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Basic treatment of QCD phase transition bubble nucleation

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Starting from the QCD Lagrangian and the surface tension of QCD bubbles we derive the critical size of bubbles, the nucleation probability and the nucleation site separation distance.

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I. INTRODUCTION

For more than two decades there have been studies of the nucleation of bubbles during the QCD phase transition (QCDPT), the transition from a universe of deconfined gluons, quarks and antiquarks (the quark/gluon plasma) to a hadronic universe. These studies, based on a first-order QCDPT, use models of QCD to estimate the critical size of bubbles, the nucleation rate, and the distance between nucleating centers [1–11]. In an attempt to find possible observational effects from this early universe phase transition, there have also been estimates of magnetic fields generated by nucleating bubble surfaces during the QCDPT [12,13], and possible observable CMBR effects arising from magnetic walls[14] which arise from collisions of nucleating bubbles [15]. Estimates of magnetic fields arising from bubble collisions during the earlier electroweak phase transition (EWPT)using effective Lagrangians also have been made [16–18]. Many of these studies make use of the Coleman model [19] of tunneling from a false to a true vacuum. Nucleation rates were estimated using standard thermodynamics/statistical mechanics [4] and using hydrodynamics [20] with the Langer formalism [21].

The knowledge that the universe evolved from a quark/gluon plasma to our hadronic universe inspired the RHIC (relativistic heavy ion collisions) program at BNL and other laboratories. The challenge for the RHIC program, as well as the early universe studies, is to identify unambiguous observables for the transition (or transitions). In a recent review [22] possible relations between the early universe and heavy ion heavy phase transitions and the status of theoretical attempts to reach thermalization on the time scale that seems to be needed at RHIC are discussed. There is also a detailed discussion of flow that is associated with RHIC. In this review, as well as in another recent report [23], the possible existence of color superconducting phases and a color glass condensate that involves low momentum fields with long time scales compared to what is considered to be natural time scales are examined. Extensive references to theoretical and experimental research in RHIC physics, and possible relationships to the early universe chiral phase transition, are given in these reviews.

Lattice gauge calculations indicate that there is no first-order EWPT transition nor consistency with baryogenesis in the standard EW model. These calculations also show that with supersymmetric fields there can be a first-order transition and consistency with baryogenesis during the EWPT . Lattice calculations are not yet able to prove definitely whether there is a first-order QCDPT. Because of the great importance of possible observational effects of a first-order QCDPT, with bubble nucleation, collisions, and possible observable magnetic and other effects, in the present note we assume that there is a first-order chiral phase transition and investigate QCD bubble nucleation.

The most notable aspect of the present work is that we start from the basic QCD Lagrangian, rather than an effective model, and use previous research on nonperturbative QCD condensates to carry out the calculations.

In Sec. II we derive the critical radius for nucleating bubbles, and in Sec. III we estimate the site separation for nucleation of bubbles during the QCDPT.

II. CRITICAL RADIUS DURING THE QCDPT

In this section we estimate the critical radius, R_c, for bubble nucleation, starting from the basic QCD Lagrangian. Physically, the critical radius is attained when force due to the pressure difference inside and outside the bubble, a volume effect, equals that of the surface tension, an area effect. In the following section we derive the nucleation probability and site separation, which requires quite different methods. First, an outline of the paper is given.

A. Outline of paper

The starting point of our work is the basic QCD Lagrangian density:

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1. QCD Lagrangian density and basic equations of motion

The QCD Lagrangian density for massless quarks is

$$\mathcal{L}^{\rm QCD} = -\frac{1}{2}Tr[G \cdot G] + \bar{q}_f\gamma^\mu(i\partial_\mu + gA_\mu)q_f, \quad (1)$$

where

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \quad A_\mu = A^a_\mu \lambda^a/2, \quad (2)$$

with λ^a the eight SU(3) Gell-Mann matrices, $[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c$. Minimizing the action, one obtains the general QCD equations of motion (EOM)

$$\partial_\mu\partial^\mu A^a_\nu - \partial_\nu\partial^\mu A^a_\mu + gf^{abc}(2A^b_\mu\partial^\mu A^c_\nu - A^b_\mu\partial_\nu A^{\mu c} - \partial^\mu A^b_\mu A^c_\nu) + g^2 f^{abc}f^{cef}A^b_\mu A^{\mu e}A^f_\nu + g\bar{q}_f\gamma_\nu\frac{\lambda^a}{2}q_f = 0. \quad (3)$$

