Determining the regimes of cold and warm inflation in the supersymmetric hybrid model

Mar Bastero-Gil[1,*] and Arjun Berera[2,†]

[1]*Departamento de Física Teórica y del Cosmos, Universidad de Granada, Granada-18071, Spain*

[2]*School of Physics, University of Edinburgh, Edinburgh, EH9 3JZ, United Kingdom*

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The SUSY hybrid inflation model is found to dissipate radiation during the inflationary period. Analysis is made of parameter regimes in which these dissipative effects are significant. The scalar spectral index, its running, and the tensor-scalar ratio are computed in the entire parameter range of the model. A clear prediction for strong dissipative warm inflation is found for $n_S - 1 \simeq 0.98$ and a low tensor-scalar ratio much below 10^{-6}. The strong dissipative warm inflation regime also is found to have no η problem and the field amplitude much below the Planck scale. As will be discussed, this has important theoretical implications in permitting a much wider variety of SUGRA extensions to the basic model.

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I. INTRODUCTION

The success of the inflationary paradigm has motivated in recent times more serious efforts in building realistic particle physics models that incorporate cosmology [1–3]. The objective of this sort of model building is to account for various cosmological features, central being inflation, but also leptogenesis, dark matter etc. to constrain the high energy properties of the model, and such that in the low-energy regime the model reduces to the standard model. Supersymmetry (SUSY) plays a central role here since, aside from its attractive features for particle physics, it also allows stabilizing very flat scalar potentials, which are essential in inflation models due to density perturbation constraints. In this respect, a widely studied SUSY model of inflation has been the hybrid model,

$$W = \kappa S(\Phi_1\Phi_2 - \mu^2), \tag{1}$$

where Φ_1, Φ_2 are a pair of charged fields[1] under some gauge group G, and S is the singlet which plays the role of the inflaton.

An important feature about the model, Eq. (1), and its various embeddings into more realistic particle physics models [2,5] is that the inflaton field generally interacts with other fields, with coupling strengths that can be fairly large. Even though the nonzero vacuum energy necessary to drive inflation will break SUSY, this underlying symmetry can still protect the very flat inflaton potential from radiative corrections arising from these perturbatively large couplings [6,7]. It has been observed in recent works [6,8] that the effect of interactions of the inflaton with other fields does not simply affect the local contributions to the inflaton effective potential, but also induces temporally nonlocal terms in the inflaton evolution equation, that in the moderate to large perturbative regime yield sizable dissipative effects. Although SUSY cancels the large local quantum effects, for the dynamical problem the nonlocal quantum effects cannot be canceled by SUSY. These dissipative effects in general can lead to warm inflationary regimes [9]. Thus, the conclusion of the works [6–8] is that, in general, models in which the inflaton has interactions with other fields with moderate to strong perturbative coupling, inflation divides into two different dynamical regimes, cold [10–12] and warm [9]. This finding is very important, since these two types of inflationary dynamics are qualitatively much different. Thus, one should expect different observable signatures in the two cases, as well as other theoretical differences in the treatment of inflation.

The purpose of this paper is to apply these recent findings about dissipative dynamics during inflation to the SUSY hybrid model, Eq. (1), and to common extensions of this model. In particular, two models will be studied in this paper, Eq. (1) and this model with a matter field $\Delta\bar{\Delta}$ coupled to it as

$$W = \kappa S(\Phi_1\Phi_2 - \mu^2) + g\Phi_2\Delta\bar{\Delta}. \tag{2}$$

The above is a toy model representing an example of how the basic hybrid model, Eq. (1), is embedded within a more complete particle physics model, in this case through the Δ fields.

In this paper we will study inflation for both models, Eqs. (1) and (2). We will show that in the above models both cold and warm inflation exist and we will determine the parameter regime for them. This will then explicitly verify the conclusions from the recent papers on dissipation [6,8], that showed both types of inflationary dynamics could exist. In both inflationary regimes, we will calculate the scalar spectral index $n_S - 1$, and its running $dn_S/d\ln k$. With this information, we will then identify the qualitative and quantitative differences arising from the warm versus cold regimes.

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*Electronic address: mbg@ugr.es

†Electronic address: ab@ph.ed.ac.uk

[1]One could also consider the fields $\Phi_{1,2}$ being gauge singlets. However, in this case the gravitino constraint on the reheating temperature translates into κ being at most of the order of 10^{-5} [4]. That constraint is avoided when instead $\Phi_{1,2}$ are nonsinglets.

We emphasize here that the main objective of this paper is to determine in an explicit and well-known model how these two dynamically very different inflationary regimes emerge. This result departs radically from current wisdom, in which it is tacitly always assumed that all regimes in any model are cold inflationary. In light of our result, many other particle physics model building issues emerge in the warm inflationary regimes that we identify. Although we make some effort in this paper to discuss these issues, it is not the purpose here to dwell on them. In cold inflation, these issues have been the subject of many years of study, and likewise a complete understanding of similar issues for warm inflationary regimes will take focused effort in future work.

The paper is organized as follows. Section II reviews the effective potential for the SUSY hybrid model, as well as some of the basic results based on this model for cold inflation. Section III reviews the main results of dissipative dynamics during inflation based on [6,8]. Included here are the main formulas for density perturbations and the scalar spectral index in the presence of dissipation and a thermal bath. Sections III A and III B compute the effect that dissipation has on inflaton evolution in the two models, Eqs. (1) and (2), respectively. The outcome of the analysis in these two subsections is a graph that divides the parameter space of the model into cold and warm inflationary regimes, and associated predictions for the scalar spectral index, its running, and the tensor-scalar ratio. Section V addresses some model building issues that emerge in the newly found warm inflationary regimes, such as the gravitino abundances and the constraint on the reheating temperature. Finally, Sec. V states our conclusions.

II. SUSY HYBRID MODEL

We briefly review first some well-known results about standard supersymmetric hybrid inflation [13–15], in order to study later the main modifications introduced when taking into account the dissipative dynamics present during inflation [6,8]. We consider the standard superpotential for the supersymmetric hybrid inflationary model, Eq. (1). Without taking into account SUSY breaking, the zero-energy global minimum in the model, Eq. (1), is located at the vacuum expectation values (VEV) $S = 0$, $\Phi_1 = \Phi_2 = \mu$. If the gauge group G of Φ_1 and Φ_2 is identified with a GUT symmetry, the scale μ would be the GUT symmetry breaking scale. On the other hand, for $|S| > \mu$, there is a local minimum at $\Phi_1 = \Phi_2 = 0$ with potential energy given by the constant term $\kappa^2\mu^4$. Inflation occurs while the system is located in this false vacuum. Here, the inflaton scalar field S and its fermionic partner remain massless at tree level, while the scalars Φ_1 and Φ_2 combine into a pair of real scalar and pseudoscalar particles with mass $m_+^2 = \kappa^2(|S|^2 + \mu^2)$, and another pair with mass $m_-^2 = \kappa^2(|S|^2 - \mu^2)$; their fermionic superpartners are degenerate with mass $m_F = \kappa|S|$.

