Nonlinear electromagnetic and gravitational actions of neutron star fields on electromagnetic wave propagation

Victor I. Denisov* and Sergei I. Svertilov†

Physics Department, Moscow State University, Moscow, 119992, Russia

(Received 29 December 2004; published 4 March 2005)

The nonlinear electrodynamic and gravitational actions on the weak electromagnetic wave propagation in the strong dipole magnetic and gravitational fields of a neutron star are discussed. The eikonal equations for an electromagnetic wave propagating in the external field as well as the motion equations of photons in the dipole magnetic and gravitational fields of a neutron star are obtained from the parametrized post-Maxwellian electrodynamics of the vacuum, which is analogous to the parametrized post-Newtonian theory of gravitation. The solution of these equations indicates that electromagnetic signals, carried by normal waves with mutually orthogonal polarization, travel along different rays and take different time to reach the detector from the same source. It is also shown that in appropriate conditions the value of this nonlinear-electrodynamic lag can be about a tenth part of a microsecond. A detailed analysis of the possibilities of observing this effect by the detection of X-rays and gamma rays from pulsars and magnetars was made.

DOI: 10.1103/PhysRevD.71.063002 PACS numbers: 95.30.Gv, 12.20.–m, 95.30.Sf

I. INTRODUCTION

As it is well known, Maxwell electrodynamics in the absence of matter is a linear theory. Its predictions in the various applications besides the subatomic level are constantly confirmed with higher and higher accuracy. Quantum electrodynamics, which is elaborated on the base of the Maxwell electrodynamics with the renormalization procedure, also describes the different subatomic processes well, and it is considered to be one of the most sound physical theories.

However, as it follows from fundamental physical reasons, electrodynamics in vacuum should be a nonlinear theory. Recent experiments [1] on the inelastic scattering of laser photons on gamma-quanta confirm this conclusion of the theory. Thus, the different models of nonlinear electrodynamics of vacuum as well as their predictions, which can be experimentally verified, are of great importance.

A number of laboratory experiments [2–10], in which such effects can be studied, were recently proposed. However, the magnetic fields available in ground laboratories $B \sim 10^6$ G are much smaller than the typical quantum electrodynamics value $B_q = m^2c^3/e\hbar = 4.41 \cdot 10^{13}$ G. Thus, the nonlinear corrections to the Maxwell equations are so small that it is extremely difficult to observe effects induced by these corrections in vacuum.

The nonlinear effects should be most pronounced in the astrophysical objects in fields $B \sim 10^{12} - 10^{16}$ G, which are typical for some pulsars and magnetars. Adler [11] was probably the first who indicated this reason.

Nonlinear effects, which can occur in this case are: photon splitting [11], second harmonic generation [12], the electromagnetic ray bending in the dipole magnetic field [13], and the lag of electromagnetic signal carried by the one normal wave in comparison with the electromagnetic signal carrying by the other normal wave [14]. This lag is caused by the so-called vacuum birefringence induced in strong magnetic fields.

It is necessary to note that, due to the presence of a matter-filled magnetosphere in the vicinity of neutron stars, these effects can be observed mainly in X- and gamma rays, for which magnetosphere is to a certain extent transparent.

However, due to the nonlinearity of equations these effects are now studied for only some particular cases, when the wave vector of electromagnetic wave, falling on the neutron star, lays in the dipole magnetic field symmetry plane.

We will consider the general case, when the electromagnetic wave vector has an arbitrary direction relatively to the vector of the neutron star dipole magnetic field.

We will analyze the main vacuum nonlinear electrodynamic effects, which are induced by the weak electromagnetic wave propagation through the dipole magnetic and gravitational fields of a neutron star.

To solve this problem, the parametrized post-Maxwellian electrodynamics with different free parameter values in different models of non-linear electrodynamics of vacuum is introduced in the Sec. II in full analogy with the parametrized post-Newtonian formalism of the theory of gravitation [15].

The eikonal equation for the electromagnetic wave propagating in the external electromagnetic and gravitational fields is obtained in Sec. III on the base of parametrized post-Maxwellian electrodynamics equations. It follows from this equation that electromagnetic waves propagate in the external fields along the geodesics of some effective pseudo-Riemannian space-time, the metric tensor of which

*Electronic address: denisov@srd.sinp.msu.ru
†Electronic address: sis@coronas.ru

depends on the metric tensor of the background space-time $g_{ik}^{(0)}$, as well as the tensor of the external electromagnetic field F_{ik} and the electromagnetic wave polarization state.

The equations of photon propagation in the dipole magnetic and gravitational fields of a neutron star are obtained in Sec. IV from this tensor and equations of geodesics. The solution of these equations are found in Sec. V and the formulas describing the effect of non-linear electromagnetic lag of electromagnetic signal carried by one normal wave in comparison with the other normal wave with polarization orthogonal to the first one are obtained in Sec. VI

The relation describing nonlinear electromagnetic lag for the different relative positions of the neutron star, source and detector of electromagnetic waves is analyzed in Sec. VII. And finally, the possibilities of observing the considered effect by the detection of X- and gamma rays from the well known pulsars and magnetars are examined in the Discussion.

