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Spontaneous Lorentz violation, Nambu-Goldstone modes, and gravity

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The fate of the Nambu-Goldstone modes arising from spontaneous Lorentz violation is investigated. Using the vierbein formalism, it is shown that up to 10 Lorentz and diffeomorphism Nambu-Goldstone modes can appear and that they are contained within the 10 modes of the vierbein associated with gauge degrees of freedom in a Lorentz-invariant theory. A general treatment of spontaneous local Lorentz and diffeomorphism violation is given for various spacetimes, and the fate of the Nambu-Goldstone modes is shown to depend on both the spacetime geometry and the dynamics of the tensor field triggering the spontaneous Lorentz violation. The results are illustrated within the general class of bumblebee models involving vacuum values for a vector field. In Minkowski and Riemann spacetimes, the bumblebee model provides a dynamical theory generating a photon as a Nambu-Goldstone boson for spontaneous Lorentz violation. The Maxwell and Einstein-Maxwell actions are automatically recovered in axial gauge. Associated effects of potential experimental relevance include Lorentz-violating couplings in the matter and gravitational sectors of the Standard-Model Extension and unconventional Lorentz-invariant couplings. In Riemann-Cartan spacetime, the possibility also exists of a Higgs mechanism for the spin connection, leading to the absorption of the propagating Nambu-Goldstone modes into the torsion component of the gravitational field.

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I. INTRODUCTION

Violations of Lorentz symmetry arising from new physics at the Planck scale offer a potentially key signature for investigations of quantum-gravity phenomenology [1]. One elegant possibility is that Lorentz symmetry is spontaneously broken in an ultimate fundamental theory [2]. The basic idea is that interactions among tensor fields in the underlying theory trigger the formation of nonzero vacuum expectation values for Lorentz tensors. The presence of these background quantities throughout spacetime implies that Lorentz symmetry is spontaneously broken.

In general, spontaneous violation of a symmetry has well-established consequences. The existence of a nontrivial vacuum expectation value directly modifies the properties of fields that couple to it and can indirectly modify them through interactions with other affected fields. For example, the vacuum value $\langle\phi\rangle$ of the Higgs field spontaneously breaks the $SU(2)\times U(1)$ symmetry of the standard model (SM), introducing effective fermion masses via couplings of the fermion fields to $\langle\phi\rangle$. Moreover, when a continuous global symmetry is spontaneously broken, massless modes called Nambu-Goldstone (NG) modes appear [3]. If instead the local symmetry of a gauge theory is spontaneously broken, then the gauge bosons for the broken symmetry become massive modes via the Higgs mechanism [4].

For spontaneous Lorentz violation, the situation is similar. The existence of a nontrivial vacuum value for a tensor field affects the behavior of particles coupling to it, either directly or indirectly through other particles. These effects can be comprehensively characterized in an effective theory for the gravitational and SM fields observed in nature [5–7], via the introduction of coefficients for Lorentz violation carrying spacetime indices. This theory, called the Standard-Model Extension (SME), describes the phenomenological implications of spontaneous Lorentz violation independently of the structure of the underlying theory. To date, Planck-scale sensitivity has been attained to the dominant SME coefficients in many experiments, including ones with photons [8,9], electrons [10–12], protons and neutrons [13], mesons [14], muons [15], neutrinos [16,17], and the Higgs [18], but a substantial portion of the coefficient space remains to be explored.

As in the case of internal symmetries, the vacuum values triggering Lorentz violation are accompanied by NG modes associated with the generators of the broken-symmetry transformations. The fate of these NG modes is relevant to gravitational and SM phenomenology. If the massless NG modes are present as such and propagate over long distances, their phenomenology must be compatible with existing or hypothetical long-range forces. For example, it has been suggested that the NG modes in a vector theory with spontaneous Lorentz violation may be equivalent to electrodynamics in a nonlinear gauge [19,20]. If instead a mass were to develop for the graviton in analogy with the usual Higgs mechanism, other issues would arise. For example, even a small mass for the graviton can modify the predictions of general relativity and disagree with experiment [21]. In fact, in general relativity with spontaneous Lorentz breaking, it is known that a conventional Higgs mechanism cannot give rise to a mass for the graviton since the analogue of the usual Higgs mass term involves derivatives of the metric [22]. Further complications occur because the usual simple counting arguments for the number of NG modes can require modification in the presence of Lorentz violation [23].

In the context of gravity in a Riemann geometry, the investigation of spontaneous Lorentz violation was initiated with a study of a class of vector theories [22], called bumblebee models, that are comparatively simple field theories in which spontaneous Lorentz violation occurs. These models and some versions with ghost modes have since been investigated in a variety of contexts [5,24,25]. There has also been recent interest in the timelike diffeomorphism NG mode that arises when Lorentz symmetry is spontaneously broken by a timelike vector. If such a mode were to appear in a theory with second-order time derivatives, it has been shown that it would have an unusual dispersion relation leading to interesting anomalous spin-dependent forces [26].

In this paper, we investigate the ultimate fate of the NG modes associated with spontaneous violation of local Lorentz and diffeomorphism symmetries. We perform a generic analysis of theories formulated in Riemann-Cartan spacetime and its limits, including the Riemann spacetime of general relativity and the Minkowski spacetime of special relativity, and we illustrate the results within the bumblebee model. The standard vierbein formalism for gravity [27] offers a natural and convenient framework within which to study the properties of the NG modes, and we adopt it here. The basic gravitational fields can be taken as the vierbein ${e_\mu}^a$ and the spin connection ${\omega_\mu}^{ab}$. The associated field strengths are the curvature and torsion tensors. In a general theory of gravity in a Riemann-Cartan spacetime [28], these fields are independent dynamical quantities. The usual Riemann spacetime of general relativity is recovered in the zero-torsion limit, with the spin connection fixed in terms of the vierbein. Our focus here is on models in which one or more tensor fields acquire vacuum values, a situation that could potentially arise in the context of effective field theories for a variety of quantum-gravity frameworks in which mechanisms exist for Lorentz violation. These include, for example, string theory [2,29], noncommutative field theories [30], spacetime-varying fields [31–33], loop quantum gravity [34], random-dynamics models [35], multiverses [36], and brane-world scenarios [37], so the results obtained in the present work are expected to be widely applicable.

The organization of this paper is as follows. A generic discussion of spontaneous Lorentz violation in the vierbein formalism is presented in Sec. II. Section III discusses basic results for the bumblebee model. The three subsequent Secs. IV, V, and VI examine the fate of the NG modes in Minkowski, Riemann, and Riemann-Cartan spacetimes, respectively. Section VII contains a summary of the results. Throughout this work, we adopt the notation and conventions of Ref. [5].

II. SPONTANEOUS LORENTZ VIOLATION

For gravitational theories with a realistic matter sector, the vierbein formalism [27] is widely used because it permits a straightforward treatment of fermions in nontrivial spacetimes. Since this formalism distinguishes cleanly between local Lorentz frames and coordinate frames on the spacetime manifold, it is also ideally suited for investigations of Lorentz and *CPT* breaking [5], including the effects of spontaneous violation.

A. General considerations

A basic object in the formalism is the vierbein ${e_\mu}^a$, which can be viewed as providing at each point on the spacetime manifold a link between the covariant components $T_{\lambda\mu\nu\cdots}$ of a tensor field in a coordinate basis and the corresponding covariant components $T_{abc\cdots}$ of the tensor field in a local Lorentz frame. The link is given by

$$T_{\lambda\mu\nu\cdots} = {e_\lambda}^a {e_\mu}^b {e_\nu}^c \cdots T_{abc\cdots}. \tag{1}$$

In the coordinate basis, the components of the spacetime metric are denoted $g_{\mu\nu}$. In the local Lorentz frame, the metric components take the Minkowski form η_{ab}, but the basis may be anholonomic. Expressions for contravariant or mixed tensor components similar to Eq. (1) can be obtained by appropriate contractions with the components $g^{\mu\nu}$ of the inverse spacetime metric.

The vierbein formalism permits the treatment of both basic types of spacetime transformations relevant for gravitation theories: local Lorentz transformations and diffeomorphisms. Consider a point P on the spacetime manifold. Local Lorentz transformations at P act on the tensor components $T_{abc\cdots}$ via a transformation matrix ${\Lambda^a}_b$ applied to each index. For an infinitesimal transformation, this matrix has the form

$${\Lambda^a}_b \approx {\delta^a}_b + {\epsilon^a}_b, \tag{2}$$

where $\epsilon_{ab} = -\epsilon_{ba}$ are the infinitesimal parameters carrying the six Lorentz degrees of freedom and generating the local Lorentz group. In contrast, a diffeomorphism is a mapping of P to another point Q on the spacetime manifold, with an associated mapping of tensors at P to tensors at Q. The pullback of a transformed tensor at Q to P differs from the original tensor at P. For infinitesimal diffeomorphisms characterized in a coordinate basis by the transformation

$$x^\mu \rightarrow x^\mu + \xi^\mu, \tag{3}$$

this difference is given by the Lie derivative of the tensor $T_{\lambda\mu\nu\cdots}$ along the vector ξ^μ. The four infinitesimal parameters ξ^μ comprise the diffeomorphism degrees of freedom.

The vierbein formalism is natural for studies of Lorentz violation. Spontaneous violation of local Lorentz invariance occurs when the Lagrangian of the theory is invariant under local Lorentz transformations but the vacuum solution violates one or more of the symmetries. The key feature is the existence of a nonzero vacuum expectation value for the components $T_{abc\cdots}$ of a tensor field in a local

Lorentz frame [5]:

$$\langle T_{abc\cdots}\rangle \equiv t_{abc\cdots} \neq 0. \tag{4}$$

The values $t_{abc\cdots}$ may be constants or specified functions, provided they solve the equations of motion of the theory. Each such expectation value specifies one or more orientations within any local frame, which is the characteristic of spontaneous Lorentz violation.