Because of the splitting in the masses, there is a nonvanishing one-loop radiative correction to the potential, ΔV, which provides the necessary slope and mass correction[2] to the inflaton potential to drive slow-roll inflation. In particular, the first and second derivatives of the effective inflaton potential are given, respectively, by

$$\Delta V' = \frac{\sqrt{2}\kappa^4\mu^3\mathcal{N}}{16\pi^2}\left(x^3\ln\frac{(x^2-1)(x^2+1)}{x^4} + x\ln\frac{x^2+1}{x^2-1}\right) = \frac{\sqrt{2}\kappa^4\mu^3\mathcal{N}}{16\pi^2}F_1[x], \tag{3}$$

$$\Delta V'' = \frac{\kappa^4\mu^2\mathcal{N}}{16\pi^2}\left(3x^2\ln\frac{(x^2-1)(x^2+1)}{x^4} + \ln\frac{x^2+1}{x^2-1}\right) = \frac{\kappa^4\mu^2\mathcal{N}}{16\pi^2}F_2[x], \tag{4}$$

where $\mathcal{N}$ is the dimensionality[3] of the $\Phi_{1,2}$ representations, $x \equiv \phi_S/(\sqrt{2}\mu)$, ϕ_S is the real part of the complex field S, and we are setting the imaginary components of all the fields to zero for simplicity. For large x, we have $F_1[x] \sim 1/x$, $F_2[x] \sim -1/x^2$.

Standard (cold) "slow-roll" inflation is characterized by having small slow-roll parameter $\epsilon_H \ll 1$ and $\eta_H \ll 1$, with

$$\epsilon_H = \frac{m_P^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{\kappa^2}{(4\pi)^2}\left(\frac{\kappa m_P}{4\pi\mu}\right)^2(\mathcal{N}F_1[x])^2 \approx \frac{\kappa^2}{(4\pi)^2}\left(\frac{\kappa m_P}{4\pi\mu}\right)^2\frac{\mathcal{N}^2}{x^2}, \tag{5}$$

$$\eta_H = m_P^2\frac{V''}{V} = \left(\frac{\kappa m_P}{4\pi\mu}\right)^2\mathcal{N}F_2[x] \approx -\left(\frac{\kappa m_P}{4\pi\mu}\right)^2\frac{\mathcal{N}}{x^2}, \tag{6}$$

where $m_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Therefore, during inflation the evolution of the inflaton field is well approximated by

$$3H\dot{\phi}_S + V' \simeq 0. \tag{7}$$

At this point one should worry about supergravity (SUGRA) corrections to the inflaton potential. Generically, those give rise to mass corrections for the scalars during inflation of the order of $O(H^2)$, with $\eta_H \sim O(1)$, spoiling inflation (the so-called η problem [13,16]). However, such corrections are not present if we take the minimal Kahler potential for the fields with a superpotential like Eqs. (1) and (2), and this is the choice we adopt in this paper. Nevertheless, they will induce a quartic term

[2]We will not add any additional SUSY breaking mass term for the inflaton during inflation, so that its mass is given purely by the radiative corrections. Given that the μ scale is typically of the order of the GUT scale, SUSY breaking masses of the order of $O(1)$ TeV are negligible unless the coupling $\kappa < O(10^{-5})$.

[3]We take $\mathcal{N} = 1$ throughout this paper unless otherwise explicitly stated.

(plus some higher order corrections) in the inflaton potential [2,13,17,18], with

$$V \simeq \kappa^2 \mu^4 \left(1 + \frac{\phi_S^4}{8m_P^4} + \cdots\right) + \Delta V. \tag{8}$$

The quartic term dominates the inflationary dynamics when $\phi_S \sim m_P$, which happens for $\kappa \sim O(1)$.

The values of the coupling κ and the scale μ consistent with the inflationary dynamics are obtained by demanding that (a) we have "enough" inflation (at least 60 e-folds), and (b) that the amplitude of the primordial spectrum generated by the inflaton vacuum fluctuations are in the range given by COBE observations. The former constraint gives the value of the inflaton field $N_e (\simeq 60)$ e-folds before the end of inflation,

$$x_N \approx \sqrt{2N_e}\left(\frac{\kappa m_P}{4\pi\mu}\right), \tag{9}$$

which is then used to evaluate the amplitude of the primordial curvature spectrum,

$$P_{\mathcal{R}}^{1/2} = \left(\frac{H}{\dot{\phi}_S}\right)\left(\frac{H}{2\pi}\right) = \sqrt{\frac{2}{3}}\left(\frac{\kappa}{4\pi}\right)^2\left(\frac{4\pi\mu}{\kappa m_P}\right)^3 \frac{1}{F_1[x_N]} \approx \sqrt{\frac{4N_e}{3}}\left(\frac{\mu}{m_P}\right)^2. \tag{10}$$

Therefore, using the COBE normalization [19,20] $P_{\mathcal{R}}^{1/2} = 5 \times 10^{-5}$ at $N_e \simeq 60$, we have[4] that $\mu \simeq 2 \times 10^{-3} m_P \simeq 5 \times 10^{15}$ GeV.

Given that implicitly we are working with a SUGRA model, at most the VEV of the inflaton field could be of the order of the Planck scale, preferably below that scale. From Eq. (9) one can see that $\phi > m_P$ for values of $\kappa > 0.8$ and so are excluded [17]. Moreover, taking into account the quartic SUGRA correction in Eq. (8), the spectrum becomes blue tilted ($n_S > 1$) [2,17,18,21] already for $\kappa \simeq 0.05$, which is not favored by the observational data on the spectral index from WMAP,[5] $n_S = 0.93 \pm 0.03$ [20].

