between the radii of the θ and β cycles at infinity

$$\frac{R_\beta}{R_\theta} = \frac{\lambda}{N}. \quad \text{(A2)}$$

From the world-volume point of view the wrapping procedure can be understood as follows. One starts with N M5 branes, whose near horizon geometry is $AdS_7 \times S^4$ and whose world-volume field theory is a six dimensional conformal gauge theory with $(0, 2)$ supersymmetry. Wrapping the branes along the β cycle of radius R_β without breaking supersymmetry, one gets a 5D SU(N) gauge theory with coupling $g_5^2 \sim R_\beta$ at low energy. Then a second compactification is taken, along the θ circle. The low energy world-volume theory will then appear four dimensional and its gauge coupling will go as $g_{YM}^2 \sim R_\beta/R_\theta$. If one takes, on the direct product of the two circles, a supersymmetry-preserving spin structure, the four dimensional theory will be $\mathcal{N} = 4$ SU(N) SYM. Otherwise, after imposing antiperiodic boundary conditions to the fermions along the θ circle, the low energy theory becomes the pure nonsupersymmetric SU(N) YM. In fact, the fermionic degrees of freedom get immediately a mass proportional to the inverse radius, while the would-be massless scalars obtain mass at one-loop level. The usual limits $N \gg 1$, $g_{YM} \to 0$, $\lambda = g_{YM}^2 N =$ fixed, are taken.

After the whole wrapping is performed, the 11 dimensional metric reads

$$ds_{11}^2 = \frac{\rho^2}{9}\left(1 - \frac{b^6}{\rho^6}\right)d\theta^2 + \frac{b^2}{\rho^2}\left(1 - \frac{b^6}{\rho^6}\right)^{-1} d\rho^2 + \rho^2 \sum_{i=1}^{4} dx_i dx_i + \frac{\rho^2\lambda^2}{9N^2} d\beta^2 + \frac{b^2}{4} d\Omega_4^2$$
$$= e^{-2\Phi/3} ds_{10}^2 + l_s^2 e^{4\Phi/3} d\beta^2, \quad \text{(A3)}$$

where l_s is the string length $l_s^2 = \alpha'$ and the ten-dimensional metric can be read as the one generated by N D4 branes in type IIA wrapping the θ circle. From the previous expression, the ten-dimensional string frame metric and dilaton read

$$ds_{10}^2 = \frac{\lambda\rho}{3Nl_s}\left[\frac{\rho^2}{9}\left(1 - \frac{b^6}{\rho^6}\right)d\theta^2 + \frac{b^2}{\rho^2}\left(1 - \frac{b^6}{\rho^6}\right)^{-1} d\rho^2 + \rho^2 \sum_{i=1}^{4} dx_i dx_i + \frac{b^2}{4} d\Omega_4^2\right],$$
$$e^{2\Phi/3} = g_s \frac{\lambda\rho}{3Nl_s}. \quad \text{(A4)}$$

The parameter g_s is the string coupling in the "unwrapped" D4 metric at infinity. In order to rewrite the whole background in a more compact form let us define

$$R = \frac{b}{2} = (\pi N g_s)^{1/3}\alpha'^{1/2}, \qquad \frac{u}{R} = \frac{\rho^2\lambda^2}{9N^2 l_s^2},$$
$$u_0 = \frac{b^3\lambda^2}{18N^2 l_s^2}. \quad \text{(A5)}$$

The ten-dimensional string frame metric and dilaton are thus given by

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[\eta_{\mu\nu}dx^\mu dx^\nu + \frac{4R^3}{9u_0} f(u)d\theta^2\right] + \left(\frac{R}{u}\right)^{3/2}\frac{du^2}{f(u)} + R^{3/2}u^{1/2}d\Omega_4^2,$$
$$f(u) = 1 - \frac{u_0^3}{u^3}, \qquad e^\Phi = g_s \frac{u^{3/4}}{R^{3/4}}, \quad \text{(A6)}$$

where u has dimensions of length and ranges in $[u_0, \infty)$, and we shifted to a Minkowski 4D metric.