As we shall see, the last term in the EOM, Eq. (3), which gives the quark-gluon interaction, plays a crucial role in determining the critical radius of nucleating bubbles during the QCD chiral phase transition.

2. Method for deriving the critical radius

As mentioned above, the force at a bubble wall from the pressure difference on the two sides of the wall, which drives the nucleation, increases for given pressure difference as one extra power of the bubble radius than the force from the surface tension, which tends to shrink the bubble. For the calculation of the surface tension, σ, and pressure difference, Δp, we start with the basic QCD Lagrangian. To determine the critical radius we minimize the action with respect to the radius, as in the models of Refs. [18–20]. These calculation are done near the critical temperature, and do not require finite-temperature field theory. The approximations used are dropping the higher-dimensional gluonic terms in calculating the surface tension and using only pions as hadrons for the effect of the quark condensate forming at the critical temperature. These approximations are discussed in the next subsections in this section.

For this investigation only the action in the vicinity of the bubble wall at a temperature near the critical temperature is needed, and we use an extension of the instanton model to SU(3) symmetry in Minkowski space. For the more general study of nucleation during the QCD chiral phase transition this model must be generalized to treat the interior and exterior of the bubbles.

3. Method for determining the nucleation probability and site separation

For the determination of the nucleation site separation, the main goal of the present paper, standard classical statistical mechanics is used to determine the rate at which nuclei of bubbles of critical radius form. For this the pressure difference between the quark/gluon and hadronic phases at the critical temperature, as well as the surface tension and critical bubble size, are needed, and are obtained from the QCD Lagrangian density within our instantonlike model.

For the final calculation of the site separation distance the temperature dependence of the pressure difference is needed. For this we use the standard finite-temperature field theory, with a reduction from four-dimensional space-time to three-dimensional space at fixed temperature by replacing real time by imaginary 1/temperature.

B. SU(3) Ansatz for the color gauge field

We use the Lorentz gauge and an SU(3) ansatz, in which we remove the color dependence of Eq. (3) by extending the color fields, A^a_μ, to SU(3) matrices:

$$\partial_\mu A^a_\mu = 0$$
$$A^a_\mu \rightarrow i\frac{\lambda^a}{2}W_\mu. \quad (4)$$

Note that this ansatz cannot be used in the Lagrangian, Eqs. (1) and (2), to define the action and obtain the EOM Eq. (3), but it is rather a method to apply a reasonable symmetry on the color field in the EOM. It can be compared to the instanton ansatz [24], with the color fields given in terms of the SU(2) quantities $\eta^a_{\mu\nu}$ [25], but our prescription defined in Minkowski space leads to Lorentz covariant equations needed for studying nucleation and other time-dependent processes. We are guided in our choice of parameters by the instanton liquid model reviewed in Ref. [26]. In using this ansatz one operates on Eq. (3) by the color SU(3) matrix λ^a, uses the matrix extension of the color field given by Eq. (4) and the properties of the SU(3) generators to obtain the EOM for the W_μ

$$\partial_\mu\partial^\mu W_\nu - \frac{3}{2}g(2W_\mu\partial^\mu W_\nu - W_\mu\partial_\nu W^\mu) + \frac{9}{4}g^2 W_\mu W^\mu W_\nu - \chi^V\langle\bar{q}q\rangle\bar{N}W_\nu = 0, \quad (5)$$

where we make use of the study [27] of the vector vacuum susceptibility defined by the three-point function

$$\langle g\bar{q}\gamma_\nu\lambda^a/2q\rangle = \chi^V\bar{N}\langle\bar{q}q\rangle A^a_\nu, \quad (6)$$

in terms of the quark condensate, $\langle\bar{q}q\rangle$. The parameter χ^V was determined from the study of the vector three-point function for a nucleon [27], while the factor $\bar{N}$, the number of hadrons in a typical hadronic volume V, is needed for our finite T study and will be discussed below.