III. DISSIPATIVE DYNAMICS DURING INFLATION

Dissipative effects can be important already during inflation, modifying the inflationary dynamics described by Eq. (7). These are related to the quantum corrections in the effective potential of the background field. When neither the decay of the inflaton nor that of the fields coupled to the inflaton are kinematically allowed, loop corrections to the propagators are real, and they are absorbed into the renormalized masses and couplings, order by order in perturbation theory. On the other hand, when the inflaton (or the fields coupled to the inflaton) can decay into other particles, the propagators in the loop have the standard Breit-Wigner form, with an imaginary contribution related to the decay rate Γ. Therefore, when computing the one-loop effective potential for the inflaton field, the contributions associated to the decay rate lead to dissipative effects [6,8]. In general, this reflects itself in the form of temporally nonlocal terms in the inflaton evolution equation. Under certain approximations this translates into a simple effective friction term Υ_S in the equation of motion for the background inflaton field [6,8],

$$\ddot{\phi}_S + (3H + \Upsilon_S)\dot{\phi}_S + \Delta V' = 0. \tag{11}$$

The emergence of this friction term due to these underlying decay channels implies the dynamics of the system is such that part of the inflaton energy is dissipated into the lighter particles produced in the decays, i.e., into radiation ρ_R, with

$$\dot{\rho}_R + 4H\rho_R = \Upsilon_S \dot{\phi}_S^2. \tag{12}$$

Although the basic idea of interactions leading to dissipative effects during inflation is generally valid, the above set of equations has strictly been derived in [6,8] only in the adiabatic-Markovian limit, i.e., when the fields involved are moving slowly, which requires

$$\frac{\dot{\phi}_S}{\phi_S} < H < \Gamma, \tag{13}$$

with Γ being the decay rate. The second inequality, $H < \Gamma$, is also the condition for the radiation (decay products) to thermalize.

Thus, in general, any inflation model could have two very distinct types of inflationary dynamics, which have been termed cold and warm [8,9]. The cold inflationary regime is synonymous with the standard inflation picture [10–12], in which dissipative effects are completely ignored during the inflation period. On the other hand, in the warm inflationary regime dissipative effects play a significant role in the dynamics of the system. A rough quantitative measure that divides these two regimes is $\rho_R^{1/4} \approx H$, where $\rho_R^{1/4} > H$ is the warm inflation regime and $\rho_R^{1/4} \lesssim H$ is the cold inflation regime. This criteria is independent of thermalization but, if such were to occur, one sees this criteria basically amounts to the warm inflation regime corresponding to when $T > H$. This is easy to understand since the typical inflaton mass during inflation is $m_\phi \approx H$ and so, when $T > H$, thermal fluctuations of the inflaton field will become important. This criteria for entering the warm inflation regime turns out to require the dissipation of a very tiny fraction of the inflaton vacuum

[4] Corrections to this estimation appear for small values of $\kappa < 0.01$, for which $x_N \sim O(1)$.

[5] From the WMAP data only, the best fit value without running of the spectral index is $n_S = 0.99 \pm 0.04$. A recent analysis of the combined Lyman-α forest spectra + WMAP [22] also gives $n_S = 0.99 \pm 0.03$ with no running, and $n_S = 0.959 \pm 0.036$ with $dn_S/d\ln k = -0.033 \pm 0.025$. However, a similar analysis in Ref. [23] gives $n_S = 0.98 \pm 0.02$ with $dn_S/d\ln k = -0.003 \pm 0.01$.

energy during inflation. For example, for inflation with vacuum (i.e., potential) energy at the GUT scale $\sim 10^{15-16}$ GeV, in order to produce radiation at the scale of the Hubble parameter, which is $\approx 10^{10-11}$ GeV, it just requires dissipating one part in 10^{20} of this vacuum energy density into radiation. Thus, energetically not a very significant amount of radiation production is required to move into the warm inflation regime. In fact the levels are so small, and their eventual effects on density perturbations and inflaton evolution are so significant that care must be taken to account for these effects in the analysis of any inflation models.

The conditions for slow-roll inflation ($\dot{\phi}_S^2 \ll V$, $\ddot{\phi}_S \ll H\dot{\phi}_S$) are modified in the presence of the extra friction term Υ_S, and we have now

$$\epsilon_\Upsilon = \frac{\epsilon_H}{(1+r)^2}, \tag{14}$$

$$\eta_\Upsilon = \frac{\eta_H}{(1+r)^2}, \tag{15}$$

where $r = \Upsilon_S/(3H)$, and ϵ_H, η_H are the slow-roll parameters without dissipation given in Eqs. (5) and (6). In addition, when the friction term Υ_S depends on the value of the inflaton field, we can define a third slow-roll parameter

$$\epsilon_{H\Upsilon} = \frac{r}{(1+r)^3}\beta_\Upsilon, \tag{16}$$

with

$$\beta_\Upsilon = \frac{V'}{3H^2}\frac{\Upsilon_S'}{\Upsilon_S}. \tag{17}$$

Similarly to the slow-roll regime without dissipation, when $\eta_\Upsilon < 1$, $\epsilon_\Upsilon < 1$, and $\epsilon_{H\Upsilon} < 1$, Eqs. (11) and (12) are well approximated by

$$\dot{\phi}_S \simeq -\frac{\Delta V'}{3H}\frac{1}{1+r}, \tag{18}$$

$$\rho_R \simeq \frac{\Upsilon_S}{4H}\dot{\phi}_S^2 \simeq \frac{1}{2}\frac{r}{(1+r)^2}\epsilon_H V, \tag{19}$$

and the number of e-folds is given by

$$N_e \simeq -\int_{\phi_{Si}}^{\phi_{Se}} \frac{3H^2}{\Delta V'}(1+r)d\phi. \tag{20}$$

Obviously, when $\Upsilon_S \ll 1$ we recover the standard "cold" hybrid inflation (CHI) scenario.

The effect of the dissipative term is twofold: On one hand, dissipation of the vacuum energy into radiation acts as an extra friction term and slows down the motion of the inflaton field, so that inflation last longer. That means that when Υ_S is non-negligible, we would require in general smaller initial values of the inflaton field in order to have enough (at least 60 e-folds) inflation, Eq. (20). On the other hand, fluctuations in the radiation background affect those of the inflaton field through the interactions, and this in turn will affect the primordial spectrum generated during inflation. Approximately, one can say that when $T > H$ the fluctuations of the inflaton field are induced by the thermal fluctuations, instead of being vacuum fluctuations, with a spectrum proportional to the temperature of the thermal bath. We notice that having $T > H$ does not necessarily require $\Upsilon_S > 3H$. Dissipation may not be strong enough to alter the dynamics of the background inflaton field, but it can be enough even in the weak regime to affect its fluctuations, and therefore the spectrum. Depending on the different regimes, the spectrum of the inflaton fluctuations $P_{\delta\phi}^{1/2}$ is given for cold inflation [24], weak dissipative warm inflation [25,26], and strong dissipative warm inflation [27], respectively, by

