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# Einstein-Yang-Mills equations in the presence of q-stars in scalar-tensor gravitational theories

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We study Einstein-Yang-Mills equations in the presence of a gravitating nontopological soliton field configuration consisting of a Higgs doublet, in Brans-Dicke and general scalar-tensor gravitational theories. The results of General Relativity are reproduced in the $\omega_{\rm BD}, \omega_0 \rightarrow \infty$ limit. The numerical solutions correspond to a soliton star with a non-Abelian gauge field. We study the effects of the coupling constant, the frequency of the Higgs field, and the Brans-Dicke field on the soliton parameters.

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## I. INTRODUCTION

Einstein-Yang-Mills (EYM) equations have been investigated in several systems [1]. Bartnik and McKinnon found particlelike non-Abelian solutions of the coupled EYM theory [2,3]. The coupling of the EYM system with a scalar field may lead to several theories. We mention gravitating Skyrmions [4], black hole solutions in dilaton [5,6], massive dilaton and axion gravity [7], and other field configurations in the EYM-Higgs theory with a Higgs doublet [8] or with a Higgs triplet [9–11].

Nontopological solitons and soliton stars have a long presence in modern physics [12]. In the present work we need the theory of q-balls and q-stars. Q-balls are nontopological solitons in Lagrangians with a global $U(1)$ symmetry [13] or an $SU(3)$ or $SO(3)$ symmetry [14]. Their relativistic generalizations may consist of one or two scalar fields [15] in a Lagrangian with a global $U(1)$ symmetry, or of a non-Abelian scalar field in the adjoint representation of $SU(3)$ [16] or of a scalar and a fermion field [17] in asymptotically flat or anti de Sitter spacetime [18]. Q-solitons with local symmetries have also been investigated. There are charged q-balls [19], charged q-stars [20], and q-type stars in the framework of EYM theory [21].

Interesting alternative gravitational theories are the scalar-tensor gravitational theories, which appeared in the original paper of Brans and Dicke [22], where the Newtonian constant $G$ was replaced by a scalar field $\phi_{\rm BD}$, and the total action contained kinetic terms for the new field times an $\omega_{\rm BD}$ quantity. $\omega_{\rm BD}$ was regarded as a constant in the original paper. The theory generalized in a series of papers [23,24], mainly in the direction of replacing the constant $\omega_{\rm BD}$ with a function of the Brans-Dicke (BD) scalar field. Within the BD gravitational framework, Gunderson and Jensen investigated the coupling of a scalar field with quartic self-interactions with the metric and the BD scalar field, $\phi_{\rm BD}$ [25]. The properties of boson stars within this framework have been extensively studied in a series of papers [26–28]. Their results generalized in scalar-tensor gravitational theories [29,30]. The case of charged boson stars in a scalar-tensor gravitational theory has been analyzed in [31].

The purpose of the present work is to find numerical solutions, resembling q-stars, of the EYM equations in the presence of a Higgs doublet in the fundamental representation of $SU(2)$ in BD theory or in a general scalar-tensor gravitational theory and to compare the above results with the solutions obtained in General Relativity. In the absence of the gauge field, the equations of motion give rise to a gravitating nontopological soliton, when using a certain potential for which $\omega_E \equiv \sqrt{U/|\Phi|^2}_{\rm min} < m$, where $m$ is the mass of the free particles, imposing a harmonic time dependence on the scalar field and equalizing the frequency to $\omega_E$. Our gravitating soliton is *nontopological* in the sense that $\Phi, U \rightarrow 0$ for $\rho \rightarrow \infty$ according to [12]. It is a q-type nontopological soliton in the sense that in the absence of both gravitational and gauge fields one can find by simple calculations that this spherically symmetric Higgs field rotates within its symmetry space with a frequency $\omega_E$ equal to the minimum of the $\sqrt{U/|\Phi|^2}$ quantity, as in q-balls. The difference between this soliton and the usual non-Abelian q-balls is that the symmetry space in the case of non-Abelian q-balls is the entire $SU(3)$ space but in our case is an Abelian $U(1)$ subgroup of the $SU(2)$ group, though both field configurations are non-Abelian. Also, we investigate the fundamental and not the adjoint representation of $SU(2)$. In any case we find an analytical solution for the scalar field within the soliton, using the approximation known by the study of q-stars.