The background also includes a constant four-form field strength

$$F_4 = 3R^3\omega_4, \quad \text{(A7)}$$

where ω_4 is the volume form of the transverse S^4

$$\int_{S^4} \omega_4 = \frac{8\pi^2}{3}, \quad \text{(A8)}$$

and the normalization of F_4 guarantees that the quantization condition

$$\int_{S^4} F_4 = 8\pi^3 \alpha'^{3/2} g_s N \tag{A9}$$

is satisfied.

1. Gauge theory parameters

The relation between the u variable and the field theory energy scale is not known explicitly for the background above. Let us only suggest that the limit $u \rightarrow u_0$ can be interpreted as an IR limit in the dual field theory. It is in this regime in fact that confinement was deduced in the string context (through the Wilson loop analysis). Also notice that in this limit the g_{00} component of the metric goes to a constant, a necessary condition for confinement being realized in the dual field theory [37].

Let us underline that it is natural to believe that the UV limit in the dual gauge theory is conversely reached when $u \rightarrow \infty$. It is clear that one of the most difficult problems in the context of the gauge/gravity correspondence is to find a supergravity description of the weakly coupled regime of gauge theories. In particular, asymptotic freedom in supergravity seems to be beyond our current reach. There is, however, an effective vanishing of the coupling in some supergravity backgrounds. For example, in the case of the Maldacena-Nùñez [25] model (dual to $\mathcal{N} = 1$ 4D SYM plus KK matter), the SYM coupling has been identified using the DBI action of wrapped D5 branes [38]. Interestingly, using a possible radius/energy relation deduced via identification of the dual of the gluino condensate, it can be shown that the SYM coupling behaves as a logarithmic function of the energy scale which is the expected UV behavior. A logarithmic behavior of the UV beta functions was found also in the Klebanov-Strassler background with fractional branes [24].

This unexpected behavior does not occur in the Witten model. From the DBI action for N D4 branes wrapped around the circle parametrized by θ in our background (A6) one obtains

$$\frac{1}{g_{YM}^2(u)} = \frac{1}{(2\pi)^2 l_s} \int d\theta e^{-\Phi} \sqrt{g_{\theta\theta}} = \frac{1}{3\pi l_s g_s} \left(\frac{R^3}{u_0}\right)^{1/2} \sqrt{1 - \frac{u_0^3}{u^3}}, \tag{A10}$$

which gives, in the large u limit, the "geometrical" constant value[15]

$$g_{YM}^2 = \frac{3\pi l_s g_s \sqrt{u_0}}{R^{\frac{3}{2}}} = 3\sqrt{\pi}\left(\frac{g_s u_0}{l_s N}\right)^{1/2}. \tag{A11}$$

So, the would-be 4D YM coupling[16] $g_{YM}(u)$ increases while going toward the IR, as expected. However, in the UV limit it does not go to zero.

[15] This is obtained using $g_{YM}^2 = 2\pi l_s g_s / R_\theta$ and the relations $R_\theta^2 = \frac{4R^3}{9u_0}$, $R^3 = \pi l_s^3 g_s N$.

[16] We denote as g_{YM}^2 and λ the UV couplings, leaving the u dependence in $g_{YM}^2(u)$ to indicate the running coupling.

The D4-brane map is fairly intricate given that the theory is not conformal [39]. It makes sense for us to revise it in the presence of a thermal circle. The supergravity regime of validity for this system spans IIA and M-theory for appropriate values of the energies. Let us investigate this topic in more details. As explained in [39], a stack of D4 branes in the Maldacena limit is better described by starting with a stack of M5 branes wrapped on the 11th dimensional circle. The world-volume theory is a (0, 2) six dimensional conformal field theory on the M-theory circle.