$$T < H: \qquad P_{\delta\phi}^{1/2}|_{T=0} \simeq \frac{H}{2\pi}, \tag{21}$$

$$\Upsilon_S < H < T: \qquad P_{\delta\phi}^{1/2}|_T \simeq \sqrt{TH} \sim \sqrt{\frac{T}{H}}P_{\delta\phi}^{1/2}|_{T=0}, \tag{22}$$

$$\Upsilon_S > H: \qquad P_{\delta\phi}^{1/2}|_\Upsilon \simeq \left(\frac{\pi\Upsilon_S}{4H}\right)^{1/4}\sqrt{TH} \sim \left(\frac{\pi\Upsilon_S}{4H}\right)^{1/4}\sqrt{\frac{T}{H}}P_{\delta\phi}^{1/2}|_{T=0}, \tag{23}$$

with the amplitude of the primordial spectrum of the curvature perturbation given by

$$P_{\mathcal{R}}^{1/2} = \left|\frac{H}{\dot{\phi}_S}\right| P_{\delta\phi}^{1/2} \simeq \left|\frac{3H^2}{\Delta V'}\right|(1+r)P_{\delta\phi}^{1/2}. \tag{24}$$

Given the different "thermal" origin of spectrum, the spectral index also changes with respect to the cold inflationary scenario [28–31], even in the weak dissipative warm inflation regime when the evolution of the inflaton field is practically unchanged. Again, for the different regimes, it is obtained:

$$T < H: \qquad n_S - 1 = -6\epsilon_H + 2\eta_H, \tag{25}$$

$$\Upsilon_S < H < T: \qquad n_S - 1 = -\tfrac{17}{4}\epsilon_H + \tfrac{3}{2}\eta_H - \tfrac{1}{4}\beta_\Upsilon, \tag{26}$$

$$\Upsilon_S > H: \qquad n_S - 1 = \left(-\tfrac{9}{4}\epsilon_H + \tfrac{3}{2}\eta_H - \tfrac{9}{4}\beta_\Upsilon\right) \times \frac{1}{(1+r)}. \tag{27}$$

In the latter case there could be appreciable departures from scale invariance in the spectrum; we notice again then in the strong dissipative case, slow-roll only demands

$\epsilon_H \ll (1+r)^2$ and $\eta_H \ll (1+r)^2$, whereas the spectral index depends on the ratios $\epsilon_H/(1+r)$ and $\eta_H/(1+r)$, which are not necessarily much smaller than 1.

The question then is not whether there is dissipation during inflation, but whether this will affect the inflationary predictions. First, how large can Υ_S be in a realistic setup? In the calculations in [6–8], a robust mechanism for dissipation during inflation has been identified. The basic interaction structure for this mechanism is

$$\mathcal{L}_I = -\tfrac{1}{2}g^2\phi^2\chi^2 - g'\phi\bar{\psi}_\chi\psi_\chi - h\chi\bar{\psi}_d\psi_d, \tag{28}$$

where ϕ is the inflaton field, χ and ψ_χ are additional fields to which the inflaton couples, and ψ_d are light fermions into which the scalar χ particles can decay $m_\chi > 2m_{\psi_d}$. This interaction structure can be identified in both models we are studying in this paper, Eqs. (1) and (2). In the next two subsections, the dissipative properties of these two models will be computed based on the results on [6,8] and then the effect of this dissipation to inflation will be studied.

A. Decay into massive fermions

First consider the model, Eq. (1), which has only the minimal matter content. In this model, the scalar with the largest mass m_+ can decay into its fermionic superpartner $\tilde{\Phi}_+$ and a massless inflatino, with decay rate[6]

$$\Gamma_+ = \frac{\kappa^2}{16\pi}m_+\left(\frac{1}{x_N^2+1}\right)^2, \tag{29}$$

where again x_N is the value of the inflaton field (normalized by μ) N_e e-folds before the end of inflation. This decay rate is always smaller than the rate of expansion during inflation:

$$\frac{\Gamma_+}{H} = \sqrt{3}\frac{\kappa^2}{16\pi}\frac{m_P/\mu}{(x_N^2+1)^{3/2}} \ll 1, \tag{30}$$

and strictly speaking the adiabatic-Markovian approximation would not apply. Nevertheless, in order to get some numbers, let us proceed and estimate the dissipative coefficient and the amount of radiation produced. The former is given by

$$\begin{aligned}\Upsilon_S(\phi_S) &= \frac{\sqrt{2}(\kappa^4/4)(\Gamma_+/m_+)}{64\pi\sqrt{1+(\Gamma_+/m_+)^2}\sqrt{\sqrt{1+(\Gamma_+/m_+)^2}+1}}\frac{\phi_S^2}{m_+}\\ &\simeq \frac{\pi^3}{2}\left(\frac{\kappa}{4\pi}\right)^5\frac{x_N^2}{(x_N^2+1)^{5/2}}\mu \simeq \frac{\pi^3}{2}\left(\frac{\kappa}{4\pi}\right)^5\frac{\mu}{x_N^3},\end{aligned} \tag{31}$$

which is always suppressed with respect to the expansion rate during inflation, $H \simeq \kappa\mu^2/(\sqrt{3}m_P)$:

$$\frac{\Upsilon_S}{3H} \simeq \frac{\pi^2}{8\sqrt{3}}\left(\frac{\kappa}{4\pi}\right)^4\frac{m_P}{\mu}\frac{x_N^2}{(x_N^2+1)^{5/2}} \ll 1. \tag{32}$$

Nevertheless, the amount of "radiation" produced, i.e., the energy density dissipated from the inflaton, could be larger than H^4,

$$\begin{aligned}\frac{\rho_R}{H^4} &\simeq \frac{9}{2}\frac{\Upsilon_S}{3H}\epsilon_H\frac{m_P^4}{\kappa^2\mu^4}\\ &\simeq \frac{9}{256\sqrt{3}}\left(\frac{\kappa}{4\pi}\right)^6\left(\frac{m_P}{\mu}\right)^7\frac{x_N^2}{(x_N^2+1)^{5/2}}F_1[x_N]^2.\end{aligned} \tag{33}$$

Given that the ratio ρ_R/H^4 goes like the inverse of x_N^5, the ratio increases as the value of the inflaton field decreases during inflation. However, it only becomes larger than 1 for $\kappa \sim 0.1$ toward the end of inflation, well after the 60 e-folds before the end. We can conclude then that dissipation in this example is too weak to affect either the spectrum of the primordial perturbations or the dynamics of the inflaton field.