II. THE POST-MAXWELLIAN EQUATIONS OF VACUUM NONLINEAR ELECTRODYNAMICS IN THE GRAVITATIONAL FIELD OF A NEUTRON STAR

Let us consider a neutron star with a strong magnetic field (gamma-pulsar or magnetar), with radius R_s, magnetic dipole moment $\mathbf{m}$ and gravitational radius r_g. For certainty, we will assume that it is not transparent for the electromagnetic emission.

The star matter as well as the dipole magnetic field with particles, existing in it, produce the gravitational field of this star. Since the rest energy of the neutron star matter is more than 6 orders higher than the magnetic field energy even in the case of a magnetar, the star matter gives the main contribution to the gravitational field. If we assume spherical symmetry, the Schwarzschild solution in isotropic coordinates can be chosen as the metric tensor of pseudo-Riemannian space-time [16]:

$$ds^2 = \frac{[4r - r_g]^2}{[4r - r_g]^2} c^2 dt^2 - \left[1 + \frac{r_g}{4r}\right]^4 \{dx^2 + dy^2 + dz^2\},$$

were we denote $r = \sqrt{x^2 + y^2 + z^2}$.

In the case of weak electromagnetic fields the general Lagrangian of the vacuum nonlinear electrodynamics can be written in the parametrized post-Maxwellian form [17], which in a certain sense is analogous to the parametrized post-Newtonian formalism of the gravitational theory [15], which is used for study different effects in the weak gravitational field of the Solar system:

$$L = \frac{\sqrt{-g}}{32\pi}\{2J_2 + \xi[(\eta_1 - 2\eta_2)J_2^2 + 4\eta_2 J_4]\} - \frac{1}{c} j^n A_n, \tag{1}$$

where $J_2 = F_{ik}F^{ki}, J_4 = F_{ik}F^{km}F_{ml}F^{li}$ are the invariants of the electromagnetic field tensor, $\xi = 1/B_q^2$, and the values of dimensionless post-Maxwellian parameters η_1 and η_2 depend on the choice of the vacuum nonlinear electrodynamic model.

For example, in the Heisenberg-Euler nonlinear electrodynamics [18], which is the consequence of quantum electrodynamics, parameters η_1 and η_2 have quite definite values $\eta_1 = \alpha/(45\pi) = 5.1 \cdot 10^{-5}$, $\eta_2 = 7\alpha/(180\pi) = 9.0 \cdot 10^{-5}$, while in the Born-Infeld theory [19] they are a function of the some unknown constant a^2: $\eta_1 = \eta_2 = a^2 B_q^2/4$.

The system of electromagnetic field equations, which can be obtained from the Lagrangian (1) has the form:

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^n}\{\sqrt{-g} H^{mn}\} = -\frac{4\pi}{c} j^m, \qquad \frac{\partial F_{mn}}{\partial x^k} + \frac{\partial F_{nk}}{\partial x^m} + \frac{\partial F_{km}}{\partial x^n} = 0, \tag{2}$$

where in order to shorten the expression, we denote:

$$H^{mn} = \{1 + \xi(\eta_1 - 2\eta_2)J_2\}F^{mn} + 4\xi\eta_2 F^{ml}F_{lk}F^{kn}.$$

There are two small parameters of the considered problem r_g/r and $\eta_{1,2}\xi B^2(\mathbf{r})$. Due to the different dependence of these parameters on $\mathbf{r}$, their ratio changes with the changing $\mathbf{r}$, and the values of these parameters are different on the surface of various neutron stars.

This circumstance produces serious problem in providing equal calculation accuracy according to these parameters series. Thus, we will calculate the main nonlinear electrodynamic and gravitational effects with accuracy, which is linear in $\eta_{1,2}\xi B^2(\mathbf{r})$ and is quadratic in r_g/r.

III. THE EIKONAL EQUATION

We will find the eikonal equation, which is satisfied by the weak high frequency electromagnetic wave propagating in the gravitational and magnetic fields of a neutron star.

We will express the tensor of electromagnetic field in Eqs. (2) as the sum of the tensor of neutron star electromagnetic field $F_{ik}^{(0)}$ and the tensor of weak electromagnetic wave field f_{ik}: $F_{ik} = F_{ik}^{(0)} + f_{ik}$.

In the linear approximation of the weak electromagnetic wave f_{ik} Eqs. (2) have the form:

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^n}\{\sqrt{-g} H_{(1)}^{mn}\} = 0, \qquad \frac{\partial f_{mn}}{\partial x^k} + \frac{\partial f_{nk}}{\partial x^m} + \frac{\partial f_{km}}{\partial x^n} = 0, \tag{3}$$

where

$$H^{mn}_{(1)} = [1 + \xi(\eta_1 - 2\eta_2)J_2^{(0)}]f^{mn} + 4\xi\eta_2[f^{ml}F^{(0)}_{lk}F^{kn}_{(0)} + F^{ml}f_{lk}F^{kn}_{(0)} + F^{ml}_{(0)}F^{(0)}_{lk}f^{kn}] + 2\xi(\eta_1 - 2\eta_2)f_{ik}F^{ki}_{(0)}F^{mn}_{(0)}.$$

To calculate the influence of gravitational and magnetic fields on the weak electromagnetic wave rays we will present the tensor f_{nm} as:

$$f_{nm} = h_{nm}(\mathbf{r}, t)e^{iS(\mathbf{r},t)}, \tag{4}$$

where, as usual, the amplitude $h_{nm}(\mathbf{r}, t)$ is a slowly changing function of space coordinates and time, and the eikonal $S(\mathbf{r}, t)$ is the fast changing function.