It is interesting that by looking at the dilaton alone we have a hierarchy of scales precisely as for the nonthermalized D4 of [39]. Recall that the decoupling limit in this case takes the form

$$\frac{u}{\alpha'} = \text{fixed}, \qquad g_5^2 = (2\pi)^2 g_s \sqrt{\alpha'} = \text{fixed}, \qquad \alpha' \rightarrow 0. \tag{A12}$$

In these quantities the dilaton takes the form

$$e^\Phi = \left(\frac{u^3 g_5^6}{2^6 \pi^7 \alpha'^3 N}\right)^{1/4}. \tag{A13}$$

For $N^{1/3} \ll g_5^2 u/\alpha'$ the appropriate description is in terms of 11D supergravity given that in this regime the dilaton is large. The decoupling limit for the 11D theory takes the form

$$\ell_p \rightarrow 0, \qquad R_\beta = g_s \sqrt{\alpha'} = g_5^2/(2\pi)^2 = \text{fixed}. \tag{A14}$$

To completely determine the regime of validity of IIA we need two conditions: small dilaton and small curvatures in string units. The Ricci scalar for the 10D background takes the form

$$\mathcal{R} = -\frac{9}{R^{3/2} u^{1/2}} \left(5 - \frac{u_0^3}{u^3}\right). \tag{A15}$$

As expected for D4 branes we obtain that [39] $N^{-1} \ll g_5^2 u/\alpha' \ll N^{1/3}$. Note that since $u \geq u_0$, the curvature is always smaller than that of a stack of D4 branes.

We are interested in the regime of validity of the solution in terms of 4D parameters. For this purpose, note that the curvature has its maximum for $u \rightarrow u_0$, where it is of order

$$\mathcal{R} \sim \frac{1}{R^{3/2} u_0^{1/2}} \sim \frac{1}{l_s^2 g_{YM}^2 N}. \tag{A16}$$

Then the large 't Hooft coupling regime $\lambda \equiv g_{YM}^2 N \gg 1$ is required in order to use the supergravity approximation. For what concerns the dilaton, imposing $e^\Phi \ll 1$ fixes for u a maximal critical value $u_{\text{crit}} = l_s^2 \sqrt{u_0} N^{1/3} / R^3 g_{YM}^2$. Then, $u_0 \ll u_{\text{crit}}$ means $N^{2/3} \gg \lambda \gg 1$ and then $g_{YM}^2 \ll N^{1/3}$.

Finally, the relevant string parameters can be expressed in terms of the gauge parameters in the following way [40]

$$R^3 = \frac{g_{YM}^2 N l_s^2}{3m_0}, \qquad g_s = \frac{g_{YM}^2}{3\pi m_0 l_s}, \qquad u_0 = \frac{1}{3} g_{YM}^2 N m_0 l_s^2. \tag{A17}$$

APPENDIX B: COMMENT ON THE SOLUTIONS AT CONSTANT RADIUS

In this appendix we make some observations on the classical solutions at constant value of the radius. Because of the nontrivial dependence of the metric on the radial coordinate u, any string with nontrivial dynamics along the radial direction [i.e., with a nonconstant $u(\tau, \sigma)$] has got complicate equations of motions. In Appendix C we consider in detail the simplest and most important example of this kind of nontrivial configuration: the Wilson loop (for other examples of classical string configurations with a nonconstant $u(\tau, \sigma)$, see [17,19]).

In this paper we consider string configurations extending and/or spinning in the four flat directions or in the internal four-sphere and satisfying the static gauge $t = k\tau$. Let us assume that a general string with this shape lies at a constant $u \neq u_0$. We will use the metric in the form given in (A6). Since we are looking for solutions with constant θ too, from the Polyakov action in conformal gauge we obtain the following equation of motion for u

$$\frac{3u^{1/2}}{2R^{3/2}} \partial^\alpha x^\mu \partial_\alpha x_\mu + \frac{R^{3/2}}{2u^{1/2}} G_{IJ}^{S^4}(\chi) \partial^\alpha \chi^I \partial_\alpha \chi^J = 0, \tag{B1}$$

where χ^I and G^{S^4} are, respectively, the (arbitrary) coordinates and the metric for the four-sphere. This equation of motion must be supplemented with the conformal constraint

$$\frac{u^{3/2}}{R^{3/2}} (\partial_\tau x^\mu \partial_\tau x_\mu + \partial_\sigma x^\mu \partial_\sigma x_\mu) + R^{3/2} u^{1/2} G_{IJ}^{S^4}(\chi)(\partial_\tau \chi^I \partial_\tau \chi^J + \partial_\sigma \chi^I \partial_\sigma \chi^J) = 0. \tag{B2}$$