B. Decay into massless fermions

As we have seen, dissipation through the decay into massive fermions does not have much effect during inflation. However, in more realistic models, one would expect the presence of other fields, which in principle are not directly relevant during inflation but can play a role during/after reheating. For example, fields coupled to either Φ_1 or Φ_2 are massless during inflation, and become massive at the global minimum. Thus, the model Eq. (2), where we have introduced a pair of matter fields, Δ and $\bar{\Delta}$, coupled to Φ_2. Because they are massless during inflation, they do not contribute to the one-loop effective potential, and radiative corrections are the same as in the previous case, with the slope and curvature of the effective potential given by Eqs. (3) and (4). But now the heaviest field with mass m_+ can decay into the massless fermionic partners of Δ and $\bar{\Delta}$, with decay rate

$$\Gamma_+ = \frac{g^2}{16\pi}m_+. \tag{34}$$

Since now there is no phase space suppression factor in the decay rate, we have $\Gamma_+ \propto \phi_S$, and this can be much larger than the Hubble rate during inflation:

$$\frac{\Gamma_+}{H} = \sqrt{3}\frac{g^2}{16\pi}\left(\frac{m_P}{\mu}\right)(x_N^2+1)^{1/2}. \tag{35}$$

Having $\Gamma_+/H > 1$, all the way up to the end of inflation, requires only $g > 0.16$ for $\kappa < 0.001$ ($g > 0.01$ for $\kappa = 0.5$). This allows us to apply the adiabatic-Markovian limit in the effective equation of motion for the inflaton background field, Eq. (11), with the dissipative coefficient given now by

[6]The interaction Lagrangian is given by $\mathcal{L} = -(\kappa/\sqrt{2})\phi_+\tilde{S}\tilde{\Phi}_+$.

$$\Upsilon_S \simeq \frac{\pi^2}{2}\left(\frac{\kappa}{4\pi}\right)^3\left(\frac{g^2}{16\pi}\right)\frac{x_N^2}{(1+x_N^2)^{1/2}}\mu, \tag{36}$$

and the ratio to the Hubble rate is given by

$$\frac{\Upsilon_S}{3H} \simeq \frac{\kappa^2}{128\sqrt{3}\pi}\left(\frac{g^2}{16\pi}\right)\frac{x_N^2}{(1+x_N^2)^{1/2}}\frac{m_P}{\mu}, \tag{37}$$

which behaves like $\Upsilon_S/(3H) \propto x_N \propto \phi_S$, and so decreases during inflation. That is, the evolution of the inflaton field may change from being dominated by the friction term Υ_S to be dominated by the Hubble rate H. Whether the transition between these two regimes happens before or after 60 e-folds will depend on the value of the parameters of the model like κ and g. The amount of radiation obtained through the dissipative term is given by

$$\frac{\rho_R}{H^4} \simeq \frac{9}{2}\frac{r}{(1+r)^2}\epsilon_H\frac{m_P^4}{\kappa^2\mu^4}, \tag{38}$$

which even when $\Upsilon_S < H$ could give rise to a thermal bath with $T > H$. In particular, we can have

(a) (a) $\Upsilon_S > 3H$, and $T > H$ ($\dot{\phi}_S \simeq -V_\phi/\Upsilon_S$):

$$\frac{\rho_R}{H^4} \simeq \frac{36\sqrt{3}}{\pi^2}\frac{1}{g^2}\left(\frac{m_P}{\mu}\right)^5\frac{1}{x_N^3}, \tag{39}$$

(b) (b) $\Upsilon_S < H$ [$\dot{\phi}_S \simeq -V_\phi/(3H)$]:

$$\frac{\rho_R}{H^4} \simeq \frac{9}{256\sqrt{3}\pi}\left(\frac{\kappa}{4\pi}\right)^4\left(\frac{g^2}{16\pi}\right)\left(\frac{m_P}{\mu}\right)^7\frac{1}{x_N}. \tag{40}$$

The values of the couplings κ and g for which we could have cold or warm inflation, and strong or weak dissipative dynamics, are plotted in Fig. 1. In order to get the different regions, we have proceeded as follows: For each pair of values in the plane $\kappa - g$, the value of the inflaton field at the end of inflation is determined. This is done in the cold and weak dissipative regimes by the condition[7] $\eta_\Upsilon = 1$, Eq. (15). In the strong dissipative regime, inflation can end either with $\eta_\Upsilon = 1$ or it may also happen that most of the vacuum energy is already transferred into radiation during inflation, and then inflation will end when $\rho_R \simeq \kappa^2\mu^4$ instead. In this case, whichever occurs first fixes the value of the inflaton field at the end of inflation. The value of the inflaton field at 60 e-folds of inflation is then obtained from Eq. (20). This in turn fixes the value of the dissipative coefficient Υ_S, Eq. (36), the temperature of thermal bath, Eq. (38), and therefore the amplitude of the spectrum, Eq. (24). The COBE normalization is then used to fix the value of the scale μ. In order to match the expressions for the spectrum across the different regimes, we have used a simple expression with

[7]The value of η_Υ becomes larger than 1 before the other two slow-roll parameters.

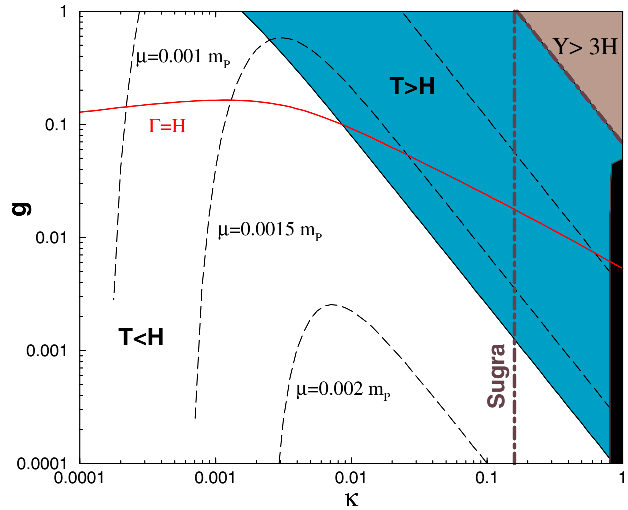

FIG. 1 (color online). Regions of cold ($T < H$), and warm ($T > H$) SUSY hybrid inflation in the $\kappa - g$ plane. The warm inflation region is divided into the weak dissipative regime with $T > H$ and $\Upsilon_S < 3H$ (lighter shaded region), and the strong dissipative regime with $\Upsilon_S > 3H$ (darker shaded region). Included also are the contour plots of constant μ and the adiabatic-Markovian limit $\Gamma_+ = H$. The black region on the right of the plot is excluded because $\phi_S > m_P$. In addition, when SUGRA corrections are taking into account, values to the left and down the wide dot-dashed line are excluded.