## II. EYM EQUATIONS IN BRANS-DICKE GRAVITATIONAL THEORY

We consider a Brans-Dicke scalar field $\phi_{\rm BD}$, a matter Higgs scalar doublet $\Phi$ in the fundamental representation of $SU(2)$, and a gauge field $A$, coupled to the metric $g_{\mu\nu}$. The total action is:

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$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi_{BD}R - \omega_{BD}g^{\mu\nu}\frac{\partial_\mu\phi_{BD}\partial_\nu\phi_{BD}}{\phi_{BD}}\right) + \int d^4x\sqrt{-g}\mathcal{L}_{matter}, \quad (1)$$

with $\omega_{BD}$ a constant in BD gravity and a certain function of the $\phi_{BD}$ field in generalized scalar-tensor gravitational theories and:

$$\mathcal{L}_{matter} = \frac{1}{4Kg^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)^\dagger(D^\mu\Phi) - U. \quad (2)$$

In the above Lagrangian we define:

$$D_\mu\Phi = \partial_\mu\Phi - \imath A_\mu\Phi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \imath[A_\mu, A_\nu]. \quad (3)$$

One may use the $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ form and reproduce Eq. (3) rescaling: $A_\mu \to A_\mu/g$ with g the gauge coupling, or field strength. The oneform gauge field $A$ is: $A \equiv A_\mu dx^\mu \equiv \mathbf{T}_a A^a_\mu dx^\mu$, with $\mathbf{T}_a = \frac{1}{2}\tau_a$ and $\tau_a$ the Pauli matrices. The factor $K$ appearing in the action is defined by the relation $\mathrm{Tr}(\mathbf{T}_a\mathbf{T}_b) = K\delta_{ab}$, reading $K = 1/2$.

In the presence of the Brans-Dicke scalar, the Einstein equations take the form:

$$G_{\mu\nu} = \frac{8\pi}{\phi_{BD}}T_{\mu\nu} + \frac{1}{\phi_{BD}}(\phi_{BD,\mu;\nu} - g_{\mu\nu}\phi_{BD;\lambda}{}^{;\lambda}) + \frac{\omega_{BD}}{\phi^2_{BD}} \times\left(\partial_\mu\phi_{BD}\partial_\nu\phi_{BD} - \frac{1}{2}g_{\mu\nu}\partial_\lambda\phi_{BD}\partial^\lambda\phi_{BD}\right), \quad (4)$$

and the equation of motion for the BD field is:

$$\frac{2\omega_{BD}}{\phi_{BD}}\phi_{BD;\lambda}{}^{;\lambda} - \omega_{BD}\frac{\partial^\lambda\phi_{BD}\partial_\lambda\phi_{BD}}{\phi^2_{BD}} + R = 0. \quad (5)$$

$G_{\mu\nu}$ is the Einstein tensor, $R$ is the scalar curvature, and $T_{\mu\nu}$ the energy momentum tensor for the matter fields (gauge and Higgs) given by the equation:

$$T_{\mu\nu} = \frac{2}{g^2}\mathrm{Tr}\left(g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right) + (D_\mu\Phi)^\dagger(D_\nu\Phi) + (D_\mu\Phi)^T(D_\nu\Phi)^* - g_{\mu\nu}[g^{\alpha\beta}(D_\alpha\Phi)^\dagger(D_\beta\Phi)] - g_{\mu\nu}U. \quad (6)$$

Tracing Einstein equations and substituting the result in the Lagrange equation for the BD field we take:

$$\phi_{BD;\lambda}{}^{;\lambda} = \frac{8\pi}{2\omega_{BD} + 3}T. \quad (7)$$

We will choose a general, spherically symmetric field configuration, defining: $n^a \equiv (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$ and $\mathbf{T}_\rho = n^a\mathbf{T}_a$, $\mathbf{T}_\vartheta = \partial_\vartheta\mathbf{T}_\rho$, and $\mathbf{T}_\varphi = (1/\sin\vartheta)\partial_\varphi\mathbf{T}_\rho$. The gauge field and the Higgs doublet are:

$$A = a\mathbf{T}_\rho + \imath(1 - \mathrm{Re}\omega)[\mathbf{T}_\rho, d\mathbf{T}_\rho] + \mathrm{Im}\omega\mathbf{T}_\rho = a\mathbf{T}_\rho + [\mathrm{Im}\omega\mathbf{T}_\vartheta + (\mathrm{Re}\omega - 1)\mathbf{T}_\varphi]d\vartheta + [\mathrm{Im}\omega\mathbf{T}_\varphi + (1 - \mathrm{Re}\omega)\mathbf{T}_\vartheta]\sin\vartheta d\varphi, \quad (8)$$