From this equality we can extract $(\partial_\tau t)^2$ in terms of the other coordinates and substituting the result into the equation of motion we arrive at the condition

$$\frac{3u^{1/2}}{R^{3/2}} \partial_\sigma x^a \partial_\sigma x^a + \frac{R^{3/2}}{u^{1/2}} G_{IJ}^{S^4}(\chi)(\partial_\tau \chi^I \partial_\tau \chi^J + 2\partial_\sigma \chi^I \partial_\sigma \chi^J) = 0. \tag{B3}$$

This implies that, at constant nonminimal radius $u \neq u_0$, we can have only solutions with the only nonconstant coordinates $x^a = x^a(\tau)$, whose dependence can be solved using the equation of motion for x^a and correspond to a collapsed string traveling with the speed of light, i.e.,

$$x^a = x_0^a + n^a \tau \quad \text{with } n^a n^a = k^2. \tag{B4}$$

More general solutions describing strings entirely lying at the minimal radius $u = u_0$ are instead allowed. This is easily understood by looking at the metric in the form (2.5), since for $r = 0$ (and so $u = u_0$) the equation of motion for r is trivially satisfied.

APPENDIX C: WILSON LOOP: STRAIGHT STRING AT NONMINIMAL RADIUS

Let us consider the more general open string configuration giving the actual Wilson loop. This corresponds to the minimal area configuration spanned by an open string having boundaries at infinite radius and penetrating for some distance in the bulk, without reaching the horizon at $u = u_0$, as we will see in a moment. In order to keep a clear interpretation in terms of our parameters, we will use the original coordinates in (2.1). Keeping the "straight" shape in t, x^1, consider the general radial dependence

$$t = \tau, \qquad x^1 = \sigma, \qquad u = u(x^1). \tag{C1}$$

It is easy to see that this gives a solution of the equations of motions and that the action reduces to [41]

$$S = \frac{T}{2\pi} \int dx^1 \sqrt{\frac{u^3}{R^3} + \frac{u'^2}{1 - u_0^3/u^3}}. \tag{C2}$$

The equation for x^1 gives the relation

$$\frac{u^3/R^3}{\sqrt{\frac{u^3}{R^3} + \frac{u'^2}{1-u_0^3/u^3}}} = \text{const}, \tag{C3}$$

from which it follows that

$$L = 2\int_{u_m}^{\infty} \frac{u_m^{3/2} R^{3/2} du}{\sqrt{(u^3 - u_0^3)(u^3 - u_m^3)}}, \tag{C4}$$

where we used the fact that the minimum of the radius, u_m, is reached, by symmetry, at $x^1 = 0$, and that the extremum of x^1 is precisely the string length L. The relation (C4) gives us L in terms of an Appell function of u_0, u_m evaluated at the extrema. One can see that the limit for $u \to \infty$ is finite, while the limit for $u \to u_m$ is an hypergeometric function of u_0, u_m. We are interested in a semiclassical regime described by large L.

As a first approximation, we can observe the following. It was shown in [10] that in the large L limit, the classical configuration of the Wilson loop tends to have a bathtub shape, with the string coming down from infinity practically straight up to u_m, then becoming suddenly (but smoothly) flat along a special direction [x^1 in (C1)] and then returning back to infinity with another straight line. This string spans the loop traveling for a time $\bar{t}$. The important point is that the deviation of this smooth configuration from the rectangular well is exponentially vanishing with L. Moreover, the lines coming down from infinity give the infinite quark masses which require renormalization. This makes the following straight line approximation of the above solution

$$t = \tau, \qquad x^1 = \sigma, \qquad \sigma \in \left[-\frac{L}{2}, \frac{L}{2} \right], \qquad u = u_m, \tag{C5}$$

an accurate one. Moreover, in the regime $L \to \infty$, one has that $u_m \to u_0$, as can be verified from (C4), where the extremum at $u = \infty$ can be ignored. More precisely, (C4) gives (see [37] for the analytic derivation)

$$L \sim -\frac{2R^{3/2}}{3u_0^{1/2}} \log \frac{u_m - u_0}{u_0}. \tag{C6}$$

For these reasons, the QCD string tension can be simply inferred by considering the string sitting at u_0 and it is classically given by (3.2).