$$P_{\mathcal{R}}^{1/2} = \left|\frac{3H^2}{\Delta V'}\right|(1+r)\left(1+\sqrt{\frac{T}{H}}\right)\left[1+\left(\frac{\pi\Upsilon_S}{4H}\right)^{1/4}\right]\left(\frac{H}{2\pi}\right). \tag{41}$$

We can see in Fig. 1 that the strong dissipative regime $\Upsilon_S > 3H$ requires large values of the couplings, $\kappa \sim g \sim O(1)$; for values $\kappa \simeq g \simeq 0.1$ we are in the weak dissipative regime; and for values $\kappa \simeq g \simeq 0.01$ we recover the cold inflationary scenario. Typically, for a fixed value of the scale μ the amplitude of the spectrum in the strong dissipative regime would be larger than the one generated at zero T. The COBE normalization implies then a smaller value of the inflationary scale μ. For example, for $\kappa = g = 1$ we have $\mu \simeq 10^{13}$ GeV, whereas pushing the coupling toward its perturbative limit, $\kappa = g = \sqrt{4\pi}$, we would get $\mu \simeq 2 \times 10^{10}$ GeV. On the other hand, going from the cold to the weak dissipative regime, the value of μ only varies by a factor of 2 or 3, and it is still in the range of the GUT scale $O(10^{15})$ GeV.

As mentioned before, the quartic term in the inflaton potential induced by SUGRA corrections becomes non-negligible for not very small values of κ. In the CHI scenario, the value of the inflaton field becomes larger than m_P for $\kappa \geq 0.15$, and consequently that region is excluded. The same constraint applies also in the weak dissipative scenario. However, in the strong dissipative regime, with $\Upsilon_S > 3H$, the extra friction term keeps the value of the inflaton field below the Planck scale, and the constraint on κ can be avoided.

In Fig. 2, we have compared the prediction for the spectral index of the scalar spectrum of perturbations in both the CHI scenario, and warm hybrid inflation (WHI). From the warm inflation scenario we can always recover the CHI prediction by taking $g \ll 1$. In standard SUSY GUT hybrid inflation, for small values of the coupling κ the spectrum is practically scale invariant, it reaches a minimum around $\kappa \simeq 0.01$, and then rises due to SUGRA corrections up to positive values, which are disfavored by WMAP results. But in the weak and the strong dissipative regime, due to the different origin of the spectrum, we get that the spectral index is still below 1 even for values of the coupling $\kappa > 0.01$. This is especially true in the strong dissipative regime, where the dynamic is such that the inflaton field is well below the Planck scale and SUGRA corrections are negligible. In that regime the departure from scale invariance is within the observational value, with $n_S - 1 \simeq -0.022$.

In Fig. 3, we have also compared the running of the spectral index, $dn_S/d\ln k$ in both scenarios. The running although negative is much smaller than the value preferred by the WMAP data, $dn_S/d\ln k \simeq -0.031^{+0.016}_{-0.015}$. Again, in the strong dissipative regime we can have larger values of the couplings κ and g compatible with observations, but the predicted running of the spectral index is still small, with $dn_S/d\ln k \simeq -4 \times 10^{-4}$. Nevertheless, the statistical relevance of this result is not yet clear, which in any case would be finally confirmed or excluded by Planck Satellite experiment.

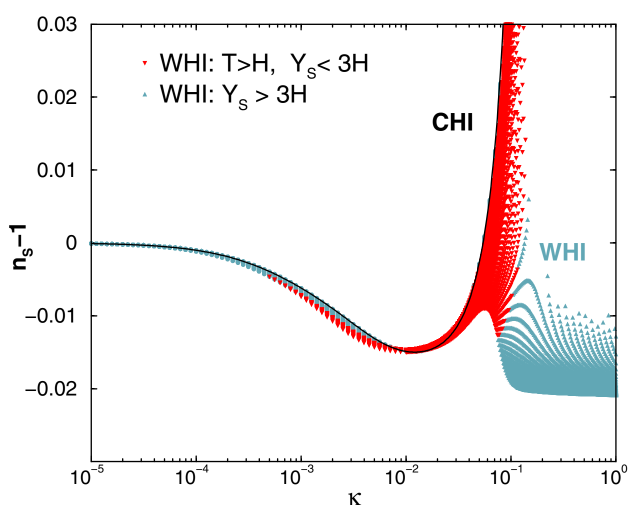

FIG. 2 (color online). Spectral index for cold SUSY hybrid inflation (solid line, CHI), and warm inflation (gray region, WHI). The weak dissipative regime ($T > H$ but $\Upsilon_S < 3H$) is given by the darker gray region (down triangle); the strong dissipative regime ($\Upsilon_S > 3H$) is given by the light gray region (up triangle).

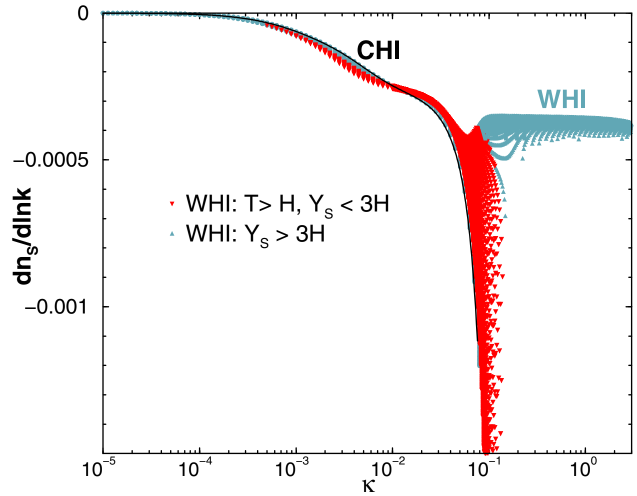

FIG. 3 (color online). Running of the spectral index for cold SUSY hybrid inflation (solid line, CHI), and warm hybrid inflation (gray region, WHI). The weak dissipative regime ($T > H$ but $\Upsilon_S < 3H$) is given by the darker gray region (down triangle); the strong dissipative regime ($\Upsilon_S > 3H$) is given by the light gray region (up triangle).