To determine the critical radius one needs to balance the forces near the surface of the bubble. The essential variable for determining the critical radius is $s = \sqrt{x^\mu x_\mu}$, and we use the form

$$W^\mu = x_\mu W(s), \quad (7)$$

with the gauge condition

$$W'(s) = -\frac{4}{s}W(s). \tag{8}$$

The resulting EOM for the function $W(s)$ is

$$W'' + \frac{5}{s}W' - \frac{3}{2}g(W^2 + sWW') + \frac{9}{4}g^2s^2W^3 - \chi^V\bar{N}\langle\bar{q}q\rangle W = 0. \tag{9}$$

C. Surface tension, pressure, and critical radius

Since the purpose of the present paper is to estimate the critical radius of nucleating bubbles and the site separation, and we are not attempting to study the general problem of bubble nucleation and collisions during the QCDPT, we shall concentrate on the bubble wall and the regions in the QGP and HP close to the bubble walls. We neglect some of the hadronic structure in the HP and quark/gluon structure in the QGP. This allows us to directly use the QCD Lagrangian for the nucleation properties in the early stages of the QCDPT, which has not been done previously, but the method must be extended for the complete treatment of the QCDPT.

The instanton representation of the color gauge field is known to give a satisfactory representation of midrange nonperturbative QCD. Although instantons cannot be defined between hadronic and quark/gluon vacua, we shall use an instantonlike representation of the bubble wall, as in our earlier work [14,15]. This will lead to a modification of the variables used in the expression of the color gauge fields, as explained below.

Let us first estimate the surface tension, one of the two parameters needed to determine the critical radius. The surface tension is obtained by integrating the energy density, T^{00}, through the bubble wall,

$$\sigma^{\text{inst}} = \int dsT^{00} = \int ds\left[\frac{\partial\mathcal{L}}{\partial(\partial_t W)}\partial_t W - g^{00}\mathcal{L}\right] \tag{10}$$

$$\simeq \int ds\left[6W^2 - \frac{9}{2}g\left(s^2W^3 + \frac{g}{8}s^4W^4\right)\right]. \tag{11}$$

Only the dominant gluonic contribution in Eq. (11) will be used in estimating the surface tension.

In Ref. [28] it was shown that an instanton model for the bubble wall is consistent with the surface tension estimated in lattice QCD calculations. Based on this we use an instantonlike form near the surface of the bubble wall. Recognizing that $W_\mu W^\mu = 0$ at $s = s_0$, the position of the bubble wall in the instanton model, and is nonzero in the region of approximately $s = s_0 \pm \rho$, we modify the variables used in Eq. (7); and for $W_\mu W^\mu$ and $W(s)$ we assume the instantonlike form

$$W_\mu W^\mu = \bar{s}^2W(\bar{s})^2$$
$$W(\bar{s}) = \frac{C_W}{(\bar{s}^2 + \rho^2)^2}, \tag{12}$$

which satisfies the gauge condition Eq. (8) for $\bar{s} > \rho$. Note that with the metric obtained with the continuation from Euclidean space, in which instantons are derived, to Minkowski space, $\bar{s} = \sqrt{r^2 - t^2} - s_0$, the most important region for $W_\mu W^\mu$ is the interval $-\rho \le \bar{s} \le \rho$. To check the validity of the this form we solve the EOM Eq. (9) for $W(s)$, as shown in Fig. 1. In Fig. 1 $s_0 = 10.2$ fm, which can be interpreted as R, the radius of the bubble if we take $t = 0$ as the time shown in the figure. One can see from the figure that the surface peak drops about a factor of 2 as s–R goes from 0 to 0.15, as expected from the form of Eq. (12). Therefore, the instantonlike form is seen to be valid near the surface of the bubble. For our calculation, we take $\rho = 0.2$ fm, which is consistent with the result of Fig. 1 and the value of the ρ for the instanton liquid model [26]. One can find the value of the parameter C_W from such solutions, but as is shown below this is not necessary in the present work, where we only derive the critical radius and the nucleation site separation. The oscillations in $W(s)$ found outside the wall would be important for collisions, which we do not treat in the present work. See Ref. [15].