The vacuum expectation value of the vierbein is also a constant or a fixed function, either given by the solution to the gravitational equations or specified as a background. For example, in a spacetime with Minkowski background the vacuum value of the vierbein is $\langle e_\mu{}^a\rangle = \delta_\mu{}^a$ in a suitable coordinate frame. It follows from Eq. (1) that the existence of a vacuum value $t_{abc\cdots}$ for a tensor in a local frame implies it additionally has a vacuum value $t_{\lambda\mu\nu\cdots}$ in the coordinate basis on the manifold. However, a nontrivial vacuum expectation value for $t_{\lambda\mu\nu\cdots}$ also implies spontaneous violation of diffeomorphism invariance. This shows that *the spontaneous violation of local Lorentz invariance implies spontaneous violation of diffeomorphism invariance.*

In fact, the converse is likewise true: *if diffeomorphism invariance is spontaneously broken, so is local Lorentz invariance.* It is immediate that any violation of diffeomorphism invariance via vacuum values of vectors or tensors breaks local Lorentz invariance, as above. An alternative source of diffeomorphism violations is possible via vacuum values of scalars provided the scalars are nonconstant over the spacetime manifold, but this also leads to violations of local Lorentz invariance because the derivatives of the scalar vacuum values provide an orientation within each local Lorentz frame.

B. Identification of NG modes

In discussing the consequences of spacetime-symmetry violations, it is useful to distinguish among several types of transformations. Treatments of Lorentz-invariant theories in the literature commonly define two classes of Lorentz transformations, called active and passive, which act on tensor components essentially as inverses of each other. In a Lorentz-violating theory, however, the presence of vacuum expectation values with distinct properties implies that there are more than two possible classes of transformations [7]. For most purposes it suffices to limit attention to two possibilities, called observer transformations and particle transformations.

Observer transformations involve changes of the observer frame. It is standard to assume that any physically meaningful theory is covariant under observer transformations, and this remains true in the presence of Lorentz violation [7]. An observer local Lorentz transformation can be viewed as a rotation or boost of the basis vectors in the local tangent space. Tensor components are then expressed in terms of the new basis. Observer coordinate transformations on the manifold are general coordinate transformations, which leave invariant the action. The statement of observer invariance therefore contains no physical information other than the assumption of observer independence of the physics.

Particle transformations are defined to act on individual particles or localized fields, while leaving unchanged vacuum expectation values. A particle Lorentz transformation involves a rotation or boost only of localized tensor fields. The components of the tensor are affected, while the basis and any vacuum values are unchanged. Similarly, particle diffeomorphisms with the pullback incorporated can be viewed as changes only in localized field distributions, with the tensor components transforming via the Lie derivative but the basis and all vacuum values unaffected.

Invariance of a system under particle transformations has physical consequences, including notably the existence of conservation laws. Local Lorentz invariance implies a condition on the antisymmetric components of the energy-momentum tensor $T^{\mu\nu}$, while diffeomorphism invariance implies a covariant conservation law for it. Thus, for example, in general relativity the laws are $T^{\mu\nu} = T^{\nu\mu}$ and $D_\mu T^{\mu\nu} = 0$. Spontaneous breaking of these spacetime symmetries leaves unaffected the conservation laws. In contrast, explicit breaking of these symmetries, which is described by noninvariant terms in the action, modifies the laws. For local Lorentz and diffeomorphism transformations, the conservation laws in the presence of spontaneous and explicit breaking are obtained in Ref. [5] in the context of a general gravitation theory.

In a theory with spontaneous breaking of a continuous symmetry, one or more NG modes are expected. The NG modes can be identified with the virtual excitations around the vacuum solution that are generated by the particle transformations corresponding to the broken symmetry. According to the above discussion, if the extremum of the action involves a nonzero vacuum expectation value $t_{abc\cdots}$ for a tensor in a local frame, both local Lorentz invariance and diffeomorphism invariance are spontaneously broken. Since these invariances involve 10 generators for particle transformations [38,39], we conclude that *up to ten NG modes can appear when an irreducible Lorentz tensor acquires a vacuum expectation value.*

In the subsequent parts of this work, it is shown that the vierbein formalism is particularly well suited for describing these NG modes. A simple counting of modes illustrates the key idea. The vierbein $e_\mu{}^a$ has 16 components. In a Lorentz- and diffeomorphism-invariant theory, 10 of these can be eliminated via gauge transformations, leaving 6 potentially physical degrees of freedom to describe the gravitational field. In general relativity, four of these six are auxiliary and do not propagate, leaving only the two usual transverse massless graviton modes; more general metric gravitational theories can have up to six graviton modes [40]. However, in a theory with local Lorentz and diffeomorphism violation, the ten additional vierbein modes cannot all be eliminated by gauge transformations and instead must be treated as dynamical fields in the theory. In short, *the ten potential NG modes from spontaneous local Lorentz and diffeomorphism breaking are contained within the ten components of the vierbein that are gauge degrees of freedom in the Lorentz-invariant limit.*

C. Perturbative analysis

In general, each NG mode can be obtained by performing on the vacuum a virtual particle transformation for a broken-symmetry generator and then elevating the corresponding spacetime-dependent parameter to the NG field. To identify the NG modes and study their basic properties, it therefore suffices to consider small excitations about the vacuum and to work in a linearized approximation.

If the vacuum solution of a given theory involves the metric $g^{\rm vac}_{\mu\nu}$, then the metric $g_{\mu\nu}$ in the presence of small excitations can be written as

$$g_{\mu\nu} = g^{\rm vac}_{\mu\nu} + h_{\mu\nu}. \tag{5}$$

In the general scenario, distinguishing the background from gravitational fluctuations requires some care. For instance, in the shortwave approximation [41] the distinction is made in terms of the amplitude of $h_{\mu\nu}$ and the scales on which $g^{\rm vac}_{\mu\nu}$ and $h_{\mu\nu}$ vary. For our purposes, however, the presence of a nontrivial background spacetime is unnecessary and serves to complicate the basic study of the properties of the NG modes. We therefore focus attention here on spacetimes in which the vacuum geometry is Minkowski.

Small metric fluctuations about the Minkowski background can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{6}$$

To linear order, the inverse metric is then $g^{\mu\nu} \approx \eta^{\mu\nu} - \eta^{\mu\alpha}\eta^{\nu\beta}h_{\alpha\beta}$. In this context, the distinction between coordinate indices μ, ν, ... on the manifold and the local Lorentz indices a, b, ... is diminished, and Greek letters can be used for both. The 16-component vierbein can be written as

$$e_{\mu\nu} = \eta_{\mu\nu} + \left(\frac{1}{2}h_{\mu\nu} + \chi_{\mu\nu}\right), \tag{7}$$

where the ten symmetric excitations $h_{\mu\nu} = h_{\nu\mu}$ are associated with the metric, while the six antisymmetric components $\chi_{\mu\nu} = -\chi_{\nu\mu}$ are the local Lorentz degrees of freedom.

The vacuum expectation value (4) of an arbitrary tensor becomes

$$\langle T_{\mu\nu\cdots}\rangle \equiv t_{\mu\nu\cdots}, \tag{8}$$

and excitations about this vacuum value are denoted as

$$\delta T_{\lambda\mu\nu\cdots} = (T_{\lambda\mu\nu\cdots} - t_{\lambda\mu\nu\cdots}). \tag{9}$$

There can be many more such excitations than NG modes. The NG modes are distinguished by the requirement that $\delta T_{\lambda\mu\nu\cdots}$ maintains the extremum of the action and corresponds to broken-symmetry generators.

For modes $\delta T_{\lambda\mu\nu\cdots}$ excited via local Lorentz transformations or diffeomorphisms, the magnitude of $T_{\lambda\mu\nu\cdots}$ at each point is preserved,

$$T^{\lambda\mu\nu\cdots}g_{\lambda\alpha}g_{\mu\beta}g_{\nu\gamma}\ldots T^{\alpha\beta\gamma\cdots} = t^2, \tag{10}$$

where $t^2 = t^{\lambda\mu\nu\cdots}\eta_{\lambda\alpha}\eta_{\mu\beta}\eta_{\nu\gamma}\ldots t^{\alpha\beta\gamma\cdots}$. This holds, for example, in a theory with potential V having the simple functional form

$$V = V(T^{\lambda\mu\nu\cdots}g_{\lambda\alpha}g_{\mu\beta}g_{\nu\gamma}\ldots T^{\alpha\beta\gamma\cdots} - t^2), \tag{11}$$

which can trigger a vacuum value $t_{\lambda\mu\nu\cdots}$ when V is extremized. For instance, V could be a positive quartic polynomial in $T^{\lambda\mu\nu\cdots}$ with minima at zero, such as $V(x) = \lambda x^2/2$, where λ is a coupling constant. The condition (10) is automatically satisfied by the choice

$$T_{\lambda\mu\nu\cdots} = e_\lambda{}^\alpha e_\mu{}^\beta e_\nu{}^\gamma \ldots t_{\alpha\beta\gamma\cdots}, \tag{12}$$

which also reduces to the correct vacuum expectation value (8) when the vierbein excitations vanish. This implies all the excitations in $\delta T_{\lambda\mu\nu\cdots}$ associated with the NG modes are contained in the vierbein through Eq. (12).

Using the expansion (7) of the vierbein in Eq. (12) yields a first-order expression for the tensor excitations $\delta T_{\lambda\mu\nu\cdots}$ in terms of the 16 fields $h_{\mu\nu}$ and $\chi_{\mu\nu}$:

$$\delta T_{\lambda\mu\nu\cdots} \approx \left(\frac{1}{2}h_{\lambda\alpha} + \chi_{\lambda\alpha}\right)t^\alpha{}_{\mu\nu\cdots} + \left(\frac{1}{2}h_{\mu\alpha} + \chi_{\mu\alpha}\right)t_\lambda{}^\alpha{}_{\nu\cdots} + \ldots. \tag{13}$$

Evidently, the combination $(\frac{1}{2}h_{\mu\nu} + \chi_{\mu\nu})$ contains the interesting dynamical degrees of freedom.

We can observe the effects of local Lorentz and diffeomorphism transformations by performing each separately. Under infinitesimal Lorentz transformations, the vierbein components transform as

$$\begin{aligned} h_{\mu\nu} &\rightarrow h_{\mu\nu}, \\ \chi_{\mu\nu} &\rightarrow \chi_{\mu\nu} - \epsilon_{\mu\nu}, \end{aligned} \tag{14}$$

while their transformations under infinitesimal diffeomorphisms are

$$\begin{aligned} h_{\mu\nu} &\rightarrow h_{\mu\nu} - \partial_\mu\xi_\nu - \partial_\nu\xi_\mu, \\ \chi_{\mu\nu} &\rightarrow \chi_{\mu\nu} - \frac{1}{2}(\partial_\mu\xi_\nu - \partial_\nu\xi_\mu). \end{aligned} \tag{15}$$

In these expressions, quantities of order (ϵh), $(\epsilon\chi)$, (ξh), $(\xi\chi)$, etc. are assumed small and hence negligible in the linearized treatment.