Concerning the primordial spectrum of tensor perturbations, as they do not couple strongly to the thermal background, the amplitude is the same as in CHI, with

$$P_{\rm tensor} = \frac{2}{m_P^2}\left(\frac{H}{2\pi}\right)^2. \quad (42)$$

As we have seen, strong dissipation [$\kappa \sim O(1)$] translates into a smaller inflationary scale μ compared to that of standard CHI, therefore a lower value of H and a smaller contribution of the gravitational waves to the spectrum. However, the same level of primordial tensor perturbations can be obtained decreasing the value of κ and no dissipation. This can be seen in Fig. 4, where we have plotted the prediction for the tensor-to-scalar ratio, defined as

$$R_g = \frac{P_{\rm tensor}}{P_{\mathcal{R}}}, \quad (43)$$

versus the prediction for the scalar spectral index $n_S - 1$, for the different regimes (cold, weak dissipation, and strong dissipation). The smallest values of κ correspond to a practically scale invariant spectrum. At present, from the cosmic microwave background polarization measurements the tensor-to-scalar ratio is poorly constraint ($R_g < 0.4$), although in the future ratios as low as 10^{-6} could be probed [32]. Still, such a gravitational background can be achieved (but not larger) in this kind of model only in the cold or weak dissipative regime, and typically for values of the coupling κ close to the maximum allowed by SUGRA corrections (blue tilted spectrum). On the other hand, in the strong dissipative regime we obtain a clear prediction that distinguishes this regime from the others: no expected tensor signal, with $R_g < 10^{-9}$, and a red tilted spectrum with $n_S \simeq 0.98$.

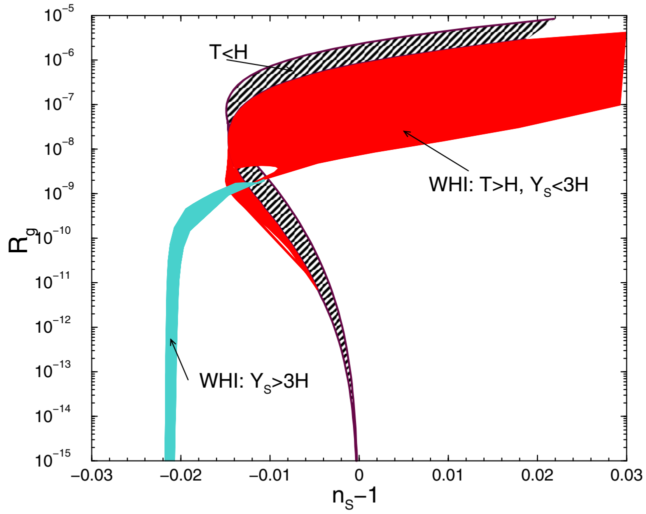

FIG. 4 (color online). Tensor-to-scalar ratio versus the predicted scalar spectral index $n_S - 1$. The cold regime, $T < H$, is given by the dark grey areas, the weak dissipative regime, $T > H$ but $\Upsilon_S < 3H$, is given by the striped area, and the strong dissipative regime, $\Upsilon_S > 3H$, is given by the light gray region.

IV. REHEATING TEMPERATURE

One of the main constraints on model building of inflationary supersymmetric particle physics models comes from the gravitino constraint on the reheating T after inflation. Gravitinos with a typical mass of the order of $O(1\text{–}100)$ TeV can be thermally produced during the radiation dominated era that follows inflation. If T is too high, we will have too many of them, and their subsequent decay will interfere with the predictions of the abundances of light elements at the time of big bang nucleosyntheses (BBN) [33]. This puts an upper bound on the reheating $T_{\rm RH}$ typically of the order of 10^9 GeV [33], and a more recent analysis on BBN has lowered this bound to $T_{\rm RH} \lesssim 10^7$ GeV for a gravitino mass $m_{3/2} \simeq O(1)$ TeV [34,35]. In cold inflation, the reheating T after inflation can be well approximated by

$$T_{\rm RH} \simeq \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{\Gamma_\phi m_P}, \qquad (44)$$

where Γ_ϕ is the decay rate of the inflaton field, and g_* is the effective number of degrees of freedom [typically of the order of $O(100)$ in a SUSY model]. Therefore, the reheating constraint translates in the inflaton not decaying too fast after inflation, which may imply some further constraint on the couplings.[8]

In this letter we have just minimally extended the inflationary sector by adding a pair of extra matter fields Δ, $\bar{\Delta}$, with generic coupling g. This allows the decay of the Φ_i fields already during inflation, and the possibility of having a *warm* regime of hybrid inflation, when the couplings are not very small, say $\kappa, g > 0.1$. The question then is whether this large coupling g could give rise to a large decay rate, and therefore a too large $T_{\rm RH}$. From the S, Φ_i sector we would have scalars and pseudoscalars with masses $m_S = \sqrt{2}\kappa\mu$ (plus a massless state if the minimum is along the D-flat direction $\Phi_1 = \Phi_2$). On the other hand, the Δ, $\bar{\Delta}$ sector gives both scalars and fermions with a mass $m_\Delta = \sqrt{2} g \mu$. In order to avoid the decay of the inflaton into these fermions, it is enough to require $\kappa < 2g$, which is just a mild constraint on the values of the couplings.

In order to complete the transfer of the energy density into radiation after inflation, the model has to include the decay of the inflaton field into other lighter fields, with coupling h_S and $\Gamma_S = m_S h_S^2/(8\pi)$. In the strong dissipative regime ($\kappa > 0.1$) we have $\mu \simeq O(10^{12}\text{–}10^{14})$ GeV, and demanding $T < 10^9$ GeV gives $h_S \lesssim 5 \times 10^{-6}$. A similar constraint is obtained in the CHI scenario and the weak dissipative regime, where now $\mu \sim O(10^{15})$ GeV, but m_S decreases with $\kappa < 0.1$

As a well-motivated example, which combines inflation with leptogenesis and light neutrino masses [37,38], the inflaton can decay into right-handed (s)neutrino fields ν_{Ri} (i = family index). The decay proceeds through the nonrenormalizable coupling $\Phi_1 \Phi_1 \nu_{Ri} \nu_{Ri}$, with decay rate

$$\Gamma_S = \frac{1}{8\pi}\left(\frac{M_i}{\mu}\right)^2 m_S, \qquad (45)$$

where M_i is the RH (s)neutrino mass. In the CHI scenario, with $\mu \simeq O(10^{15})$ GeV, $\kappa \simeq 10^{-2}$, and $m_S \simeq 10^{13}$ GeV, the gravitino constraint $T_{\rm RH} \leq 10^9$ GeV translates roughly into $M_i \simeq 10^{-3}\mu \sim O(10^{12})$ GeV. Those values are also consistent with baryogenesis and light neutrino masses [38]. This kind of scenario is also viable in the warm inflationary regime. Being consistent with the observed baryon asymmetry and the atmospheric neutrino oscillations does not directly constrain the value of κ but the value of $m_S \sim 10^{13}$ GeV. In the warm inflationary regime the value of the scale μ required for successful inflation reduces as we moved into the strong dissipative regime, m_S is of the order of 10^{13} GeV for $\kappa \simeq O(1)$, and the gravitino constraint gives now $M_i \simeq 10^{-3}\mu \sim O(10^{10})$ GeV. Therefore, a model of *warm* inflation and leptogenesis without the need of small couplings would be viable and compatible with observations, in the strong dissipative regime.