Recognizing that the lowest-dimensional term is dominant, the surface tension is

$$\sigma \simeq \int ds6W(s)^2 = \frac{15\pi C_W^2}{2^4\rho^7}. \tag{13}$$

To determine the pressure difference, we exploit the results of the Higgs model [18,19], noting that the expectation value of the Higgs field is replaced in our work by the quark condensate, which also goes from zero to a finite value during the phase transition. In the Higgs model, in the thin-wall approximation [18,19],

$$T^{00}(|\phi| = \eta) - T^{00}(|\phi| = 0) = \epsilon\lambda\eta^4 \tag{14}$$

which is the negative of the last term in the Lagrangian

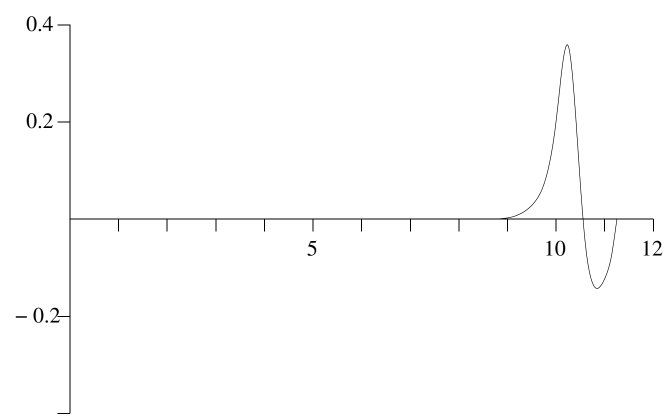

FIG. 1. Solution for $W(s)$ for $t = 0$ near the surface at $s_0 = r \simeq R = 10.2$ fm.

density used in Ref. [18]. Note, however, that

$$\epsilon\lambda\eta^4 = \Delta p(\text{Higgs Model}). \tag{15}$$

Thus, in the Higgs model, as well as for our Lagrangian, Δp is given by the difference in T^{00} across the bubble wall. With this observation, we are able to use all the familiar results of finite-temperature field theory for obtaining the consequences of our Lagrangian density for nucleation.

Note that at this stage of the nucleation process we are working in the brief time interval between t_c and t_f, the time following t_c when the universe has reheated (see the following section on nucleation). Since the gluonic condensate does not vanish rapidly with $T \simeq T_c$, the difference in the gluonic free energy on the two sides of the bubble wall is negligible during the first stages of nucleation of interest here. For similar reasons we also neglect the free energy difference of the noninteracting fermionic terms within and outside of the bubble, which play an important part of the QCDPT after the time t_f when there is an equilibrium nucleation process from the quark/gluon to the hadronic universe. For example, in a more general treatment of nucleation and collisions for the QCDPT, the perturbative hadronic and quark kinetic energy terms in the QCD Lagrangian given by $\bar{q}_f(i\gamma^\mu\partial_\mu)q_f$ must be considered [4,5].

Since the quark condensate is the parameter for the chiral QCD phase transition being treated in the present work, as one can see from the expression for T^{00}, given in Eq. (10), in our theory starting with the QCD Lagrangian the pressure difference is given approximately by the term in T^{00} that is the negative of the quark-gluon interaction, $\mathcal{L}^{q(\text{int})} = -\langle g\bar{q}\gamma_\mu(\lambda^a/2)q\rangle A^{a\mu}$, the last term in the Lagrangian density (1). Having determined the surface tension and Δp from our Lagrangian, the critical radius can be obtained by minimizing the four-dimensional action [19]

$$S_E = 2\pi^2\sigma R^3 + \frac{\pi^2}{2}\Delta p R^4, \tag{16}$$

with respect to the bubble radius, R.