$$\Phi = \sigma\exp(i\xi\mathbf{T}_\rho)|b\rangle, \quad (9)$$

with $\sigma = \sigma(\rho, t)$, $\xi = \xi(\rho, t)$, $|b\rangle$ a constant unit vector of the internal $SU(2)$ space of the scalar (Higgs) field and $a = a_0dt + a_\rho d\rho$. In order to form a field configuration corresponding to a charged q-type soliton we choose: $a_0 = a_0(\rho)$, $\sigma(\rho, t) = \sigma(\rho)$, and $\xi = \omega_E t$, when we choose $a_\rho = 0$ for simplicity. The ansatz $\sigma(\rho, t) = \sigma(r)$ and $\xi = \omega_E t$ is the obvious generalization to the $\phi(\rho, t) = \sigma(\rho)e^{\imath\Omega t}$ ansatz, known from q-solitons. So, the role of the eigenfrequency $\Omega$ is now played by $\omega_E$. The choice $a_\rho = 0$ implies that our field configuration is not very general, but our purpose is not to find the more general solution, but a proper one, with the above features and resulting to stable solitons. The BD scalar field is supposed to be static and to posses a spherical symmetry, as the matter field configuration. With the above assumptions, we can write a static, spherically symmetric metric:

$$ds^2 = -\frac{1}{B}dt^2 + \frac{1}{A}d\rho^2 + \rho^2d\vartheta^2 + \rho^2\sin^2\vartheta d\varphi^2. \quad (10)$$

The matter action takes now the form:

$$S_{matter} = \int\frac{\rho^2\sin\vartheta}{\sqrt{AB}} \times\left[-\frac{1}{2g^2}\left(a_0'^2AB + 2\frac{|\omega|^2a_0^2}{\rho^2} + \frac{(|\omega|^2-1)^2}{\rho^4}\right) + \sigma'^2A - \frac{1}{4}(\omega_E - a_0)^2\sigma^2B + \frac{\sigma^2}{2\rho^2} \times[(\mathrm{Re}\omega - \cos(\omega_Et))^2 + (\mathrm{Im}\omega - \sin(\omega_Et))^2] - U\right]. \quad (11)$$

In order for the action to be time-independent we may choose $\mathrm{Re}\omega = \cos(\omega_Et)$ and $\mathrm{Im}\omega = \sin(\omega_Et)$, but this choice is not a solution to the equation of motion for $\omega$, or $\omega = 0$ which is a solution of the equation of motion, so our solution is embedded Abelian.

If $m$ is the mass of the free particles, we make the following rescalings:

$$\tilde{\rho} = 2m\rho, \quad \tilde{\omega}_E = \frac{\omega_E}{2m}, \quad \tilde{a}_0 = \frac{a_0}{2m}, \quad \tilde{\sigma} = \frac{\sigma}{m/2}, \quad \tilde{r} = \epsilon\tilde{\rho}, \quad \tilde{g} = g\epsilon^{-1}, \quad \Phi_{BD} = \frac{2\omega_{BD} + 4}{2\omega_{BD} + 3}G\phi_{BD}, \quad (12)$$

with:

$$\epsilon \equiv \sqrt{8\pi G m^2}. \tag{13}$$

We define:

$$W = \left(\frac{d\Phi}{dt}\right)^\dagger \left(\frac{d\Phi}{dt}\right), \qquad V = \left(\frac{d\Phi}{d\rho}\right)^\dagger \left(\frac{d\Phi}{d\rho}\right). \tag{14}$$

Gravity becomes important when $R \sim 8\pi GM$, with $<\phi_{BD}> = 1/G$. With our rescalings and because the energy density within the soliton is $\sim m^4$, we find that $\tilde{r} \sim 1$ and if $\sigma$ varies very slowly within the soliton, from a $\sigma(0)$ value at $\tilde{r} = 0$ to a zero value at the outer edge of the soliton surface, then $V \sim \epsilon^2 m^4$. For $m \sim$ GeV the $O(\epsilon)$ quantities are negligible. We choose a simple rescaled potential, admitting q-ball type solutions in the absence of gravity and gauge fields, namely:

$$U = m^2 \Phi^\dagger \Phi \left(1 - \frac{4}{m^2}\Phi^\dagger\Phi + \frac{16}{3m^4}(\Phi^\dagger\Phi)^2\right), \tag{15}$$

which with our rescalings and after some algebra takes the form:

$$\tilde{U} = \frac{\tilde{\sigma}^2}{4}\left(1 - \tilde{\sigma}^2 + \frac{\tilde{\sigma}^4}{3}\right), \tag{16}$$

where we set $m = 1$. From now on we drop the tildes and the $O(\epsilon)$ quantities. From the equation of motion for the Higgs field we find:

$$\begin{aligned} &\sigma^2 = 1 + \theta_0 B^{1/2}, \qquad U = \frac{1}{12}(1 + \theta_0^3 B^{3/2}), \\ &W = \theta_0^2 B(1 + \theta_0 B^{1/2}), \qquad T = 2W - 4U, \end{aligned} \tag{17}$$

with:

$$\theta_0 = \omega_E - a_0. \tag{18}$$

The equation of motion for the Higgs field within the surface gives a boundary condition for the gauge field $\theta_0$, which reduces to an eigenvalue equation for the frequency in the case of global $SU(2)$ symmetry (i.e. when $a_0 = 0$). The surface width is of $O(m^{-1})$. The Higgs field $\sigma$ varies rapidly from a $\sigma_0$ value at the inner edge of the surface, to a zero value at the outer one. Dropping form the Lagrange equation the $O(\epsilon)$ terms and integrating the resulting equation, we find that within the surface:

$$V + W - U = 0. \tag{19}$$

In order to match the interior with the surface solution we set $\sigma' = 0$ at the inner edge of the surface. Then, using Eqs. (17) and (19) we find:

$$\theta_{0\mathrm{sur}} = \frac{A_{\mathrm{sur}}^{1/2}}{2} = \frac{B_{\mathrm{sur}}^{-1/2}}{2}, \tag{20}$$

where $\theta_{0\mathrm{sur}}$ is the value of $\theta_0$ within the thin surface. In the absence of gauge fields we take: $\omega_E = A_{\mathrm{sur}}^{1/2}/2$ which in the absence of gravity gives $\omega_E = 1/2$, which is the correct eigenvalue equation for the q-soliton frequency.

With the above definitions, the independent Einstein equations take the following form:

$$\begin{aligned} \frac{A-1}{r^2} + \frac{1}{r}\frac{dA}{dr} = {}& \frac{2\omega_{BD}+3}{(2\omega_{BD}+4)\Phi_{BD}} \times \left(-W - U - \frac{2W-4U}{2\omega_{BD}+3}\right) \\ & - \frac{\omega_{BD}A}{2\Phi_{BD}^2}\left(\frac{d\Phi_{BD}}{dr}\right)^2 - \frac{A}{2\Phi_{BD}B}\frac{dB}{dr}\frac{d\Phi_{BD}}{dr}, \end{aligned} \tag{21}$$

$$\begin{aligned} \frac{A-1}{r^2} - \frac{A}{B}\frac{1}{r}\frac{dB}{dr} = {}& \frac{2\omega_{BD}+3}{(2\omega_{BD}+4)\Phi_{BD}} \times \left(W - U - \frac{2W-4U}{2\omega_{BD}+3}\right) \\ & + \frac{\omega_{BD}A}{2\Phi_{BD}^2}\left(\frac{d\Phi_{BD}}{dr}\right)^2 + \frac{A}{\Phi_{BD}} \times \left(\frac{d^2\Phi_{BD}}{dr^2} + \frac{1}{2A}\frac{dA}{dr}\frac{d\Phi_{BD}}{dr}\right), \end{aligned} \tag{22}$$

the Euler-Lagrange equation for the BD scalar is:

$$A\left[\frac{d^2\Phi_{BD}}{dr^2} + \left(\frac{2}{r} + \frac{1}{2A}\frac{dA}{dr} - \frac{1}{2B}\frac{dB}{dr}\right)\frac{d\Phi_{BD}}{dr}\right] = \frac{2W-4U}{2\omega_{BD}+4}, \tag{23}$$

and the equation of motion for the new gauge field $\theta_0$:

$$\theta_0'' + \left(\frac{2}{r} + \frac{A'}{2A} + \frac{B'}{2B}\right)\theta_0' - \frac{g^2\theta_0(1+\theta_0 B^{1/2})}{2A} = 0, \tag{24}$$

with boundary conditions:

$$A(0) = 1, \quad \theta'(0) = 0, \quad \theta(R) = \theta_{\mathrm{sur}}, \quad \Phi_{BD}' = 1, \tag{25}$$

and for $r \rightarrow \infty$:

$$A(r) = 1/B(r) = 1, \quad \Phi_{BD} = 1, \quad \theta(r) = \omega_E. \tag{26}$$

The first condition of Eq. (24) reflects our freedom to redefine $A(r)$, the second and fourth result from the spherical symmetry of the configuration and the third is the eigenvalue equation for the new gauge field $\theta_0$, with $R$ the soliton radius. The first condition of Eq. (25) is a straightforward consequence of the Einstein equations for localized matter configurations, the second is the boundary condition for the BD field with the proper rescalings, and the third denotes the absence of gauge fields at infinity. We numerically solve the coupled system of Eqs. (21)–(24).