Nevertheless, in the presence of a thermal bath already during inflation, one could worry about the value of T at the end of inflation, especially in the strong dissipative regime. It would be premature to impose the gravitino constraint directly on that temperature. Taking into account the decay of the inflaton field, the entropy production during the reheating phase can dilute the abundance of the gravitinos thermally produced at the end of inflaton [39]. Roughly

[8]This can be avoided if the gravitino abundance is diluted by the entropy produced during the late-decay of some other particle [36].

speaking, the entropy dilution factor would be $\gamma = S_{RH}/S_{end} \sim T_{end}/T_{RH}$, where the subindices "RH" and "end" refer to the end of reheating and the end of inflation, respectively. One should study in more detail the reheating phase after "warm" hybrid inflation before drawing any conclusion, taking into account in addition that production of gravitinos during reheating does not take place in a pure radiation dominated universe. Inflation will end before the vacuum energy has been completely dissipated into radiation, and the singlets may still oscillate around the global minimum, with their energy density on average behaving like matter. The initial production of gravitinos would proceed initially in a mixture of radiation and matter, but would be later diluted by the entropy produced by the decay of the singlets.

V. CONCLUSION

The key result of this paper is that the SUSY hybrid model, in particular Eq. (2), has regimes of warm inflation. Until now, it has been assumed that this model in all parameter regimes has only cold inflationary dynamics. However, Fig. 1 firmly dispels this belief, as it shows that the parameter regime divides into regions of both warm and cold inflation. In light of this finding, the scalar spectral index, its running, and the tensor-scalar ratio have been computed in the entire parameter space of these models. We find a clean prediction for strong dissipative warm inflation with $n_S - 1 = 0.98$ and a tensor-scalar ratio that is effectively zero. As shown in Fig. 4, this prediction is very clearly separated from the cold results, which until now have been the expected predictions from these models. Also, these predictions for strong dissipative warm inflation are clearly separated from those of weak dissipative warm inflation.

Theoretically, the effects of dissipation in these models also present distinctive features. In particular, in the strong dissipative regime there is no η problem. Moreover, even for large coupling $\kappa \sim 1$, the inflaton field amplitude is well below the Planck scale. A consequence of these features is that SUGRA corrections are insignificant. One interpretation of this would be that a much richer variety of supergravity extensions of the basic model, Eqs. (1) and (2), are permissible in comparison to the cold inflation case. This could have important model building applications, especially when identifying viable inflation models in low-energy limits of string theory.

The main purpose of this paper was to highlight the dissipative dynamics inherent in this very popular SUSY hybrid model and then to outline the variety of new features this implies. There remains a great deal about our results that must be studied in further detail in future work. For example, as shown in [7] the process of radiation production will also induce small temperature dependent corrections to the inflaton effective potential. These effects will alter predictions for density perturbations. In fact, as shown in [29], temperature dependent effects could introduce qualitatively new features to the scalar power spectrum, such as oscillations. Thus, a more accurate treatment of density perturbations and a thorough examination of their evolution is important to consider in future work. Along similar lines, a deeper issue is that of thermalization. In the analysis in this paper, we followed the results in [6,8], which treat thermalization based on some simple criteria. As stated in those works, a proper dynamical treatment of thermalization is still needed, and the consequences of such work could make significant changes regarding the predictions for density perturbations in some parameter regimes. However, this paper has outlined the basic result that there are vast parameter regimes in this model, in which there is particle production during inflation, and thus in these regimes the statistical state of the system is substantially altered from the ground state.

For the parameter values of the strong dissipative regime, reheating may start with a fraction of the vacuum energy already converted into background radiation. The decay products of the inflaton acquire plasma (temperature dependent) masses which will affect the reheating process [40], kinematically blocking the inflaton decay until the temperature falls below the inflaton mass. Processes involving different particle production (thermal or out-of-equilibrium) mechanisms during reheating should then be reexamined, such as production of RH (s)neutrinos and leptogenesis. In any case, reheating is completed in the warm inflation scenario through new decay channels different from those active during inflation. Inflation *per se* does not force the couplings entering in Eq. (2) to be small as we have seen, and neither does reheating and the T_{RH} constraint.

One general result that can be taken away from this paper is that warm inflation regimes can be expected in SUSY inflationary models. There are many other models aside from the one studied in this paper in which we expect to find warm inflation regimes. An interesting case, that is worth mentioning, are SUSY models which lead to monomial inflaton potentials. One important example of such a model is the next-to-minimal supersymmetric standard model (NMSSM) [41], which has the superpotential,

$$W = \lambda \Phi H_u H_d - \kappa \Phi^3 + h_t Q_3 U_3^c H_u + \cdots,$$

where Φ is a singlet superfield, H_u, H_d are the Higgs doublets, Q_3 the third generation left-handed quarks, and U_3^c the corresponding right-handed up quark, and h_t the top Yukawa coupling. Identifying the singlet field Φ with the inflaton leads to an inflaton potential $\sim \kappa^2 \phi^4$. In standard inflation, such a possibility for the NMSSM has not been of great interest, since it is well known for such a chaotic inflation potential that the amplitude of the inflaton is greater than the Planck scale, $\langle \phi \rangle > m_P$, thus leading to the problem of an infinite number of unsuppressed higher dimensional contributions entering the potential. However, in warm inflation, it is known [8] that monomial potentials like this one yield observationally consistent warm inflation for field amplitudes *below* the Planck scale $\langle\phi\rangle < m_P$. Thus, in warm inflation such potentials have no trouble with higher dimensional contributions, and so are completely consistent. As such, this fact implies that NMSSM is a model, with no further modifications, that can support inflation. This is one of the simplest and may even be the minimal SUSY model that is consistent with the standard model and yields inflation. In future work we plan to do a detailed analysis of warm inflation in the NMSSM.

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