Analogously, it could be concluded that in this regime it is a good approximation to go to the $u = u_0$ configuration also for the quantization procedure. Let us see why this is the case.

Since for large L most of the string is almost straight and reaches its minimum u_m at its middle point, we can solve the equation of motion for u in the approximation $y = (u - u_m)/u_0 \ll 1$ and $a_m \equiv u_m/u_0 - 1 \ll 1$. Up to subleading terms in y or a_m, this reads

$$y' \simeq 3\left(\frac{u_m}{R^3}\right)^{1/2} \sqrt{y(y + a_m)}, \tag{C7}$$

which can be easily integrated. The result in terms of u is the following

$$\begin{aligned} u - u_m &\simeq \frac{1}{2}(u_m - u_0)\left\{\cosh\left[3\left(\frac{u_m}{R^3}\right)^{1/2}(\sigma - \sigma_0)\right] - 1\right\} \\ &\simeq \frac{u_m}{2}\left\{\cosh\left[3\left(\frac{u_m}{R^3}\right)^{1/2}(\sigma - \sigma_0)\right] \right. \\ &\quad \left. - 1\right\} e^{-(3L/2)(u_m/R^3)^{1/2}}, \end{aligned} \tag{C8}$$

where σ_0 is the middle point of the string, i.e., $u(\sigma_0) = u_m$. As one can immediately see, $u \to u_m$ as $L \to \infty$. We can now check to what extent we can use the above solution. If we take the general constraint $\sigma - \sigma_0 \leq \alpha L/2$ with $\alpha \leq 1$, we obtain the following upper bound

$$y \sim \leq \frac{1}{2} a_m^{1-\alpha}. \tag{C9}$$

Then, in the large L limit, we can use the above approximation for any $\alpha < 1$. In particular, the approximation is still valid if

$$1 - \alpha \sim (\log a_m)^{-\beta}, \qquad 0 < \beta < 1, \tag{C10}$$

and, at the leading order in the large L limit, the above approximation can be extended to the whole string.

We can now turn to the study of the quadratic fluctuations. The only nontrivial terms are those for θ and u, while all the other modes are massless in the above approximation. Then, we can focus on the following terms[17] of the metric

$$ds^2 = \ldots + \frac{4R^3}{9u_0}\left(\frac{u}{R}\right)^{3/2} f(u) d\theta^2 + \left(\frac{R}{u}\right)^{3/2} \frac{du^2}{f(u)} + \ldots, \tag{C11}$$

which are clearly degenerate in the large L limit. Adding a bar to denote the above classical solution as $\bar{u}(\sigma)$, the general dynamical field u can be expanded around $\bar{u}$. By rescaling the fluctuating fields in the following way:

$$u = \bar{u} + \frac{1}{R^{3/4}} \sqrt{\bar{u}^{3/2} f(\bar{u})} \zeta, \qquad \theta = \frac{3}{2}\left(\frac{u_0}{R^{\frac{3}{2}}}\right)^{1/2} \frac{\chi}{\sqrt{\bar{u}^{3/2} f(\bar{u})}}, \tag{C12}$$

the quadratic Lagrangian for ζ and χ is given by

$$\begin{aligned} \mathcal{L}_{\zeta,\chi} \sim{}& \partial_\alpha \chi \partial^\alpha \chi + \partial_\alpha \zeta \partial^\alpha \zeta + \frac{(\bar{u}')^2}{2\bar{u}^{3/2} f(\bar{u})} \partial_u^2 [u^{3/2} f(u)]_{|u=\bar{u}} \chi^2 + \left\{ \frac{\bar{u}''}{2\bar{u}^{\frac{3}{2}} f(\bar{u})} \partial_u [u^{3/2} f(u)]_{|u=\bar{u}} \right. \\ & \left. - \frac{(\bar{u}')^2}{4[\bar{u}^{\frac{3}{2}} f(\bar{u})]^2} [\partial_u [u^{\frac{3}{2}} f(u)]_{|u=\bar{u}}]^2 \right\} (\chi^2 + \zeta^2). \end{aligned} \tag{C13}$$