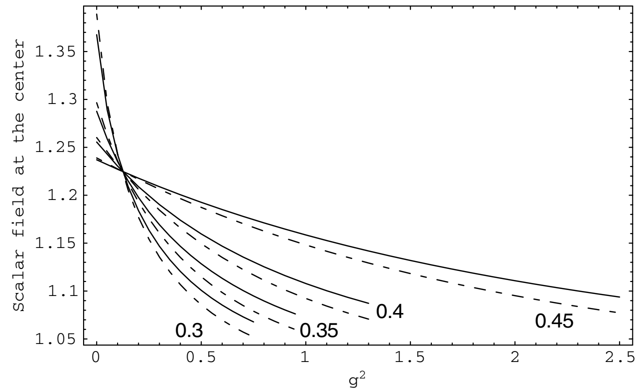


FIG. 1. The value of the Higgs field $\sigma$ at the center of the soliton as a function of the coupling constant $g^2$. The numbers within the Figs. 1–5 denote the eigenvalue $\theta_{0\mathrm{sur}}$, which for $\mathrm{g}^2 = 0$ reduces to $\omega_E$. Dashed lines correspond to $\omega_{\mathrm{BD}} = 5$ and solid lines to $\omega_{\mathrm{BD}} = 500$. The results of General Relativity almost coincide with the BD theory for $\omega_{\mathrm{BD}} = 500$.

The Noether currents corresponding to the generators of the $SU(2)$ algebra are given by the relation:

$$j_{0\alpha} = \begin{pmatrix} \frac{\partial L}{\partial(\partial_0 \Phi)} & \frac{\partial L}{\partial(\partial_0 \Phi^*)} \end{pmatrix} \begin{pmatrix} iT_\alpha & 0 \\ 0 & -iT_\alpha \end{pmatrix} \begin{pmatrix} \Phi \\ \Phi^* \end{pmatrix}. \tag{27}$$

We can find that:

$$j_{01} = \frac{1}{2}\sigma^2\theta_0 \sin\vartheta \cos\varphi, \qquad j_{\overline{02}}\ \tfrac{1}{2}\sigma^2\theta_0 \sin\vartheta \sin\varphi, \qquad j_{03} = \frac{1}{2}\sigma^2\theta_0 \cos\vartheta, \tag{28}$$

and

$$j_0 \equiv \sqrt{j_{01}^2 + j_{02}^2 + j_{03}^2} = \frac{1}{2}\sigma^2\theta_0. \tag{29}$$

The particle number is:

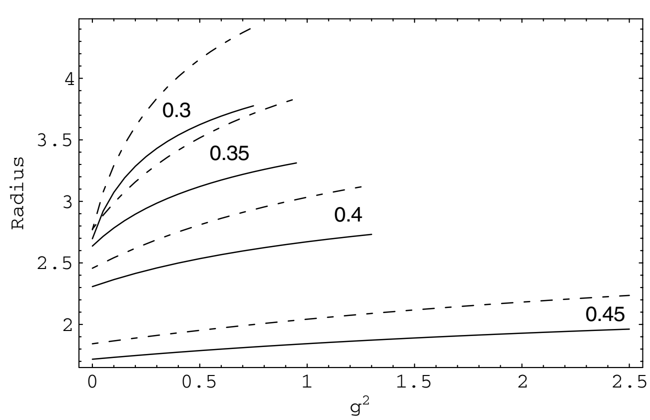


FIG. 2. The radius of the soliton as a function of $g^2$.

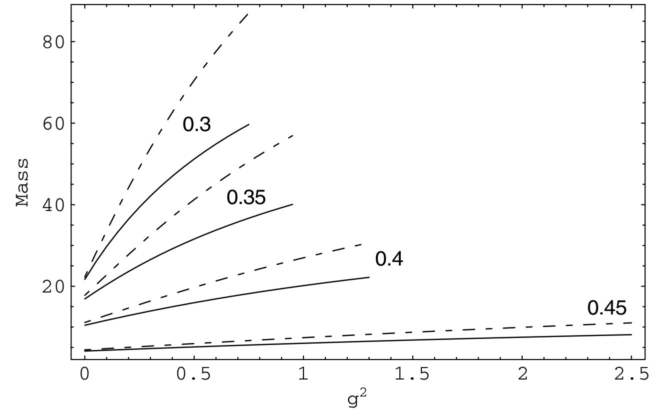


FIG. 3. The asymptotically anti de Sitter mass $M$ as a function of $g^2$.

