

**Coupling quintessence to inflation in supergravity**

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The evolution of the quintessence field during a phase of chaotic inflation is studied. The inflaton  $\phi$  and the quintessence field  $Q$  are described in a supergravity framework where the coupling between the inflaton and quintessence is induced by nonrenormalizable operators suppressed by the Planck mass. We show that the resulting quintessence potential during inflation possesses a time-dependent minimum playing the role of an attractor. The presence of this attractor forces the quintessence field to be small during inflation. These initial conditions are such that the quintessence field is on track now.

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**I. INTRODUCTION**

A host of recent cosmological observations, the anisotropy of the cosmic microwave background (CMB) [1,2], the large scale structures of the Universe [3], and type Ia supernovae [4], indicate that the Universe has experienced two stages of cosmic acceleration. The first one is the inflationary era which occurred in the early Universe [5]. It is responsible for the almost flatness of the Universe and primordial density fluctuations [6] (see also Ref. [7]). The second one, which started in the recent past, leads to the present acceleration of the expansion of the Universe. Various explanations for this last phenomenon have been proposed in the literature: a pure cosmological constant [8], quintessence [9–15],  $k$  essence [16], modified gravity theories [17], the Chaplygin gas [18], bulk viscosity [19], or quantum cosmological effects [20]. In this paper, we focus on the quintessence hypothesis. In this case, the two phenomena described above are modeled as resulting from the presence of two scalar fields whose energy densities drive the acceleration of the expansion. The quintessence hypothesis has been further investigated in Ref. [21]. In particular, finding a natural candidate for the quintessence field in the realm of high energy physics has been a major goal for a lot of authors [12,13,15,22] as well as studying some aspects of its interaction with the “rest of the world” [23]. Since, contrary to a cosmological constant, the quintessence field can develop some inhomogeneities, the theory of cosmological perturbations has also been studied in detail [14,24] and has been used in order to constrain various models observationally [25]. Of course, the prospect of utilizing the fact that the quintessence equation of state is no longer time (or redshift) independent as a tool

for discriminating among the various possibilities has been widely discussed in the recent literature [26].

As a low energy description of string theory [27], supergravity captures prominent features of physics beyond the standard models of particle physics and cosmology. Supergravity is the best framework within which both quintessence and inflation can be described. Indeed inflation (in its most common models like chaotic inflation) involves high energies as the inflaton rolls down its potential with values exceeding the Planck mass. Similarly, in quintessence models with a rolling scalar, the quintessence field reaches values of the order of the Planck mass now, hence the necessity for a treatment where nonrenormalizable interaction terms suppressed by the Planck mass are under control. In supergravity, such nonrenormalizable corrections to supersymmetric models are taken into account and play an important role. This justifies the use of supergravity models both in inflation and quintessence model building. In the following we will concentrate on both quintessence and inflation as described in supergravity.

One of the commonly used models of quintessence, first devised by Ratra and Peebles [9], requires an inverse power-law behavior  $V(Q) = M^{4+\alpha} Q^{-\alpha}$  with an attractor mechanism at large time. It was soon realized that this type of potential can be generated in supersymmetric theories when a strongly interacting sector is present [11]. In particular, the value of the quintessence field becomes of the order of the Planck scale which prompts the necessity of a supergravity treatment. A simple embedding of the previous model in supergravity fails as the potential is highly modified by supergravity corrections and can become negative [12,13]. Hence a more phenomenological approach may be required where one postulates the form of the Kähler potential and the superpotential which leads to quintessence in supergravity. This was done in Refs. [12,13] and subsequent work.

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Similarly, as a high energy phenomenon occurring in the early universe, inflation must be described within supergravity. Recently, there has been an upsurge of inflation models in supergravity motivated by string theory [28–30]. It seems natural to study the influence of quintessence on inflation and vice versa.

Quintessence must be almost decoupled from ordinary matter; otherwise the quintessence field would lead to observable fifth-force signals [23]. On the contrary the inflaton field must couple quite strongly to ordinary matter in order to have a reheating period at the end of inflation where the oscillations of the inflaton result in a radiation bath. Hence the coupling of quintessence and the inflaton cannot be large. A natural way of realizing this criterion is to consider that the inflaton and the quintessence field are decoupled in the Kähler potential and the superpotential of supergravity. This implies that the only possible interactions between both fields spring from nonrenormalizable interactions suppressed by the Planck mass. Here we provide such a supergravity description of the coupling between inflation and quintessence. Finally, we will comment on the compatibility of quintessence with fifth-force constraints and tests of the equivalence principle in the general context considered here, namely, supergravity extensions of the standard model of particle physics.

The paper is arranged as follows. In a first part (Sec. II), we analyze the supergravity coupling between a particular quintessence model, the so-called SUGRA model [12,13,15], and a generic inflationary model. We then (Sec. III) apply this analysis to the specific example of chaotic inflation where we show that the quintessence field develops a potential with a rolling minimum during inflation. The rolling minimum is an attractor such that the values of the quintessence field remain small throughout the inflationary era. In particular, these values are much smaller than the values of a free quintessence field during inflation. The smallness of the quintessence field during inflation implies that it is on track now, i.e. it reaches its long time attractor. Finally, in Sec. IV, we discuss the limitations of our approach, try to indicate what possible improvements could be made, and present our conclusions.

## II. QUINTESSENCE AND INFLATION IN SUPERGRAVITY

### A. Quintessence in supergravity

Let us now briefly review a simple model of quintessence in supergravity often dubbed the SUGRA model in the literature. We assume that the Kähler potential and the superpotential are given by [12,13]

$$K_{\text{quint}}(X, Y, Q) = XX^\dagger + QQ^\dagger + \kappa^p YY^\dagger (QQ^\dagger)^p, \quad (1)$$

$$W_{\text{quint}}(X, Y, Q) = \mu X^2 Y, \quad (2)$$

with  $\kappa \equiv 8\pi/m_{\text{pl}}^2$ . Here  $X$  and  $Y$  are two charged fields

under an (anomalous)  $U(1)$  symmetry with charges 1 and  $-2$ , while  $Q$  is the neutral quintessence field. Notice the direct coupling between  $Q$  and  $Y$ . The constant  $\mu$  is a dimensionless coupling constant and  $p$  is a free coefficient. It is worth mentioning that one can derive the SUGRA model from more general Kähler potentials but we will not need them in this article; see Ref. [15]. At this stage, we assume that

$$\langle X \rangle = \xi, \quad \langle Y \rangle = 0. \quad (3)$$

As a specific example,  $\xi$  can be realized as a Fayet-Ilioupoulos term arising from the Green-Schwarz anomaly cancellation mechanism. When  $\mu = 0$ , the  $Q$  direction is flat. It is lifted by the superpotential leading to the quintessence potential. In supergravity, negative contributions to the scalar potential arise from the vacuum expectation value (vev) of the superpotential. Notice that we have here

$$\langle W_{\text{quint}} \rangle = 0, \quad (4)$$

implying that no negative contribution appears in the scalar potential.

We are now in a position where the scalar potential can be computed. In supergravity, it is given by

$$V = \frac{1}{\kappa^2} e^G (G^A G_A - 3), \quad (5)$$

where the matrix  $G_{A\bar{B}}$  which is used to raise and lower the index  $A$  is defined by

$$G_{A\bar{B}} = \frac{\partial^2}{\partial \varphi^A \partial (\varphi^{\bar{B}})^\dagger} [\kappa K_{\text{quint}} + \ln(\kappa^3 |W_{\text{quint}}|^2)], \quad (6)$$

where  $\varphi^A = \{X, Y, Q\}$  are the fields in the quintessence sector. Straightforward calculations leads to a matrix which is block diagonal, namely,

$$G_{A\bar{B}} = \kappa \begin{bmatrix} 1 & 0 & 0 \\ 0 & (\kappa Q Q^\dagger)^p & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (7)$$

Then the complete SUGRA potential becomes

$$V_{\text{quint}}(Q) = e^{\kappa Q^2 + \kappa \xi^2} \frac{M^{4+2p}}{Q^{2p}}, \quad (8)$$

where the mass scale  $M$  characterizing the potential can be expressed as  $M^{4+2p} \equiv \mu^2 \xi^4 \kappa^{-p}$ . It is easy to find that  $(K_{\text{quint}})_{QQ^\dagger} = 1$  which means that the real part field  $Q$  is in fact not correctly normalized. Therefore, one has to redefine the field  $Q$  according to  $Q \rightarrow Q/\sqrt{2}$  and this gives

$$V_{\text{quint}}(Q) = e^{\kappa Q^2/2 + \kappa \xi^2} \frac{M^{4+2p}}{Q^{2p}}, \quad (9)$$

where we have slightly redefined the mass scale  $M$  such that  $M^{4+2p} \rightarrow M^{4+2p} \times 2^p$ . The main feature of the above potential is that supergravity corrections have been exponentiated and appear in the prefactor. Phenomenologically, this potential has the nice feature that the equation of state

$\omega \equiv p_Q/\rho_Q$  can be closer to  $-1$  than with the Ratra-Peebles potential.