The excitation due to infinitesimal Lorentz transformations is

$$\delta T_{\lambda\mu\nu\cdots} \approx -\epsilon_{\lambda\alpha} t^{\alpha}{}_{\mu\nu\cdots} - \epsilon_{\mu\alpha} t_{\lambda}{}^{\alpha}{}_{\nu\cdots} - \ldots . \tag{16}$$

Depending on the properties of the vacuum value $t_{\mu\nu\cdots}$, up to six independent excitations associated with broken Lorentz generators can appear in this expression. Their nature is determined by the six parameters $\epsilon_{\mu\nu}$. It follows that the corresponding NG modes $\mathcal{E}_{\mu\nu}$ for the broken Lorentz symmetries stem from the antisymmetric components $\chi_{\mu\nu}$ of the vierbein.

The excitation due to infinitesimal diffeomorphisms is

$$\delta T_{\lambda\mu\nu\cdots} \approx -(\partial_\lambda \xi_\alpha) t^{\alpha}{}_{\mu\nu\cdots} - (\partial_\mu \xi_\alpha) t_{\lambda}{}^{\alpha}{}_{\nu\cdots} - \ldots - \xi^\alpha \partial_\alpha t_{\lambda\mu\nu\cdots}. \tag{17}$$

This can contain up to four independent excitations associated with broken diffeomorphisms, depending on the properties of $t_{\mu\nu\cdots}$. Except for the case of a scalar T, the four potential NG modes Ξ_μ corresponding to the four parameters ξ_μ enter the vierbein accompanied by derivatives. This can potentially alter their dispersion relations and couplings to matter currents.

As a simple example, consider spontaneous breaking due to a nonzero vector vacuum value t_μ, which could be timelike, spacelike, or lightlike. Introduce a quantity $\mathcal{E}^\mu = \mathcal{E}^{\mu\nu} t_\nu$ obeying $t_\mu \mathcal{E}^\nu = 0$. The 3 degrees of freedom of $\mathcal{E}^\mu$ correspond to the three Lorentz NG modes associated with the three Lorentz generators broken by the direction t_ν. Similarly, one can introduce a scalar $\Xi^\mu t_\mu$, which plays the role of the NG mode corresponding to the diffeomorphism broken by t_μ. In this example, which is studied in more detail in the next section, there are four potential NG modes. If a second orthogonal vacuum value t'_μ is also present, an additional two Lorentz NG modes appear because two additional Lorentz generators are broken. All six Lorentz NG modes $\mathcal{E}_{\mu\nu}$ enter the theory once a third orthogonal vacuum value exists. Similarly, as additional vacuum values are added, more components of the fields Ξ_μ enter as NG modes for diffeomorphisms, until all four are part of the broken theory.

Examples with more complicated tensor representations also provide insight. For instance, consider a theory with an expectation value for a two-index symmetric tensor $T^{\mu\nu} = T^{\nu\mu}$. In this case, the choice of vacuum value $t_{\mu\nu}$ can crucially affect the number and type of NG modes. There is a choice among many possible scenarios. A subset of the space of possible vacuum values consists of those $t_{\mu\nu}$ that can be made diagonal by a suitable choice of coordinate basis, but even if attention is restricted to this subset there are many possibilities. For instance, a vacuum value with diagonal elements (3,1,1,1) breaks three boosts and four diffeomorphisms for a total of seven NG modes, one with diagonal elements (4,1,1,2) breaks five Lorentz transformations and four diffeomorphisms for a total of nine NG modes, while one with diagonal elements (6,1,2,3) breaks all ten symmetries for a total of ten NG modes.

In the general case, up to ten NG modes can appear when a tensor acquires a vacuum expectation value $t_{\mu\nu\cdots}$. The fluctuations of the tensor about the vacuum under virtual particle transformations are given as the sum of the right-hand sides of Eqs. (16) and (17). The associated NG modes consist of up to six Lorentz modes $\mathcal{E}_{\mu\nu}$ and up to four diffeomorphism modes Ξ_μ.

The ultimate fate of the NG modes, and, in particular, whether some or all of them propagate as physical massless fields, depends on the specific dynamics of the theory. At the level of the Lagrangian, the linearized approximation involves expanding all fields around their vacuum values and keeping terms of quadratic order or less. The dominant terms in the effective Lagrangian for the NG modes can then be obtained by replacing the tensor excitation with the appropriate NG modes $\mathcal{E}_{\mu\nu}$ and Ξ_μ according to Eqs. (16) and (17). The resulting dynamics of the modes are determined by several factors, including the basic form of the terms in the original action and the type of spacetime geometry in the theory. Disentangling these issues is the subject of the following sections.

III. BUMBLEBEE MODELS

To study the behavior of the NG modes and distinguish dynamical effects from geometrical ones, it is valuable to consider a class of comparatively simple models for Lorentz and diffeomorphism violation, called bumblebee models, in which a vector field B^μ acquires a constant expectation value b^μ [5,22]. These models contain many of the interesting features of cases with more complicated tensor vacuum values. For example, all the basic types of rotation, boost, and diffeomorphism violations can be implemented, and the existence and properties of the corresponding NG modes can be studied for various spacetime geometries. In this section, some general results for these models are presented.

A. Projectors for NG modes

The characteristic feature of bumblebee models is that a vector field B^μ acquires a vacuum expectation value b^a in a local Lorentz frame. This breaks three Lorentz transformations and one diffeomorphism, so there are four potential NG modes. According to Eq. (12) and the associated discussion, the vector field B^μ can be written in terms of the vierbein as

$$B^\mu = e^\mu{}_a b^a, \tag{18}$$

which holds in any background metric. The vierbein degrees of freedom include the NG modes of interest.

As before, we proceed under the simplifying assumption that the background spacetime geometry is Minkowski. The vacuum solution then takes the form

$$\langle B^\mu \rangle = b^\mu, \qquad \langle e_{\mu\nu} \rangle = \eta_{\mu\nu} \tag{19}$$

in a suitable coordinate frame. The vierbein can be expanded in terms of $h_{\mu\nu}$ and $\chi_{\mu\nu}$, as in Eq. (7). The fluctuations about the vacuum can therefore be written as

$$\delta B^\mu = (B^\mu - b^\mu) \approx \left(-\frac{1}{2}h^{\mu\nu} + \chi^{\mu\nu}\right) b_\nu. \tag{20}$$

The results of the previous subsection imply that three of the four potential NG modes are contained in fields $\mathcal{E}^\mu$ obeying $b_\mu \mathcal{E}^\mu = 0$, while one appears in a combination $\Xi^\mu b_\mu$. To identify these modes, it is convenient to separate the excitations (20) into longitudinal and transverse components using projection operators. Focusing for definiteness on the nonlightlike case ($b^2 \neq 0$), we define the projectors

$$(P_\parallel)^\mu{}_\nu = \frac{b^\mu b_\nu}{b^\sigma b_\sigma}, \qquad (P_\perp)^\mu{}_\nu = \delta^\mu{}_\nu - (P_\parallel)^\mu{}_\nu. \tag{21}$$

The transverse and longitudinal projections of the fluctuations δB^μ can then be identified as

$$\mathcal{E}^\mu = (P_\perp)^\mu{}_\nu \delta B^\nu \approx \left(-\frac{1}{2}h^{\mu\nu} + \chi^{\mu\nu}\right) b_\nu - b^\mu \rho, \tag{22}$$

and

$$\rho^\mu = (P_\parallel)^\mu{}_\nu \delta B^\nu \approx b^\mu \rho, \tag{23}$$

respectively, where we have introduced the quantity

$$\rho = -\frac{b^\mu h_{\mu\nu} b^\nu}{2 b^\sigma b_\sigma}. \tag{24}$$

In terms of these fluctuation projections, the field B^μ is

$$B^\mu \approx (1 + \rho) b^\mu + \mathcal{E}^\mu. \tag{25}$$

The reader is warned that at the same level of approximation the covariant components B_μ are given by

$$B_\mu \equiv g_{\mu\nu} B^\nu \approx (1 + \rho) b_\mu + \mathcal{E}_\mu + h_{\mu\nu} b^\nu. \tag{26}$$

One effect of these projections is to disentangle in B^μ the NG modes associated with Lorentz and diffeomorphism breaking. To see this, start in the vacuum and perform a virtual local particle Lorentz transformation with parameters $\epsilon^{\mu\nu}$ satisfying $\epsilon^{\mu\nu} b_\nu \neq 0$. This leaves unchanged the metric $\eta_{\mu\nu}$ and the projection ρ. However, nonzero transverse excitations $\epsilon^{\mu\nu} b_\nu$ are generated. When $-\epsilon^{\mu\nu}$ is promoted to a field $\mathcal{E}^{\mu\nu}$, these become the Lorentz NG modes $\mathcal{E}^\mu \equiv \mathcal{E}^{\mu\nu} b_\nu$. Note that they automatically obey an axial-gauge condition, $b_\mu \mathcal{E}^\mu = 0$, the significance of which is elaborated in subsequent sections.

Similarly, the excitations ρ^μ about the vacuum contain the diffeomorphism NG mode. To verify this, note that the components ξ^μ of the broken diffeomorphism obey

$$\xi^\mu = (P_\parallel)^\mu{}_\nu \xi^\nu \approx \frac{b^\mu \xi^\nu b_\nu}{b^\sigma b_\sigma}, \tag{27}$$

with $\xi^\mu b^\nu = \xi^\nu b^\mu$. Then, a virtual diffeomorphism generates a nonzero value for ρ, but the field $\mathcal{E}^\mu$ is unaffected. Promoting ξ^μ to the NG field Ξ^μ, the expression for ρ becomes

$$\rho = \frac{b^\mu \partial_\mu \Xi_\nu b^\nu}{b^\sigma b_\sigma} = \partial_\mu \Xi^\mu. \tag{28}$$

We see that the longitudinal excitation of B^μ can indeed be identified with the diffeomorphism NG mode, as claimed above. Note that an associated fluctuation in the metric, given by

$$\eta_{\mu\nu} \to g_{\mu\nu} \approx \eta_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu, \tag{29}$$

is also generated by the diffeomorphism.

B. Bumblebee dynamics

The dynamical behavior of B^μ and the associated NG modes is determined by the structure of the action that defines the specific model under study. In general, the Lagrangian $\mathcal{L}_B$ for a single bumblebee field B^μ coupled to gravity and matter can be written as a sum of terms

$$\mathcal{L}_B = \mathcal{L}_g + \mathcal{L}_{gB} + \mathcal{L}_K + \mathcal{L}_V + \mathcal{L}_J. \tag{30}$$

Here, $\mathcal{L}_g$ is the gravitational Lagrangian, $\mathcal{L}_{gB}$ describes the gravity-bumblebee coupling, $\mathcal{L}_K$ contains the kinetic terms for B^μ, $\mathcal{L}_V$ contains the potential, including terms triggering the spontaneous Lorentz violation, and $\mathcal{L}_J$ determines the coupling of B^μ to the matter or other sectors in the model.