In applying this formalism to our model we use the following concepts:

(i) Our bubble wall is instantonlike, not instantons. This model follows from the success in obtaining an instantonlike interior wall after bubble collisions with QCD bubbles having such an instantonlike bubble wall [15].

(ii) The wall is very thin, so a thin-wall approximation is justified. We just need the difference between T^{00} inside and outside the bubble wall.

(iii) Making use of the fact that the quark condensate vanishes for $T \geq T_c$ outside the bubble wall and that during the phase transition when $T \simeq T_c$ inside the bubble pions dominate the hadronic phase, we assume that all of the nonvanishing quark condensate is in the pions inside the bubble. Using Eq. (6), the quark-gluon interaction Lagrangian density inside the bubble is

$$\mathcal{L}^{q(\text{int})} \simeq -\chi^V n_\pi\langle\bar{q}q\rangle A^a_\nu A^{a\nu}, \tag{17}$$

while outside the bubble

$$\mathcal{L}^{q(\text{int})} \simeq 0, \tag{18}$$

where the color field A^a_ν is not of the instantonlike form given by Eqs. (4), (7) and (12) except at the bubble wall. A more general theory of gluonic structure in the hadronic phase is needed to discuss the interior, but for the purposes of the present paper we only need the pressure difference on the two sides of our bubble wall.

(iv) From this we conclude that the difference in T^{00} inside and outside the bubble wall is

$$\Delta[T^{00}] \simeq \frac{\chi^V}{2} n_\pi\langle\bar{q}q\rangle\langle A_\mu A^\mu\rangle, \tag{19}$$

with $\langle A_\mu A^\mu\rangle$ the mean value of A^2 in the wall. Using Eqs. (4), (7), and (12), this gives for the pressure difference inside and outside the bubble surface in the thin-wall approximation

$$\Delta p \simeq \frac{\chi^V}{2} n_\pi\langle\bar{q}q\rangle\bar{s}^2 W(\bar{s})^2 \tag{20}$$

$$\simeq \frac{\chi^V}{2} n_\pi\langle\bar{q}q\rangle\frac{C_W^2}{16}\frac{1}{\rho^6}, \tag{21}$$

where we have taken $\bar{N} = n_\pi$, the pion number, and have evaluated $A^{a\mu}$ at $\bar{s} = \rho$. The fact that the pions dominate the hadronic density at the phase transition, and thus that the effective density of quark condensate is given by the pion density, is well-known.

To estimate the error in evaluating $\bar{s}^2W(\bar{s})^2$ at $\bar{s} = \rho$, we can calculate the mean value by calculating $I_a \equiv \int_{-a}^{a}\bar{s}^2W(\bar{s})^2/2a$. The most reasonable value for a is $a = \rho$. One finds $I_\rho = \frac{C_W^2}{16}\frac{1}{\rho^6}\times 1.12$. This would give a 12% decrease in the critical radius and a larger nucleation site separation. If one takes $a = 2\rho$, which is larger than expected for the instantonlike wall, $I_\rho = \frac{C_W^2}{16}\frac{1}{\rho^6}\times 0.82$. Therefore we conclude that the estimate in Eq. (21) is correct to about 10%.

Recognizing that the bubble wall thickness is given by $\rho = 0.2$ fm, we use the thin-wall approximation in minimizing the action [18,20] to get the classical thin-wall equation for the critical radius. From Eqs. (13), (21), and (16) we obtain

$$R_c = \frac{3\sigma}{\Delta p} \simeq \frac{15\pi}{\rho}\frac{1}{\chi^V\langle\bar{q}q\rangle/2}\frac{1}{n_\pi}. \tag{22}$$

Using the standard value for the pion volume density [29], $n_\pi/V = 0.365T^3$, with $V = (4\pi/3)R_n^3 =$ the nucleon volume ($R_n = 1.1$ fm), and using the value of the vector susceptibility from Ref. [27], $\chi^V\langle\bar{q}q\rangle/2 \simeq 3$ GeV^2, with $T = 150$ MeV, we obtain from Eq. (22)

$$R_c = 11 \text{ fm}. \tag{23}$$

There are a number of approximations which could change our value for R_c, and due to the sensitivity of the nucleation site separation to this parameter, this could change the evaluation in the next section. We show that this value is consistent with the evaluation of Ref. [5] at the critical time of freeze-out in the next section. The innovation of the present work is that we estimate R_c directly from the QCD Larangian, rather than using models as in previous estimates.