### B. Inflation in supergravity

Let us now give a brief description of the inflation models we will concentrate on. We will consider a class of models described by the following Kähler potential:

$$K_{\text{inf}} = -\frac{3}{\kappa} \ln[\kappa^{1/2}(\rho + \rho^\dagger) - \kappa \mathcal{K}(\phi - \phi^\dagger)] + \mathcal{G}(\phi - \phi^\dagger), \quad (10)$$

where  $\mathcal{K}$  and  $\mathcal{G}$  are arbitrary functions. The field  $\phi$  is the inflaton while  $\rho$  represents, for instance, a moduli of a string compactification. The superpotential  $W_{\text{inf}} = W_{\text{inf}}(\rho, \phi)$  is not specified at this stage. This form is justified by the fact that one can obtain flat enough potentials in supergravity by requiring that a shift symmetry  $\phi \rightarrow \phi + c$ , where  $c$  is a real constant, is a symmetry of the Kähler potential, later broken mildly. The Kähler potential given by Eq. (10) obviously possesses this symmetry. Indeed, a striking feature of  $F$ -term inflation in supergravity is the natural presence of  $\mathcal{O}(H_{\text{inf}})$  corrections to the inflaton mass which would spoil the flatness of the potential. These problems can be avoided by considering the above class of models.

To go further, one must specify the functions  $\mathcal{K}$ ,  $\mathcal{G}$ , and the superpotential. We choose an example of chaotic inflation as can be found in Ref. [31] where a similar case is treated. Explicitly, one assumes

$$\mathcal{K} = -\frac{1}{2}(\phi - \phi^\dagger)^2, \quad \mathcal{G} = +\frac{1}{2}(\phi - \phi^\dagger)^2, \quad (11)$$

and for the superpotential

$$W_{\text{inf}}(\rho, \phi) = \frac{\alpha}{2} m \phi^2. \quad (12)$$

The factor  $\alpha$  in the superpotential is free and can be chosen for future convenience. Notice that the shift symmetry is preserved by our choice of the functions  $\mathcal{K}$  and  $\mathcal{G}$  while, on the contrary, the superpotential breaks this symmetry explicitly. Then, straightforward calculations lead to

$$V_{\text{inf}}(\rho, \phi) = \frac{1}{\Delta^2(3 - \Delta)} \alpha^2 m^2 \phi^2, \quad (13)$$

where  $\Delta = \kappa^{1/2}(\rho + \rho^\dagger)$ . It is easy to see that the moduli can be stabilized if  $\Delta = 2$ . Furthermore, one can check that the normalization of the inflaton is given by  $(K_{\text{inf}})_{\phi\phi^\dagger} = 3/\Delta - 1 = 1/2$  and, hence, is correct. In this case, the potential takes the form

$$V_{\text{inf}}(\phi) = \frac{\alpha^2}{4} m^2 \phi^2, \quad (14)$$

which is nothing but the usual chaotic inflation potential if one chooses  $\alpha = \sqrt{2}$ .

Of course, it is possible to discuss more complicated and/or general inflationary models. The one considered here has the advantage to lead to the prototypical single field inflationary model, namely, chaotic inflation. Since our main goal is not to study inflation itself but the coupling of the inflaton with the quintessence field, this model is sufficient. However, it is clear that the next step would be to study how the form of the coupling term that we are going to derive below depends on the assumed inflationary model.

### C. Coupling the inflaton to the quintessence field

We now turn to our main goal, namely, the calculation of the coupling between the inflaton field and the quintessence field. Our basic assumption is that the quintessence and inflation sectors are decoupled, i.e. that the total Kähler potential and superpotential can be written as

$$K = K_{\text{quint}}(X, Y, Q) + K_{\text{inf}}(\rho, \phi), \quad (15)$$

$$W = W_{\text{quint}}(X, Y, Q) + W_{\text{inf}}(\rho, \phi), \quad (16)$$

where the quintessential Kähler potential and superpotential have been given before but where, at least at this stage, the inflationary part is still arbitrary. However, later, we will restrict our considerations to the (chaotic) inflation model studied in the preceding section. From the above equations, one deduces that the matrix  $G_{A\bar{B}}$ , where now  $\varphi^A = \{X, Y, Q, \rho, \phi\}$ , is diagonal by blocks. Explicitly, one has

$$G_{A\bar{B}} = \begin{bmatrix} G_{\rho\rho^\dagger} & G_{\rho\phi^\dagger} & 0 & 0 & 0 \\ G_{\phi\rho^\dagger} & G_{\phi\phi^\dagger} & 0 & 0 & 0 \\ 0 & 0 & \kappa & 0 & 0 \\ 0 & 0 & 0 & \kappa(\kappa Q Q^\dagger)^p & 0 \\ 0 & 0 & 0 & 0 & \kappa \end{bmatrix}. \quad (17)$$

Then, the scalar potential takes the form (recall that the  $D$ -terms contribution vanishes)

$$V = e^{\kappa\xi^2} \left[ e^{\kappa Q^2/2} V_{\text{inf}}(\rho, \phi) + e^{\kappa K_{\text{inf}}} V_{\text{quint}}(Q) + \kappa^2 \left( \xi^2 + \frac{Q^2}{2} \right) |W_{\text{inf}}|^2 e^{\kappa(K_{\text{inf}} + Q^2/2)} \right], \quad (18)$$

where

$$V_{\text{inf}}(\rho, \phi) = \frac{1}{\kappa^2} e^{G_{\text{inf}}} [G_{\text{inf}}^A (G_{\text{inf}})_A - 3], \quad (19)$$

$$V_{\text{quint}}(Q) = e^{\kappa Q^2/2} \frac{M^{4+2p}}{Q^{2p}}. \quad (20)$$

Let us notice that, for convenience, we have slightly changed the notation for  $V_{\text{quint}}(Q)$ . Now, we no longer include the factor  $\exp(\kappa\xi^2)$  in its definition; see Eq. (8). As explained before, we have also redefined the quintessence field according to  $Q \rightarrow Q/\sqrt{2}$  in order to work with correctly normalized fields. The above expression repre-

sents the general form of the coupling between the SUGRA model of quintessence and inflation in supergravity.

We now specify the inflaton model and consider the model described in the previous subsection with  $\alpha = \sqrt{2}$ . It is convenient to work in terms of dimensionless quantities. In particular, we define the dimensionless potential  $f(\phi, Q)$  by  $V(\phi, Q) \equiv m_{\text{Pl}}^4 f(\phi, Q)$  and this quantity can be written as

$$f(\phi, Q) = f_{\text{inf}} + f_{\text{quint}} + f_{\text{inter}}, \quad (21)$$

where

$$\begin{aligned} f_{\text{inf}} &= \frac{1}{2} \left( \frac{m}{m_{\text{Pl}}} \right)^2 \left( \frac{\phi}{m_{\text{Pl}}} \right)^2 e^{8\pi\xi^2/m_{\text{Pl}}^2 + 4\pi Q^2/m_{\text{Pl}}^2}, \\ f_{\text{quint}} &= \frac{1}{8} \left( \frac{M}{m_{\text{Pl}}} \right)^{4+2p} \left( \frac{Q}{m_{\text{Pl}}} \right)^{-2p} e^{8\pi\xi^2/m_{\text{Pl}}^2 + 4\pi Q^2/m_{\text{Pl}}^2}, \\ f_{\text{inter}} &= 4\pi^2 \left( \frac{m}{m_{\text{Pl}}} \right)^2 \left[ \left( \frac{\xi}{m_{\text{Pl}}} \right)^2 + \frac{1}{2} \left( \frac{Q}{m_{\text{Pl}}} \right)^2 \right] \\ &\quad \times \left( \frac{\phi}{m_{\text{Pl}}} \right)^4 e^{8\pi\xi^2/m_{\text{Pl}}^2 + 4\pi Q^2/m_{\text{Pl}}^2}. \end{aligned} \quad (22)$$

