

Cosmology and the S matrix

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We study conditions for the existence of asymptotic observables in cosmology. With the exception of de Sitter space, the thermal properties of accelerating universes permit arbitrarily long observations, and guarantee the production of accessible states of arbitrarily large entropy. This suggests that some asymptotic observables may exist, despite the presence of an event horizon. Comparison with decelerating universes shows surprising similarities: Neither type suffers from the limitations encountered in de Sitter space, such as thermalization and boundedness of entropy. However, we argue that no realistic cosmology permits the global observations associated with an S matrix.

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I. INTRODUCTION

One problem with quantum gravity is that we do not know what the theory should compute. In particle physics, the most precise observable is the S matrix. But this quantity seems ill-suited to cosmology, where the observer is not outside the system, initial states cannot be set up, and experiments cannot be arbitrarily repeated to gain statistically significant results.

This ignorance is not especially unusual or embarrassing. It is rarely clear at the outset what a theory should compute. For example, the insight that gravity is a theory of a symmetric, diffeomorphism-invariant tensor field in itself already constituted a significant part of the development of general relativity. But once a theory is in its final form, the observables should be apparent.

If string theory is the correct quantum theory of gravity, then whatever it computes presumably are the observables. But string theory—perhaps because it is *not* in its final form—has so far sidestepped the problem of cosmological observables. It defines quantum gravity for certain classes of geometries characterized by asymptotic conditions, such as asymptotically flat or anti-de Sitter spacetimes. In these geometries an S matrix happens to make sense, and string theory computes its matrix elements. (In the case of AdS, it computes boundary correlators, which are a close analogue of the S matrix.)

However, we have yet to learn how to apply string theory to cosmology or to an observer inside a black hole, with the same level of rigor as in anti-de Sitter space. Hence, it would be premature to conclude that the S matrix will remain the only well-defined object. It is too early to know what, if anything, string theory has to say about cosmological observables.

Fortunately, classical and quantum properties of cosmological solutions impose significant constraints on possible observables, and may even hint at some of the principles on which a theory computing them must be based. De Sitter

space is a case in point. Semiclassical analysis has provided overwhelming evidence that *no* exact observables exist in eternal de Sitter space—at least, none that correspond to experiments that can be performed by an observer inside the universe. This is related to the presence of a cosmological event horizon in de Sitter space, which limits the accessible information and emits pernicious thermal radiation.

In this paper we use similar semiclassical reasoning to characterize constraints on exact observables in other cosmological solutions. Does the universe contain regions where fluctuations, including those of the gravitational field, become arbitrarily weak? Accurate measurements take a long time,¹ and they require devices with a large number of states. Does the universe last long enough, i.e., does it contain geodesics of infinite proper time in the future? Does the causally accessible region have enough quantum states? According to entropy bounds [1–3], this translates into a minimum size for the region. By asking whether such requirements are met, one can investigate whether exact quantum mechanical observables exist in a given cosmology, without knowledge of the full theory.

By an observable we mean a quantity or limit of quantities that can actually be measured by an observer inside the universe, without violating laws of physics such as causality or entropy bounds. For example, it may turn out that an S matrix for de Sitter space can be formally computed as a useful “meta-observable” [4], from which predictions for true, operationally defined observables can be extracted by further processing. The restrictions derived below apply only to the latter, operationally meaningful quantities.

Our conclusions for different classes of universes vary in their details, but they do strike two common chords. First: Aside from de Sitter space and the obvious case of crunching universes, our necessary conditions for exact observ-

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¹For this reason, we shall use the terms “asymptotic observable” and “exact observable” interchangeably.

ables are satisfied in all the other cases considered. Surprisingly, this includes universes with a cosmological event horizon. Second: Observables that invoke a global out-state (such as the S matrix) do not seem to describe any experiment in cosmology. We find that the information content of a generic out-state is causally inaccessible even in a universe with a null infinity and no horizon.

We present a number of intermediate results that are of interest in their own right: an analysis of the thermodynamics and the fluctuation spectrum of quintessence universes; an argument demonstrating that an open universe resides inside the Farhi-Guth solution; and entropic reasoning suggesting that the global state of a noncompact universe is not accessible to experiment, independently of event horizons.

This paper does not tackle the actual definition of any asymptotic cosmological observables (see Refs. [5,6] for recent approaches). Even that challenge, in turn, will only be an intermediate goal. In our view, asymptotic observables are at best a crutch. The description of a real experiment involving gravity requires well-defined (but necessarily imprecise?) local observables. This is a famously difficult problem in the presence of gravity. It is further complicated, but perhaps also helpfully constrained, by the counter-intuitive holographic restriction on bulk degrees of freedom [1–3,7–11]. This task will have to be confronted eventually.

Relation to other work—For a review of the difficulties with physics in de Sitter space, see, e.g., Ref. [12]. A broad discussion of the problem of observables in cosmologies with a nonpositive cosmological constant was given by Banks and Fischler [5], who noted that in a noncompact universe, an S matrix description must restrict to states with a finite number of extra particles, and that those states are very special. While this restriction is necessary, it is not sufficient: as shown below, the unobserved region can have infinite entropy even if no particles are added, because of the internal states of the matter already present.

Our analysis of the thermodynamics of Q-space builds on Refs. [13,14], who derived its global structure and pointed out that its event horizon obstructs the definition of an S matrix. We do not question this conclusion; indeed, we find that the difficulties with an S matrix are quite general in cosmology. We do argue, however, that other asymptotic observables may exist in Q-space. This possibility was first raised by Witten [4], who noted that observers will not be thermalized in Q-space. The existence of accessible high entropy states was not demonstrated there.

The problem of defining cosmological observables is closely related to the challenge of describing physics from the point of view of an observer falling into a black hole. In both cases, some type of local observables will eventually be required, but in both cases, one can hope to make progress by asking how some of the information in

the gravity-dominated region may be encoded in asymptotic data [15,16].

The recent discovery that the universe is accelerating has turned the cosmological constant problem into the (worse) problem of small positive vacuum energy. Its possible resolution by a discretuum of metastable vacua in string theory [17], populated by cosmological dynamics, makes it all the more urgent to understand string theory observables in cosmology. Explicit constructions of de Sitter vacua have been proposed (e.g., Ref. [18]), and sophisticated counting arguments (e.g., Refs. [19,20]) broadly confirm the original estimates of the vast number of such vacua. The present discussion does not address specifically the development of a theoretical framework [6,21,22] describing this “landscape” [23]. But the question of observables is a part of this challenge, so our results may have some implications in this context.

Outline—The paper is structured as follows. In the first sections we mainly consider spatially flat Friedmann-Robertson-Walker (FRW) universes with fixed equation of state $w = p/\rho$. They are especially simple and suffice for deriving our main results. Moreover, their late-time behavior is a good approximation to other classes of cosmologies, including some we discuss at the end of the paper.

Section II, aside from a review of the flat FRW solutions and their causal structure, contains our main observation about decelerating universes ($w > -1/3$): All observers, at all times, lack information about infinitely large regions of the universe, even though there is no event horizon. If such regions contain any nonredundant information, then the global out-state computed by an S matrix cannot be measured.

Next, we turn to eternally accelerating universes, de Sitter space ($w = -1$) and “Q-space” ($-1 < w < -1/3$) [24],² which have a cosmological event horizon [13,14]. We show in Sec. III that Q-space exhibits thermodynamic properties similar to those of the de Sitter horizon. The horizon radius in Q-space grows linearly with time, and consequently the temperature slowly decreases. We find that this behavior is consistent with the first law of thermodynamics: the temperature and entropy respond appropriately to the flux of quintessence stress-energy across the horizon.

Section IV contains our main results for accelerating universes. They support the existence of asymptotic observables in Q-space. We study specific aspects of the thermal spectrum emitted by the horizon. The time-dependence of the temperature leads to significant differences between de Sitter space and Q-space. In the semiclassical theory, an infinite number of Hawking quanta are

²For a subset of this range, quintessence has been proposed as a model of dark energy [25]. Here we study these solutions simply as instructive examples to understand conditions for asymptotic observables.

produced (and reabsorbed) by the horizon. In de Sitter space, the total energy thus emitted diverges, whereas in Q-space the energy per quantum decreases rapidly enough to render the total energy finite. Hence, observers in Q-space will not be thermalized.