Various forms for each of these partial Lagrangians are possible, and for certain purposes some can be set to zero. As an explicit example containing all types of terms, consider the Lagrangian

$$\mathcal{L}_B = \frac{1}{2\kappa}(eR + \xi e B^\mu B^\nu R_{\mu\nu}) - \frac{1}{4} e B_{\mu\nu} B^{\mu\nu} - eV(B_\mu B^\mu \pm b^2) - e B_\mu J^\mu, \tag{31}$$

where $\kappa = 8\pi G$ and $e \equiv \sqrt{-g}$ is the determinant of the vierbein. The Lorentz-invariant limit of this theory has been studied previously in the context of alternative theories of gravity in Riemann spacetime [42]. The simplified Lorentz-violating limit of the theory with $\xi = 0$ was introduced in Ref. [2], while the theory (31) and some versions including ghost vectors have been explored more recently in Refs. [5,24,25].

In the model (31) and its subsets, which are among those used in the sections below, the gravitational Lagrangian $\mathcal{L}_g$ is that of general relativity. The specific nonminimal gravity-bumblebee interaction $\mathcal{L}_{gB}$ in this example is controlled by the coupling constant ξ. The last term in Eq. (31)

represents a matter-bumblebee interaction $\mathcal{L}_J$, involving the matter current J^μ. The theory (31) is written for a Riemann or Riemann-Cartan spacetime, but the limit of Minkowski spacetime is also of interest below, and the corresponding Lagrangian can be obtained by eliminating the first two terms and setting $e = 1$.

The partial Lagrangian $\mathcal{L}_V$ in the model (31) involves a potential V of the general form in Eq. (11), inducing the spontaneous Lorentz and diffeomorphism violation. The quantity b^2 is a real positive constant, related to the vacuum value b^a of the bumblebee field by $b^2 = |b^a \eta_{ab} b^b|$, while the $\mp$ sign in V determines whether b^a is timelike or spacelike. As before, V can be a polynomial such as $V(x) = \lambda x^2/2$, where λ is a coupling constant. An alternative explicit form for V of particular value for studies of NG modes is the sigma-model potential

$$V(B_\mu B^\mu \pm b^2) = \lambda(B_\mu B^\mu \pm b^2), \tag{32}$$

where the quantity λ is now a Lagrange-multiplier field. The Lagrange multiplier acts to constrain the theory to the extrema of V obeying $B_\mu B^\mu \pm b^2 = 0$, thereby eliminating fields other than the NG modes. This model is a limiting case of the previous polynomial one in which the massive mode is frozen. Note that in any case the potential V excludes the possibility of a U(1) gauge invariance involving B^μ, whatever the form of the bumblebee kinetic term in the action.

The kinetic partial Lagrangian $\mathcal{L}_K$ in the model of Eq. (31) involves a field strength $B_{\mu\nu}$ for B_μ, defined as

$$B_{\mu\nu} = D_\mu B_\nu - D_\nu B_\mu, \tag{33}$$

where D_μ are covariant derivatives appropriate for the chosen spacetime geometry. In a Riemann or Minkowski spacetime, where the torsion vanishes, this field strength reduces to $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. In either case, with B^μ given by Eq. (25), it follows that the diffeomorphism mode contained in ρ cancels in the kinetic partial Lagrangian $\mathcal{L}_K$ in Eq. (31), and $\mathcal{L}_K$ propagates two modes. If an alternative kinetic partial Lagrangian $\mathcal{L}_K$ is adopted instead an additional mode can propagate. A simple example is given by the choice [43]

$$\mathcal{L}_{K,\mathrm{ghost}} = \frac{1}{2} e B_\mu D^\nu D_\nu B^\mu, \tag{34}$$

which propagates three modes, although this kinetic partial Lagrangian can be unphysical as such because it can contain ghost dynamics.

The above discussion demonstrates that the fate of the Lorentz and diffeomorphism NG modes depends on the geometry of the spacetime and on the dynamics of the theory. To gain further insight into the nature and fate of the NG modes, we consider in turn the three cases of Minkowski, Riemann, and Riemann-Cartan spacetimes, each in a separate section below.

IV. MINKOWSKI SPACETIME

A. Role of the vierbein

In Minkowski spacetime, the curvature and torsion vanish by definition, and global coordinate systems exist such that

$$g_{\mu\nu} = \eta_{\mu\nu}. \tag{35}$$

Particle Lorentz transformations can be performed using

$$\Lambda_\mu{}^\nu \approx \delta_\mu{}^\nu + \epsilon_\mu{}^\nu \tag{36}$$

to rotate and boost tensor components. These transformations are global if $\epsilon_\mu{}^\nu$ is independent of the spacetime point; otherwise, they are local. In any case, they maintain the metric in the form $\eta_{\mu\nu}$ at each point. It is instructive to compare the expression (36) with the form of the vierbein when $h_{\mu\nu} = 0$:

$$e_\mu{}^\nu \approx \delta_\mu{}^\nu + \chi_\mu{}^\nu. \tag{37}$$

It follows that a vierbein with $h_{\mu\nu} = 0$ in Minkowski spacetime can be identified with a local particle Lorentz transformation. Also, starting from a vacuum solution with $e_\mu{}^\nu = \delta_\mu{}^\nu$, a local Lorentz transformation (36) generates $\chi_\mu{}^\nu = \epsilon_\mu{}^\nu$.

In Minkowski spacetime, the diffeomorphisms maintaining $h_{\mu\nu} = 0$ are global spacetime translations, corresponding to constant ξ^μ in Cartesian coordinates. Under local diffeomorphisms with nonconstant ξ^μ, the metric transforms as [44]

$$\eta_{\mu\nu} \rightarrow g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{38}$$

where $h_{\mu\nu} = -\partial_\mu \xi_\nu - \partial_\nu \xi_\mu$. A corresponding term is generated in the vierbein, given by Eq. (15).

To study the NG modes in Minkowski spacetime, consider first a theory with tensor field $T_{\lambda\mu\nu\cdots}$ satisfying the condition (10). As before, the NG modes can be identified with small excitations $\delta T_{\lambda\mu\nu\cdots}$ maintaining this constraint. The constraint is invariant under global Lorentz transformations and translations, as is evident by writing Eq. (10) in a coordinate frame in which Eq. (35) holds. More importantly, the constraint also is invariant under local Lorentz transformations and diffeomorphisms. It follows that even in Minkowski spacetime there are up to ten NG modes when spontaneous Lorentz violation occurs.

As a more explicit example, consider a bumblebee model with the vacuum solution of the form (19). Local Lorentz transformations and local diffeomorphisms change the components B^μ and the metric $g_{\mu\nu}$ while maintaining the equality $B^\mu g_{\mu\nu} B^\nu = \mp b^2$, but some of these transformations are spontaneously broken by the vacuum values. As before, there are potentially four NG modes that can appear, consisting of three Lorentz NG modes $\mathcal{E}^\mu \equiv \mathcal{E}^{\mu\nu} b_\nu$ obeying $b_\mu \mathcal{E}^\mu = 0$, and one diffeomorphism NG mode contained in ρ and given by Eq. (28).

In a bumblebee model, the vacuum solution can break global Lorentz transformations while preserving translations, so that energy-momentum is conserved. This type of assumption is sometimes adopted within the broader context of the SME as a useful simplification for experimental studies. Translation symmetry holds if b^μ is a constant in a coordinate frame in which the metric takes the form (35). However, the vacuum solution b^μ in this frame in principle also could be a smoothly varying function of spacetime obeying $b^\mu \eta_{\mu\nu} b^\nu = \mp b^2$, such as a soliton solution. Then, b^μ has constant magnitude but different orientation at different spacetime points, and both global Lorentz and translation symmetries are broken. In this case, a vierbein can be introduced at each point that transforms b_μ into a local field b_a obeying $b_a b^a = \mp b^2$ at each point but having components that are constant over the spacetime. The role of the vierbein in this context can be regarded as a link to a convenient anholonomic basis in which b_a appears constant. Whatever the fate of the global transformations, however, local Lorentz transformations and diffeomorphisms are broken by the vacuum solution (19), and the behavior of the four potential NG modes contained in $\mathcal{E}^\mu$ and ρ is determined by the dynamics of the theory.

B. Fate of the NG modes

Since gravitational excitations are absent in Minkowski spacetime, no kinetic terms for $h_{\mu\nu}$ can appear and there is no associated dynamics. Any propagation of NG modes must therefore originate from Lagrangian terms involving $T^{\lambda\mu\nu\cdots}$. Diffeomorphisms produce infinitesimal excitations of the vacuum solution given by (17), which generate NG modes in the combination $\partial_\mu \Xi_\alpha$. It might therefore seem that even nonderivative terms for $T^{\lambda\mu\nu\cdots}$ in the Lagrangian could generate derivative terms for some NG modes and hence possibly lead to their propagation. However, when a potential V drives the breaking, any nonderivative term in $T^{\lambda\mu\nu\cdots}$ is intrinsically part of V, so its presence may affect the specific form of the vacuum solution (8) but cannot contribute to the propagator for the NG modes. Indeed, no contributions from V arise in the effective action for the NG modes because this action is obtained via virtual particle transformations leaving V invariant and at its extremum. This result can equivalently be obtained using the vierbein decomposition (13) of $T^{\lambda\mu\nu\cdots}$, since this expansion automatically extremizes V and also contains the NG modes as shown before. It follows in this case that any propagation of NG modes must be determined by kinetic or derivative-coupling terms for $T^{\lambda\mu\nu\cdots}$.

Next, we illustrate some of the possibilities for generating propagators for the NG modes using kinetic terms in a bumblebee model. Consider first a special Minkowski-spacetime limit of the theory (31), for which the Lagrangian is

$$\mathcal{L}_B = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \lambda(B_\mu B^\mu \pm b^2) - B_\mu J^\mu. \quad (39)$$

The ghost-free kinetic term chosen involves the zero-torsion limit of the field strength $B_{\mu\nu}$ in Eq. (33), and the adoption of the sigma-model potential (32) ensures a focus on the NG modes.