Let us substitute the relation (4) in Eqs. (3). We now take into account that the function $S(\mathbf{r}, t)$ strongly varies on distances of about one wave length and on time intervals of about period. Thus, derivatives of S in coordinates and time satisfy the condition:

$$|h_{nm}(\mathbf{r}, t)\partial S/\partial x^l| >> |\partial h_{nm}(\mathbf{r}, t)/\partial x^l|.$$

Besides, for points located outside the star the following estimates of the order of magnitude are valid:

$$\partial F^{(0)}_{nm}/\partial r \sim F^{(0)}_{nm}/R_s,\ \partial S/\partial r \sim S/\lambda,$$

where R_s—the star radius, λ—the electromagnetic emission wave length.

Because for X-rays and gamma rays $R_s/\lambda \gg 1$, it is necessary to retain by the differentiation of $H^{nm}_{(1)}$ in Eqs. (3) the derivatives on the eikonal S only.

As a result, the linearly-independent equations of the system (3) can be expressed as:

$$[1 + \xi(\eta_1 - 2\eta_2)J_2^{(0)}]f^{\mu n}\frac{\partial S}{\partial x^n} + 4\xi\eta_2[f^{\mu l}F^{(0)}_{lk}F^{kn}_{(0)} + F^{\mu l}_{(0)}f_{lk}F^{kn}_{(0)} + F^{\mu l}_{(0)}F^{(0)}_{lk}f^{kn}]\frac{\partial S}{\partial x^n} + [2\xi(\eta_1 - 2\eta_2)f_{ik}F^{ki}_{(0)}]F^{\mu n}_{(0)}\frac{\partial S}{\partial x^n} = 0,$$
$$f_{\alpha\beta}\frac{\partial S}{\partial x^0} + f_{\beta 0}\frac{\partial S}{\partial x^\alpha} + f_{0\alpha}\frac{\partial S}{\partial x^\beta} = 0. \tag{5}$$

Let us multiply the first equation of system (5) by $\partial S/\partial x^0$ and exclude from it the components $f_{\alpha\beta}$ using the second equation (5). As the result, we obtain the homogeneous system of three linear algebraic equations relatively to three components $f_{0\beta}$ of tensor f_{nm}

$$\Pi^{\mu\beta}f_{0\beta} = 0,$$

where the three-dimensional tensor $\Pi^{\mu\beta}$ has the form:

$$\Pi^{\mu\beta} = \{[1 + \xi(\eta_1 - 2\eta_2)J_2^{(0)}][g^{nm}_{(0)}g^{\mu\beta}_{(0)} - g^{\mu n}_{(0)}g^{\beta m}_{(0)}] - 4\xi[(\eta_1 - 2\eta_2)F^{\mu n}F^{\beta m} + \eta_2 g^{\beta n}_{(0)}F^{\mu k}F^{\cdot m}_{k\cdot} + \eta_2 g^{\mu n}_{(0)}F^{\beta k}F^{\cdot m}_{k\cdot} - \eta_2[g^{nm}_{(0)}F^{\mu k}F^{\cdot\beta}_{k\cdot} + g^{\mu\beta}_{(0)}F^{nk}F^{\cdot m}_{k\cdot} - F^{\mu n}F^{\beta m}]]\}\frac{\partial S}{\partial x^n}\frac{\partial S}{\partial x^m}.$$

In this expression and in the following ones we omit index (0) in the $F^{nk}_{(0)}$ tensor of the neutron star magnetic field.

As it is well-known, the eikonal equation is the consequence of condition

$$det||\Pi^{\mu\beta}|| = 0. \tag{6}$$

Using the tensor formalism, which was developed in [20,21], we reduce Eq. (6) to the form:

$$\left[g^{ik}_{(1)}\frac{\partial S}{\partial x^i}\frac{\partial S}{\partial x^k}\right] \times \left[g^{mn}_{(2)}\frac{\partial S}{\partial x^m}\frac{\partial S}{\partial x^n}\right] = 0, \tag{7}$$

where the notations

$$g^{ik}_{(1,2)} = g^{ik}_{(0)} + 4\eta_{1,2}\xi F^{ip}F^{\cdot k}_{p\cdot} \tag{8}$$

are introduced and the tensor F_{pi} indexes can be raised up using the metric tensor $g^{kp}_{(0)}$.

Thus, at $\eta_1 \neq \eta_2$ nonlinear electrodynamics predicts the appearance of the vacuum birefringence even in the post-Maxwellian approximation, as the result of which the laws of electromagnetic wave propagation in the external electromagnetic field will depend on their polarization. Hence, any electromagnetic wave propagating through the neutron star magnetic field will be split into two normal waves with orthogonal polarization. Due to Eq. (7) the first one will propagate along the geodesics of the effective pseudo-Riemannian space-time with metric tensor $g^{ik}_{(1)}$, and the second—along the geodesics of the pseudo-Riemannian space-time with metric tensor $g^{ik}_{(2)}$.