We ask whether observers will be destroyed by rare massive fluctuations, such as black holes. We consider objects of fixed energy and compute the rate at which they are emitted, according to standard statistical mechanics. If the energy is much larger than the temperature, the rate will be miniscule. However, in de Sitter space the rate is constant, so all fluctuations that are not completely forbidden will occur. This guarantees that any observer who survives the thermal radiation long enough will eventually be swallowed by a large black hole emitted by the horizon. In Q-space, the rate of such violent processes decreases exponentially with time. The integrated probability is therefore finite and can be exceedingly small. It follows that experiments in Q-space can last for an arbitrarily long time.

But the classical supply of matter in Q-space is bounded, seemingly ruling out exact measurements. Yet, we show in Sec. IV E that arbitrarily complex matter configurations are quantum mechanically *produced* by the Q-space horizon: The rate for a fluctuation of a given fixed entropy—no matter how large—is constant and nonvanishing at late times. This contrasts pleasantly with de Sitter space, where the entropy is strictly bounded by the inverse of the (fixed) cosmological constant.

In Sec. V we draw conclusions on the nature of observables in the universes we have studied. In particular, we argue that no direct analogue of an S matrix can be defined in any flat FRW universe unless the set of allowed states is severely restricted.

In Sec. VI we extend the discussion to open and closed FRW solutions. We also study composite universes that feature an asymptotically flat region on the far side of a black hole. We show that the Farhi-Guth solution can be regarded as an example of this setup in which the black hole resides inside an open universe produced by the decay of metastable de Sitter space. Because an open universe has infinite entropy, one would not expect generic microstates to be represented on the far side of the black hole, or on the asymptotic boundary.

II. SPATIALLY FLAT UNIVERSES

In this section we review various classical properties of flat FRW universes—in particular, the results of Refs. [13,14] on the causal structure of accelerating cosmologies. We ask how much matter and information is causally accessible to an observer in the classical evolution. We find that this amount is finite in accelerating universes and unbounded in decelerating universes. However, even in the latter case, no more than an infinitely small fraction of the matter is ever observable.

A. Metric and causal structure

The metric of a spatially flat FRW universe is given by

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2d\Omega^2). \quad (1)$$

A quick way to obtain its causal structure is to transform to conformal time, defined by $d\eta = dt/a(t)$. This shows that the metric is conformal to Minkowski space: $ds^2 = a(\eta)^2d\bar{s}^2$, where

$$d\bar{s}^2 = -d\eta^2 + dr^2 + r^2d\Omega^2. \quad (2)$$

Hence, the conformal diagram is a subset of the Minkowski space Penrose diagram (Fig. 1), selected by the range of η . The existence of horizons is determined as follows.

If and only if η is bounded from above ($\eta \rightarrow \eta_{\max} < \infty$ as $t \rightarrow \infty$), then an observer at $r = 0$ is surrounded by a future event horizon.³ The horizon is located at $r = \eta_{\max} - \eta$. Light-rays originating beyond this hypersurface never reach the observer. Similarly, if η is bounded from below, then there exists a past horizon.⁴ Events beyond this horizon cannot be influenced by the observer.

The dynamical evolution of the scale factor and the matter density is determined by the equations

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi\rho}{3}, \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p). \quad (4)$$

We will assume that the energy density ρ and pressure p obey the equation of state

$$p = w\rho \quad (5)$$

with constant w . It will be more convenient to work with the parameter

$$\epsilon = \frac{3}{2}(w + 1). \quad (6)$$

Thus one obtains a family of solutions parametrized by ϵ ,

$$a(t) = t^{1/\epsilon}, \quad (7)$$

$$\rho(t) = \frac{3}{8\pi\epsilon^2 t^2}. \quad (8)$$

except for $\epsilon = 0$, which corresponds to a cosmological constant Λ . In that case a solution is given by $a(t) = \exp[(\Lambda/3)^{1/2}t]$, $\rho = \Lambda/8\pi$.

³By homogeneity, all comoving observers are equivalent, so we consider an observer at $r = 0$. Any noncomoving observer whose spatial position remains finite at late times has the same horizon as a comoving observer located at the same asymptotic spatial position.

⁴This assumes that the FRW solution in question is past inextendible. Hence this analysis does not apply to the flat slicing of de Sitter space.

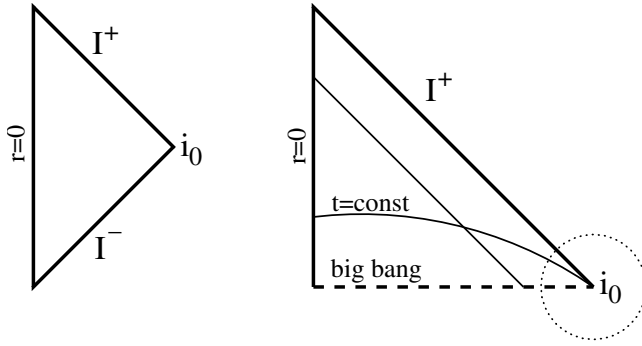


FIG. 1. Conformal diagrams of Minkowski space (left) and a decelerating flat FRW universe (right). In the FRW case, any infinitesimal neighborhood of spatial infinity (circle) contains an infinite amount of matter and potentially an infinite amount of information, whereas the observer's causal past is a finite region.

We assume the dominant energy condition, which restricts ϵ to the range $0 \leq \epsilon \leq 3$. From Eq. (4) we can see directly that for $\epsilon > 1$, the expansion of the universe decelerates: $\ddot{a} < 0$. This includes the familiar cases of matter domination ($\epsilon = 3/2$) and radiation domination ($\epsilon = 2$). For $\epsilon < 1$, on the other hand, the scale factor grows increasingly rapidly: $\ddot{a} > 0$. The degenerate case $\epsilon = 1$ will not be considered here.

As discussed more generally above, we transform to conformal time,

$$\eta = \frac{\epsilon}{\epsilon - 1} t^{(\epsilon-1)/\epsilon}, \quad (9)$$

to reveal the causal structure. For decelerating universes ($\epsilon > 1$), this expression shows that conformal time is bounded below but unbounded above. There is no future event horizon. The conformal diagram is given by the upper half ($\eta > 0$) of the Penrose diagram of Minkowski space (Fig. 1).

For accelerating universes with $0 < \epsilon < 1$, the situation is reversed. Conformal time ranges from $-\infty$ to 0, and so is bounded above but not below. Hence, the conformal diagram is the lower half of the Minkowski wedge (Fig. 2). There is a future event horizon at $r + \eta = 0$, whose area A_E grows quadratically with time. The proper horizon area-radius, $R_E = (A_E/4\pi)^{1/2}$, is given by

$$R_E = -\frac{\epsilon}{\epsilon - 1} t. \quad (10)$$

In the case of de Sitter space, $\epsilon = 0$, the metric (1) is geodesically incomplete and extendible. The maximal extension has closed spatial slices and is given by

$$ds^2 = \frac{3/\Lambda}{\sin^2 \eta} (-d\eta^2 + d\chi^2 + \sin^2 \chi d\Omega^2). \quad (11)$$

Hence, the conformal diagram is a square (Fig. 2), and de Sitter space has both past and future event horizons of

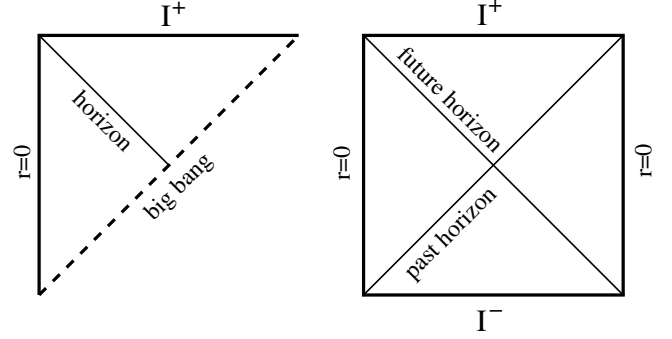


FIG. 2. Conformal diagrams of flat Q-space [13,14] (left) and de Sitter space (right). Past and future cosmological event horizons are shown. The area of the de Sitter horizon is constant, whereas the area of the Q-space horizon grows without bound at late times [13]. The Q-space initial singularity is not really null, since the curvature already becomes Planckian on a nearby spacelike slice (see Fig. 3).

constant radius $\sqrt{3/\Lambda}$.

B. Classical observable matter content

The maximum spacetime region probed by an experiment is called the causal diamond [26]. It is generally defined as the causal past of the future endpoint of the observer's world line, intersected with the causal future of the past endpoint. (Note that the latter is crucial: events lying in the observer's past but outside the bottom cone cannot be probed directly and may not send any signals in the observer's direction. If a signal is sent, then what information can be gleaned about the event is precisely what passes through the bottom cone.)