The first χ^2 term is subleading in the large L limit and then the mass for χ and ζ is the following:

[17] There is also another mass term for u, but it is exponentially vanishing with L.

$$M_{\chi,\zeta}^2 \simeq \frac{\bar{u}''}{2\bar{u}^{3/2}f(\bar{u})}\partial_u[u^{3/2}f(u)]_{|u=\bar{u}} - \frac{(\bar{u}')^2}{4[\bar{u}^{3/2}f(\bar{u})]^2}[\partial_u[u^{3/2}f(u)]_{|u=\bar{u}}]^2 \simeq \frac{3\bar{u}''}{2u_m f(\bar{u})} - \frac{9(\bar{u}')^2}{4u_m^2[f(\bar{u})]^2}$$
$$\simeq \frac{9u_m}{4R^3}\left\{\frac{2\cosh[3(\frac{u_m}{R^3})^{1/2}(\sigma-\sigma_0)]}{\cosh[3(\frac{u_m}{R^3})^{1/2}(\sigma-\sigma_0)]-1} - \frac{\sinh^2[3(\frac{u_m}{R^3})^{1/2}(\sigma-\sigma_0)]}{\{\cosh[3(\frac{u_m}{R^3})^{1/2}(\sigma-\sigma_0)]-1\}^2}\right\} = \frac{9u_m}{4R^3}. \tag{C14}$$

To summarize, at the leading order the string can be considered as an almost straight string lying at u_m for almost all its length and with two massive modes with constant, σ-independent mass $M_{\chi,\zeta}$. The corresponding frequencies reduce to the ones found in Sec. III A [see Eq. (3.6)] in the large L ($u_m \to u_0$) limit.

APPENDIX D: REGGE TRAJECTORY: ANOMALY CANCELLATION

In this appendix we show why the quantized configuration of Sec. IV is free of conformal anomalies. Let us start by observing how the σ dependent mass term in the bosonic action (4.6) can be reabsorbed as a curvature effect for 2D scalar fields with constant mass on a curved world sheet with metric given by the induced metric $h_{\alpha\beta} = k^2\cos^2\sigma\eta_{\alpha\beta}$. Indeed, (4.6) can be rewritten as

$$S = -\frac{1}{3\pi}\int d\tau d\sigma\sqrt{-h}\sum_{a=1,2}\left[h^{\alpha\beta}(\nabla_\alpha y_a\nabla_\beta y_a) + \frac{9}{4}(y_a)^2\right], \tag{D1}$$

where we have introduced the 2D world-sheet covariant derivative ∇_α to put the action in an explicitly covariant form. Then the equations of motion become

$$\left[h^{\alpha\beta}\nabla_\alpha\nabla_\beta - \frac{9}{4}\right]y_a = 0. \tag{D2}$$

The two descriptions with flat and curved world sheet in (4.6) and (D1), which are related by a classical Weyl transformation, can be really considered equivalent only if the conformal anomaly vanishes, a necessary condition for the consistency of the theory which is also related to the finiteness of the theory (and as a consequence, of the one-loop correction to the energy).

In the same way, let us write the fermionic action (4.17) as an action for fermions on a curved world sheet with metric equal to the induced metric $h_{\alpha\beta}$. If we organize the spinors Θ^I in eight 2D Majorana spinors $\psi^i = \psi_i$, the action (4.17) can be written as

$$S_F = \frac{i}{4\pi}\int d\tau d\sigma\sqrt{-h}\left[h^{\alpha\beta}\bar{\psi}^i\tau_\alpha D_\beta^{(2)}\psi_i - \frac{3}{4}\bar{\psi}^i\tilde{\gamma}_{ij}\psi^j\right], \tag{D3}$$

where τ_α are the curved 2D gamma matrices and $D_\alpha^{(2)}$ is the 2D spinorial covariant derivative. Then the equations of motion become

$$h^{\alpha\beta}\tau_\alpha D_\beta^{(2)}\psi^i - \frac{3}{4}\tilde{\gamma}_j^i\psi^j = 0. \tag{D4}$$