For this theory, a coordinate frame can be chosen in which the vacuum solution is

$$\langle B^\mu \rangle = b^\mu, \qquad \langle e_{\mu\nu} \rangle = \eta_{\mu\nu}, \qquad \langle \lambda \rangle = 0. \quad (40)$$

For simplicity, in what follows we take b^μ to be constant in this frame. The relevant virtual fluctuations of the bumblebee field around the vacuum solution, generated by the broken particle Lorentz transformations and diffeomorphisms, can be decomposed using the projector method. The result is Eq. (25), where $\mathcal{E}_\mu$ and ρ contain the NG modes. As before, the Lorentz NG modes satisfy an axial-gauge condition $b_\mu \mathcal{E}^\mu = 0$ and so represent 3 degrees of freedom.

With the vacuum (40) and the choice of kinetic term in Eq. (39), the diffeomorphism mode contained in ρ cancels in $B_{\mu\nu}$. It therefore cannot propagate and is an auxiliary mode. In fact, the kinetic term in Eq. (39) reduces to the form of a U(1) gauge theory in an axial gauge, so one of the three Lorentz modes is auxiliary too. Adopting the suggestive notation $\mathcal{E}_\mu \equiv A_\mu$ and denoting the corresponding field strength by $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, we find that the Lagrangian (39) reduces at leading order to

$$\mathcal{L}_B \to \mathcal{L}_{\rm NG} \approx -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\mu J^\mu - b_\mu J^\mu + b^\mu \partial_\nu \Xi_\mu J^\nu. \quad (41)$$

This is the effective quadratic Lagrangian determining the propagators of the NG modes in the theory (39). Note that the axial-gauge condition $b_\mu A^\mu = 0$ includes the special cases of temporal gauge ($A^0 = 0$) and pure axial gauge ($A^3 = 0$), and it ensures the constraint term is absent in $\mathcal{L}_{\rm NG}$. Note also that varying with respect to Ξ_μ yields the current-conservation law, $\partial_\mu J^\mu = 0$.

We see that the NG modes for the Minkowski-spacetime bumblebee theory (39) have the basic properties of the massless photon, described as a U(1) gauge theory in an axial gauge. This result is consistent with an early analysis by Nambu [20], who investigated the constraint $B_\mu B^\mu = \mp b^2$ as a nonlinear gauge choice that spontaneously breaks Lorentz invariance. In the linearized limit with $B^\mu \approx b^\mu + \mathcal{E}^\mu$, this gauge choice reduces to an axial-gauge condition $b_\mu \mathcal{E}^\mu = 0$ at leading order. The discussion here involves a Lagrange-multiplier constraint rather than a direct gauge choice and so the theories differ, but the result remains unaffected.

The masslessness of the photon in the effective theory (41) is a consequence of the spontaneously broken Lorentz symmetry in the original theory (39). An interesting question is whether this idea has experimentally verifiable consequences. Indeed, a version of the theory (39) with an explicit matter sector has been presented in Ref. [24] as a model for quantum electrodynamics (QED) that generates a Lorentz-violating term in the SME limit. The latter appears in Eq. (41) as the Lorentz-violating term $b_\mu J^\mu$, along with a conventional charged-current interaction $A_\mu J^\mu$.

If the current J^μ represents the usual electron current in QED, then the term b_μ can be identified with the coefficient a^e_μ for Lorentz violation in the QED limit of the SME [7]. If this coefficient is spacetime independent, it is known to be unobservable in experiments restricted to the electron sector [45], but coefficients of this type can generate signals in the quark [14,46] and neutrino [7,17] sectors. Moreover, various other possible sources of experimental signals can be considered, such as spacetime dependence of the coefficients, the presence of multiple fields and other types of current, and interference between several sources of Lorentz violation. Nonminimal couplings to other sectors, including the gravitational couplings discussed in the next section, also can produce experimental signals. All these effects are contained within the SME. More radical options for interpretation of the NG modes from Lorentz and diffeomorphism breaking in a general theory likewise can be envisaged, ranging from new long-range forces weakly coupled to matter with possible implications for dark matter and dark energy to the identification of many or all massless modes in nature with the NG modes. A careful investigation of these possibilities lies outside our present scope but would be of definite interest.

Another interesting issue is whether a consistent theory exists in Minkowski spacetime in which the diffeomorphism mode contained in ρ propagates. Consider, for example, substituting an alternative kinetic term of the form (34) in the Lagrangian (39), yielding the model

$$\mathcal{L}_{B,\text{ghost}} = \frac{1}{2} B_\mu \partial^\nu \partial_\nu B^\mu - \lambda (B_\mu B^\mu \pm b^2) - B_\mu J^\mu \qquad (42)$$

in Cartesian coordinates. This model may have a ghost, but the behavior of the NG modes can nonetheless be examined. Proceeding via the projector method as before, we find the kinetic term becomes $\frac{1}{2}\mathcal{E}^\mu \partial^\nu \partial_\nu \mathcal{E}_\mu$, so the diffeomorphism NG mode contained in ρ is auxiliary while the three $\mathcal{E}_\mu$ modes propagate. More generally, in the covariant derivative $D_\mu B_\nu$ relevant for a general coordinate system in Minkowski spacetime, the NG excitations reduce to $\partial_\mu \mathcal{E}_\nu$ when b_ν is constant in Cartesian coordinates, so kinetic terms contain no propagation of the diffeomorphism mode Ξ_μ in this case.

This example illustrates a general difficulty in forming a covariant kinetic term that permits propagation of the diffeomorphism modes for the case of constant vacuum value $t_{\lambda\mu\nu\cdots}$. To be observer independent, the Minkowski-spacetime Lagrangian must be formed from contractions of a tensor $T^{\lambda\mu\nu\cdots}$ and its derivatives. Only terms with one or more derivatives can contribute to the propagation, as shown above. However, for covariant derivatives of $T^{\lambda\mu\nu\cdots}$ with constant $t_{\lambda\mu\nu\cdots}$, the diffeomorphism modes always enter combined with a derivative, $\partial_\mu \Xi_\alpha$, while the connection acquires a corresponding change induced by the metric fluctuation (29) that cancels them. Other possibilities would therefore need to be countenanced, such as a nonconstant $t_{\lambda\mu\nu\cdots}$. In any case, the structure of terms containing derivatives of Ξ_μ in the effective Lagrangian for the NG modes represents a major difference between spontaneous breaking of internal and spacetime symmetries. In the former, the relevant fields carry internal indices that are independent of spacetime derivatives, and so theories with compact internal symmetry groups can be constructed that propagate all the NG modes without generating ghosts. In contrast, the spontaneous violation of spacetime symmetries involves fields with spacetime indices, and for the diffeomorphism NG modes this changes the derivative structure in the effective Lagrangian.

V. RIEMANN SPACETIME

In this section, we revisit for Riemann spacetimes the results obtained in the Minkowski-spacetime case. The general features obtained in the previous section apply to a nondynamical Riemann spacetime with fixed background metric. The primary interest here therefore lies instead with Riemann spacetimes having a dynamical metric.

A. Vierbein and spin connection

In a Riemann spacetime with dynamical metric $g_{\mu\nu}$, the nature and properties of the NG modes for spontaneous Lorentz and diffeomorphism violation still can be analyzed following the general approach of Secs. II and III. We assume that the solutions to a theory for a tensor $T_{\lambda\mu\nu\cdots}$ satisfy the condition (10), thereby inducing a nonzero vacuum value $t_{\lambda\mu\nu\cdots}$. This condition is automatically satisfied by writing $T_{\lambda\mu\nu\cdots}$ in terms of the vierbein,

$$T_{\lambda\mu\nu\cdots} = e_\lambda{}^a e_\mu{}^b e_\nu{}^c \ldots t_{abc\cdots}, \qquad (43)$$

where $t_{abc\cdots}$ is the vacuum value of the tensor in a local Lorentz frame. For definiteness in what follows, we suppose that $t_{abc\cdots}$ is constant over the spacetime manifold in the region of interest. This assumes appropriate smoothness of $t_{\lambda\mu\nu\cdots}$ and compatibility with any boundary conditions. For example, if the Riemann spacetime is asymptotically Minkowski and the vacuum value of the vierbein $e_{\mu\nu}$ is taken as $\eta_{\mu\nu}$, then the components of $t_{\lambda\mu\nu\cdots}$ must be asymptotically constant.

A primary difference in Riemann spacetime is that up to six of the 16 independent components of the vierbein $e_\mu{}^a$ can represent dynamical degrees of freedom of the gravitational field. The Lagrangian for the theory must therefore contain dynamical terms for the vierbein. This raises the issue of the effect of these additional terms on the other 10 components of the vierbein, all of which are potential NG modes for the spontaneous violation of spacetime symmetries.

At first sight the situation might seem to be further complicated by the existence of the spin connection $\omega_\mu{}^{ab}$, which permits the construction of the covariant derivative and in principle can have up to 24 independent components. However, the requirement that the connection be metric,

$$D_\lambda e_\mu{}^a = 0, \tag{44}$$

and the vanishing of the torsion tensor in a Riemann spacetime imply that the spin connection $\omega_\mu{}^{ab}$ can be specified completely in terms of the vierbein and its derivatives according to

$$\omega_\mu{}^{ab} = \frac{1}{2}e^{\nu a}(\partial_\mu e_\nu{}^b - \partial_\nu e_\mu{}^b) - \frac{1}{2}e^{\nu\nu}(\partial_\mu e_\nu{}^a - \partial_\nu e_\mu{}^a) - \frac{1}{2}e^{\alpha a}e^{\beta b}e_\mu{}^c(\partial_\alpha e_{\beta c} - \partial_\beta e_{\alpha c}). \tag{45}$$

It is therefore sufficient to consider the behavior of the vierbein in studying the properties of the NG modes. For example, a Higgs mechanism is excluded for the spin connection in a Riemann spacetime, since no independent propagating massless modes for $\omega_\mu{}^{ab}$ exist to absorb the NG degrees of freedom. In the next section, we revisit this issue in the context of the more general Riemann-Cartan geometry, for which the spin connection is an independent dynamical field.

Covariant derivatives acting on $T_{\lambda\mu\nu\cdots}$ in the Lagrangian can also generate propagators for the vierbein and hence for the NG modes. The covariant derivative $D_\alpha T_{\lambda\mu\nu\cdots}$ is given by

$$D_\alpha T_{\lambda\mu\nu\cdots} = e_\lambda{}^a e_\mu{}^b e_\nu{}^c \ldots D_\alpha t_{abc\cdots}. \tag{46}$$

The term $D_\alpha t_{abc\cdots}$ in this equation contains products of the spin connection with the vacuum value $t_{abc\cdots}$, which according to Eq. (45) generates expressions involving a single derivative of the vierbein. In the presence of spontaneous violation of spacetime symmetries, it follows that any piece of the Lagrangian involving a quadratic power of $D_\alpha T_{\lambda\mu\nu\cdots}$ can produce quadratic-derivative terms involving the vierbein.