How much matter enters an observer's causal diamond? We restrict for now to the classical evolution of the cosmological fluid, and postpone the inclusion of the thermal properties of the horizon until Sec. IV.

1. De Sitter space

In eternal de Sitter space ($\epsilon = 0$), the causal diamond is the region limited by both the past and future event horizons. The maximum amount of matter that can enter is the largest black hole allowed in asymptotically de Sitter space, the Nariai black hole. Its entropy is one third that of the empty de Sitter horizon. But to arrange for matter to enter, one must either include thermal effects, or set up appropriate initial conditions in the infinite past.

It is more interesting to consider a universe such as ours, which contains an era of matter- or radiation-domination before the cosmological constant takes over. The Penrose diagram for this type of solution is shown in Fig. 3. In that case, the bottom cone of the causal diamond is the future light cone of a point at the big bang (usually called the particle horizon). Its structure depends on the details of the matter content. But as long as the universe is asymptoti-

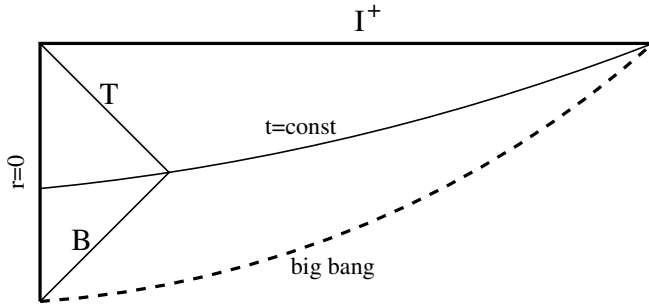


FIG. 3. This conformal diagram can be interpreted in three ways. It represents pure Q-space, with a spacelike singularity reflecting a Planck scale cutoff of the classical metric (see Fig. 2). It also corresponds to a big bang universe initially dominated by matter or radiation, which asymptotes to Q-space or de Sitter space at late times. The causal diamond of the observer at $r = 0$ is shown. The bottom cone (B) has finite maximal area, indicating that only a finite amount of entropy enters the observable region by classical evolution. In asymptotically Q-space, however, the top cone (T) allows arbitrarily large entropy. Indeed, an unbounded number of states can be accessed by quantum fluctuations of the horizon (Sec. IV D).

cally de Sitter in the future, the amount of information inside the causal patch is bounded by the entropy at late times, which is that of empty de Sitter space.

To summarize, an observer in asymptotically de Sitter space can access at most an entropy of order the inverse cosmological constant [27]. This conclusion is independent of whether thermal effects are included, and may extend to a larger class of universes with positive cosmological constant [28].⁵

2. Q-space

In an accelerating universe with $\epsilon > 0$, the largest possible causal diamond is the intersection of the past of the point $t = t_{\text{late}}, r = 0$ with the future light cone of the point $t = 1, r = 0$, in the limit $t_{\text{late}} \rightarrow \infty$ (Fig. 3). (We follow Ref. [30] in excising the high curvature region prior to the Planck time. This replaces the null singularity with a more standard, spacelike big bang.) The lower cone is again the particle horizon. The upper cone, in the limit taken, is the future event horizon (and so is a cone only conformally).

The amount of information entering the causal diamond from the past, S_{in} , is bounded by the maximal area of the lower cone [3,8]. One thus finds that

$$S_{\text{in}} \leq \pi \left(\frac{\epsilon}{1 - \epsilon} 2^{\epsilon/(1-\epsilon)} \right)^2. \quad (12)$$

Unless ϵ is very close to 1, this is at most of order unity, indicating that virtually no information enters the observ-

⁵If the requirement of a future asymptotic region dominated by the vacuum energy is dropped, examples with greater entropy are known in more than four spacetime dimensions [29].

er's causal diamond. Note that this result applies strictly to an accelerating $\epsilon > 0$ fluid with no other matter present.

The conclusion changes somewhat if other types of matter dominate at early times. If quintessence were the source of the vacuum energy in our universe, for example, our particle horizon would intersect our future event horizon about now (Fig. 3). Its maximal area would be quite large: about 10^{123} in Planck units. Still, like in de Sitter space, and unlike the decelerating universes, only a finite amount of matter and information ever enter the causal diamond by conventional evolution [31–33]. To show that Q-space exhibits unbounded complexity, one needs to include thermal fluctuations (Sec. IV).

3. Decelerating FRW

In a decelerating universe, the bottom cone extends all the way to future infinity and has infinite maximal area. There is no bound on the entropy that can enter. Indeed any comoving particle *will* enter it sooner or later. Thus, in decelerating universes any observer has access to arbitrarily large amounts of matter and entropy.

However, there is an important order-of-limits issue. Let us ask how much of the universe is seen by an observer at some finite time t . One finds that the sphere at the edge of the causal diamond has area

$$A_{\text{edge}} = \pi \left(\frac{\epsilon t}{\epsilon - 1} \right)^2. \quad (13)$$

This area is a bound on the amount of entropy that has entered the region observed by the time t . Note that this does not diverge at finite t . But finite t is all an observer can ever attain.

Hence, the number of accessible degrees of freedom is, at all times, an infinitely small fraction of the total number of degrees of freedom in the universe. This is shown in Fig. 1; only the past light cone is shown (rather than the stronger restriction to the causal diamond), since this already suffices to illustrate the problem. In Sec. V we will argue that this limitation is an important criterion distinguishing the observations made in a decelerating FRW universe from the S matrix of asymptotically flat space.

III. TEMPERATURE AND ENTROPY OF ACCELERATING UNIVERSES

In this section we obtain the basic thermodynamic properties of Q-space: entropy, energy, and temperature. We demonstrate that they satisfy the first law of thermodynamics. We begin by reviewing the thermodynamics of de Sitter space.

A. Thermodynamics in de Sitter space

De Sitter space has an event horizon of radius $R_0 = \sqrt{3/\Lambda}$. Its area is $A = 4\pi R_0^2 = 12\pi/\Lambda$. It is also a Killing

horizon with surface gravity $\kappa = R_0^{-1}$, with respect to the usual timelike Killing vector field normalized at the origin.

Consider an object of mass M in an otherwise empty asymptotically de Sitter universe. In the presence of this object, the cosmological horizon will be smaller than that of empty de Sitter space. One way to estimate its size is to model the object as a small black hole. For this case an exact solution is known: the Schwarzschild-de Sitter black holes, with metric

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_2^2, \quad (14)$$

where

$$V(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2. \quad (15)$$

For $0 < M < 1/(3\sqrt{\Lambda})$, $V(r)$ has two positive roots. (The maximal case $M = 1/(3\sqrt{\Lambda})$ is known as the Nariai solution; larger black holes do not exist in de Sitter space.) The smaller root is the black hole horizon; it obeys $R_B \approx 2M$ for small M . The larger root is the cosmological event horizon. For small M , it obeys

$$R_C^2 \approx R_0^2 - 2R_0M, \quad (16)$$

and it decreases monotonically over the whole range of M .

Now suppose that a black hole, or any other object of small mass M , falls across the cosmological horizon, restoring the observer's patch to empty de Sitter space. (This can be achieved simply by the observer moving away from the object.) By Eq. (16), this process increases the cosmological horizon area by $\Delta A = -8\pi R_0 M$. Thus, the cosmological horizon satisfies the usual first law of horizon dynamics [34]:

$$-dE = \frac{\kappa dA}{8\pi}, \quad (17)$$

where we have defined dE to be the change in the mass of the matter present on the observer's side of the horizon.⁶ As in the case of black holes, this classical relation betrays the semiclassical thermodynamic properties exhibited by the de Sitter horizon. Analysis of quantum field theory in a de Sitter background [35,36] shows that a freely falling detector will measure a temperature proportional to the surface gravity

$$T = \frac{\kappa}{2\pi}. \quad (18)$$

Moreover, in order to avoid a decrease of observable entropy in the above process, it is natural to propose that the horizon area represents a true contribution to the total

⁶In black hole mechanics [34], dE thus corresponds to the change in mass of matter remaining *outside* the black hole, which is minus the change of black hole mass, and hence is negative when matter is added to the black hole. Hence Eq. (17) takes the same form for black holes and for de Sitter space.

entropy, as originally suggested for black holes [37]:

$$S = \frac{A}{4}. \quad (19)$$

Consistency requires that these quantities satisfy the first law of thermodynamics, which is ensured by Eq. (17).