III. NUCLEATION PROBABILITY AND NUCLEATION SITE SEPARATION

For applications to astrophysical observations the distance between nucleation sites, d_n, is a critical parameter, since with large separation between nucleation sites the hadronic universe can be formed via the collision between a few large nucleating bubbles, and this could lead to interesting large-scale structure. For example, it was shown with such a scenario that large magnetic walls would form during the QCDPT [14], which could lead to observable effects in cosmic microwave background polarization.

One obtains the nucleation site separation, d_n, from the probability of nucleation per volume-time, $p(t)$. In this section we estimate the nucleation probability using the critical radius derived in the previous section. In classical statistical theory the rate at which nuclei form per unit volume-time is given by

$$P(T(t)) = P_0 e^{-S(t)}, \tag{24}$$

where in a Coleman-type model of tunneling from the false to the true vacuum the action, $S(t)$, is treated in Euclidean space, and for a nucleus of critical radius in four dimensions is $S = 2\pi^2\sigma R_c^3 - \frac{\pi^2}{2}\Delta p R_c^4$, the extremum of which gives the relationship of Eq. (22). Most of the work that has been done during the past two decades has assumed homogeneous nucleation, for which the form

$$P(t) = P(t_f)e^{-\alpha(t_f - t)}, \tag{25}$$

with t_f the time at which the universe has reheated after the time t_c, at which time T is the critical temperature T_c for the phase transition (assumed first order). The parameters of the theory are determined from the energy-momentum tensor in a field theory or from the thermodynamic potential in a theory using classical statistical mechanics. In the following subsection we discuss the application of our QCD approach assuming homogeneous nucleation, and in the next subsection briefly consider inhomogeneous nucleation.

A. Homogeneous nucleation

From statistical mechanics one knows [4] that the probability for a fluctuation producing a bubble of radius R_c for $T < T_c$ satisfies

$$P(T) \propto e^{-\Delta F/T}, \tag{26}$$

with ΔF the bulk free energy difference between the two phases plus the contribution from the surface free energy,

$$\Delta F = \frac{4\pi}{3} R_c^3 (P_q - P_h) + 4\pi\sigma R_c^2, \tag{27}$$

with (P_q, P_h) the pressure in the (quark,hadronic) phases, and in our previous notation $\Delta p = P_q - P_h$. In the estimate of nucleation in the bag model [4,5] the quark pressure at $T < T_c$, with B the bag constant, is

$$P_q = \frac{37\pi^2}{90} T^4 - B. \tag{28}$$

The main difference between our present work and that of the earlier work is that we use the QCD Lagrangian directly and the vanishing of the quark condensate at $T = T_c$, rather than the bag model to obtain ΔF.

In the approach of [5] Csernai and Kapusta, who use the Langer model [21], and take $P_h = (3\pi^2/90)T^4$, find a singularity in $R_c(T)$ as $T = T_c$, and obtain a value of R_c in agreement with our value of $R_c \simeq 11$ fm for $(T_c - T)/T_c \simeq 1\%$. Noting that $(T_c - T_f)/T_c$ is of the order of 1%, one sees from Eq. (25) that our result for the critical radius R_c is not inconsistent with this work.