$$N = 2\pi \int \sigma^2\theta_0 \sqrt{\frac{A}{B}} r^2 dr. \tag{30}$$

The total energy of the field configuration results from the relation:

$$A(\rho) = 1 - \frac{2GM}{\rho} + \frac{G\mathrm{g}^2 N^2}{4\pi\rho^2}, \qquad \rho \to \infty, \tag{31}$$

which gives with our rescalings:

$$M = 4\pi r\left(1 - A(r) + \frac{\mathrm{g}^2 N^2}{32\pi^2 r^2}\right), \qquad r \to \infty. \tag{32}$$

For $\omega_{\mathrm{BD}} \to \infty$ the Einstein equations take the simple form:

$$\frac{A-1}{r^2} + \frac{A'}{r} = -U - W - \frac{\theta_0'^2}{2\mathrm{g}^2} AB, \tag{33}$$

$$\frac{A-1}{r^2} - \frac{A}{B}\frac{B'}{r} = W - U - \frac{\theta_0'^2}{2\mathrm{g}^2} AB, \tag{34}$$

when the other relations remain the same and the results of General Relativity are reproduced.

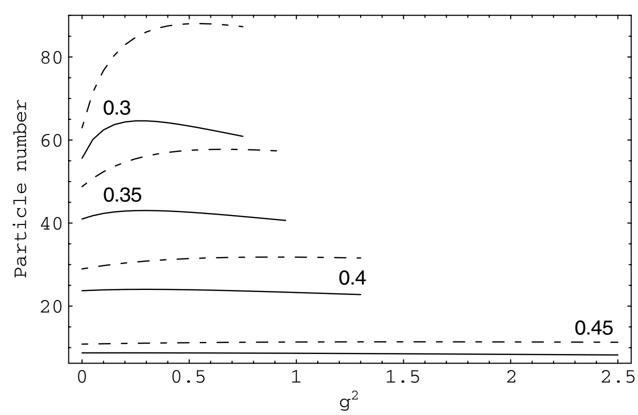


FIG. 4. The particle number $N$ of the soliton as a function of $g^2$.

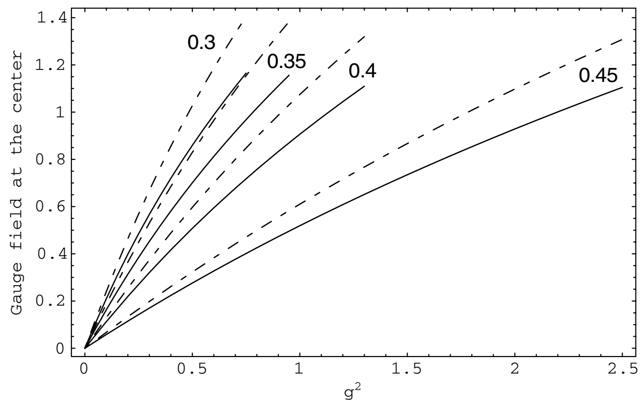


FIG. 5. The value of the gauge field $a_0$ at the center of the soliton as a function of $g^2$.

## III. EYM EQUATIONS IN GENERAL SCALAR-TENSOR THEORY

In the original BD gravitational theory $\omega_{\rm BD}$ is a constant. In a more general theory it may be regarded as a function, usually of the BD field. We will use one of the forms investigated in a cosmological framework [32,33], namely:

$$2\omega_{\rm BD} + 3 = \omega_0 \phi_{\rm BD}^n, \tag{35}$$

with $\omega_0$ and $n$ constants. The Lagrange equation for the BD field is:

$$\phi_{{\rm BD};\lambda}^{;\lambda} = \frac{1}{\omega_0 \phi_{\rm BD}^n}\left(8\pi T - \frac{d\omega_{\rm BD}}{d\phi_{\rm BD}}\phi_{\rm BD}^{;\rho}\phi_{{\rm BD},\rho}\right). \tag{36}$$

We rescale:

$$\tilde{\omega}_0 = \left(\frac{2\omega_{\rm BD}+3}{2\omega_{\rm BD}+4}\right)^n G^n \omega_0, \tag{37}$$