B. Thermodynamics in Q-space

Slow-roll inflation can be thought of as a de Sitter-like era with a slowly decreasing effective cosmological constant. It is well-known that the apparent cosmological horizon during inflation has thermodynamic properties akin to those of de Sitter space [35]. Indeed, this temperature is the origin of density fluctuations and so is responsible for all structure in the universe.

One would expect similar considerations to apply to an eternally accelerating universe with w sufficiently close to -1 . In this case, the universe is also locally similar to de Sitter space, with slowly decreasing vacuum energy. In fact, since a $w > -1$ fluid can be modeled by a scalar field with custom-designed potential [24], it can be thought of as a special case of slow-roll inflation. Thus, horizon thermodynamics should apply in Q-space.

We will now verify this expectation. Our arguments will be rigorous only for w very close to a cosmological constant:

$$0 < w + 1 \ll 1, \quad (20)$$

though we expect our results to be qualitatively correct at least in the range $-1 < w < -2/3$. The parameter $\epsilon = \frac{3}{2}(w + 1)$ will be small and positive for the accelerating universes studied here. However, all classical formulas below are exact in ϵ .

The radius of the event horizon was given in Eq. (10). We will also be interested in the apparent horizon.⁷ In any FRW universe, its proper radius is directly related to the energy density:

$$R_A = \left(\frac{3}{8\pi\rho}\right)^{1/2}. \quad (21)$$

For a flat universe, the apparent horizon radius is thus equal to the Hubble scale, $t_H = a/\dot{a}$, and is given by

$$R_A = t_H = \epsilon t. \quad (22)$$

The two horizons satisfy the following key properties. First, they are approximately equal in the regime we study. More precisely, the apparent horizon is smaller than the event horizon by a fixed ratio close to unity:

$$\frac{R_A}{R_E} = 1 - \epsilon, \quad (23)$$

⁷On each constant time slice, the apparent horizon of an observer at $r = 0$ is the sphere whose orthogonal ingoing future-directed light-rays have vanishing expansion.

Second, neither horizon changes significantly over one Hubble time:

$$\frac{t_H \dot{R}_X}{R_X} = \epsilon \ll 1; \quad X = A, E. \quad (24)$$

Hence, a thermodynamic description of the horizon will be approximately valid, and it will not matter much whether we use the apparent or the event horizon for this purpose.

We will work with the apparent horizon, since this approach is more general. (For example, in slow-roll inflation, there may be no event horizon, but one would still like to describe the approximate thermal state during inflation.) Hence, an observer at $r = 0$ will perceive a thermal heat bath with slowly time-dependent temperature

$$T = \frac{1}{2\pi R_A} \quad (25)$$

and will ascribe to the apparent horizon a Bekenstein-Hawking entropy

$$S = \pi R_A^2. \quad (26)$$

As a consistency check, let us verify that the first law of thermodynamics is satisfied. We follow Ref. [38], where a similar check was performed for slow-roll inflation. Consider an infinitesimal time interval dt . The amount of energy crossing the horizon during this time is obtained by integrating the flux of the stress tensor across the surface, contracted with the (approximate) generators of the horizon, the future directed ingoing null vector field k^a :

$$-dE = 4\pi R_A^2 T_{ab} k^a k^b dt = 4\pi R_A^2 \rho (1+w) dt = \epsilon dt. \quad (27)$$

In the last equality we have used Eq. (21). By Eqs. (26) and (22), the horizon entropy increases by

$$dS = (2\pi R_A) \dot{R}_A dt = (2\pi R_A) \epsilon dt. \quad (28)$$

The term in parentheses is the inverse temperature, Eq. (25). Thus we confirm the first law,

$$-dE = T dS. \quad (29)$$

IV. THERMAL FLUCTUATIONS IN ACCELERATING UNIVERSES

Both in Q-space and in de Sitter space, the thermal horizon produces fluctuations—but as we shall see in this section, their implications are quite different in the two cases. Fluctuations in de Sitter space are fatal to experiments. We show, however, that in Q-space fluctuations are benign: entropic enough to produce complex systems, but not energetic enough to destroy an observer measuring them.

A. Typical quanta

We begin by asking: What is the total number of quanta emitted by the horizon? For de Sitter as for Q-space, the expected rate is one quantum (typically with wavelength of order R_A), per Hubble time R_A . In de Sitter space, $R_A = R_0$ is a constant, and an infinite number of quanta are emitted in total. (No observer will last long enough to notice more than a finite number, however, as we shall see shortly.) In Q-space, we see from Eq. (22) that R_A grows linearly with time. However, the integrated number of quanta still diverges (though only logarithmically, not linearly as in the de Sitter case):

$$\int \frac{dt}{R_A} \sim \log t \rightarrow \infty. \quad (30)$$

What is the total energy radiated? In de Sitter space, the typical energy of each quantum is fixed, so the radiated power integrates to infinite energy, suggesting that it will erode any physical structure. Any observer in de Sitter space will be thermalized by the steady stream of radiation from the horizon.

In Q-space, the rate of emission of quanta and the energy per quantum each go like R_A^{-1} . Hence, the radiated power drops off like the inverse square of time, and it integrates to a finite total radiated energy. Quantitatively the total energy radiated after the time $t = t_0$ is

$$E \approx \int_{t_0}^{\infty} \frac{dt}{R_A} = \frac{1}{\epsilon} \int_{R_A(t_0)}^{\infty} \frac{dR_A}{R_A^2} = \frac{1}{\epsilon R_A(t_0)}. \quad (31)$$

For example, taking t_0 to be the time at which dark energy began to dominate the evolution of our universe, the total energy (to be) radiated by the cosmological horizon would be comparable to that of a single quantum with wavelength of order the present Hubble scale. Thus, the Q-space horizon falls far short of thermalizing the matter it contains, in stark contrast with the de Sitter horizon.

B. Large energy fluctuations in de Sitter space

What is the probability for a state of specified energy E to be radiated by the horizon? Aside from a slow death by thermalization, observers in de Sitter space also face the threat of collisions with objects of greater energy than the typical Hawking quanta. Though exponentially suppressed, such objects will eventually appear as rare fluctuations in the thermal spectrum. A particularly destructive example is that of a nearly maximal Schwarzschild-de Sitter black hole, which will swallow the observer.

For small energy, $E \ll R_0$, the problem is approximately equivalent to that of a hot cavity at temperature $T = (2\pi R_0)^{-1}$. The horizon provides the heat bath. For larger energies, gravitational backreaction can change the volume of the cavity and the temperature of the horizon by factors of order unity. In particular, there is a largest possible energy, corresponding to a black hole that just fits inside the cosmological horizon. We will take these finiteness

effects into account but we begin by considering small energies.

The probability to find in the cavity a particular state $|i\rangle$, of energy E_i , is given by

$$P(|i\rangle) = \frac{1}{Q} e^{-E_i/T} = \frac{1}{Q} \exp(-2\pi E_i R_0). \quad (32)$$

For a cavity with radius of order the inverse temperature (and a reasonable number of species), we can neglect factors of the partition function,

$$Q = \sum_{|i\rangle} \exp(-E_i/T), \quad (33)$$

since it is dominated by a few states of energy T and so is of order unity. Note that the probability $P(|i\rangle)$ is really a rate per time interval of order the interaction time of the heat bath, R_0 .

The probability to find an arbitrary state with energy E is larger than (32) by a factor of the number of such states, $N(E) = e^{S(E)}$:

$$\frac{P_E}{R_0} = \exp[S(E) - 2\pi E R_0]. \quad (34)$$

A de Sitter space variant of the Bekenstein bound [7,39], the D-bound [26], guarantees that the exponent will be nonpositive. For high energies compared to the thermal energy R_0^{-1} , the second term in the exponent is large. Thus the rate of the corresponding fluctuations will be exponentially suppressed, unless the entropy enhancement factor $e^{S(E)}$ nearly cancels the suppression term, leaving an exponent of order unity. We now estimate $S(E)$ to argue that this is not the case.