At this point, some remarks are in order. *A priori*, there is no clear separation between the inflaton and the quintessence fields in the term  $f_{\text{inf}}$  because of the presence of the exponential term. However, in the regime we will be studying (during inflation),  $Q \ll m_{\text{Pl}}$  and, therefore, the exponential term will be very close to 1. In this case, one recovers the simple chaotic model  $V_{\text{inf}} = m^2 \phi^2/2$ . The term  $f_{\text{quint}}$  is nothing but the SUGRA potential studied in Refs. [12–15] but, during inflation, it will reduce to the Ratra-Peebles case. Let us notice that we have an extra factor 1/8 originating from the term  $\exp(\kappa K_{\text{inf}})$ . This comes from the fact that  $\kappa K_{\text{inf}} = -3 \ln(2)$  since the moduli is stabilized at  $\Delta = 2$ . Finally, in the regime  $Q \ll m_{\text{Pl}}$ , the interaction term reads  $V_{\text{inter}} \propto m^2 \phi^4 Q^2/m_{\text{Pl}}^4$ . This is due to the fact that we have  $Q \gg \xi$  as will be discussed below. The coupling constant between the inflaton and the quintessence fields reads  $m^2/m_{\text{Pl}}^4$ . The Planck mass appears in this expression because the coupling between the two fields has been entirely fixed by the supergravity. Notice also that the quintessence field picks up an inflaton dependent mass term during inflation. The competition between this mass term and the Ratra-Peebles potential will be studied in the next section. Finally, using the fact that

$$\begin{aligned} \frac{\partial f}{\partial (\phi/m_{\text{Pl}})} &= e^{8\pi\xi^2/m_{\text{Pl}}^2 + 4\pi Q^2/m_{\text{Pl}}^2} \left( \frac{m}{m_{\text{Pl}}} \right)^2 \\ &\quad \times \frac{\phi}{m_{\text{Pl}}} \left\{ 1 + 16\pi^2 \left[ \left( \frac{\xi}{m_{\text{Pl}}} \right)^2 \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \left( \frac{Q}{m_{\text{Pl}}} \right)^2 \right] \left( \frac{\phi}{m_{\text{Pl}}} \right)^2 \right\}, \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{\partial f}{\partial (Q/m_{\text{Pl}})} &= e^{8\pi\xi^2/m_{\text{Pl}}^2 + 4\pi Q^2/m_{\text{Pl}}^2} \frac{1}{8} \left( \frac{M}{m_{\text{Pl}}} \right)^{4+2p} \left( \frac{Q}{m_{\text{Pl}}} \right)^{-2p} \\ &\quad \times \left[ 8\pi \frac{Q}{m_{\text{Pl}}} - 2p \left( \frac{Q}{m_{\text{Pl}}} \right)^{-1} \right] \\ &\quad + e^{8\pi\xi^2/m_{\text{Pl}}^2 + 4\pi Q^2/m_{\text{Pl}}^2} \left( \frac{m}{m_{\text{Pl}}} \right)^2 \left( \frac{\phi}{m_{\text{Pl}}} \right)^2 \\ &\quad \times \frac{Q}{m_{\text{Pl}}} \left\{ 4\pi + 4\pi^2 \left[ 1 + 8\pi \left( \frac{\xi}{m_{\text{Pl}}} \right)^2 \right. \right. \\ &\quad \left. \left. + 4\pi \left( \frac{Q}{m_{\text{Pl}}} \right)^2 \right] \left( \frac{\phi}{m_{\text{Pl}}} \right)^2 \right\}, \end{aligned} \quad (24)$$

it is easy to show that this potential possesses an absolute minimum given by

$$\frac{\phi}{m_{\text{Pl}}} = 0, \quad \frac{Q}{m_{\text{Pl}}} = \sqrt{\frac{2p}{8\pi}}. \quad (25)$$

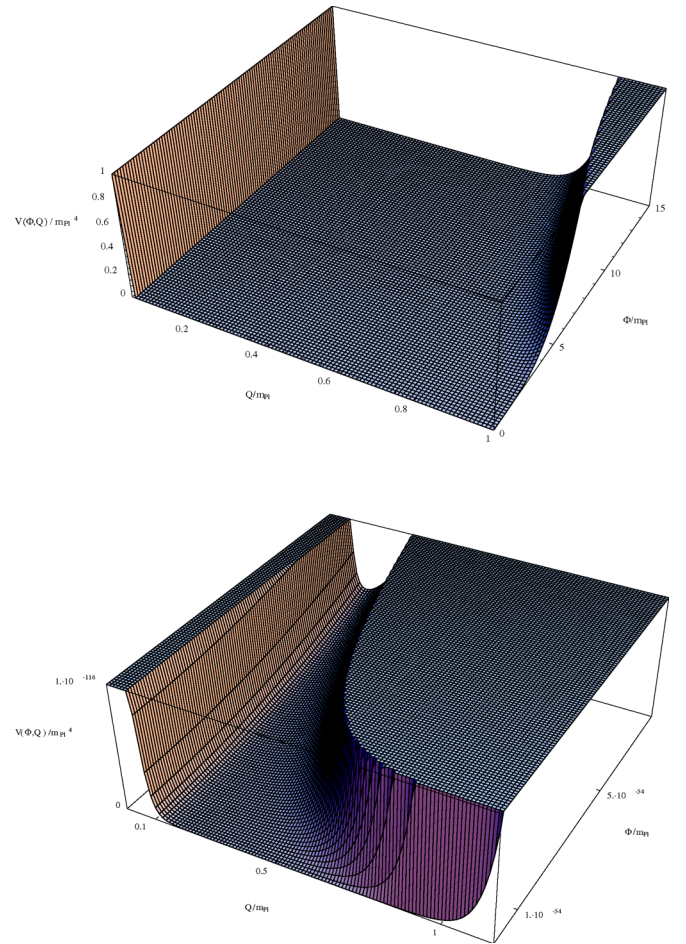


FIG. 1 (color online). Upper panel: Potential  $V(\phi, Q)$  for the following choice of parameters:  $p = 3$ ,  $m = 10^{-5} m_{\text{Pl}}$ ,  $\xi = 10^{-30} m_{\text{Pl}}$ , and  $(M/m_{\text{Pl}})^{4+2p} = 10^{-122}$ . The absolute minimum located at  $Q \approx 0.4886 \times m_{\text{Pl}}$  and  $\phi = 0$  cannot be viewed with the scales used. Bottom panel: zoom in the region of the potential where the minimum is located. It is clear that the tiny values of the inflaton field are, in this panel, not interesting from a physical point of view (there is no inflation for such small values).

The potential is represented in Fig. 1. The value  $\phi = 0$  is the minimum of the inflaton potential without interaction while  $Q = \sqrt{p/(4\pi)}$  is the minimum of the SUGRA potential. At the absolute minimum, the value of the potential is nonvanishing and given by  $V = V_{\text{quint}} = (m_{\text{Pl}}^4/8) \times (M/m_{\text{Pl}})^{4+2p} (4\pi/p)^p \exp(\kappa\xi^2) \exp(p)$ .

### III. COSMOLOGICAL EVOLUTION

#### A. Fixing the free parameters

Let us now discuss the values of the free parameters that appear in Eqs. (21) and (22). If we assume that the quintessential part is responsible for the acceleration now then one should have

$$e^{\kappa\xi^2 + \kappa Q_0^2/2} \frac{M^{4+2p}}{Q_0^{2p}} \simeq m_{\text{Pl}}^2 H_0^2, \quad (26)$$

where  $Q_0$  and  $H_0$  denote the values of the quintessence field and of the Hubble parameter (respectively) now, at vanishing redshift. In order to have a successful model of quintessence, the field should be on track today which in turn implies that  $Q_0 = \mathcal{O}(m_{\text{Pl}})$ . Strictly speaking, this conclusion is valid for the Ratra-Peebles potential only, but it has been shown in Refs. [12,13] that this is also valid for the SUGRA potential despite the presence of the exponential correction (this is simply because, except at small redshifts, we have  $Q \ll m_{\text{Pl}}$  and the exponential SUGRA correction does not play an important role). This gives

$$\left(\frac{M}{m_{\text{Pl}}}\right)^{4+2p} \simeq \frac{H_0^2}{m_{\text{Pl}}^2} \simeq 10^{-122}. \quad (27)$$

Using the fact that  $M^{4+2p} \simeq \mu^2 \xi^4 \kappa^{-p}$  and assuming no fine-tuning of the coupling constant, i.e.  $\mu = \mathcal{O}(1)$ , one deduces that

$$\frac{\xi}{m_{\text{Pl}}} \simeq \sqrt{\frac{H_0}{m_{\text{Pl}}}} \simeq 10^{-30}. \quad (28)$$

In a sense this is the usual fine-tuning of the cosmological constant; it reappears here in the guise of the tuning of the vev of a field leading to the quintessence potential. However, if one works with an effective model valid up to a cutoff scale  $m_C$ , then it has been shown in Ref. [15] that the previous problem can be solved provided the scale is chosen such that  $m_C \ll m_{\text{Pl}}$ . Then the value of the Fayet-Iliopoulos term can even be above the weak scale. However, again, our purpose here is not to study the details of the dark energy model and, therefore, in the following, we will ignore these subtleties and work with the value of  $\xi$  derived before. Let us notice that even with a Fayet-Iliopoulos term above the weak scale, in general, we still have  $Q > \xi$  and then the form of the coupling is not modified when one works with a cutoff scale much below the Planck scale; see also the discussion after Eq. (22).