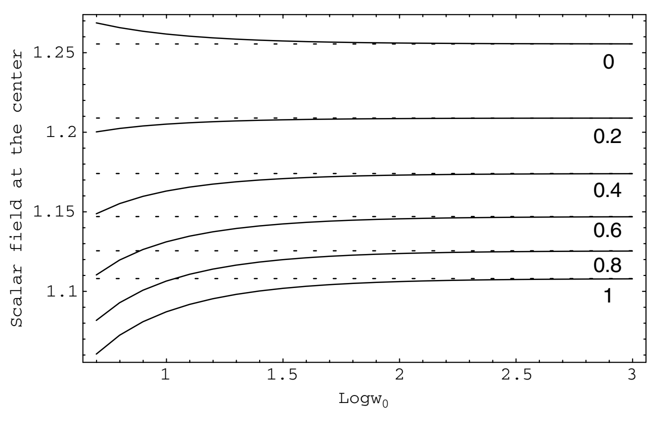


FIG. 6. The value of the field $\sigma$ at the center of the soliton as a function of $\omega_0$. The numbers within the Figs. 6–10 denote the $g^2$ value. We use $\theta_{0\rm sur} = 0.4$. In Figs. 6–10 solid lines are the numerical results produced in the framework of scalar-tensor gravitational theory, when dashed lines are the results from General Relativity.

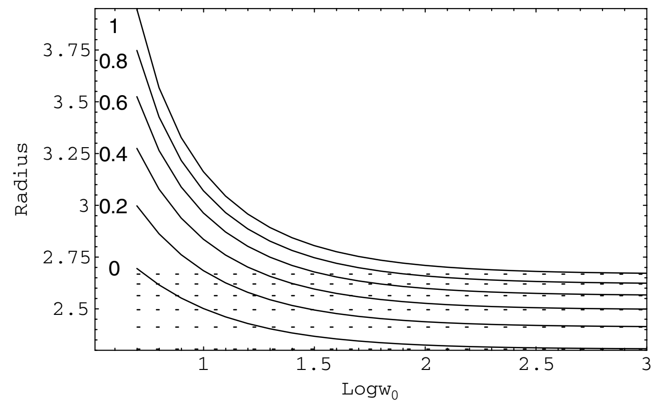


FIG. 7. The radius of the soliton as a function of $\omega_0$.

and the other quantities as in (12) and drop the tildes and the $O(\epsilon)$ quantities. We will use $n = 1$. For other values of $n$ the behavior of the soliton parameters is very similar. The Einstein equations take the following form:

$$\frac{A-1}{r^2} + \frac{1}{r}\frac{dA}{dr} = \frac{\omega_0}{\omega_0\Phi_{\rm BD}+1}\left[-W - U - \frac{1}{\omega_0\Phi_{\rm BD}}\times\left(2W - 4U - \frac{A\Phi_{\rm BD}'^2}{2}\frac{\omega_0\Phi_{\rm BD}+1}{\Phi_{\rm BD}}\right)\right] - \frac{\omega_0\Phi_{\rm BD}-3}{2}\frac{A\Phi_{\rm BD}'^2}{2\Phi_{\rm BD}^2} - \frac{AB'\Phi_{\rm BD}'}{2\Phi_{\rm BD}B}, \tag{38}$$

$$\frac{A-1}{r^2} - \frac{A}{B}\frac{1}{r}\frac{dB}{dr} = \frac{\omega_0}{\omega_0\Phi_{\rm BD}+1}\left[W - U - \frac{1}{\omega_0\Phi_{\rm BD}}\times\left(2W - 4U - \frac{A\Phi_{\rm BD}'^2}{2}\frac{\omega_0\Phi_{\rm BD}+1}{\Phi_{\rm BD}}\right)\right] + \frac{\omega_0\Phi_{\rm BD}-3}{2}\frac{A\Phi_{\rm BD}'^2}{2\Phi_{\rm BD}^2} + \frac{A\Phi_{\rm BD}''}{\Phi_{\rm BD}} + \frac{A'\Phi_{\rm BD}'}{2\Phi_{\rm BD}}. \tag{39}$$

The equation of motion for the BD field is:

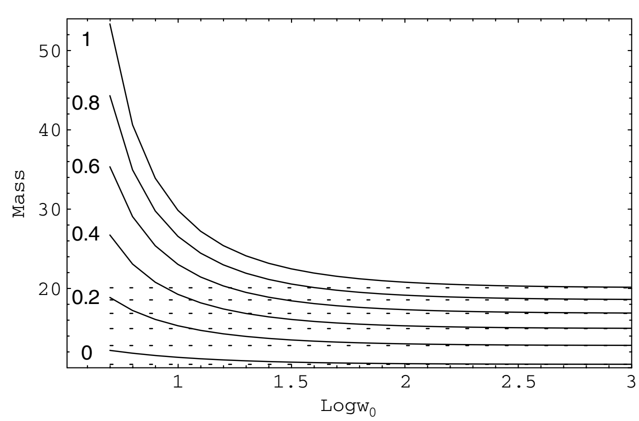


FIG. 8. The asymptotically anti de Sitter mass $M$ of the soliton as a function of $\omega_0$.