In a quantum field theory coupled to gravity (a description which should be locally valid at late times), the objects of highest entropy for a given energy E are either a black hole, or a radiation gas with temperature τ and radius χ such that $E \approx \chi^3 \tau^4$. (We assume that the number of species with mass less than τ is not significant, i.e., less than 10^4). The entropy of the black hole is of order E^2 . The entropy of the thermal radiation is $\chi^3 \tau^3 \approx (E\chi)^{3/4}$. This is maximized by choosing the radius occupied by the gas as large as possible, $\chi = R_0$. Thus the maximal entropy of thermal radiation is

$$S_{\text{therm}} \approx (E R_0)^{3/4}. \quad (35)$$

Whether this is larger than the black hole entropy E^2 depends on the size of the horizon.

For $R_0^{-1} \lesssim E \lesssim R_0^{3/5}$, thermal radiation wins. Hence, in this regime we obtain the following upper bound for the logarithm of the production rate:

$$S(E) - 2\pi E R_0 \leq -2\pi(1 - \delta) E R_0 \quad (36)$$

for some small number $\delta \approx (E R_0)^{-1/4} \ll 1$. At the level

of accuracy required below, the $S(E)$ term can clearly be dropped altogether ($\delta \approx 0$).

For $R_0^{3/5} \lesssim E \lesssim R_0$, a black hole dominates the ensemble. Near the lower end of this range, the black hole will have radius $R_B \approx 2E$. For larger energy, however, the backreaction on the cosmological horizon is significant, and the definition of energy itself becomes ambiguous. We will simply use the black hole radius, R_B , as an energylike parameter and abandon the estimate (34) in favor of a direct computation of the rate of black hole nucleation [40,41]:

$$\frac{P_B}{R_0} = \exp[S_{\text{SdS}}(R_B) - S_{\text{dS}}]. \quad (37)$$

$S_{\text{SdS}}(R_B)$ is the total entropy of a Schwarzschild-de Sitter geometry with a black hole of radius R_B . It is given by a quarter of the sum of the black hole and the cosmological horizon area. $S_{\text{dS}} = \pi R_0^2$ is the entropy of the empty de Sitter solution with the same cosmological constant. Einstein's equation implies for any static spherically symmetric vacuum solution [41]:

$$R_B^2 + R_C^2 + R_B R_C = R_0^2. \quad (38)$$

Here, R_C is the radius of the cosmological horizon. Hence the creation rate (37) is simply

$$\frac{P_B}{R_0} = \exp[-\pi R_B R_C]. \quad (39)$$

The exponent agrees well with Eq. (36) in a large region of overlap: For $R_B \ll R_0$, one can take $R_B \approx 2E$. Moreover, the contribution $S(E)$ from the black hole entropy is subleading.

Already the smallest black holes, with $R_B \approx 1$ and $P_B \sim \exp(-\pi R_0)$, are exponentially suppressed and thus very unlikely to arise in the thermal spectrum. At fixed cosmological constant, one finds from Eq. (38) that $R_C(R_B)$ is a monotonically decreasing function: the cosmological horizon gets smaller for larger black holes. But $R_B R_C(R_B)$ grows monotonically, so larger black holes are more and more unlikely. The biggest black hole allowed by Eq. (38) has $R_B = R_0/\sqrt{3}$ and is suppressed by $\exp(-\pi R_0^2/3)$.

However, no matter how small the rate of such fluctuations, in de Sitter space it is independent of time. Hence, even the most unlikely fluctuation will eventually occur, on a timescale of order R_0/P .

C. Large energy fluctuations in Q-space

In a $w > -1$ accelerating universe, Eqs. (32), (34), and (37) still describe the probability for the corresponding fluctuations, if we substitute R_A for R_0 . But as we shall see now, violent events of a specified energy E are not likely to ever occur at late times, no matter how long one waits.

First consider a fluctuation of less than the Planck energy, $E \leq 1$. Its rate is given by Eq. (34). Let t_0 be a

sufficiently late time so that the temperature of the horizon has become small compared to the energy of the fluctuation: $R_A(t_0) = \epsilon t_0 \gg E^{-1}$. What is the total probability $\mathcal{P}(E)$ for the fluctuation of energy E to occur after the time t_0 ?

$$\mathcal{P}(E) = \int_{t_0}^{\infty} dt \frac{P_E(t)}{R_A(t)} \quad (40)$$

$$\leq \frac{1}{R_0} \int_{t_0}^{\infty} dt \exp[S(E) - 2\pi E R_A(t)]. \quad (41)$$

$$\leq \frac{1}{\epsilon R_0} \int_{R_A(t_0)}^{\infty} dR_A \exp[-(2\pi - \delta) E R_A] \quad (42)$$

$$= \frac{1}{(2\pi - \delta)\epsilon E R_A(t_0)} \exp[-(2\pi - \delta) E R_A(t_0)]. \quad (43)$$

(Here $\delta \approx [E R_A(t_0)]^{-1/4}$.) Since $E R_A(t_0) \gg 1$, the total probability is exponentially small.

Fluctuations greater than the Planck energy cannot be considered until the horizon has grown large enough to contain a black hole of energy E . During the period $E/\epsilon \leq t \leq E^{5/3}/\epsilon$, a fluctuation of energy E is most likely to occur in the form of a black hole. But this power-law time interval is insufficient to overcome the exponential suppression in Eq. (37), so the fluctuation is extremely unlikely to occur during this period. Thereafter, the thermal ensemble begins to dominate, and the fluctuation rate is given by Eq. (34). Then the analysis of the previous paragraph applies, with $t_0 = E^{5/3}/\epsilon$. Since $E R_A(t_0)$ is again large, the integrated probability remains negligible for all times.

We conclude that large energy fluctuations inevitably occur in de Sitter space (if only after an exponentially large time), guaranteeing the destruction of any observer. In Q-space, however, the temperature falls monotonically. After it drops below a given energy E , fluctuations of that energy become virtually impossible.

D. Large entropy fluctuations in de Sitter space

What is the probability for a state of specified entropy S to be radiated by the de Sitter horizon? We have seen in the previous subsection that the probability of fluctuations is mainly determined by their energy; the entropy factor in Eq. (34) turned out to be negligible. Hence the question is, what is the lightest object with entropy S ?

For $S \leq R_0^{6/5}$ the lightest object is a thermal state with temperature $\tau \approx S^{1/3}/R_0$ and energy $E \approx S^{4/3}/R_0$.⁸ It is radiated with a probability derived from Eq. (34):

⁸At least it is the lightest object that we are sure exists. If a lighter object has the same entropy, Eqs. (44), (47), and (49) still provide lower bounds on its rate of production, leaving our conclusions intact.

$$\frac{P_S}{R_0} = \exp(S - \alpha S^{4/3}). \quad (44)$$

(Here α is a numerical coefficient involving Stefan's constant and the effective number of species with mass below τ ; for small species number, it will be on the order of 10.) For $S \geq R_0^{6/5}$ the lightest object is a black hole with radius $R_B = (S/\pi)^{1/2}$. Thus Eq. (39) implies

$$\frac{P_S}{R_0} = \exp[-(\pi S)^{1/2} R_C]. \quad (45)$$

From Eq. (38) it follows that the suppression becomes stronger if S is increased at fixed cosmological constant. Since these rates are constant, the situation is similar to the case of large-energy fluctuations: All events that can occur in de Sitter space, will occur.

However, there is an absolute entropy bound in de Sitter space [27,28]. There are no states with entropy greater than that of the horizon of empty de Sitter, πR_0^2 . This bound refers to the combined entropy of the cosmological horizon and of the matter it encloses. If we ask about the entropy only of systems contained *within* the horizon, the limit is more stringent: there can be no objects with entropy greater than that of the Nariai black hole ($\pi R_0^2/3$). Fluctuations with greater entropy cannot occur; their probability is exactly zero. This fundamentally limits the complexity and accuracy of experiments in de Sitter space.

E. Large entropy fluctuations in Q-space

In Q-space the horizon grows linearly. A fluctuation of entropy S first becomes possible (in the form of a maximal black hole) when the horizon reaches a radius of order $S^{1/2}$. Thus, the rate begins at e^{-S} , the suppression of a Nariai black hole. Thereafter the required black hole radius remains constant. But the corresponding radius of the cosmological horizon R_C increases as the effective cosmological constant decreases, according to Eq. (45). Hence the fluctuation becomes more and more unlikely. Eventually the horizon radius satisfies

$$R_A \geq S^{5/6}. \quad (46)$$

In this regime, the lightest object of entropy S is an ordinary thermal state. For all remaining time, the rate of such a fluctuation is given by Eq. (44).