We now discuss the constraint on the parameter characterizing the inflaton sector, i.e. the mass  $m$  of the field. In order to simplify the discussion, we will assume that the

initial conditions are such that the quintessence field is always subdominant. In this situation the quantum fluctuations of the inflaton field are at the origin of the CMB anisotropy observed today. As a consequence, and as is well known, the Cosmic Background Explorer (COBE) and Wilkinson Microwave Anisotropy Probe (WMAP) normalizations fix the coupling constant of the inflaton potential, namely, the mass  $m$  in the present context. More precisely, for small  $\ell$ , the multipole moments are given by

$$C_\ell = \frac{2H_{\text{inf}}^2}{25\epsilon m_{\text{Pl}}^2} \frac{1}{\ell(\ell+1)} \quad (29)$$

and what has been actually measured by the COBE and WMAP satellites is  $Q_{\text{rms-PS}}^2/T^2 = 5C_2/(4\pi) \simeq (18 \times 10^{-6}/2.7)^2 \simeq 36 \times 10^{-12}$ . The quantity  $H_{\text{inf}}$  is the Hubble parameter during inflation and is related to the potential by the slow-roll equation  $H_{\text{inf}}^2 \simeq \kappa V_{\text{inf}}/3$  evaluated at the Hubble radius crossing. Putting everything together, we find that the inflaton mass is given by

$$\left(\frac{m}{m_{\text{Pl}}}\right)^2 \simeq 45\pi \left(N_* + \frac{1}{2}\right)^{-2} \frac{Q_{\text{rms-PS}}^2}{T^2}, \quad (30)$$

that is to say

$$m \simeq 1.3 \times 10^{-6} \times m_{\text{Pl}}. \quad (31)$$

All the parameters of the potential are now specified.

Let us now discuss in more detail the conditions under which the inflaton field is always dominant. The quintessence energy density must be smaller than the inflaton energy density. This gives a lower bound on the possible values of the field  $Q$  which can be expressed as

$$\frac{Q_{\text{low}}}{m_{\text{Pl}}} \simeq \left(\frac{m}{H_0} \frac{\phi}{m_{\text{Pl}}}\right)^{-1/p} \simeq 10^{-55/p} \left(\frac{\phi}{m_{\text{Pl}}}\right)^{-1/p}, \quad (32)$$

where we have used the value of  $m \simeq 10^{-6} \times m_{\text{Pl}}$  obtained before. The fact that we obtain a lower bound is consistent with the fact that the potential is an inverse power law of the quintessence field: the smaller the field, the larger the corresponding energy density. Second, there exists also an upper bound coming from the fact that the interaction energy density must be smaller than the inflaton energy density. Concretely, this gives

$$\frac{Q_{\text{up}}}{m_{\text{Pl}}} \simeq \sqrt{\frac{1}{4\pi^2} \left(\frac{\phi}{m_{\text{Pl}}}\right)^{-2} - 2 \left(\frac{\xi}{m_{\text{Pl}}}\right)^2} \simeq \left(\frac{\phi}{m_{\text{Pl}}}\right)^{-1}, \quad (33)$$

where we have used the fact that the maximal value of the inflaton field is  $\phi \simeq 10^6 \times m_{\text{Pl}}$ ; see below.

Under the condition that  $Q_{\text{low}} < Q < Q_{\text{up}}$ , the behavior of the background is determined by the energy density of the inflaton and it is well known that, in this case, the slow-roll approximation is valid. The slow-roll approximation is controlled by two parameters (in fact, at leading order, there are three relevant slow-roll parameters but we will not need the third one) defined by [7]

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \delta = -\frac{\dot{\epsilon}}{2H\epsilon} + \epsilon. \quad (34)$$

In the present context, where the inflaton potential is proportional to  $m^2\phi^2$ , the slow-roll parameters are given by

$$\epsilon = \frac{1}{2N_* + 1}, \quad \delta = 0, \quad (35)$$

where  $N_* \simeq 60$  is the number of  $e$  folds between the time at which scales of astrophysical interest today left the Hubble radius during inflation and the end of inflation. In the situation where these parameters are small, namely,  $\epsilon \ll 1$  and  $\delta \ll 1$ , the equation of motion of the inflaton field can be easily integrated. For this purpose, it is convenient to express everything in terms of the number of  $e$  folds (not to be confused with  $N_*$ ) defined by

$$N \equiv \ln\left(\frac{a}{a_{\text{ini}}}\right), \quad (36)$$

such that, at the beginning of inflation, one has  $N = 0$ . Then, in the slow-roll approximation, one obtains that the evolution of the field is given by

$$\frac{\phi}{m_{\text{Pl}}} = \sqrt{\left(\frac{\phi_{\text{ini}}}{m_{\text{Pl}}}\right)^2 - \frac{N}{2\pi}}, \quad (37)$$

where  $\phi_{\text{ini}}$  is the initial value of the field. This value is related to the total number of  $e$  folds given by

$$N_{\text{T}} = 2\pi\left(\frac{\phi_{\text{ini}}}{m_{\text{Pl}}}\right)^2 - \frac{1}{2}. \quad (38)$$

If  $N_{\text{min}}$  is the minimum number of  $e$  folds required in order to solve the problems of the hot big-bang model ( $N_{\text{min}} \simeq 60$ ) then one has

$$\phi_{\text{ini}} > m_{\text{Pl}} \sqrt{\frac{1}{2\pi}\left(N_{\text{min}} + \frac{1}{2}\right)} \simeq 3.1 \times m_{\text{Pl}}. \quad (39)$$

There exists also an upper bound for the value of the inflaton field which corresponds to the situation where the potential energy density  $m^2\phi^2/2$  is Planckian. Using that  $m \simeq 10^{-6} \times m_{\text{Pl}}$  this immediately gives that  $\phi_{\text{max}} \simeq 10^6 \times m_{\text{Pl}}$ . This is the value of  $\phi_{\text{max}}$  that we considered before.

## B. Analytical study of the Klein-Gordon equation

We now turn to the resolution of the quintessence equation of motion, i.e. the Klein-Gordon equation. It can be written as

$$\ddot{Q} + 3H\dot{Q} + \frac{\partial}{\partial Q} V(\phi, Q) = 0, \quad (40)$$

where  $H \equiv \dot{a}/a$  is the Hubble parameter which depends only on the inflaton energy density. A dot denotes a derivative with respect to cosmic time. We now work with the new time variable introduced before, namely, the number of  $e$  folds. One gets

$$\frac{d^2}{dN^2} \left(\frac{Q}{m_{\text{Pl}}}\right) + \left(3 + \frac{1}{H} \frac{dH}{dN}\right) \frac{d}{dN} \left(\frac{Q}{m_{\text{Pl}}}\right) + \left(\frac{m_{\text{Pl}}}{H}\right)^2 \frac{\partial f}{\partial(Q/m_{\text{Pl}})} = 0. \quad (41)$$

The main feature of the above equation is that the potential is now explicitly time dependent because of the interaction of  $Q$  with the inflaton, i.e. we have  $f(\phi, Q) = f(N, Q)$ . This renders this equation very difficult to solve exactly. Therefore, in order to get some analytical approximate solution, it is necessary to make some assumptions. With a very good accuracy, the fact that  $Q_{\text{low}} < Q < Q_{\text{up}}$  implies that the potential contains two dominant terms and can be approximated as  $\sim Q^{-2p} + Q^2$ . As a consequence, its derivative can be expressed as

$$\frac{\partial f}{\partial(Q/m_{\text{Pl}})} \simeq -\frac{p}{4} \left(\frac{M}{m_{\text{Pl}}}\right)^{4+2p} \left(\frac{Q}{m_{\text{Pl}}}\right)^{-2p-1} + 4\pi^2 \left(\frac{m}{m_{\text{Pl}}}\right)^2 \frac{Q}{m_{\text{Pl}}} \left(\frac{\phi}{m_{\text{Pl}}}\right)^4. \quad (42)$$

This means that the quintessence field evolves in a time-dependent potential which possesses a minimum. Of course, this minimum is explicitly time dependent and can be expressed as

$$Q_{\text{min}}(N) = m_{\text{Pl}} \left\{ \frac{p}{16\pi^2} \left(\frac{H_0}{m_{\text{Pl}}}\right)^2 \left(\frac{m}{m_{\text{Pl}}}\right)^{-2} \times \left[\frac{\phi(N)}{m_{\text{Pl}}}\right]^{-4} \right\}^{1/[2(p+1)]}. \quad (43)$$

As shown below, this equation turns out to be one of the main results of the present article. Indeed, we will demonstrate that, after a period of rapid oscillations, the quintessence field always tends toward the above solution. Therefore, in the case where the interaction between the inflaton and the quintessence field is important,  $Q_{\text{min}}$  can be viewed as a kind of attractor solution since, regardless of the initial conditions, the final value of the field is always given by  $Q_{\text{min}}$ .

Several remarks are in order at this stage. First, let us evaluate the typical time of evolution of the minimum. It is given by  $\Delta N \simeq Q_{\text{min}}/(dQ_{\text{min}}/dN) = [d \ln Q_{\text{min}}/dN]^{-1}$ . From Eq. (43), one has  $Q_{\text{min}} \propto H^{-2/(p+1)}$ . Therefore, this implies that

$$\Delta N_{\text{min}} \simeq \left| \frac{p+1}{2\epsilon} \right| \gg 1, \quad (44)$$

where  $\epsilon = -(dH/dN)/H$  is the first slow-roll parameter.