These equations can be squared into the equations

$$\left[h^{\alpha\beta}\hat{\nabla}_\alpha\hat{\nabla}_\beta - \frac{1}{4}\mathcal{R}^{(2)} - \frac{9}{16}\right]\psi^i, \tag{D5}$$

where $\hat{\nabla}_\alpha$ is the complete covariant derivative, containing both the spin connection and the Christoffel symbols, and $\mathcal{R}^{(2)}$ is the 2D scalar curvature and is equal to

$$\mathcal{R}^{(2)} = \frac{2}{k^2\cos^2\sigma}(1+\tan^2\sigma). \tag{D6}$$

As we will see in a moment, this action has the right form in order to combine well with the bosonic and ghost contributions, giving a finite (at least at one-loop) theory with vanishing conformal anomaly.

In fact, following the analysis of [23] and starting from the actions with the curved induced metric above, it is possible to extract the one-loop contribution to the partition function computing the determinants of the second order operators which enter the bosonic and fermionic equations of motions (D2) and (D5), supplemented by the ghost contribution. Working in the Euclidean formulation and regularizing the determinants with an UV cutoff, it is possible to show in general how these determinants have quadratically, linearly and logarithmically divergent terms. The quadratic and linear divergences trivially drop out in our case due to the matching between the bosonic and fermionic degrees of freedom. The logarithmic term is more delicate and can be expressed in term of the integral of the following Seeley coefficients for bosons, fermions and ghosts

$$b_{2B} = 10\times\frac{\mathcal{R}^{(2)}}{6} - \sum_B m_B^2, \qquad b_{2F} = 8\times\frac{\mathcal{R}^{(2)}}{3} + \sum_F m_F^2,$$
$$b_{2gh} = -2\times\frac{\mathcal{R}^{(2)}}{6} - \mathcal{R}^{(2)}. \tag{D7}$$

Since in our case we have only two massive bosons with equal masses $m_B^2 = \frac{9}{4}m_0^2$ and eight massive fermions with equal masses $m_F^2 = \frac{9}{16}m_0^2$ (reintroducing the m_0 dependence), we have that the mass matching condition

$$\sum_B m_B^2 = \sum_F m_F^2, \tag{D8}$$

implies that the resulting total divergence coefficient is then the integral of

$$b_2^{(\mathrm{tot})} = 3\mathcal{R}^{(2)}, \tag{D9}$$

which is proportional to the Euler character. As explained in [23], this term is exactly canceled by the cutoff dependent factors in the conformal Killing vector and/or Teichmüller measure. Then the theory is finite and since the above Seeley coefficients are exactly the same ones appearing in the Weyl anomaly, the theory is also anomaly free at one-loop. This is indeed what we have verified explicitly in the other computations of the one-loop energy considered in this paper where the world-sheet induced metric is constant and the finiteness of the one-loop correction is due to the same mass matching condition (D8). The same happens for the Regge trajectories. Going to the flat world-sheet metric gauge, we obtain an analogous (σ-dependent) mass matching condition

$$\sum_B m_B^2(\sigma) = \sum_F m_F^2(\sigma), \tag{D10}$$

which again checks the UV finiteness of the 2D theory, as, for example, happens for the folded strings considered in [30].

APPENDIX E: PENROSE LIMIT

We perform here the Penrose limit of the Witten background along the great circle of the four-sphere. Since we are interested in the IR regime, we will use the metric expanded around $u = u_0$ (2.5)

$$ds^2 \approx \left(\frac{u_0}{R}\right)^{3/2}\left[1 + \frac{3r^2}{2}\right](\eta_{\mu\nu}dx^\mu dx^\nu) + \frac{4}{3}R^{3/2}\sqrt{u_0}(dr^2 + r^2 d\theta^2) + R^{3/2}u_0^{1/2}\left[1 + \frac{r^2}{2}\right]d\Omega_4^2. \tag{E1}$$

We will also use the fact that in $u = u_0$ the dilaton goes to a constant $e^\Phi \to g_s u_0^{3/4}/R^{3/4}$.