What matters about this asymptotic rate is not its (miniscule) value, but that it is both constant and nonzero, however large one chooses S . It depends on R_A only in that R_A is the time interval for which P_S represents the probability of one fluctuation. Hence the integrated probability \mathcal{P} for a fluctuation of entropy S diverges logarithmically:

$$\mathcal{P}(S) = \int_{t_0}^{\infty} dt \frac{P_S}{R_A} \quad (47)$$

$$\leq \epsilon^{-1} \exp(S - \alpha S^{4/3}) \int_{S^{5/6}}^{\infty} \frac{dR_A}{R_A} \rightarrow \infty. \quad (48)$$

This is a remarkable result: objects of any complexity, no matter how large, will eventually be emitted by the horizon. The key observation is that at fixed entropy, there are many “scaling states” whose energy is inversely proportional to their linear size. As the horizon grows, these states become energetically cheaper at the same rate as the temperature drops, leaving their probability invariant. This restricts consideration to massless fields at late times.⁹

Indeed, the stronger statement holds that each scaling microstate individually is produced with certainty:

$$\mathcal{P}(|i\rangle) = \epsilon^{-1} \exp(-\alpha S^{4/3}) \int \frac{dR_A}{R_A} \rightarrow \infty. \quad (49)$$

This includes highly structured, irregular configurations.

V. ASYMPTOTIC OBSERVABLES

In this section we will compare the various cosmological solutions studied above, with an eye on the complexity and precision of measurements that can be achieved, and on the possibility of defining exact asymptotic observables or an S matrix.

By an asymptotic observable, we mean any quantity that can be measured with arbitrary precision at sufficiently late times. We expect that asymptotic observables exist only in spacetimes where experiments of arbitrarily long duration can be made and an arbitrarily large amount of entropy can be accessed. An S matrix is a special case of an asymptotic observable, consisting of matrix elements between the complete initial and final asymptotic states of a closed isolated system.

The limitations on observation discussed here are imposed by fundamental aspects of the cosmological solution, such as its causal and thermal properties, and its information content. This allows us to proceed without any assumptions about the nature of experiments or observers.¹⁰

⁹In our estimates, this restriction is implemented by using the number of *massless* species in the thermodynamic formulas for the energy and entropy of a thermal cavity. Note that other types of universes will also have only massless particles at late times, if massive particles are unstable or processed by black holes.

¹⁰Additional restrictions may arise, for example, from a limited supply of free energy or inability to harvest this energy for experiments (see, e.g., Refs. [4,42–44]). To the extent that they are insurmountable, they may further constrain the asymptotic observables.

A. De Sitter

Asymptotically de Sitter spacetimes ($w = -1$) are particularly hostile to observers. There is no S matrix, since the observer’s causal diamond misses almost all of the asymptotic regions, $\eta \rightarrow 0$ and $\eta \rightarrow \pi$ in the global metric (11), on which the global in and out-states might be defined. An S matrix between such states would, at best, be a “meta-observable” [4]: It would relate a state no one can set up (because of the past event horizon $\eta = \chi$) to a state no one can measure (because of the future event horizon $\eta = \pi - \chi$). This conclusion does not improve if the past asymptotic region is replaced by a big bang; this only trades the past event horizon for a particle horizon, and does not affect the future event horizon.

Nor are there any other asymptotic observables in asymptotically de Sitter space. The total accessible entropy is bounded (Sec. II B), and the duration of any experiment fundamentally limited by thermal erosion (Sec. IV A) and by collisions with black holes (Sec. IV B).

B. Q-space

Like de Sitter space, quintessence dominated universes ($-1 < w < -1/3$) have a cosmological event horizon [13,14] (see Sec. II). Hence, the global state in the asymptotic future cannot be measured, and there is no S matrix.

However, some other asymptotic observables may well exist. We have seen in the previous section that Q-space is significantly more welcoming to physicists than de Sitter space. Thermal fluctuations¹¹ are present but are too weak to terminate experiments by erosion (Sec. IV A) or by black hole production (Sec. IV C). Because the cosmological horizon becomes arbitrarily large [13], there is no absolute entropy bound. What we showed in Sec. IV E is that an unbounded number of different states are actually produced at late times. Thus, observers can experience arbitrarily complex events (and, one might imagine, store large amounts of information for long times).¹²

C. Decelerating FRW

Decelerating universes ($-1/3 < w < 1$) clearly satisfy important conditions for the existence of asymptotic observables, as noted by several authors [4–6,13,14]. As the particle horizon grows, the amount of entropy allowed in the causal diamond increases without bound, as does its

¹¹We should emphasize again that the thermal properties of Q-space discussed in Secs. III and IV were rigorously derived only in the limit of small ϵ ($w \rightarrow -1$). But we expect no qualitative transitions at least in the range $-1 < w < -2/3$.

¹²Whether these fluctuations, which involve only massless fields, can give rise to an apparatus capable of precise measurements is another question, and we do not claim to have proven that this will happen.

actual matter content (Sec. II B). This may include massive particles, if they are stable and if they are not converted to radiation by black holes.

But is there an S matrix? At first sight, the situation looks promising. There is no future event horizon (Sec. II). Every timelike geodesic eventually enters the causal diamond of the observer. But this does not mean that the global state of the universe is observable.

At any finite time, only a finite portion of the universe is in the observer's causal diamond, by Eq. (13). Beyond lies a noncompact region, which has *at all times* infinite volume (as measured on the homogeneous spacelike slices). This unobserved region contains an infinite amount of matter and, potentially, an infinite amount of information. Thus, the decelerating universe never reveals more than an infinitely small fraction of itself to the observer (see Fig. 1).

Whether all or only part of a system is measured, makes an enormous difference. Page [45] has shown that in order to obtain at least 1 bit of information about a system in a typical pure state, one must perform a measurement on more than half of the degrees of freedom constituting the system. Thus, even a measurement of one half of a finite system by the other half will reveal practically no information whatsoever about the global state. The situation in a flat FRW universe is far more problematic yet: the number of degrees of freedom available for measurement are finite, and the total system is infinite.

Let us compare this to a real S matrix experiment, such as the scattering of particles in an accelerator. Here, the entire system hits the detector by some finite time.¹³ The key difference is that in asymptotically flat space, there exists a region near spatial infinity (i.e., outside a sufficiently large sphere) that is devoid of matter and energy. Entropy bounds, such as the Bekenstein bound and the generalized covariant bound [7,11,46], imply that this region contains no information. In a FRW solution, on the other hand, the density at fixed time is asymptotically constant and nonzero. In this case, entropy bounds permit an arbitrarily large information content. Therefore, the asymptotic structure of a FRW universe does not guarantee that an S matrix exists.

The situation could be improved by restricting to a set of states such that the (infinite) exterior of some finite region contains either no information or only redundant information. (See, e.g., Ref. [6] for an approach to constructing an appropriate reference state.) We emphasize that this is a strong additional constraint. The appropriate states would form a set of measure zero in the Hilbert space of states of

¹³The fact that the complete system is observable does not rely on any limiting procedure. The usual limit of late times and large detectors is taken only in order to refine the separation of particles and make sure that they have stopped interacting. This is a separate requirement related to our preference for expressing the out-state in a convenient Hilbert space basis (the Fock space of the noninteracting theory).

the FRW universe. It is possible, but not obvious, that suitable states are selected as initial conditions by theory.

D. Discussion: Cosmology vs the S matrix

We have found that the entropy of observable matter is unbounded in any flat FRW universe dominated by a $w > -1$ fluid in the asymptotic future, accelerating or not. In particular, we conclude that a future event horizon does not in itself impose a significant restriction. Its absence is neither necessary for the existence of asymptotic observables nor sufficient for the existence of an S matrix. Indeed, we argued that an S matrix is not a natural observable in any of the cosmologies considered.

We based our argument on the combination of two observations: the late-time global state of the universe is never fully contained in any observer's causal past; and unlike asymptotically flat space, the unobserved portion of a cosmological universe can contain information—in the case of decelerating universes, it contains an infinite number of degrees of freedom. Page's theorem [45] then implies that no information can be gleaned about a generic global pure quantum state, if by information we mean finding a density matrix of submaximal entropy.

This is just a particularly bad version of a more general problem that arises whenever one part of a closed system measures another part. This includes any measurement of the global state of the universe, independently of causal restrictions. Obviously, the apparatus must have at least as many degrees of freedom as the system whose quantum state it attempts to establish (in practice it usually has orders of magnitude more). This means that at most half of the degrees of freedom can be observed, just below the Page cutoff for obtaining the first bit of information about the complete system.