Second, it is interesting to calculate the effective mass of the quintessence field at the minimum of its time-dependent potential. Using Eq. (43), one obtains

$$\frac{m_{\text{eff}}^2}{m_{\text{Pl}}^2} = \frac{\partial^2 f}{\partial(Q/m_{\text{Pl}})^2} \Big|_{\text{min}} \simeq 8\pi^2(p+1) \left(\frac{m}{m_{\text{Pl}}}\right)^2 \left[\frac{\phi(N)}{m_{\text{Pl}}}\right]^4. \quad (45)$$

Therefore, one has

$$\frac{m_{\text{eff}}^2}{H^2} = 6\pi(p+1) \left[\frac{\phi(N)}{m_{\text{Pl}}}\right]^2 > 1, \quad (46)$$

and we conclude that, at its minimum, the quintessence field is not a light field.

Third, we are now in a position where one can study how small fluctuations behave around the time-dependent mini-

mum. The fluctuations are given by  $\delta Q \equiv Q - Q_{\min}$  and their evolution is governed by the equation

$$\frac{d^2}{dN^2} \left( \frac{\delta Q}{m_{\text{Pl}}} \right) + 3 \frac{d}{dN} \left( \frac{\delta Q}{m_{\text{Pl}}} \right) + \left( \frac{m_{\text{eff}}}{H} \right)^2 \frac{\delta Q}{m_{\text{Pl}}} = 0, \quad (47)$$

where, in the damping term, we have neglected the derivative of the Hubble parameter which is nothing but the slow-roll parameter  $\epsilon$ . Using the expression of the effective mass established before and the expression of the inflaton in the slow-roll approximation, one finds that the solution can be expressed as

$$\frac{\delta Q}{m_{\text{Pl}}} = e^{-3N/2} [A_1 \text{Ai}(-x) + A_2 \text{Bi}(-x)], \quad (48)$$

where  $A_1$  and  $A_2$  are two constants determined by the initial conditions. The functions Ai and Bi are the Airy functions [32]<sup>1</sup> and the quantity  $x$  is defined by

<sup>1</sup>Another method to solve the Klein-Gordon equation, under the assumption that the slow-roll hypothesis is valid for the inflaton, is the following. Instead of directly neglecting  $\epsilon = -(dH/dN)/H$  in the damping term of Eq. (41) as we did before, one works with the dimensionless field  $q(N)$  defined by

$$\frac{Q}{m_{\text{Pl}}} \equiv g(N)q(N) \equiv \left( \frac{H}{m_{\text{Pl}}} \right)^{-1/2} e^{-3N/2} q(N). \quad (49)$$

Then, from Eq. (41), it is easy to show that the field  $q(N)$  obeys

$$\frac{d^2 q}{dN^2} + \left[ -\frac{1}{2H} \frac{d^2 H}{dN^2} + \frac{1}{4H^2} \left( \frac{dH}{dN} \right)^2 - \frac{15}{4H} \frac{dH}{dN} - \frac{9}{4} \right] q + \frac{1}{g(N)} \left( \frac{H}{m_{\text{Pl}}} \right)^{-2} \frac{\partial f}{\partial (Q/m_{\text{Pl}})} \Big|_{Q/m_{\text{Pl}}=g(N)q(N)} = 0. \quad (50)$$

In the second term between squared brackets, the various derivatives of the Hubble parameter can be expressed in terms of the slow-roll parameters. Explicitly, this term reads  $\epsilon(3\epsilon + 2\delta)/2 + \epsilon^2/4 + 15\epsilon/4 - 9/4 \simeq -9/4$ . Therefore, the Klein-Gordon equation can be simplified further and we obtain

$$\frac{d^2 q}{dN^2} - \frac{9}{4} q + \frac{1}{g(N)} \left( \frac{H}{m_{\text{Pl}}} \right)^{-2} \frac{\partial f}{\partial (Q/m_{\text{Pl}})} \Big|_{g(N)q(N)} \simeq 0. \quad (51)$$

Using the fact that the potential is given by  $m_{\text{eff}}^2 \phi^2/2$ , hence we are now studying  $\delta Q \equiv g(N)q(N)$ , the above equation takes the form

$$\frac{d^2 q}{dx^2} + xq = 0, \quad (52)$$

where  $x$  is defined in Eq. (54). As before, this equation can be solved in terms of Airy functions and this gives

$$\frac{\delta Q}{m_{\text{Pl}}} = \left( \frac{H}{m_{\text{Pl}}} \right)^{-1/2} e^{-3N/2} [B_1 \text{Ai}(-x) + B_2 \text{Bi}(-x)]. \quad (53)$$

This solution should be compared with Eq. (48). We see that the equations are similar up to the factor  $(H/m_{\text{Pl}})^{-1/2}$ . As our approximation is valid during a few  $e$  folds only, the Hubble parameter can be considered as a constant and then the two solutions are identical.

$$x \equiv 3^{-2/3} (p+1)^{-2/3} \left\{ 6\pi (p+1) \left[ \frac{\phi(N)}{m_{\text{Pl}}} \right]^2 - \frac{9}{4} \right\}. \quad (54)$$

Initially, and during a few  $e$  foldings, one has  $x \gg 1$ . In this case, one can use the asymptotic expression of the Airy function [32] and one obtains

$$\frac{\delta Q}{m_{\text{Pl}}} \simeq e^{-3N/2} \pi^{-1/2} x^{-1/4} \left[ A_1 \sin\left( \frac{2}{3} x^{3/2} + \frac{\pi}{4} \right) + A_2 \cos\left( \frac{2}{3} x^{3/2} + \frac{\pi}{4} \right) \right]. \quad (55)$$

From this expression, one sees that one has damped oscillations. The period of the oscillations can be very easily estimated. One has

$$\Delta N_{\text{osci}} \simeq \frac{2\pi}{\sqrt{3(p+1)}} N_{\text{T}}^{-1/2}, \quad (56)$$

where we recall that  $N_{\text{T}}$  is the total number of  $e$  folds during inflation. For a typical model with  $p = 3$  and  $N_{\text{T}} \simeq 60$ , one gets  $\Delta N_{\text{osci}} \simeq 0.24$ . The previous equation also means that if the inflaton field starts at large values, then the period of the oscillations will be extremely rapid. For instance, if inflation starts at Planckian density, then  $\phi_{\text{ini}} \simeq 10^6 m_{\text{Pl}}$  which implies that  $N_{\text{T}} \simeq 10^{12}$ . As a consequence, one can get values as small as  $\Delta N_{\text{osci}} \simeq 10^{-6}$ .

Therefore, from the above considerations, one reaches the conclusion that

$$\frac{\Delta N_{\text{min}}}{\Delta N_{\text{osci}}} = \mathcal{O}(1) \frac{\sqrt{N_{\text{T}}}}{\epsilon} \gg 1. \quad (57)$$

This means that the oscillatory phase is very quick in comparison with the typical time scale of evolution of the minimum. To put it differently, the minimum can be considered as motionless or as ‘‘adiabatic’’ as the field rapidly oscillates and quickly joins its minimum. Therefore,  $Q_{\min}(N)$  can be viewed as an attractor since it does not depend on the initial conditions for the quintessence field.

We have established the above result under the assumption that the initial deviation from the attractor  $Q_{\min}$  is not too large (or, in other words, that  $\delta Q$  is not too large). What happens if this is not the case, i.e. if  $|Q_{\text{ini}} - Q_{\min}| \gg 1$  or  $\delta Q_{\text{ini}} \gg 1$ ? Is the attractor still joined rapidly enough (i.e. before the end of inflation)? *A priori*, to answer this question requires a full integration of the equation of motion (or a numerical integration, see the next subsection) which is not possible. However, we can gain some partial insights using the following considerations. If one has  $Q_{\text{ini}} \ll Q_{\min}$  then the term proportional to  $Q^{-2p}$  dominates in the potential and the Klein-Gordon equation remains nonlinear and hence difficult to integrate. But if we now assume that we start from a situation where  $Q_{\text{ini}} \gg Q_{\min}$ , then the term proportional to  $Q^2$  dominates in the potential. As a con-

sequence, the derivative of the potential can be written as, see Eq. (42),

$$\frac{\partial f}{\partial(Q/m_{\text{Pl}})} \simeq 4\pi^2 \left(\frac{m}{m_{\text{Pl}}}\right)^2 \left(\frac{\phi}{m_{\text{Pl}}}\right)^4 \frac{Q}{m_{\text{Pl}}}. \quad (58)$$

Therefore, the Klein-Gordon equation is now linear and can be integrated. In fact, one obtains the same potential as before for  $\delta Q$ , up to an unimportant factor  $2(p+1)$ , except that now  $Q - Q_{\min} > 0$  need not be small. As a consequence the solution will read the same, that is to say, roughly speaking,  $Q \simeq Q_{\text{ini}} \exp(-3N/2)$ , this solution being valid provided  $Q \gg Q_{\min}$ .