The above discussion shows that in a Riemann spacetime the fate of the NG modes can depend on both the gravitational terms in the Lagrangian and the kinetic or other derivative-coupling terms for the tensor field. In what follows, we consider implications of these results for bumblebee models.

B. Bumblebee and photon

For a bumblebee model in an asymptotically flat Riemann spacetime, the vacuum structure is similar to the Minkowski case. We take the vacuum values for B^μ and the vierbein to be those of Eq. (19). The projector method can be applied, leading to the decomposition (25) of B^μ. There are four potential NG modes contained in the fields $\mathcal{E}^\mu$ and ρ, and the axial-gauge condition $b_\mu\mathcal{E}^\mu = 0$ holds. Note that the field strength $B_{\mu\nu}$ in Eq. (33) can be rewritten to give

$$B_{\mu\nu} = (\partial_\mu e_\nu{}^a - \partial_\nu e_\mu{}^a)b_a, \tag{47}$$

where b_a is taken constant as in the previous subsection.

The properties of the NG modes depend on the kinetic terms for B^μ and the gravitational terms in the Lagrangian. To gain further insight, consider the Lagrangian (31) with a Lagrange-multiplier potential,

$$\mathcal{L}_B = \frac{1}{2\kappa}(eR + \xi eB^\mu B^\nu R_{\mu\nu}) - \frac{1}{4}eB_{\mu\nu}B^{\mu\nu} - \frac{1}{2}e\lambda(B_\mu B^\mu \pm b^2) - eB_\mu J^\mu. \tag{48}$$

The vacuum solution for this theory is that of Eq. (40).

The effective Lagrangian $\mathcal{L}_{\rm NG}$ determining the properties of the NG modes can be obtained by expanding the Lagrangian $\mathcal{L}_B$ to quadratic order, keeping couplings to matter currents and curvature, and using the decomposition (25) of B^μ. Disregarding total derivative terms, we find

$$\mathcal{L}_{\rm NG} \approx \frac{1}{2\kappa}[eR + \xi eb^\mu b^\nu R_{\mu\nu} + \xi eA^\mu A^\nu R_{\mu\nu} + \xi e\rho(\rho+2)b^\mu b^\nu R_{\mu\nu} + 2\xi e(\rho+1)b^\mu A^\nu R_{\mu\nu}] - \frac{1}{4}eF_{\mu\nu}F^{\mu\nu} - eA_\mu J^\mu - eb_\mu J^\mu + eb^\mu\partial_\nu\Xi_\mu J^\nu, \tag{49}$$

Here, we again relabel $A_\mu \equiv \mathcal{E}_\mu$, which obeys $b_\mu A^\mu = 0$, and write $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, which is the field strength of a gauge-fixed U(1) field. In Eq. (49), the gravitational excitations $h_{\mu\nu}$ obey $h_{\mu\nu}b^\mu = 0$. In the absence of curvature sources, Eq. (49) reduces to the Minkowski-spacetime result (41).

The form of $\mathcal{L}_{\rm NG}$ reveals that only two of the four potential NG modes propagate. The propagating modes are transverse Lorentz NG modes, while the longitudinal Lorentz and the diffeomorphism NG modes are auxiliary. In particular, with the kinetic term given by the square of the field strength $B_{\mu\nu}$ in Eq. (47), the diffeomorphism NG mode in ρ again cancels, as in the Minkowski-spacetime case. Moreover, the curvature terms in $\mathcal{L}_B$ also yield no contributions for ρ in $\mathcal{L}_{\rm NG}$. This is because, in an asymptotically Minkowski spacetime, metric excitations of the required NG form $h_{\mu\nu} = -\partial_\mu\Xi_\nu - \partial_\nu\Xi_\mu$ produce only a vanishing contribution to the curvature tensor at linear order and contribute only as total derivatives at second order when contracted with $\eta_{\mu\nu}$ or b_μ.

As in the Minkowski-spacetime case, it is interesting in the present context with gravity to consider the possibility that the photon observed in nature can be identified with the NG mode resulting from spontaneous Lorentz violation. We see that, in a Riemann spacetime, the theory of the form (49) produces a free propagator for the Lorentz NG mode consistent with this idea at the linearized level. Also, the interaction with the charged current J_μ has an appropriate form. Indeed, the effective action $\mathcal{L}_{\rm NG}$ contains as a subset the standard Einstein-Maxwell electrodynamics in axial gauge.

The issue of possible experimental signals can be revisited for the present Riemann-spacetime case. The discussion in the previous section about potential SME and related signals in Minkowski spacetime still applies, but further possibilities exist. In particular, there are interesting unconventional couplings of the curvature with A^μ, ρ, and b^μ. The photon acquires Lorentz-invariant curvature couplings of the form $eA^\mu A^\nu R_{\mu\nu}$, which are forbidden by gauge invariance in conventional Einstein-Maxwell electrodynamics but are consistent here with the axial-gauge condition. The term $\xi e b^\mu b^\nu R_{\mu\nu}/2\kappa$ corresponds to a nonzero coefficient of the $s^{\mu\nu}$ type in the pure-gravity sector of the SME [5]. The other terms with curvature also represent Lorentz-violating couplings. This Lagrangian therefore gives rise to additional effects that could serve to provide experimental evidence for the idea that the photon is an NG mode for spontaneous Lorentz violation. The analysis of the associated signals is evidently of interest but lies outside our present scope.

It also is of interest to ask whether there exists a theory in Riemann spacetime with a nontrivial propagator for the diffeomorphism mode contained in ρ. Indeed, for a purely timelike coefficient b_μ, for which $\rho = \partial_0 \Xi^0$, it has been shown that if a kinetic term for the diffeomorphism NG mode were to appear with second-order time derivatives, then an unusual dispersion relation would follow with potentially interesting phenomenological consequences [26]. In general, the presence of curvature makes this question more challenging than in Minkowski spacetime.

In the context of the bumblebee model (48) with the field strength (47) having constant b_a, we have seen that the diffeomorphism NG mode fails to propagate. Attempting to change this by modifying the gravitational terms in the Lagrangian (48) to any combination of covariant contractions of the curvature tensor $R^\kappa{}_{\lambda\mu\nu}$, including theories with general quadratic curvature terms [47,48], also fails to yield a nontrivial propagator for the diffeomorphism NG mode for the same reason as above. However, possibilities exist that might overcome this difficulty, such as allowing for nonconstant b_a. Another interesting option is to relax the requirement of an asymptotically Minkowski spacetime, perhaps by adding a cosmological-constant term to the theory. This leads to modifications in the projector analysis and changes in the effective action for the NG modes. For example, in a curved background a term of the form $\xi e\rho(\rho+2) b^\mu b^\nu R_{\mu\nu}$ in Eq. (49) would generate quadratic terms for ρ in the effective Lagrangian, as needed for the propagation of Ξ^μ. A cosmological term $e\Lambda$ also contains quadratic terms $h_{\mu\nu}h^{\mu\nu} - \frac{1}{2}h^2$ in the weak-field limit, which might serve as a suitable source of quadratic terms because a virtual diffeomorphism generates time derivatives for the spacelike components of Ξ^μ. Exploration of these issues is of definite interest but lies beyond the scope of this work.

VI. RIEMANN-CARTAN SPACETIME

In a Riemann-Cartan spacetime, the vierbein $e_\mu{}^a$ and the spin connection $\omega_\mu{}^{ab}$ represent independent degrees of freedom determined by the dynamics [28]. It follows that the effects of spontaneous Lorentz breaking can be strikingly different from the cases examined above. In particular, we focus in this section on the possibility that the NG modes are absorbed into the spin connection via a Higgs mechanism.

A. Higgs mechanism for the spin connection

As in the Riemann-spacetime case, we suppose that a tensor $T_{\lambda\mu\nu\cdots}$ obeys the condition (10) and acquires a nonzero vacuum value. This condition can be satisfied by expressing $T_{\lambda\mu\nu\cdots}$ in terms of the vierbein according to Eq. (43). This result can be used to calculate the covariant derivative of the field $T_{\lambda\mu\nu\cdots}$, which enters the kinetic Lagrangian for $T_{\lambda\mu\nu\cdots}$ and therefore affects the NG modes for the spontaneous Lorentz violation. A key feature of Riemann-Cartan spacetime is that this covariant derivative now involves the spin connection as an independent degree of freedom. For instance, assuming constant $t_{\lambda\mu\nu\cdots}$, the linearized expression in a Minkowski background is

$$D_\alpha T_{\lambda\mu\nu\cdots} \approx \omega_\alpha{}^\rho{}_\lambda t_{\rho\mu\nu\cdots} + \omega_\alpha{}^\rho{}_\mu t_{\lambda\rho\nu\cdots} + \ldots . \qquad (50)$$

It follows that a kinetic term involving a quadratic expression in the covariant derivative of $T_{\lambda\mu\nu\cdots}$ generates a nonderivative quadratic expression in the spin connection. This could represent a mass for the spin connection, so the spontaneous violation of Lorentz symmetry in Riemann-Cartan spacetime could incorporate a gravitational version of the Higgs mechanism. Note that this Higgs mechanism cannot occur in a Riemann spacetime, where the spin connection is identically the derivative expression (45) for the vierbein, because the same calculation produces instead a kinetic term for the NG modes, as shown in the previous section.

The Lagrangian for a generic theory with spontaneous Lorentz violation in Riemann-Cartan spacetime can be written as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm SSB}, \qquad (51)$$

where $\mathcal{L}_0$ describes the unbroken theory and $\mathcal{L}_{\rm SSB}$ induces spontaneous Lorentz violation. We suppose for simplicity that $\mathcal{L}_0$ contains only gravitational terms formed from the curvature and torsion, while the Lagrangian $\mathcal{L}_{\rm SSB}$ for a tensor field $T_{\lambda\mu\nu\ldots}$ contains a kinetic piece and a potential driving the spontaneous Lorentz violation.

A priori, it might seem that a large range of models could implement this Higgs mechanism, since numerous types of tensors could acquire a vacuum expectation value. However, a complete Higgs mechanism requires a theory $\mathcal{L}_0$ that has a massless propagating spin connection prior to the spontaneous Lorentz violation. A fully satisfactory example also requires the theory to be free of ghosts. It turns out that these conditions severely restrict the possibilities for model building.