We now continue with the standard statistical mechanics approach [4]. To obtain the nucleation site number density one carries out the integral

$$N_n = \int_{t_c}^{\infty} p(T(t)) f(t) dt \simeq \int_{t_c}^{t_f} p(T(t)), \tag{29}$$

where $f(t)$ is the fraction of the universe which has not nucleated at time t, and Eq. (29) uses the result of Ref. [3] that $f(t)$ is a step function quickly disappearing at the time t_f shortly after t_c. In this picture one obtains for the site separation distance [4]

$$d_n \simeq 0.3 \frac{\sigma^{3/2} t_c}{T_c^{1/2} L}. \tag{30}$$

The latent heat density is obtained from the free energy difference between the phases by [4]

$$L = T_c \frac{\partial}{\partial T} \Delta p. \tag{31}$$

Since the T dependence of Δp is needed, one cannot use the result of Eq. (21), but must return to the thin-wall relationship between Δp and $W^\mu W_\mu$, Eq. (20). Using our observation that in the vicinity of the bubble walls the solution for $W^\mu W_\mu$ has the instantonlike form of Eq. (12), we find that

$$\Delta p = K \frac{\bar{s}_3^2}{(\bar{s}_3^2 + \rho^2)^2}, \tag{32}$$

with K a constant. In real time Euclidean space $\bar{s}_3^2 = (r - R)^2 + t^2$. Making the reduction from 4- Euclidean space to equilibrium, $t^2 \rightarrow -1/T^2$, and $\bar{s}_3^2 = (r - R)^2 - (1/T)^2$. Using this form and the fact that for $\bar{s}_3^2 = \rho^2$, $K = 4\rho^2 \Delta F = \frac{3\sigma}{R_c}$, we find that

$$L = \frac{4}{T_c^2 \rho^2} \frac{3\sigma}{R_c}. \tag{33}$$

Using the values $\rho = .2$ fm, the lattice gauge value for σ, and our result that $R_c = 11.0$ fm, we find from Eq. (30)

$$d_n = 5.23 \text{ m}. \tag{34}$$

As anticipated, this value is about an order of magnitude larger than previous bag model results.

B. Inhomogeneous nucleation

The idea that impurities can have an important effect on the probability of nucleation for cosmological phase transitions has been known for over two decades [30]. Recently this has been considered for the QCDPT [9,11]. As pointed out in [9], impurities in the universe, with number density n_{in} at a time t_i, $t_c < t_i < t_f$, can be represented by a $P(T(t))$ which differs from the homogeneous value of Eq. (25) by

$$P(t) = n_{in} \delta(t - t_i) + P(t_f) e^{-\alpha(t_f - t)}. \tag{35}$$

Using models for this form the nucleation site separation can be orders of magnitude larger than the value for homogeneous nucleation.

We would of course obtain similar model-dependent results, but since this does not follow directly from our QCD picture we do not consider it further here.

IV. CONCLUSIONS

In this work we have started from the energy-momentum tensor derived from the basic QCD Lagrangian with quark and gluonic color fields and assumed a first-order QCD phase transition when the temperature of the universe is $T_c \simeq 150$ MeV. From this T^{00} the surface tension of gluonic walls in an instantonlike SU(3) treatment of the color field is derived. The free energy difference between the two phases on the opposite sides of the bubble wall arises mainly from the vanishing of the quark chiral condensate in the quark/gluon phase, and we derive this ΔF using previous work on the nonperturbative vector quark three-point function. Recognizing that the wall is only 0.2 fm thick, the classical action is minimized in the thin-wall approximation to derive the critical radius for nucleation, R_c, for which the pressure difference driving nucleation overcomes the surface tension contracting the bubble. We find that $R_c \simeq 11$ fm, which is satisfactory for possible astrophysical observations following from the QCDPT nucleation and bubble collisions. Assuming standard homogeneous nucleation, using our results for the critical radius, and deriving the latent heat difference density from the instanton model of QCD and the quark condensate free energy difference, we find that the separation of nucleation sites is of the order of several meters, more than an order of magnitude larger than the standard QCD bag model that has been used by previous authors. As research by other investigators has shown, the possibility of inhomogeneous nucleation could lead to quite large distances between nucleation sites and a QCDPT with only a few active bubbles. In our next research on this topic we shall investigate possible observable magnetic structures that would arise from QCD nucleation processes.

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