Equipped with this solution, one can now estimate how many  $e$  folds are necessary for the field to roll down the potential from a given initial condition and to reach the region of the minimum. When the field enters this region, the previous solution is no longer valid because the term  $Q^{-2p}$  starts playing a role but, on the other hand, since the field is now close to  $Q_{\min}$  the calculation of  $\delta Q$  applies. In order to get an upper bound on the number of  $e$  folds, let us assume that  $Q$  is initially as far as possible from the minimum, i.e.  $Q_{\text{ini}} = Q_{\text{up}}$ ; see Eq. (33). Then the number of  $e$  folds  $N$  is the solution of the algebraic equation

$$\frac{Q_{\min}(N)}{m_{\text{Pl}}} = \left(\frac{\phi_{\text{ini}}}{m_{\text{Pl}}}\right)^{-1} e^{-3\Delta N/2}. \quad (59)$$

The solution can be expressed in terms of the Lambert function  $W_0$  [33] defined by the relation  $W(z) \exp[W(z)] = z$ . Explicitly, one obtains

$$\begin{aligned} \Delta N = & -(p+1)N_{\text{T}} + \frac{2}{3}W_0\left[\frac{3}{2}(p+1)N_{\text{T}}\left(\frac{\phi_{\text{ini}}}{m_{\text{Pl}}}\right)^{(3+p)/(1+p)}\right. \\ & \left. \times \left(\frac{p}{16\pi^2} \frac{H_0^2}{m_{\text{Pl}}^2} \frac{m_{\text{Pl}}^2}{m^2}\right)^{-1/[2(p+1)]} e^{3(p+1)N_{\text{T}}/2}\right]. \end{aligned} \quad (60)$$

Since the argument of the Lambert function  $W_0$  is very large, one can use the approximation  $W_0(z) \simeq \ln(z)$ . In this case, one gets

$$\begin{aligned} \Delta N \sim & \frac{2}{3} \ln\left[\frac{3}{2}(p+1)N_{\text{T}}\right] + \frac{2(3+p)}{3(1+p)} \ln\left(\frac{\phi_{\text{ini}}}{m_{\text{Pl}}}\right) \\ & - \frac{2}{3(p+1)} \ln\left(\frac{H_0}{m_{\text{Pl}}}\right) + \frac{2}{3(p+1)} \ln\left(\frac{m}{m_{\text{Pl}}}\right). \end{aligned} \quad (61)$$

For the fiducial model with  $p = 3$ ,  $\phi_{\text{ini}} = 3.1m_{\text{Pl}}$  and  $N_{\text{T}} = 60$  one obtains  $\Delta N \simeq 26 < 60$ . Therefore, even in the extreme case where the quintessence field starts at  $Q_{\text{up}}$ , inflation lasts enough  $e$  folds so that  $Q$  has time to reach the attractor.

In conclusion, in this subsection, we have shown that, during inflation, the evolution of the quintessence field is characterized by two very different time scales. One scale describes the evolution of the adiabatic time-dependent minimum while the second one represents the period of the rapid oscillations around this minimum. We have demonstrated that  $Q_{\min}$  is in fact an attractor and that, regard-

less of the initial conditions, the quintessence field always has enough  $e$  folds during inflation to join this attractor. Although the above conclusion has been established in the quadratic part of the potential, it is in fact true even in the regime where the potential is proportional to  $Q^{-2p}$  as confirmed by a numerical study of the Klein-Gordon equation.

### C. Numerical study of the Klein-Gordon equation

We have just seen that the equation of motion cannot be analytically integrated with the complete potential. As a consequence, the part where the potential is proportional to  $Q^{-2p}$  has not been explored for values of the initial conditions far from the minimum. In this subsection, we perform the integration numerically. The difficulty is that we have to deal with very small quantities. It is therefore necessary to absorb these small quantities into a redefinition of the quintessence field which greatly facilitates the numerical integration. For this purpose, we write

$$\frac{Q}{m_{\text{Pl}}} = \lambda \mathcal{Q}, \quad (62)$$

where  $\lambda$  is a constant. It is easy to show that, if  $\lambda$  is chosen to be

$$\lambda = \left(\frac{m}{m_{\text{Pl}}}\right)^{-1/(p+1)} \left(\frac{H_0}{m_{\text{Pl}}}\right)^{1/(p+1)}, \quad (63)$$

then we can remove the dangerous coefficients from the equation of motion which now reads

$$\begin{aligned} \frac{d^2 \mathcal{Q}}{dN^2} + \left(3 + \frac{1}{H} \frac{dH}{dN}\right) \frac{d\mathcal{Q}}{dN} + \frac{3}{4\pi} \left(\frac{\phi}{m_{\text{Pl}}}\right)^{-2} \\ \times \left[-\frac{p}{4} \mathcal{Q}^{-2p-1} + 4\pi^2 \left(\frac{\phi}{m_{\text{Pl}}}\right)^4 \mathcal{Q}\right] = 0. \end{aligned} \quad (64)$$

In particular, it is interesting to evaluate how the time-dependent minimum looks after the rescaling. From Eq. (43), one gets

$$\mathcal{Q}_{\min}(N) = \left(\frac{p}{16\pi^2}\right)^{1/(2p+2)} \left(\frac{\phi}{m_{\text{Pl}}}\right)^{-2/(p+1)}. \quad (65)$$

The results of the numerical integration are presented in Figs. 2 and 3 for two different initial conditions. We always assume that the inflaton field starts at  $\phi_{\text{ini}} = 3.1m_{\text{Pl}}$  which, as already mentioned, means that the total number of  $e$  folds during inflation is  $N_{\text{T}} = 60$ . The quintessence potential has been chosen such that  $p = 3$ . This also completely specifies  $Q_{\min}$  and, in particular, we have  $Q_{\min}(N=0) \simeq 0.34$  as can be checked directly on the figures. The evolution of  $Q_{\min}(N)$  is represented by the red dotted curve in Figs. 2 and 3. Moreover, with the values of  $H_0$  and  $m$  discussed before, the rescaling constant  $\lambda$  is equal to  $\lambda \simeq 1.6 \times 10^{-14}$  and, therefore, the initial value of the attractor is in fact  $Q_{\min}(N=0) \simeq 5.6 \times 10^{-15} m_{\text{Pl}}$ .



In Fig. 2, one has  $Q_{\text{ini}} = 0.05$  or  $Q_{\text{ini}} \approx 8.3 \times 10^{-16} m_{\text{Pl}}$ . Initially, the field is therefore in the region where the potential is proportional to  $Q^{-2p}$ , i.e. the region which was explored analytically before. We see that the evolution is very similar to what was discussed before. We have a period of rapid oscillations and then, after a few  $e$  folds, the attractor is joined. We notice that the period of these oscillations is in full agreement with the estimate of Eq. (56). One can check that the amplitude of the oscillations decreases as  $\exp(-3N/2)$  as demonstrated in the previous subsection. We conclude that all the properties established before are confirmed by the numerical study, even in the part of the potential where it is proportional to  $Q^{-2p}$ . However, one should also notice that, if the field is

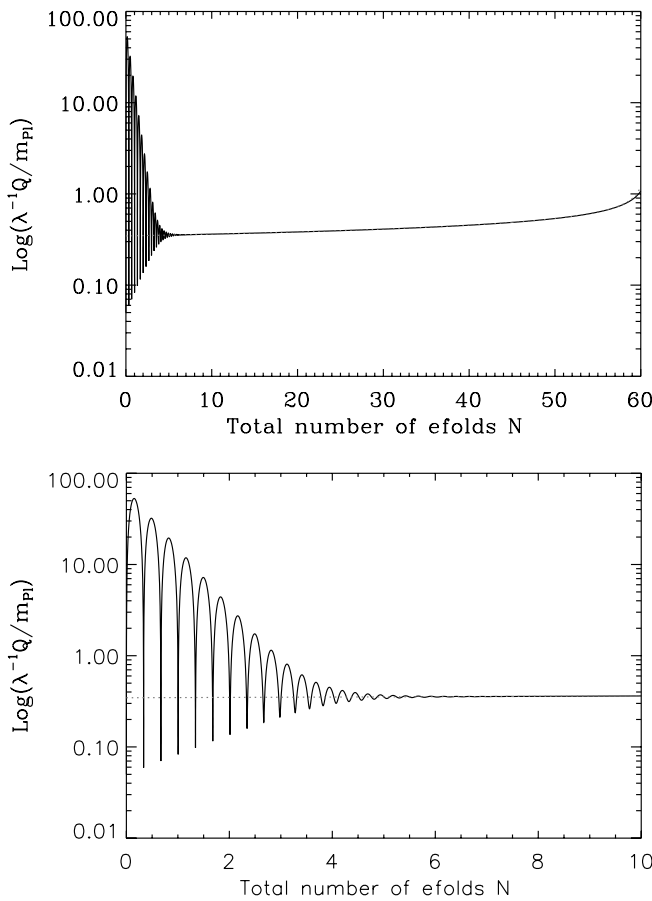


FIG. 2. Upper panel: Evolution of the quintessence field during inflation (solid black line). The model of inflation is chaotic inflation with a massive potential and the initial value of the inflaton is chosen to be  $\phi_{\text{ini}} = 3.1 \times m_{\text{Pl}}$  corresponding to a total number of  $e$  folds  $N_{\text{T}} = 60$ . The potential of the quintessence field is of the Ratra-Peebles type with  $p = 3$ . The initial value of the quintessence field is taken to be  $Q_{\text{ini}} = 0.05$  or  $Q_{\text{ini}} \approx 8.3 \times 10^{-16} m_{\text{Pl}}$ . This initial value is such that  $Q_{\text{ini}} < Q_{\text{min}}$  where  $Q_{\text{min}}$  is the time-dependent minimum of the effective potential. The evolution of  $Q_{\text{min}}(N)$  is given by the dotted red curve. Bottom panel: a zoom of the upper figure at the beginning of inflation.