Aside from the problem of measuring the out-state, an S matrix description of cosmology is emptied of operational meaning by our inability to control the initial state and to repeat experiments that extend over cosmological time and distance scales.

It is conceivable that these problems could be circumvented.¹⁴ Suppose, for example, that theory restricts to states with a high degree of spatial symmetry and no entanglement between distant degrees of freedom. In an infinite universe without event horizons, it might then be possible to perform independent but equivalent measurements on an arbitrary number of disentangled identical subsystems. At all times, however, the construction of a global state would still require infinite extrapolation. In any case, this procedure will not resemble an S matrix experiment.

If not an S matrix, what asymptotic observables should one expect to find? In any large spacetime, local high-energy scattering experiments will admit an S matrix de-

¹⁴I would like to thank L. Susskind for discussions of this issue.

scription to a good approximation. But we are interested in measurements probing the state and the evolution of the universe. We are fortunate to inhabit a fairly symmetric (part of the) universe, and we can learn aspects of its early quantum fluctuations by measuring correlations in the cosmic microwave background. They are limited by cosmic variance, but as our horizon grows we can obtain more and more data points. In decelerating universes such correlations may become exact quantum observables in the asymptotic future. In accelerating universes, one would measure correlations in fluctuations of the approximately thermal radiation from the horizon.

Whether or not asymptotic observables exist, they do not correspond precisely to the experiments we perform *today*. Even in an asymptotically empty spacetime (suppose, for instance, that our universe is a mere resonance in a giant scattering event in asymptotically flat space), one would require approximate local observables to describe measurements by parts of the system on one another at finite time. They would seem just as likely to be computable from an initial state directly, than to be derived from S matrix. (Indeed, the former option has a chance of applying in general spacetimes, whereas the latter requires assumptions about the asymptotic structure.) We do not even know whether such bulk observables are quantum mechanical.

VI. OTHER UNIVERSES

The discussion above was restricted to flat FRW universes, which may admit some asymptotic observables though not, in any operational sense, an S matrix. In this section, we extend the discussion to other universes. We focus, in particular, on the Farhi-Guth solution, which connects a cosmological region, through the interior of a black hole, to an asymptotically flat or AdS spacetime, raising anew the question of S-matrix observables for cosmology.

A. Crunching universes

Many other cosmological solutions do not admit asymptotic observables at all. This includes all universes with a big crunch, such as closed FRW solutions with decelerating matter content. Open and flat FRW solutions can also crunch, even if they are initially expanding, if they contain negative vacuum energy [30]. In this case, spatial infinity exists. However, the largest causally accessible region has finite entropy [3,9], so there will be no exact observables.

There are also spacetimes that do not crunch globally but in which all observers must end up inside a black hole. In a dust-dominated flat universe, for example, gravitational collapse can occur at arbitrarily large scales. Our own universe was probably produced by a period of inflation. Then the assumption of an infinite homogeneous flat FRW universe is not appropriate at very large scales. Assuming a generic chaotic inflation potential, the density fluctuations produced by inflation grow logarithmically with the scale.

Exponentially many years from now, the fluctuations entering the horizon will be of order unity. On some scales this will lead to large voids, on other scales to overdensities. Sooner or later, any given observer will find themselves in a large region bound to collapse into a black hole.

B. Asymptotically expanding, nonflat FRW universes

Open universes have a similar causal structure to flat universes. The main difference is that in the accelerating (decelerating) cases, the future conformal boundary is isomorphic only to a portion of the infinity of de Sitter (Minkowski) space. Hence, the spatial infinity of an open universe is a conformal two-sphere rather than a point.

The discussion of observables in flat FRW universes, in Sec. V, applies to open universes as well. In the accelerating case ($w < -1/3$), this is because the spatial curvature vanishes at late times anyway. In the decelerating case, asymptotic observables may exist, since the particle horizon diverges; but the global state of the universe is again unobservable. Closed Q-space also asymptotes locally to flat Q-space, as the curvature inflates away.

C. Coleman-De Luccia

One might hope to gain control over a poorly behaved universe, such as de Sitter space, by grafting onto it another solution which possesses asymptotic observables or perhaps even an S matrix. An example of such a hybrid is the Coleman-De Luccia (CDL) solution [47], which describes a bubble of true vacuum expanding inside a false vacuum. Of particular interest is the case where the true vacuum energy vanishes (so the false one is positive). The expanding portion of this solution describes an open expanding FRW universe joined to a metastable de Sitter space (Fig. 4). Unlike pure de Sitter space, which appears to

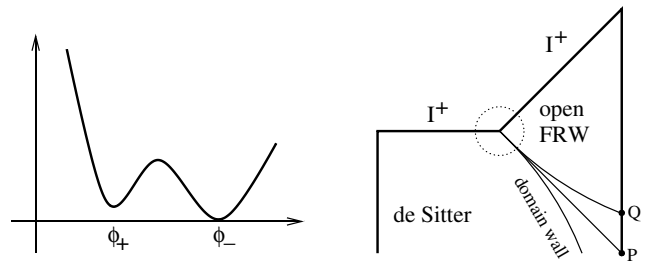


FIG. 4. A scalar field potential involving a false vacuum (left) gives rise to the Coleman-De Luccia solution, which describes a de Sitter region joined to an open FRW universe, in our example with vanishing vacuum energy. A conformal diagram of the expanding portion of this spacetime is shown on the right. Some examples of the orbits of the symmetry group $O(3,1)$ are shown: the domain wall, the light cone starting at P , and the hyperbolic time slice that includes Q . The region enclosed in the dotted circle is asymptotically identical with a corresponding region of the Farhi-Guth solution (Fig. 5).

allow no exact observables, the CDL solution should therefore admit the same asymptotic observables that can be defined in an open FRW universe.¹⁵

It is important that the $\Lambda = 0$ region of the CDL solution is an FRW cosmology [47] and not, as one might have expected, empty Minkowski space. That is, it contains infinite hyperbolic slices with constant positive energy density. For later use, we briefly review where this energy comes from.

The true and false vacuum can be modeled by a scalar field potential with a local minimum at ϕ_+ and a global minimum at ϕ_- (Fig. 4). The domain wall of the CDL solution is a spherical shell in which the field ϕ crosses the barrier between the vacua. Consider the closed, time-symmetric slice on which the domain wall radius takes its minimum value. If the energy of the false vacuum is small compared to the height of the barrier, then the thickness of the domain wall will be small compared to its radius. This limit is known as the thin-wall approximation.

Inside the wall, the field value is approximately ϕ_- , differing from the exact vacuum value only by an amount exponentially small in the distance from the wall. The wall is only of finite size, so at its center (at the event P in Fig. 4) the field value ϕ_0 still differs from ϕ_- by an exponentially small amount. (This can be avoided only by infinite fine-tuning of the potential.) Hence, the energy density at P is not exactly zero. By continuity, the energy density will also be nonzero at some point Q infinitesimally later than P . The $O(3, 1)$ symmetry of the CDL solution guarantees that Q is equivalent to all other events on the spatial hyperbolic slice generated as its orbit. Hence, the entire infinite slice has constant positive energy density. This is what distinguishes a cosmology from asymptotically flat space.

By the same token, if the true vacuum has negative cosmological constant, the bubble interior will not be anti-de Sitter space. It will again be an open FRW solution [47,48]. Like any FRW solution with negative cosmological constant and an admixture of $w > -1/3$ matter [30], it will only expand for a finite amount of time and then collapse in a big crunch.

D. Farhi-Guth

The Coleman-DeLuccia solution is a limiting case of a larger class of domain wall solutions with spherical symmetry, found by Blau, Guendelman, and Guth [49] (see also Refs. [50,51]). This class includes another composite

¹⁵One can also consider the fully extended CDL solution, which contains a collapsing FRW universe in the past. Freivogel and Susskind [6] recently proposed that asymptotic observables defined in the two FRW regions encode information not only about the metastable de Sitter region, but even about other stable and metastable vacua [17,18] in causally disconnected regions. This raises a number of subtle questions [22] which are outside the scope of this paper.

cosmology, the Farhi-Guth solution,¹⁶ which does contain a true asymptotically flat region. This suggests that it may allow the description of cosmology using an S matrix. A similar solution can be constructed with anti-de Sitter asymptotics; it has been suggested that aspects of the cosmological regions could thus be described via the AdS/CFT correspondence [53].¹⁷

This possibility has met with some scepticism (see, e.g., Ref. [48]). Here we demonstrate a feature of the Farhi-Guth solution which has not, to our knowledge, been previously noted in the literature: the fact that it contains an open, asymptotically FRW universe with a black hole. We will argue that this exacerbates the difficulties with using the Farhi-Guth solution for an S matrix description of cosmology.