IV. THE EQUATIONS OF ELECTROMAGNETIC SIGNAL PROPAGATION IN THE DIPOLE MAGNETIC AND GRAVITATIONAL FIELDS OF A NEUTRON STAR

Let us consider a given neutron star with strong magnetic field (gamma pulsar or magnetar [22]). We suppose that X-rays and gamma rays pass through it magnetic and gravitational fields and then propagate to the detector placed in the vicinity of the Earth.

We place the coordinate origin in the neutron star center. The neutron star dipole magnetic moment in the general case can be presented in the form:

$$m_1 = |\mathbf{m}|\sin\theta_1\cos\varphi_1, \qquad m_2 = |\mathbf{m}|\sin\theta_1\sin\varphi_1, \qquad m_3 = |\mathbf{m}|\cos\theta_1, \tag{9}$$

where the angles θ_1 and φ_1 as we assume do not depend on time.

To resolve Eq. (2) in a zero approximation in r_g/r and $\xi F^{in}F_{n\cdot}^{\cdot k}$, we can to find the nonzero components of the tensor F^{pk}, which describes the neutron star magnetic field with dipole moment $\mathbf{m}$, described by expression (9):

$$F_{32} = \frac{3(\mathbf{mr})x - r^2 m_1}{r^5}, \qquad F_{13} = \frac{3(\mathbf{mr})y - r^2 m_2}{r^5},$$
$$F_{21} = \frac{3(\mathbf{mr})z - r^2 m_3}{r^5},$$

where $(\mathbf{mr}) = m_1 x + m_2 y + m_3 z$.

The propagation of photons in the pseudo-Riemannian space-time with the tensor (8) occurs along the geodesics of this space and, depending on their polarization, can be described by the equation:

$$\frac{dk^m}{d\sigma} + \Gamma^m_{pn} k^p k^n = 0, \; g^{(1,2)}_{nm} k^n k^m = 0, \tag{10}$$

where $k^m = dx^m/d\sigma$, σ is some affine parameter, and Γ^m_{pn} are the Christoffel symbols of the effective pseudo-Riemannian space-time with the metric tensor $g^{(1)}_{nm}$ for normal waves of the first type and $g^{(2)}_{nm}$—for the normal waves of the second type.

It is necessary to note that Eq. (10) and all other expressions, which can be obtained from it, are valid only in the space area, where $m^2/r^6 < B_q^2$ and the expansion of Lagrangian (1) is permissible. Thus, without particular stipulation, we will study only those rays of weak electromagnetic wave, which are entirely in that space area.

It is quite convenient to replace the differentiation in σ in equations of the system (10) by differentiation in coordinate $x^0 = ct$ in accordance with the equality:

$$\frac{d}{d\sigma} = \frac{dx^0}{d\sigma}\frac{d}{dx^0} = k^0 \frac{d}{dx^0}.$$

To combine equations of the system (10), we rewrite them in a more suitable form:

$$\begin{aligned}
&\frac{d^2x}{(dx^0)^2} + \left\{\Gamma^1_{mi} - \frac{dx}{dx^0}\Gamma^0_{mi}\right\}\frac{dx^i}{dx^0}\frac{dx^m}{dx^0} = 0,\\
&\frac{d^2y}{(dx^0)^2} + \left\{\Gamma^2_{mi} - \frac{dy}{dx^0}\Gamma^0_{mi}\right\}\frac{dx^i}{dx^0}\frac{dx^m}{dx^0} = 0,\\
&\frac{d^2z}{(dx^0)^2} + \left\{\Gamma^3_{mi} - \frac{dz}{dx^0}\Gamma^0_{mi}\right\}\frac{dx^i}{dx^0}\frac{dx^m}{dx^0} = 0.
\end{aligned}\tag{11}$$

It is necessary to note that in these equations the time $t = x^0/c$, is the time measured by the clock of the observer located at a great distance from the neutron star.

If we expand Eqs. (11) in the series on small parameters r_g/r and $\xi m^2/r^6$ with accuracy, which is quadratic on r_g/r and linear on $\xi m^2/r^6$, they will be completely equivalent to the following system of equations, which are written in the Cartesian coordinates of the three-dimensioned Euclidean space:

$$\begin{aligned}
\ddot{\mathbf{r}} = &-\frac{r_g}{r^3}[\mathbf{r} - 2(\dot{\mathbf{r}}\mathbf{r})\dot{\mathbf{r}}] + \frac{r_g^2}{8r^4}[17\mathbf{r} - 2(\dot{\mathbf{r}}\mathbf{r})\dot{\mathbf{r}}]\\
&+ \frac{12\xi\eta}{r^{12}}\{[5r^2(\mathbf{mr})(\dot{\mathbf{r}}\mathbf{r})^2 - 2r^4(\mathbf{m}\dot{\mathbf{r}})(\dot{\mathbf{r}}\mathbf{r})]\mathbf{m}\\
&+ [2r^4m^2(\dot{\mathbf{r}}\mathbf{r}) - 2r^4(\mathbf{m}\dot{\mathbf{r}})(\mathbf{mr}) + 8r^2(\mathbf{mr})^2(\dot{\mathbf{r}}\mathbf{r})]\dot{\mathbf{r}}\\
&+ [6r^2(\mathbf{m}\dot{\mathbf{r}})(\mathbf{mr})(\dot{\mathbf{r}}\mathbf{r}) - r^2(\mathbf{mr})^2\\
&- m^2r^4 - 15(\mathbf{mr})^2(\dot{\mathbf{r}}\mathbf{r})^2]\mathbf{r}\},
\end{aligned}\tag{12}$$

where the point denotes the derivative on $x^0 = ct$.