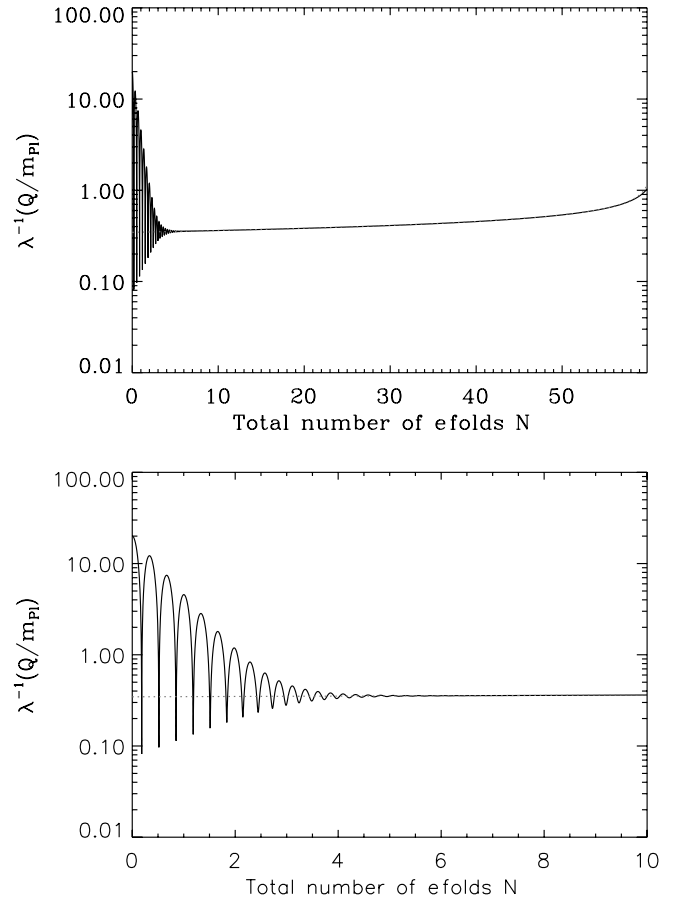


FIG. 3. Upper panel: Evolution of the quintessence field during inflation (solid black line) with the initial condition  $Q_{\text{ini}} = 20$  or  $Q_{\text{ini}} \approx 3.3 \times 10^{-13} m_{\text{Pl}}$ . This initial condition corresponds to a situation where  $Q_{\text{ini}} > Q_{\text{min}}$ . The parameters characterizing the model are identical to those used in Fig. 2. The dotted red curve represents the time-dependent minimum  $Q_{\text{min}}(N)$ . Bottom panel: a zoom of the upper figure at the beginning of inflation.

initially very displaced from its minimum such that  $Q_{\text{ini}} \ll Q_{\text{min}}$ , then the simple Fortran code used to integrate the equation of motion can quickly run into numerical problems. This is probably due to the fact that  $Q^{-2p}$  is a very steep potential. Despite this remark, one sees no reason why, in this regime, the evolution of  $Q$  should be different from what has been described before.

In Fig. 3, one has  $Q_{\text{ini}} = 20$  or  $Q_{\text{ini}} \approx 3.3 \times 10^{-13} m_{\text{Pl}}$ . This time, one starts from the other part of the potential, where  $Q_{\text{in}} > Q_{\text{min}}$  and  $V(Q) \propto Q^2$ . The remarks made before also apply to this case which appears to be in full agreement with the analytical estimates of the previous subsection.

#### IV. DISCUSSION AND CONCLUSIONS

In order to study the influence of the interaction term and to compare its effect with the standard case, it is interesting

to give the evolution of the quintessence field when this one does not interact with the inflaton (and when  $Q$  remains a test field). In particular, we are interested in calculating, for a given initial condition at the beginning of inflation, the value of  $Q$  at the end of inflation (or at the beginning of the radiation dominated era) in both cases (i.e. with and without interaction). The case without interaction can be easily treated because the Klein-Gordon equation can be integrated in the slow-roll approximation; see Ref. [34]. In fact, this equation can be rewritten as

$$\frac{\ddot{Q}}{H(\phi)\dot{Q}} = -\frac{V''_{\text{RP}}(Q)}{3H^2(\phi)} + \epsilon, \quad (66)$$

where a dot denotes a derivative with respect to cosmic time. In the above equation  $V_{\text{RP}}(Q)$  now means the Ratra-Peebles potential, namely,  $V_{\text{RP}}(Q) \propto Q^{-2p}$  since this is the potential for the quintessence field in the absence of any interaction with the inflaton field. Because of the smallness of the parameter  $\epsilon$ , the slow-roll approximation can be applied to the equations describing the motion of the

quintessence field without interaction if the following condition is satisfied:

$$\frac{V''_{\text{RP}}(Q)}{3H^2(\phi)} \ll 1. \quad (67)$$

If one applies this condition to the Ratra-Peebles potential, one gets

$$\left(\frac{Q}{m_{\text{Pl}}}\right)^{2(p+1)} \gg \frac{p(2p+1)}{2\pi} \left(\frac{M}{m_{\text{Pl}}}\right)^{4+2p} \left(\frac{m}{m_{\text{Pl}}}\right)^{-2} \left(\frac{\phi}{m_{\text{Pl}}}\right)^{-2}. \quad (68)$$

This formula is similar to Eq. (48) of Ref. [34]. It can also be rewritten as

$$\lambda^{-1} \frac{Q}{m_{\text{Pl}}} > \mathcal{F}(p) \left(\frac{\phi}{m_{\text{Pl}}}\right)^{-1/(p+1)}, \quad (69)$$

where  $\mathcal{F}(p) = [p(2p+1)/(2\pi)]^{1/[2(p+1)]} \simeq \mathcal{O}(1)$ . Then, integrating the Klein-Gordon equation leads to the following expression:

$$Q_{\text{no inter}}(N) = Q_{\text{ini}} \left[ 1 - 4p(p+1) \left(\frac{m}{m_{\text{Pl}}}\right)^{-2} \left(\frac{M}{m_{\text{Pl}}}\right)^{4+2p} \left(\frac{Q_{\text{ini}}}{m_{\text{Pl}}}\right)^{-2(p+1)} \ln \frac{\phi(N)}{\phi_{\text{ini}}} \right]^{1/2(p+1)} \quad (70)$$

$$\simeq Q_{\text{ini}} \left[ 1 + 2p(p+1) \left(\frac{m}{m_{\text{Pl}}}\right)^{-2} \left(\frac{M}{m_{\text{Pl}}}\right)^{4+2p} \left(\frac{Q_{\text{ini}}}{m_{\text{Pl}}}\right)^{-2(p+1)} \frac{N}{N_{\text{T}}} \right]^{1/2(p+1)}, \quad (71)$$

where in the last equality we have used the fact that the evolution of the inflaton field can be approximated by  $\phi \simeq \phi_{\text{ini}}[1 - N/(2N_{\text{T}})]$ . The subscript “no inter” just reminds that the above equation gives  $Q$  in the case where there is no interaction between  $Q$  and  $\phi$ . From this expression, we deduce that the quintessence field is frozen if

$$\lambda^{-1} \frac{Q_{\text{ini}}}{m_{\text{Pl}}} \gtrsim 1. \quad (72)$$

If this condition is satisfied, then obviously the condition (69) is also satisfied. The contrary is not necessarily true but, as shown for instance in Fig. 3 of Ref. [34], this concerns only a small range of initial conditions. Therefore, we can consider that  $Q_{\text{no inter}} \simeq Q_{\text{ini}}$ .