General studies exist of theories $\mathcal{L}_0$ with a propagating spin connection [47,48], including ones with both massive and massless propagating modes. However, the subset of ghost-free models is relatively small, especially for the case of a massless spin connection. The total number of propagating modes in these models depends on the presence of certain accidental symmetries. Our investigations reveal that the symmetry-breaking Lagrangian $\mathcal{L}_{\rm SSB}$ typically breaks one or more of the accidental symmetries of $\mathcal{L}_0$ when the tensor field acquires a vacuum expectation value. This significantly complicates the analysis of models, but also offers interesting new avenues by which spontaneous Lorentz violation could affect the physical modes in a realistic theory.

We are primarily interested in ghost-free Lagrangians $\mathcal{L}_0$ formed from powers of the curvature and torsion with at most two derivatives in the equations of motion for the vierbein and spin connection. In this case, up to 18 of the 24 components of the spin connection $\omega_\mu{}^{ab}$ in principle can behave as propagating degrees of freedom. The six components ω_{0ab} are auxiliary fields, irrespective of any gauge choices imposed for the Lorentz symmetry or any simplifications from accidental symmetries.

The behavior of the 18 modes can be studied by assuming a background Minkowski spacetime and linearizing the equations of motion along the lines discussed in Sec. II. Note that, at linear order in infinitesimal quantities, the spin connection transforms under Lorentz transformations according to

$$\omega_\mu{}^{ab} \to \omega_\mu{}^{ab} - \partial_\mu \epsilon^{ab}, \qquad (52)$$

while infinitesimal diffeomorphisms leave $\omega_\mu{}^{ab}$ invariant at lowest order. The vacuum solution now takes the form

$$\langle T_{\mu\nu\ldots}\rangle = t_{\mu\nu\ldots}, \qquad \langle e_{\mu\nu}\rangle = \eta_{\mu\nu}, \qquad \langle \omega_\mu{}^{ab}\rangle = 0 \qquad (53)$$

in a suitable coordinate frame. The vanishing of $\langle \omega_\mu{}^{ab}\rangle$ and the invariance of $\omega_\mu{}^{ab}$ under diffeomorphisms suggests that the fate of the diffeomorphism NG mode is unlikely to be appreciably altered by the new role of the spin connection in the present context, and this is confirmed in what follows.

B. Decompositions

The investigation of various models is facilitated by introducing two different decompositions of the 24 fields $\omega_{\lambda\mu\nu}$. The first is a decomposition according to Lorentz indices,

$$\omega_{\lambda\mu\nu} = A_{\lambda\mu\nu} + M_{\lambda\mu\nu} + \frac{1}{3}(\eta_{\lambda\mu}T_\nu - \eta_{\lambda\nu}T_\mu), \qquad (54)$$

where $A_{\lambda\mu\nu}$ is totally antisymmetric, $M_{\lambda\mu\nu}$ has mixed symmetry, and T_μ is the trace piece. The antisymmetric components define a dual $V_\kappa = \epsilon_{\kappa\lambda\mu\nu}A^{\lambda\mu\nu}/2$ that has four independent components. The mixed components $M_{\lambda\mu\nu}$ satisfy eight identities, which can be written $M_{\nu\lambda}{}^\nu = 0$ and $M_{\lambda\mu\nu} - M_{\nu\mu\lambda} = M_{\mu\lambda\nu}$, and they therefore contain 16 degrees of freedom. The trace T_μ contains the remaining 4 degrees of freedom. The reader is cautioned that Eq. (54) is *not* a Lorentz-irreducible decomposition in the usual sense because the field being decomposed is a connection rather than a tensor.

The second useful decomposition involves the spin-parity projections J^P of the fields $\omega_{\lambda\mu\nu}$. These are particularly appropriate for the case of timelike Lorentz violation induced by $t_{\mu\nu\ldots}$, such as a bumblebee vacuum value of the form $b_\mu = (b, 0, 0, 0)$. The 18 dynamical fields in this case include the projections 2^+, 2^-, 1^+, 1^-, 0^+, 0^-, while the six auxiliary fields $\omega_{0\mu\nu}$ contain two triplet projections we denote by $\tilde{1}^+$, $\tilde{1}^-$. Again, we stress that these projections involve the connection rather than a tensor, so the notation fails to reflect the true transformation properties. For example, the 1^+ projection yields a triplet of scalars under spatial rotations.

Explicit expressions for each of these projections can be found. For example, we find

$$\begin{aligned}
&\omega^{[0^+]} = \omega_{j0j}, \qquad \omega^{[0^-]} = \frac{1}{2}\epsilon_{jkl}\omega_{jkl},\\
&\omega_l^{[1^+]} = \epsilon_{jkl}\omega_{j0k}, \qquad \omega_k^{[1^-]} = \omega_{jkj},\\
&\omega_k^{[\tilde{1}^+]} = \frac{1}{2}\epsilon_{klm}\omega_{0lm}, \qquad \omega_k^{[\tilde{1}^-]} = \omega_{00k},\\
&\omega_{jk}^{[2^+]} = \frac{1}{2}(\omega_{j0k} + \omega_{k0j}) - \frac{1}{3}\delta_{jk}\omega_{l0l},\\
&\omega_{jk}^{[2^-]} = \frac{1}{4}(\epsilon_{klm}\omega_{jlm} + \epsilon_{jlm}\omega_{klm}) - \frac{1}{6}\delta_{jk}\epsilon_{lmn}\omega_{nlm},
\end{aligned} \qquad (55)$$

where spatial components are denoted by $j, k, \ldots$.

The two sets of projections can be related. We find

$$\begin{aligned}
&\omega_k^{[1^+]} = \omega_k^{[\tilde{1}^+]} - V_k = \frac{1}{2}\epsilon_{k0lm}M^{0lm} - \frac{2}{3}V_k,\\
&\omega_k^{[1^-]} = -\omega_k^{[\tilde{1}^-]} - T_k = -M_{00k} - \frac{2}{3}T_k,\\
&\omega^{[0^+]} = -T_0, \qquad \omega^{[0^-]} = V_0.
\end{aligned} \qquad (56)$$

C. Bumblebee

To gain further insight, we investigate a definite form for the Lagrangian $\mathcal{L}_{\rm SSB}$, namely, the simple bumblebee model

$$\mathcal{L}_{\rm SSB} = -\frac{1}{4} e B_{\mu\nu} B^{\mu\nu} - e\lambda (B_\mu B^\mu \pm b^2) \tag{57}$$

with a Lagrange-multiplier potential freezing any non-NG modes. In a Riemann-Cartan spacetime, the field strength $B_{\mu\nu}$ in the kinetic term of this theory is defined in Eq. (33). Its expression in terms of the vierbein and spin connection is

$$B_{\mu\nu} = (e_\mu{}^b \omega_\nu{}^a{}_b - e_\nu{}^b \omega_\mu{}^a{}_b) b_a. \tag{58}$$

Note that this form reduces to Eq. (47) in the limits of Riemann and Minkowski spacetimes, for which the spin connection is given in terms of derivatives of the vierbein by Eq. (45).

When $B_{\mu\nu}$ is squared to yield the kinetic term, quadratic terms in $\omega_\mu{}^a{}_b$ appear in the Lagrangian $\mathcal{L}_{\rm SSB}$. For example, for a Minkowski background we find

$$\begin{aligned}\mathcal{L}_K &\equiv -\frac{1}{4} e B_{\mu\nu} B^{\mu\nu} \\ &\approx -\frac{1}{4} (\omega_{\mu\rho\nu} - \omega_{\nu\rho\mu})(\omega^{\mu\sigma\nu} - \omega^{\nu\sigma\mu}) b^\rho b_\sigma. \end{aligned} \tag{59}$$

The appearance of these quadratic terms again suggests that a Higgs mechanism can occur involving the absorption of the NG modes by the spin connection.

In terms of the Lorentz decomposition in the previous subsection, the kinetic term $\mathcal{L}_K$ for B^μ becomes

$$\begin{aligned}\mathcal{L}_K \approx{}& \frac{2}{9}(b_\mu b^\mu V_\nu V^\nu - b_\mu V^\mu b_\nu V^\nu) - \frac{1}{4} M^\rho{}_{\mu\nu} M^{\sigma\mu\nu} b_\rho b_\sigma \\ & - \frac{1}{18}(b_\mu b^\mu T_\nu T^\nu - b_\mu T^\mu b_\nu T^\nu) \\ & + \frac{1}{3} \epsilon_{\lambda\mu\nu\rho} V^\rho M^{\sigma\lambda\mu} b^\nu b_\sigma \\ & - \frac{1}{6} M^\lambda{}_{\mu\nu} (b^\mu T^\nu - b^\nu T^\mu) b_\lambda. \end{aligned} \tag{60}$$

This result holds for any vacuum value b^μ, but its physical interpretation can be involved in the general case.

For the special case of timelike Lorentz violation induced by a vacuum expectation value $b_\mu = (b, 0, 0, 0)$, the J^P decomposition provides a more convenient expression. With this assumption, we find

$$\mathcal{L}_{\rm K} = -\frac{1}{2} b^2 \omega_j^{[1^+]} \omega^{[1^+]j} + \frac{1}{2} b^2 \omega_j^{[\tilde{1}^-]} \omega^{[\tilde{1}^-]j}. \tag{61}$$

We see that this expression contains an apparent physical mass term for the 1^+ and a wrong-sign mass term for the $\tilde{1}^-$. Since the $\tilde{1}^-$ is an auxiliary field, it cannot propagate independently. However, the 1^+ is an independent dynamical field, so interpreting its apparent mass term requires a study of the dynamical content of $\mathcal{L}_0$.

D. Illustrative models

Next, we present three different sample models $\mathcal{L}_0$, all containing dynamical terms for the spin connection, to illustrate some of the possible effects and issues emerging from the presence of the Lorentz-breaking term $\mathcal{L}_{\rm SSB}$. In the first model, denoted $\mathcal{L}_{0,1}$, ghosts are present but an analysis shows that a Higgs mechanism occurs when $\mathcal{L}_{\rm SSB}$ is added. The second $\mathcal{L}_{0,2}$ initially has only auxiliary or gauge degrees of freedom, but the addition of the Lorentz-violating term $\mathcal{L}_{\rm SSB}$ breaks some accidental symmetries and hence causes some modes to propagate. The third model $\mathcal{L}_{0,3}$ is ghost free and has a massless propagating spin connection.