In these notations the first integral $g^{(1,2)}_{nm} k^n k^m = 0$ of the system of equations (12) becomes:

$$\begin{aligned}
\dot{\mathbf{r}}^2 = 1 &- \frac{2r_g}{r} + \frac{17r_g^2}{8r^2} + \frac{4\xi\eta}{r^{10}}\{[(\mathbf{m}\dot{\mathbf{r}})^2 - m^2]r^4\\
&+ 6[(\mathbf{mr}) - (\mathbf{m}\dot{\mathbf{r}})(\dot{\mathbf{r}}\mathbf{r})](\mathbf{mr})r^2\\
&+ 9[(\dot{\mathbf{r}}\mathbf{r})^2 - r^2](\mathbf{mr})^2\}.
\end{aligned}\tag{13}$$

It is necessary to note, once more, that due to the vacuum birefringence in the gamma pulsar or magnetar strong magnetic field any electromagnetic signal will propagate, in general case, as two electromagnetic signals polarized mutually orthogonal to each other. These signals will be carried by the normal waves of two types, which will propagate along different rays and with different velocity. To describe the laws of the first type normal wave propagation, it is necessary to put $\eta = \eta_1$, in Eqs. (12) and (13) and for the normal waves of the second type $\eta = \eta_2$.

It is assumed that in Eqs. (12) and (13) the vector of neutron star magnetic dipole moment (9) does not depend on time. However, the obtained equations and their solutions are valid also in the case, when the neutron star rotates quasi-stationary with the frequency of Ω_1 around the axis, which passes through the center of mass, but does not coincide with the vector of it dipole moment $\mathbf{m}$. Besides, the rotational axis undergoes regular precession with the frequency of Ω_2. To satisfy the condition of quasi-stationary rotational motion we will assume that the frequencies Ω_1 and Ω_2 are small enough to guarantee that the linear velocity of points on the neutron star surface is much less than the velocity of light: $\Omega_{1,2}R_s \ll c$. Then we can neglect the fields of star magneto-dipole emission, and this means that the star dipole magnetic field gives the main contribution to the nonlinear electrodynamic influence on the propagation of weak electromagnetic waves.

Let us assume that in the moving coordinate system the neutron star vector $\mathbf{m}$ is tilted relatively to the x_3 axis by angle α_0 and the angle between its projection on the plane $X_1O'X_2$ and the positive direction of the axis x_1 is β_0:

$$\mathbf{m} = |\mathbf{m}|\{\sin\alpha_0\cos\beta_0, \sin\alpha_0\sin\beta_0, \cos\alpha_0\}.$$

Then in x, y, z coordinates vector $\mathbf{m}$ will have the following components:

$$\begin{aligned} m_x &= |\mathbf{m}|\{\sin\alpha_0[\cos\psi\cos(\Phi+\beta_0) \\ &\quad -\cos\Theta\sin\psi\sin(\Phi+\beta_0)]+\sin\Theta\sin\psi\cos\alpha_0\}, \\ m_y &= |\mathbf{m}|\{\sin\alpha_0[\cos\Theta\cos\psi\sin(\Phi+\beta_0) \\ &\quad +\sin\psi\cos(\Phi+\beta_0)]-\sin\Theta\cos\psi\cos\alpha_0\}, \\ m_z &= |\mathbf{m}|\{\cos\alpha_0\cos\Theta+\sin\Theta\sin\alpha_0\cos(\Phi+\beta_0)\}. \end{aligned} \quad (14)$$

Since the neutron star besides its own rotation undergoes precession, the angles Φ and ψ in those relations are the following functions of time (nutation of neutron stars are very small):

$$\Phi = \Omega_1 t + \Phi_0, \psi = \Omega_2 t + \psi_0, \quad (15)$$

where Φ_0 and ψ_0 are constant and $\Omega_1 > \Omega_2$.

Because the periods $T_1 = 2\pi/\Omega_1$ and $T_2 = 2\pi/\Omega_2$ for the neutron star quasi-stationary rotation are much larger than the time $T \approx 2R_s/c$ of electromagnetic signal propagation in the strong magnetic field area, we can assume that the angles Φ and ψ do not depend on time when we resolve Eqs. (12) and (13) of our problem of nonlinear electrodynamic and gravitational bending of rays and electromagnetic signal lags, and only in the final solution we can take into account relations (14) and (15).

V. SOLUTION OF THE EQUATIONS OF ELECTROMAGNETIC SIGNAL PROPAGATION

Thus, solving the system of equations (12) and (13) we can find the laws of motion of electromagnetic signals in the neutron star dipole magnetic and gravitational fields. As one of the initial conditions we assume that electromagnetic signals carried by both normal waves are emitted at the point $\mathbf{r} = \mathbf{r}_0$ at the same moment of time $t = 0$.