We are now in a position where the values of the quintessence field with and without interaction can be compared at the end of inflation. Inflation stops when the slow-roll parameter  $\epsilon$  is equal to unity corresponding to  $\phi_{\text{end}}/m_{\text{Pl}} = 1/(2\sqrt{\pi})$ . With the interaction term taken into account, the field will be on the attractor  $Q_{\text{min}}$  and, therefore, its value at the end of inflation is just the value of  $Q_{\text{min}}$  at the end of inflation, namely,

$$\frac{Q_{\text{min}}}{m_{\text{Pl}}} \Big|_{N=N_{\text{T}}} = \lambda \left(\frac{p}{16\pi^2}\right)^{1/(2p+2)} \left(\frac{1}{2\sqrt{\pi}}\right)^{-2/(p+1)} \quad (73)$$

$$= \mathcal{H}(p)\lambda. \quad (74)$$

As already mentioned, the striking feature of this expression is that it does not depend on  $Q_{\text{ini}}$ . For orders of magnitude estimate, one can consider that  $\mathcal{H}(p) = \mathcal{O}(1)$ . For  $p = 3$ , which is our fiducial model, one has  $Q_{\text{min}}(N = N_{\text{T}}) \simeq 1.9 \times 10^{-14} m_{\text{Pl}}$ . Therefore the ratio of  $Q$  at the end of inflation without the interaction term taken into account to  $Q$  at the end of inflation with the interaction term into account is given by

$$\frac{Q_{\text{no inter}}}{Q_{\text{inter}}} \Big|_{\text{end}} \simeq \frac{Q_{\text{ini}}}{m_{\text{Pl}}} \left(\frac{m}{m_{\text{Pl}}}\right)^{1/(p+1)} \left(\frac{H_0}{m_{\text{Pl}}}\right)^{-1/(p+1)} \quad (75)$$

$$\sim 10^{55/(p+1)} \frac{Q_{\text{ini}}}{m_{\text{Pl}}}. \quad (76)$$

This ratio is necessarily greater than 1, see Eq. (72), and for “large” initial conditions can be much bigger than 1. This means that, generically,  $Q_{\text{no inter}} \gg Q_{\text{inter}}$ , i.e. the effect of the interaction is to force the quintessence field to remain small during inflation.

We are now in a position to comment on the role of the attractor during inflation as initial conditions for the late time behavior of the quintessence field, which was one of the main motivations of the present study. As is well known, the main property of quintessence models in the radiation or matter era is that there is a tracking behavior with an attractor [9]. This means that there is no need to fine-tune the initial conditions and that the solution will be

on track today for a large range of initial conditions. If one fixes the initial conditions at the end of inflation, i.e.  $z \simeq 10^{28}$ , then, the allowed initial values for the energy density are approximately such that [12,13]  $10^{-37} \leq \rho_Q \leq 10^{61} \text{ GeV}^4$ , where  $10^{-37} \text{ GeV}^4$  is the background energy density at equality whereas  $10^{61} \text{ GeV}^4$  represents the background energy density (i.e. the radiation energy density) at the initial redshift. If, for instance,  $p = 3$  and if the scalar field starts at rest, this means that the initial values of the field are such that  $10^{-18} m_{Pl} \leq Q_{in} \leq 10^{-2} m_{Pl}$  just after inflation. As can be seen, we have found that the quintessence attractor during inflation leads to initial values within this range, hence such that the quintessence field is on track now. Therefore, the quintessence attractor during inflation guarantees natural initial conditions for the subsequent evolution of the quintessence field.

Two loopholes could modify the above conclusion. First, we have seen that the numerical integration, on which our study is based, is valid only if the initial value of  $Q$  is not too far from the initial value of the minimum. We have used the results obtained under this condition and have extrapolated them for any initial conditions. If the initial value of the quintessence field is far from the time-dependent minimum, we still expect a phase of oscillations. However, since we have noticed that the amplitudes of the oscillations tend to be quite big even if the initial displacement from the minimum remains reasonable (this is not surprising for a potential like  $Q^{-2p}$  which is very ‘‘abrupt’’), it could actually happen that the amplitudes of the oscillations, in the case where the initial displacement is large, are so big that the assumption that the quintessence field is a test field becomes violated. This is a regime that has not been studied in the present article. Some new interesting effects could occur in this case. However, we suspect that the numerical study of this situation could be quite tricky.

Second, it has been shown in Ref. [34] that the quantum effects could strongly modify the evolution of  $Q$  during inflation. These quantum effects have been calculated in Ref. [34] by means of the stochastic inflation formalism for a free field. Therefore, what should now be done is to compute these effects in the case where the interaction term is present. This is clearly a difficult task which is beyond the scope of the present paper. In Ref. [34], it has been shown that the quantum effects can push the quintessence field to quite large values at the end of inflation. As a consequence, the attractor solution could be joined only at a late time and even after the present time. In this case the quintessential scenario would lose an attractive feature, namely, its insensitivity to the initial conditions. In this respect, the results reached in the present article are good news since the effect of the interaction term seems to retain the field to quite small values. One could speculate that this could maybe compensate the influence of the quantum effects.

Finally, a last comment is in order. In this paper, we have investigated the coupling of quintessence to inflation in the very early universe. In particular, we have shown that nonrenormalizable operators suppressed by the Planck mass play a crucial role. Later in the evolution of the Universe, one may inquire about couplings between quintessence and matter in supergravity models. As is well known, the quintessence field cannot couple strongly to matter fields. Indeed, this would lead to a possible fifth force or violation of the equivalence principle [23]. Such couplings have to be extremely suppressed. There are two dangerous types of couplings. First of all, the electromagnetic gauge coupling constant is allowed to become field dependent in supergravity  $1/g^2 = f$  where  $f$  is a holomorphic function of the fields. However, in the minimal supergravity framework, the time variation of the fine structure constant is suppressed since  $f$  has no dependence on the quintessence field (but, of course, it would be possible to work in a nonminimal supergravity framework). This guarantees that the quintessence field plays no role in the possible time variation of the fine structure constant. Similarly, from the form of the SUGRA Lagrangian, it is easy to see that particle masses such as the fermions of the standard model pick a quintessence dependent part

$$m(Q) = e^{\kappa Q^2/2} |\delta(Q + i\tilde{Q})| v, \quad (77)$$

where  $v \sim \langle H \rangle$  is one of the Higgses vevs (which is field independent here). The overall exponential prefactor is similar to the one appearing in the SUGRA potential, and  $\delta(Q + i\tilde{Q})$  is a Yukawa coupling which may be field dependent. Supersymmetry requires this coupling to be a holomorphic function of the field, therefore involving both the quintessence field  $Q$  and its associated imaginary part  $\tilde{Q}$  in the quintessence chiral multiplet. To avoid fifth-force experiments such as the recent Cassini spacecraft experiment, one must require that the Eddington (post-Newtonian) parameter  $|\gamma - 1| \leq 5 \times 10^{-5}$ ; see Ref. [35]. This can be realized by imposing that  $\alpha_Q^2 \leq 10^{-5}$  and  $\alpha_{\tilde{Q}}^2 \leq 10^{-5}$ , where  $\alpha_Q \equiv \kappa^{-1/2} d[\ln m(Q)]/dQ$  and  $\alpha_{\tilde{Q}} \equiv \kappa^{-1/2} d[\ln m(Q)]/d\tilde{Q}$ . Explicitly, the coefficients  $\alpha_Q$  and  $\alpha_{\tilde{Q}}$  are given by

$$\alpha_Q = \kappa^{1/2} Q + \kappa^{-1/2} \frac{d \ln |\delta|}{dQ}. \quad (78)$$

As  $Q$  goes to  $m_{Pl}$  in the recent past, the first term becomes of order 1. It is precisely at this point that model building issues show up. The possibility of violating the above constraints depends on the  $Q$  dependence of the Yukawa couplings. Although it is not the main purpose of the present paper, as we focus on the behavior of  $Q$  in the very early universe, it is worth presenting a model where no problem occurs. If the Yukawa couplings possess the following explicit dependence on  $Q$ :

$$\delta(Q) = e^{-\kappa Q^2/2} \delta_0, \quad (79)$$

where  $\delta_0$  is a constant coinciding with the standard model Yukawa coupling, then one has  $\alpha_Q = 0$ . Now, the imaginary part of the quintessence chiral multiplet  $\tilde{Q}$  has no potential, i.e. it is a flat direction. As a consequence, we find that

$$\alpha_{\tilde{Q}} = -\kappa^{1/2} \tilde{Q}. \quad (80)$$

Compatibility with experiments implies that the value of  $\tilde{Q}$  now must be almost vanishing. As this field does not have any dynamics, this can be achieved by putting  $\tilde{Q} = 0$  in the early universe. Hence, we conclude that one can satisfy the fifth-force experiment constraints in quintessence models based on supergravity. Of course, justifying the above Yukawa couplings would necessitate a thorough study and a well-motivated model.

The results obtained in this article concern the early universe and are independent of the above discussion on

the variation of masses in the recent past of the universe. On the contrary, nonrenormalizable couplings between the quintessence field and cold dark matter may have a crucial impact on the late time physics of the quintessence field, i.e. on the coincidence problem. This is left for future work.

In conclusion, we have shown that the nonrenormalizable interactions between the inflaton and the quintessence field have drastic consequences during inflation. The quintessence field follows an attractor and remains small compared to the Planck scale at the end of inflation. This sets initial conditions for the quintessence field such that  $Q$  is on track today.

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