1. Global structure

The Farhi-Guth solution describes an expanding bubble of de Sitter space topologically “inside” an asymptotically flat universe. The only way [52] this can be achieved is to place the de Sitter bubble on the far side of a black hole/white hole region, as shown in Fig. 5. To allow for quick orientation, let us call this the “cosmological side,” separated by an Einstein-Rosen bridge from the “asymptotic side.” Points on opposite sides are necessarily spacelike separated, so one cannot travel between them. Note that the cosmological side is very similar to the CDL solution: it contains a metastable de Sitter region separated by a domain wall from a region of vanishing cosmological constant.

At late times, the Farhi-Guth bubble grows large, and its wall will be far from the black hole. Thus, it can be expected to behave asymptotically like the CDL domain wall. We will now verify this. The dynamics of the Farhi-Guth wall is governed by the Israel junction conditions, which yield the equation [55]

$$\left(\frac{dr}{d\tau}\right)^2 + V(r, q) = -1. \quad (50)$$

Here, r is the radius of the bubble, τ is the proper time on the bubble trajectory, and q is the radius of the black hole. The potential $V(r, q)$ is given by

$$V(r, q) = -\frac{q}{r} - \frac{[q - (\chi^2 + \kappa^2)r^3]^2}{4\kappa^2 r^4}, \quad (51)$$

where χ is the Hubble scale of the metastable de Sitter region, and $\kappa/4\pi$ is the surface tension of the domain wall.

¹⁶We refer to it by the authors of Ref. [52], who investigated features of this solution, to distinguish it from the larger class of which it is a special case.

¹⁷I have enjoyed discussions with B. Freivogel, V. Hubeny, M. Rangamani, and S. Shenker, who are independently investigating questions that overlap with some of the topics in this subsection [54].

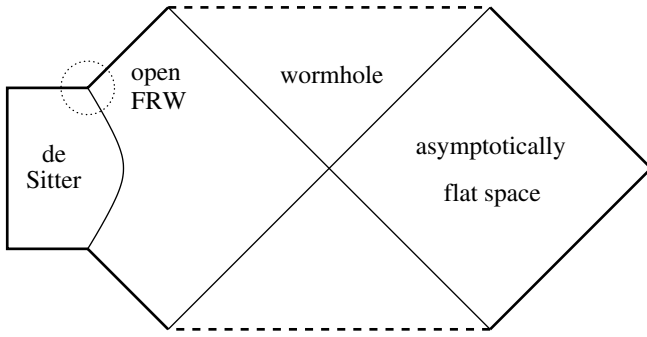


FIG. 5. Conformal diagram of the fully extended Farhi-Guth solution. In the text it is shown that the region marked by the dotted circle agrees asymptotically with the marked region in Fig. 4; this implies that it contains an open universe. The above diagram corresponds to the case of vanishing cosmological constant in the true minimum; for negative cosmological constant, the same argument shows that the open universe crunches.

These latter quantities are determined by the shape of the scalar field potential.

The CDL solution corresponds to setting $q = 0$, so $V(r) = -\frac{(\chi^2 + \kappa^2)^2 r^2}{4\kappa^2}$. This admits a growing and a decaying exponential solution. The particular, time-symmetric initial conditions of CDL select the linear combination

$$r = r_0 \cosh \frac{\tau}{r_0}, \quad (52)$$

where $r_0 = 2\kappa/(\chi^2 + \kappa^2)$.

The Farhi-Guth solution is more complicated but we are interested only in the large radius limit. For $r \rightarrow \infty$ one finds that $V(r, q) \rightarrow -\frac{(\chi^2 + \kappa^2)^2 r^2}{4\kappa^2}$, which coincides with the $q \rightarrow 0$ limit. Hence, growing bubbles are all attracted to the CDL solution at late times, independently of the black hole mass:

$$r \rightarrow C \exp \frac{\tau}{r_0}, \quad (53)$$

Differences in the prefactor can be absorbed into a shift of the time variable τ .

This universality has an important consequence: the formation of an expanding FRW universe outside the bubble will also be universal. In the CDL case, we exploited the full $O(3, 1)$ symmetry to show that the domain wall dumps constant energy density into the hyperbolic slices. The details of this mechanism, and the character of the matter it produces, will vary depending on the potential and couplings. The Farhi-Guth solution has only spherical symmetry. But at large radius the initial conditions for the hyperbolic slices are provided mainly by the domain wall. They are identical to those in the CDL solution, so the same mechanism will operate, creating asymptotically hyperbolic slices with asymptotically constant, positive energy density.

Hence, the region of vanishing vacuum energy on the cosmological side of the Farhi-Guth solution is not empty flat space but is again a cosmology. It is asymptotic to an open FRW universe. The only difference to the CDL solution is that the asymptotic FRW universe contains a black hole, through which it connects to an asymptotically flat region.

2. Discussion

The Farhi-Guth solution has it all: an asymptotically flat or AdS region, metastable de Sitter space, a black hole, an open FRW cosmology, and in the case of AdS asymptotics, even a big crunch. Could it be that all these interesting regions are described by an S matrix, or conformal field theory (CFT) correlators, defined on the asymptotic side?¹⁸

The region behind the horizon is causally inaccessible to an observer at infinity. To argue that information about the cosmological side can be retrieved on the asymptotic side, one would have to appeal either to black hole complementarity [53] or to subtle effects of analyticity [15,16]. But complementarity is a stronger conjecture here than in the case of classical black hole formation, since the matter on the cosmological side was never present on the asymptotic side.

Let us restate this in the language of holographic screens [9]. For a black hole formed in asymptotically flat space by the collapse of matter, the past null infinity is a screen whose light-sheets reach inside the black hole, covering all of the spacetime. They are appropriate for a description of the infalling observer; all information that went into the black hole can unambiguously be stored there. If this trivial fact was not true, then the assumption that the S matrix is unitary would not require that the same information be present at future infinity, and we would not be led to complementarity. In the Farhi-Guth solution, the light-sheets off of the asymptotic boundary enter the black hole/white hole region, but they cannot reach into the cosmological regions on the far side of the black hole.

A signal that complementarity may not work here is the fact that the black hole area can be made arbitrarily small, while the de Sitter region on the cosmological side can have large entropy. This contrasts with the usual case, where the black hole area grows large in response to matter crossing the horizon, and becomes small only as it returns matter to the asymptotic region in the form of Hawking radiation.

The infinite open FRW universe, which we have argued is always present on the cosmological side, exacerbates the entropy mismatch, leaving little hope that it could be resolved by some unknown macroscopic constraint on the solutions. The cosmological side would have to be in one of a small number of very special microstates, if it were to be described by boundary data defined on the asymptotic

¹⁸See Refs. [21,48] for other discussions of this issue.

side. Another logical possibility would be that the boundary theory makes more degrees of freedom available for the cosmological side of the black hole than it does for the black hole itself; this would lead to a breakdown of the UV/IR correspondence of AdS/CFT [56].

These entropic considerations are complemented by an aesthetic objection: If nature had wanted us to use a boundary theory to describe cosmology, it would have given the universe a nicer boundary. After all, the Farhi-Guth solutions are rather artificial constructs. Their description reads like a cocktail recipe: A de Sitter region separated by a domain wall from an open universe containing an eternal black hole, on the far side of whose Einstein-Rosen bridge resides the desired asymptotically flat region. (The idea of the Einstein-Rosen bridge is logically independent of the de Sitter region; for example, one could connect any FRW universe to an asymptotic region this way.)

Unlike the CDL solution, which arises naturally from the decay of false vacuum, spacetimes containing an eternal black hole are not of obvious physical relevance. They cannot be created classically from regular initial conditions [52]. It has been argued that the Farhi-Guth geometry

might arise semiclassically [55,57,58] from a small bubble of false vacuum, but no regular instanton exists for this process. Moreover, it is not clear how an observer would distinguish this type of transition from other exponentially rare events resulting in the spontaneous formation of a black hole.

In summary, it is questionable whether Farhi-Guth solutions exist in a full quantum theory of gravity; and even if they do, holographic considerations suggest that the asymptotic and the cosmological side will be completely decoupled. That said, we are unable to rule out that some aspects of cosmological evolution are encoded in boundary data via the Farhi-Guth solution.

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