We will find the solution of the system of equation (12) in the form of series on small parameters:

$$\mathbf{r} = \mathbf{f}_0(x^0) + r_g\mathbf{f}_1(x^0) + r_g^2\mathbf{f}_2(x^0) + \xi\eta\mathbf{f}_3(x^0). \quad (16)$$

Eqs. (12) can be integrated most simply in the zero approximation:

$$\mathbf{f}_0(x^0) = \mathbf{n}ct + \mathbf{r}_0, \quad (17)$$

where $\mathbf{n}$ is some constant vector.

The system of equations (12) gives in the first order:

$$\begin{aligned} \mathbf{f}_1(x^0) &= \mathbf{v}_1 ct - \mathbf{n}\ln\left[\frac{[ct+(\mathbf{n}\mathbf{r}_0)+\sqrt{c^2t^2+2(\mathbf{n}\mathbf{r}_0)ct+r_0^2}]}{r_0+(\mathbf{n}\mathbf{r}_0)}\right] \\ &\quad + \left[ct + r_0 - \sqrt{c^2t^2+2(\mathbf{n}\mathbf{r}_0)ct+r_0^2}\right] \\ &\quad \times \frac{[\mathbf{r}_0-(\mathbf{n}\mathbf{r}_0)\mathbf{n}]}{\rho^2}, \end{aligned} \quad (18)$$

where $\mathbf{v}_1$ is the constant vector, $\rho^2 = r_0^2 - (\mathbf{n}\mathbf{r}_0)^2$ is the square of impact distance.

After the integration of the equations of system (12) in the next approximation, we obtain:

$$\begin{aligned} \mathbf{f}_2(x^0) &= \mathbf{v}_2 ct + \mathbf{r}_2 + \frac{15}{16\rho^3}\{[r_0^2 - 2\rho^2 + ct(\mathbf{n}\mathbf{r}_0)]\mathbf{n} - [ct+(\mathbf{n}\mathbf{r}_0)]\mathbf{r}_0\}arcctg\left(\frac{ct+(\mathbf{n}\mathbf{r}_0)}{\rho}\right) \\ &\quad + \frac{1}{\rho^2}\left\{(\mathbf{n}\mathbf{r}_0)\mathbf{n} - \mathbf{r}_0 - \rho^2\mathbf{v}_1 - \frac{[ct(\mathbf{n}\mathbf{r}_0)+r_0^2-2\rho^2]\mathbf{n}-[ct+(\mathbf{n}\mathbf{r}_0)]\mathbf{r}_0}{\sqrt{c^2t^2+2(\mathbf{n}\mathbf{r}_0)ct+r_0^2}}\right\}\ln\left\{\frac{[ct+(\mathbf{n}\mathbf{r}_0)+\sqrt{c^2t^2+2(\mathbf{n}\mathbf{r}_0)ct+r_0^2}]}{[r_0+(\mathbf{n}\mathbf{r}_0)]}\right\} \\ &\quad - \frac{1}{\rho^4}\sqrt{c^2t^2+2(\mathbf{n}\mathbf{r}_0)ct+r_0^2}[[(\mathbf{n}\mathbf{r}_0)r_0 + (\rho^2-2r_0^2)(\mathbf{v}_1\mathbf{r}_0) - r_0^2]\mathbf{n} + [(2(\mathbf{v}_1\mathbf{r}_0)+1)(\mathbf{n}\mathbf{r}_0) - r_0]\mathbf{r}_0 - (\mathbf{n}\mathbf{r}_0)\rho^2\mathbf{v}_1] \\ &\quad + \frac{r_0[ct+2(\mathbf{n}\mathbf{r}_0) - r_0(\mathbf{v}_1\mathbf{r}_0) - r_0]\mathbf{n}}{\rho^2\sqrt{c^2t^2+2(\mathbf{n}\mathbf{r}_0)ct+r_0^2}} - \frac{[(1+(\mathbf{v}_1\mathbf{r}_0))ct+r_0]\mathbf{r}_0}{\rho^2\sqrt{c^2t^2+2(\mathbf{n}\mathbf{r}_0)ct+r_0^2}}, \end{aligned} \quad (19)$$

where $\mathbf{v}_2$ is the arbitrary constant vector, and the vector $\mathbf{r}_2$ due to the initial condition has the form, which is presented in Appendix A.

And finally, after integration of the system of equations (12) in the nonlinear electrodynamic approximation, we obtain:

$$
\begin{aligned}
\mathbf{f}_3(x^0) = {} & \mathbf{v}_3 ct + \mathbf{r}_3 - \frac{3}{64\rho^9}\{2\mathbf{m}\rho^2[25ct[(\mathbf{nr}_0)(\mathbf{nm}) - (\mathbf{r}_0\mathbf{m})] + 25r_0^2(\mathbf{nm}) - 26\rho^2(\mathbf{nm}) - 25(\mathbf{nr}_0)(\mathbf{r}_0\mathbf{m})] \\
& + 5\mathbf{r}_0[ct[35(\mathbf{r}_0\mathbf{m})^2 + 35r_0^2(\mathbf{nm})^2 + 16\mathbf{m}^2\rho^2 - 36\rho^2(\mathbf{nm})^2 - 70(\mathbf{nr}_0)(\mathbf{nm})(\mathbf{r}_0\mathbf{m})] + 16m^2\rho^2(\mathbf{nr}_0) \\
& + 60\rho^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) + 35(\mathbf{nr}_0)(\mathbf{r}_0\mathbf{m})^2 + 35r_0^2(\mathbf{nm})^2(\mathbf{nr}_0) - 26\rho^2(\mathbf{nr}_0)(\mathbf{nm})^2 - 70r_0^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m})] \\
& - \mathbf{n}[5ct[16m^2\rho^2(\mathbf{nr}_0) + 35(\mathbf{nr}_0)(\mathbf{r}_0\mathbf{m})^2 - 26\rho^2(\mathbf{nr}_0)(\mathbf{nm})^2 + 35r_0^2(\mathbf{nr}_0)(\mathbf{nm})^2 + 60\rho^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) \\
& - 70r_0^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m})] + 300\rho^2(\mathbf{nr}_0)(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) + 80m^2\rho^2r_0^2 - 350r_0^2(\mathbf{nr}_0)(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) + 104\rho^4(\mathbf{nm})^2 \\
& - 280\rho^2r_0^2(\mathbf{nm})^2 - 96m^2\rho^4 - 200\rho^2(\mathbf{r}_0\mathbf{m})^2 + 175r_0^4(\mathbf{nm})^2 + 175r_0^2(\mathbf{r}_0\mathbf{m})^2]]\} \\
& \times arctg\left(\frac{ct + (\mathbf{nr}_0)}{\rho}\right) - \frac{9}{4[c^2t^2 + 2(\mathbf{nr}_0)ct + r_0^2]^4}\{\mathbf{n}[ct[(\mathbf{r}_0\mathbf{m})^2 - 4(\mathbf{nr}_0)(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) - 4\rho^2(\mathbf{nm})^2 + 3r_0^2(\mathbf{nm})^2] \\
& + 2r_0^2(\mathbf{nm})[(\mathbf{nr}_0)(\mathbf{nm}) - (\mathbf{r}_0\mathbf{m})]] - \mathbf{r}_0[2ct(\mathbf{nm})[(\mathbf{nr}_0)(\mathbf{nm}) - (\mathbf{r}_0\mathbf{m})] + r_0^2(\mathbf{nm})^2 - (\mathbf{r}_0\mathbf{m})^2]\} \\
& + \frac{1}{8\rho^2[c^2t^2 + 2(\mathbf{nr}_0)ct + r_0^2]^3}\{10\mathbf{m}\rho^2[ct(\mathbf{nm}) + (\mathbf{r}_0\mathbf{m})] - \mathbf{n}[ct[18\rho^2(\mathbf{nm})^2 - 5r_0^2(\mathbf{nm})^2 + 5(\mathbf{r}_0\mathbf{m})^2] \\
& - 36\rho^2(\mathbf{nr}_0)(\mathbf{nm})^2 + 10(\mathbf{nr}_0)(\mathbf{r}_0\mathbf{m})^2 + 36\rho^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) - 10r_0^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m})] + \mathbf{r}_0[10ct(\mathbf{nm})[(\mathbf{r}_0\mathbf{m}) - (\mathbf{nr}_0)(\mathbf{nm})] \\
& - 18\rho^2(\mathbf{nm})^2 - 5r_0^2(\mathbf{nm})^2 + 5(\mathbf{r}_0\mathbf{m})^2]\} - \frac{1}{32\rho^4[c^2t^2 + 2(\mathbf{nr}_0)ct + r_0^2]^2}\{\mathbf{n}[ct[16m^2\rho^2 + 26\rho^2(\mathbf{nm})^2 \\
& - 25r_0^2(\mathbf{nm})^2 + 25(\mathbf{r}_0\mathbf{m})^2] + 32m^2\rho^2(\mathbf{nr}_0) + 10r_0^2(\mathbf{nr}_0)(\mathbf{nm})^2 + 60(\mathbf{nr}_0)(\mathbf{r}_0\mathbf{m})^2 + 60\rho^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) \\
& - 70r_0^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m})] + \mathbf{r}_0[50ct[(\mathbf{nm})[(\mathbf{nm})(\mathbf{nr}_0) - (\mathbf{r}_0\mathbf{m})] - 16m^2\rho^2 + 20(\mathbf{nr}_0)(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) - 14\rho^2(\mathbf{nm})^2 \\
& + 15r_0^2(\mathbf{nm})^2 - 35(\mathbf{r}_0\mathbf{m})^2] - 2\mathbf{m}\rho^2[ct(\mathbf{nm}) + 6(\mathbf{nr}_0)(\mathbf{nm}) - 5(\mathbf{r}_0\mathbf{m})]\} - \frac{1}{64\rho^6[c^2t^2 + 2(\mathbf{nr}_0)ct + r_0^2]} \\
& \times \{\mathbf{n}[3ct[16m^2\rho^2 + 26\rho^2(\mathbf{nm})^2 - 25r_0^2(\mathbf{nm})^2 + 25(\mathbf{r}_0\mathbf{m})^2] + 2[64m^2\rho^2(\mathbf{nr}_0) - 26\rho^2(\mathbf{nr}_0)(\mathbf{nm})^2 \\
& + 50r_0^2(\mathbf{nr}_0)(\mathbf{nm})^2 + 125(\mathbf{nr}_0)(\mathbf{r}_0\mathbf{m})^2 + 150\rho^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) - 175r_0^2(\mathbf{nr}_0)(\mathbf{nm})]] \\
& - 2\mathbf{m}\rho^2[3ct(\mathbf{nm}) + 28(\mathbf{nr}_0)(\mathbf{nm}) - 25(\mathbf{r}_0\mathbf{m})] + 5\mathbf{r}_0[30ct(\mathbf{nm})[(\mathbf{nr}_0)(\mathbf{nm}) - (\mathbf{r}_0\mathbf{m})] \\
& - 16m^2\rho^2 + 40(\mathbf{nr}_0)(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) + 6\rho^2(\mathbf{nm})^2 - 5r_0^2(\mathbf{nm})^2 - 35(\mathbf{r}_0\mathbf{m})^2]\},
\end{aligned} \tag{20}
$$