The Lagrangian for the first example is

$$\mathcal{L}_{0,1} = \frac{1}{4} R_{\lambda\kappa\mu\nu} R^{\lambda\kappa\mu\nu}. \tag{62}$$

To lowest order in the spin connection, the curvature tensor becomes $R_{\lambda\kappa\mu\nu} \approx \partial_\kappa \omega_{\lambda\mu\nu} - \partial_\lambda \omega_{\kappa\mu\nu}$. In this model, all the fields $\omega_{\lambda\mu\nu}$ with $\lambda \neq 0$ propagate as massless modes. However, when resolved into J^P projections, the second-derivative terms in the equations of motion for the even- and odd-parity states have opposite signs, so the theory contains ghosts. When $\mathcal{L}_{0,1}$ is combined with $\mathcal{L}_{\rm SSB}$, the linearized equations of motion become

$$\begin{aligned}\partial_\rho \partial^\rho \omega_{\lambda\mu\nu} - \partial_\lambda \partial^\rho \omega_{\rho\mu\nu} ={}& -\frac{1}{2}(\omega_{\lambda\sigma\nu} - \omega_{\nu\sigma\lambda}) b_\mu b^\sigma \\ & + \frac{1}{2}(\omega_{\lambda\sigma\mu} - \omega_{\mu\sigma\lambda}) b_\nu b^\sigma. \end{aligned} \tag{63}$$

These 24 equations can be diagonalized to determine the nature of the modes in the combined theory, and we find that among the propagating modes is a massive one. This confirms the existence of a Higgs mechanism for the spin connection in this model.

The idea behind the second model is to start with a special theory $\mathcal{L}_0$ in which accidental symmetries exclude all propagating physical modes, but chosen such that physical propagating modes emerge when the Lagrangian $\mathcal{L}_{\rm SSB}$ triggering spontaneous Lorentz violation is added. The appearance of the physical modes via this "phoenix" mechanism can be traced to the breaking of some accidental symmetries of $\mathcal{L}_0$ by $\mathcal{L}_{\rm SSB}$.

A number of models in which all modes are auxiliary or gauge are known [48]. Here, we consider one explicit example, with Lagrangian given by

$$\mathcal{L}_{0,2} = \frac{1}{2} R_{\mu\nu} R^{\mu\nu} - \frac{1}{2} R_{\mu\nu} R^{\nu\mu}, \tag{64}$$

where $R_{\mu\nu}$ is the Ricci tensor in Riemann-Cartan spacetime. Since our focus is on the spin connection, we restrict attention for simplicity to solutions in background

Minkowski spacetime. With this choice, the vierbein disappears from the linearized theory, so the spin connection is the only relevant dynamical field.

The unbroken Lagrangian can be written in terms of the Lorentz decomposed fields as

$$\mathcal{L}_{0,2} = \frac{1}{9} F_{\mu\nu} F^{\mu\nu} - \frac{1}{9} G_{\mu\nu} G^{\mu\nu} + \frac{1}{4} \partial_\rho M^\rho{}_{\mu\nu} \partial_\sigma M^{\sigma\mu\nu} - \frac{1}{3}\Big(F_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}\Big) \partial_\lambda M^{\lambda\mu\nu}. \tag{65}$$

The corresponding equations of motion are

$$\partial_\lambda \Big[\partial^\sigma M_{\sigma\mu\nu} - \frac{2}{3}\Big(F_{\mu\nu} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}\Big)\Big] = 0, \tag{66}$$

$$\partial^\mu F_{\mu\nu} = \frac{3}{2} \partial^\mu \partial^\sigma M_{\sigma\mu\nu}, \tag{67}$$

$$\partial^\mu G_{\mu\nu} = -\frac{3}{4} \epsilon_{\mu\nu\sigma\rho} \partial^\sigma \partial^\lambda M_\lambda{}^{\rho\mu}. \tag{68}$$

In these equations, $F_{\mu\nu} = \partial_\mu T_\nu - \partial_\nu T_\mu$ and $G_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ are the field strengths for T_μ and V_ν, respectively.

A cursory inspection might suggest that this theory has at least two sets of massless fields, T_μ and V_ν, which correspond to the 1^+ and the 1^- modes in the J^P decomposition. However, there are a number of accidental symmetries in this theory associated with the projection operators for the 2^+, 2^-, 0^+, and 0^- fields. These and Lorentz transformations can be used to remove all physical propagating degrees of freedom [48]. In particular, the 1^- mode can be gauged away using only rotations, while the 1^+ mode can be gauged away using only boosts:

$$\begin{aligned} \omega_j^{[1^-]} &\to \omega_j^{[1^-]} - \partial^k \varepsilon_{jk}, \\ \omega_j^{[1^+]} &\to \omega_j^{[1^+]} + \epsilon_{j0}{}^{lm} \partial_l \varepsilon_{0m}. \end{aligned} \tag{69}$$

The net result is that the Lorentz-invariant theory (64) has no physical content.

Suppose now the term $\mathcal{L}_{\rm SSB}$ in Eq. (61) for the case of a timelike vacuum expectation value b_μ is added to the Lagrangian (64). This spontaneously breaks boosts while maintaining rotation symmetry. The 1^- mode can still be gauged away via rotations, but the 1^+ mode can no longer be removed using boosts and so might be expected to propagate as a massive mode. However, the mass term in $\mathcal{L}_{\rm SSB}$ also affects the structure of the gauge and auxiliary fields in the theory by breaking some of the accidental symmetries, so other field combinations now become physical. We find two such massless modes, involving superpositions of the J^P projections.

For our third example, we take for $\mathcal{L}_0$ a ghost-free model with a massless propagating spin connection. A general analysis under the assumption of Lorentz invariance finds only four ghost-free possibilities [48]. All share the property of the previous example that the propagating modes consist of mixtures of J^P projections. In two models, the massless propagating mode incorporates contributions from the 1^+ projections, while in the other two it includes contributions from the 1^- projections. It is therefore of interest to adopt for $\mathcal{L}_0$ either of the first two models and investigate the effect on the propagating modes of adding the Lorentz-violating term $\mathcal{L}_{\rm SSB}$.

As an explicit example, consider the Lagrangian [48]

$$\mathcal{L}_{0,3} = R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2. \tag{70}$$

It turns out that the propagating modes of this Lorentz-invariant model are a mixture of 1^+ and 2^+ projections. In contrast, $\mathcal{L}_{\rm SSB}$ contains quadratic terms for the 1^+ and the auxiliary $\tilde{1}^-$ states. In the theory resulting from the combination of the two, the nature of the modes can be determined by diagonalizing the 24 linearized equations for the spin connection. We find that the propagation of the massless modes is altered, but there is no massive propagating 1^+. The incompatibility between the mixture of J^P states appearing in $\mathcal{L}_{0,3}$ and that appearing in $\mathcal{L}_{\rm SSB}$ prevents the occurrence of a clean Higgs mechanism for the J^P modes in this example.

In the context of these ideas, a number of issues of interest remain open for future investigation. Studies of the large variety of possible Lorentz-invariant Lagrangians $\mathcal{L}_0$ could lead to additional features beyond those identified in the three examples above. It also would be of interest to explore more explicitly the effects of lightlike and spacelike b_μ in the Riemann-Cartan spacetimes. The J^P decomposition is less appropriate for these cases, so alternative decompositions with respect to the corresponding little group are likely to be useful. Different choices for $\mathcal{L}_{\rm SSB}$, including ones in which the spontaneous Lorentz violation involves one or more tensor fields, similarly can be expected to affect the dynamics of the NG modes. From a broader theoretical perspective, the incorporation of Lorentz violation opens an arena for the search for ghost-free theories with dynamical curvature and torsion.

Various implications for phenomenology in the context of Riemann-Cartan spacetime also merit exploration. The scale of the emergent mass in the models considered here is set by b^2. Even if this is of order of the Planck mass, the existence of fields with Lorentz-violating physics could have effects on cosmology and in regions with strong gravitational fields such as black holes. The couplings to other known fields also merit attention and could lead to interesting signals for experiments. All relevant terms associated with gravitational and SM fields are included in the gravitational couplings of the Lorentz- and CPT-violating SME in Riemann-Cartan spacetime [5], which therefore provides the appropriate framework for investigating phenomenological implications of these models.

VII. SUMMARY

In this paper, we have examined the fate of the Nambu-Goldstone modes when Lorentz symmetry is spontaneously broken. The analysis is performed in the context of the vierbein formalism, which is well suited for this purpose because it admits a clear separation between local Lorentz and coordinate frames on the spacetime manifold. Within this formalism, we have demonstrated in Sec. II that spontaneous particle Lorentz violation is accompanied by spontaneous particle diffeomorphism violation and vice versa, and that up to 10 NG modes can appear. These modes can naturally be matched to those 10 of the 16 modes of the vierbein that in a Lorentz-invariant theory are gauge degrees of freedom. This match provides further evidence for the value of the vierbein formalism in studies of spontaneous violations of spacetime symmetries. We have also provided a generic treatment for background Minkowski spacetimes. The fate of the NG modes is found to depend both on the spacetime geometry and also on the dynamics of the tensor field triggering the spontaneous violation of local Lorentz and diffeomorphism symmetries.

As illustrative models for the analysis, we have adopted a general class of bumblebee models, involving vacuum values for a vector field that break some of the local Lorentz and diffeomorphism symmetries. Some properties of these models have been presented in Sec. III, where projectors are constructed that permit separation of the Lorentz and diffeomorphism NG modes.

In the later sections of this work, we have studied the behavior of the NG modes in Minkowski, Riemann, and Riemann-Cartan spacetimes. Each of these offers distinctive general features, which can be illustrated within bumblebee models. In Minkowski and Riemann spacetimes, Lorentz NG modes exist that can propagate as massless modes, with effective Lagrangians containing the Maxwell and Einstein-Maxwell theories in axial gauge. Suitable bumblebee models thereby provide dynamical methods of generating the photon as a Nambu-Goldstone boson for spontaneous Lorentz violation. Various possibilities exist for experimental signals in these models, including both unconventional Lorentz-invariant couplings and Lorentz-breaking couplings in the matter and gravitational sectors of the SME. In Riemann-Cartan spacetimes, the interesting possibility exists that the spin connection could absorb the propagating NG modes in a gravitational version of the Higgs mechanism. This unique feature of gravity theories with torsion may offer another phenomenologically viable route for constructing realistic models with spontaneous Lorentz violation.

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