Using the parametrization $\chi, \psi, \phi_-, \phi_+$ introduced in (5.1) for the S^4, the null geodesic we want to zoom in is determined by the following conditions on the coordinates

$$t = \phi_+, \qquad x^i = r = \psi = \chi = 0. \tag{E2}$$

As usual we first shift the 3 + 1 coordinates as[18] $x^\mu \to (R^{3/4}L/u_0^{3/4})x^\mu$ and send $L \to \infty$ while keeping

$$m_0^2 \equiv \frac{L^2}{\sqrt{u}R^{3/2}0} \quad \text{fixed.} \tag{E3}$$

This would ensure that, in the limit, we are keeping fixed the matter field and glueball masses, while taking the string tensions, proportional to $\alpha' L$, very large. Notice that $R^{3/2} \approx \sqrt{N}$. Also, recall from (A5) that $u_0 = b^3\lambda^2/(18N^2l^2)$, so that taking λ very large, namely, of order N, we get $u_0 \approx N$ and $L^2 \approx N$ in the limit. The dilaton in the IR goes like $e^{\Phi_0} \approx g_s N^{1/2}$. To be consistent with the notations in the rest of the paper, we can choose $L^2 = u_0^{3/2}/R^{3/2} \approx g_{YM}^2 N$ so that indeed $m_0^2 = u_0/R^3$.

[18]Clearly this L has nothing to do with the string length in Sec. III.

In order to perform the Penrose limit we reshift $x^i \to x^i/L$ and take

$$\psi = \frac{m_0}{L}v_3, \qquad \chi = \frac{m_0}{L}y, \qquad r = \frac{\sqrt{3}m_0}{2L}\rho. \tag{E4}$$

Expanding the whole metric near the above defined null geodesic gives

$$ds^2 = \left[1 + \frac{9m_0^2\rho^2}{8L^2}\right]\left(-L^2dt^2 + \frac{L^2}{m_0^2}d\phi_+^2\right) + dx^i dx^i + d\rho^2 + \rho^2 d\theta^2 + dy^2 + y^2 d\phi_-^2 - \left[y^2 + v_3^2 + \frac{3}{4}\rho^2\right]d\phi_+^2 + dv_3 dv_3. \tag{E5}$$

Now, let us introduce the complex variables

$$u = \rho e^{i\theta}, \qquad v = y e^{i\phi_-}, \tag{E6}$$

and the light-cone coordinates

$$t = x^+, \qquad x^- = \frac{L^2}{2}\left(t - \frac{\phi_+}{m_0}\right). \tag{E7}$$

This way we get the plane-wave metric

$$ds^2 = -4dx^+dx^- - m_0^2\left[v_3^2 + v\bar{v} + \frac{3}{4}u\bar{u}\right]dx^+dx^+ + dx^i dx^i + dud\bar{u} + dvd\bar{v} + dv_3^2. \tag{E8}$$

Written explicitly in terms of the angular variables introduced above the four-form RR field strength is

$$F_4 = 3R^3\cos^3\psi\sin\chi\cos\chi d\psi \wedge d\chi \wedge d\phi_- \wedge d\phi_+. \tag{E9}$$

After taking the Penrose limit we get

$$F_4 = \frac{3i}{2u_0^{3/4}}R^{3/4}m_0 dx^+ \wedge dv_3 \wedge dv \wedge d\bar{v}. \tag{E10}$$

Notice that this object still scales in the limit: in fact $R^{3/4}/u_0^{3/4} \approx N^{-1/2}$. In the very same way as in the D2 and fractional D2 case of [16], the correct nonscaling object is

$$e^{\Phi_0}F_4 = \frac{3i}{2}m_0 dx^+ \wedge dv_3 \wedge dv \wedge d\bar{v}. \tag{E11}$$

It is now easy to check that the only nontrivial equation of motion for the whole pp-wave background at hand

$$R_{++} = \frac{1}{12}e^{2\Phi_0}F_{+abc}\bar{F}_{+abc} \tag{E12}$$

is satisfied.

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