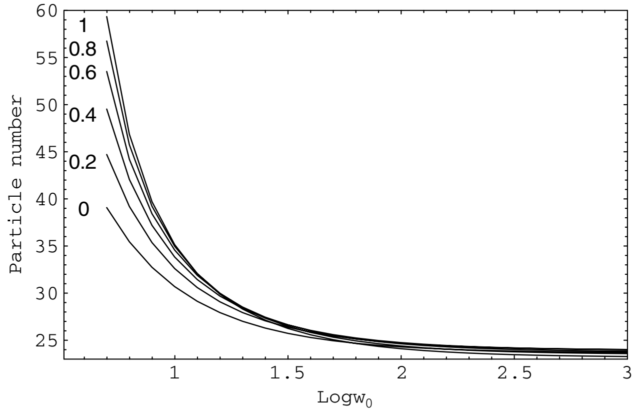


FIG. 9. The particle number of the soliton as a function of $\omega_0$.

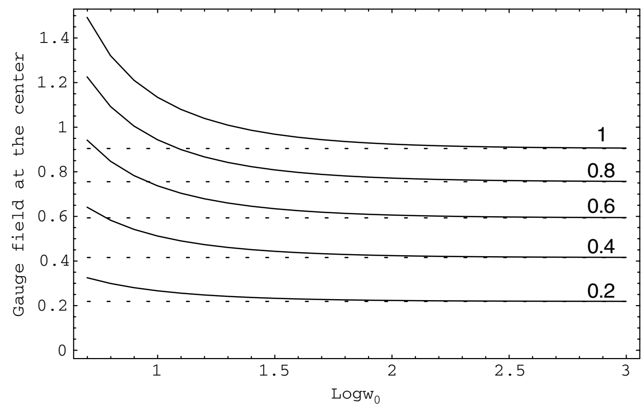


FIG. 10. The value of the gauge field at the center of the soliton as a function of $\omega_0$.

$$A\left[\Phi_{\rm BD}'' + \left(\frac{2}{r} + \frac{A'}{2A} - \frac{B'}{2B}\right)\Phi_{\rm BD}'\right] = \frac{1}{\omega_0\Phi_{\rm BD}+1}\left[2W - 4U - \frac{A\Phi_{\rm BD}'^2}{2}\frac{\omega_0\Phi_{\rm BD}+1}{\Phi_{\rm BD}}\right]. \tag{40}$$

The equations of motion for the gauge and Higgs field remain unchanged. We solve numerically the coupled system of Eqs. (37)–(40) and (24), when the relations (30) and (32) for the soliton mass and particle number hold true.

## IV. CONCLUSIONS

We studied EYM equations in the presence of a Higgs doublet in Brans-Dicke and a simple scalar-tensor gravitational theory and compared our results with the solutions of General Relativity. The Higgs doublet is characterized by a potential admitting q-star type and q-ball type solutions in the absence of gauge fields and gravity and gauge fields, respectively. It is a matter of simple algebra to verify this claim. So, the EYM-Higgs equations *reduce* to a system of equations corresponding to (charged) soliton stars. These objects are *stars* and not black holes, having no horizon or other anomalies.

There are two crucial parameters, resulting from the soliton star itself, the field strength g and the eigenvalue $\theta_{0\rm sur}$ which in the absence of gauge fields reduces to the usual soliton eigenfrequency. The above eigenvalue is straightforward connected to gravity strength on the surface, through Eq. (20). So, a soliton with small $\theta_{0\rm sur}$ shows a stronger gravitational force on its surface, which corresponds to a more massive, or denser soliton, and this can be verified by our figures. Also, larger value for the field strength increases the energy and radius of the field configuration due to the electrostatic repulsion between the different parts of the soliton, when the value of the scalar field within the soliton decreases for the same reason. These results hold true in General Relativity as well as in scalar-tensor theories.

In Figs. 1–5 the results of General Relativity are not depicted because they almost coincide with the $\omega_{\rm BD} = 500$ (solid lines) case. $\omega_{\rm BD} \simeq 500$ is the lower experimental limit. The results of general relativity are exactly reproduced when $\omega{\rm BD} \to \infty$. In Figs. 6–10 we study the behavior of the soliton parameters for $5 \leq \omega_0 \leq 1000$. The results of General Relativity are practically reproduced for $\omega_0 = 1000$ as one can see from the dashed lines in Figs. 6–10, which correspond to the results of General Relativity.

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