where $\mathbf{v}_3$ is an arbitrary constant vector, and vector $\mathbf{r}_3$ due to the initial condition has the form, which is presented in Appendix A.

To substitute the expressions (16)–(20) in the first integral (13), we obtain the relations, which the constant vectors $\mathbf{n}$, $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ should satisfy:

$$
\mathbf{n}^2 = 1, (\mathbf{nv}_1) = (\mathbf{nv}_3) = 0, (\mathbf{nv}_2) = -\frac{1}{2}\mathbf{v}_1^2 - \frac{[1 + (\mathbf{r}_0\mathbf{v}_1)]}{\rho^2}. \tag{21}
$$

VI. THE EFFECT OF NONLINEAR ELECTRODYNAMIC LAG

Expressions (16)–(20) define the laws of photon propagation along any ray beginning at the point $\mathbf{r} = \mathbf{r}_0$ in the dipole magnetic and gravitational fields of gamma ray pulsars and magnetars for arbitrary constant vectors $\mathbf{n}$, $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$, which satisfy relations (21). Let us choose from this family of rays the ray, which passes through the point $\mathbf{r} = \mathbf{r}_d$, in which the detector of X-rays and gamma rays is located. We will then determine the moment of time $t = t_d$ when the electromagnetic signal emitted at $t = 0$ from the point $\mathbf{r} = \mathbf{r}_0$, arrives at the point $\mathbf{r} = \mathbf{r}_d$. For this we present $t = t_d$ as the expansion in the degrees small parameters of our task:

$$
t_d = t_0 + r_g t_1 + r_g^2 t_2 + \xi\eta t_3, \tag{22}
$$

where $\eta = \eta_1$ for electromagnetic signal carried by the first normal wave and $\eta = \eta_2$ for the electromagnetic signal carried by the second normal wave.

If we substitute this expression in relation (16) and take into account that $\mathbf{r}(t_d) = \mathbf{r}_d$, we obtain the following set of algebraic equations:

$$
\begin{aligned}
\mathbf{n}ct_0 + \mathbf{r}_0 = \mathbf{r}_d, \mathbf{n}ct_1 + \mathbf{f}_1(ct_0) &= 0, \\
\mathbf{n}ct_2 + \mathbf{f}_2(ct_0) + \dot{\mathbf{f}}_1(ct_0)ct_1 &= 0, \\
\mathbf{n}ct_3 + \mathbf{f}_3(ct_0) &= 0.
\end{aligned} \tag{23}
$$

It follows from the first equation of this system that $\mathbf{n} = (\mathbf{r}_d - \mathbf{r}_0)/(ct_0)$. Taking into account relation (21), we obtain:

$$
\mathbf{n} = \frac{\mathbf{r}_d - \mathbf{r}_0}{|\mathbf{r}_d - \mathbf{r}_0|}, ct_0 = |\mathbf{r}_d - \mathbf{r}_0|.
$$

If we solve the rest of the equations of the system (23) and take into account relations (21), we obtain in a similar way:

$$t_d = \frac{|\mathbf{r}_d - \mathbf{r}_0|}{c} + \frac{r_g}{c}\ln\left\{\frac{(r_d + ct_0 + (\mathbf{n}\mathbf{r}_0))}{r_0 + (\mathbf{n}\mathbf{r}_0)}\right\} + \frac{r_g^2}{c}\left\{\frac{2(r_d^2 - r_0^2)}{r_d\rho^2} + \frac{15}{16\rho}\left[arctg\left(\frac{ct_0 + (\mathbf{n}\mathbf{r}_0)}{\rho}\right) - arctg\left(\frac{(\mathbf{n}\mathbf{r}_0)}{\rho}\right)\right]\right.$$
$$+ \frac{2[2r_0^3 - 2r_d r_0^2 + ct_0(3r_0 - r_d)(\mathbf{n}\mathbf{r}_0) + c^2t_0^2(r_0 - (\mathbf{n}\mathbf{r}_0) - r_d) - c^3t_0^3]}{ct_0 r_d\rho^2} + \frac{4r_0^2 r_d - 3r_0 r_d^2 + 2r_d^3 - (r_d + 2r_0)(\mathbf{r}_d\mathbf{r}_0)}{ct_0\rho^2 r_d}$$
$$\left. - \frac{2(\mathbf{n}\mathbf{r}_0)ct_0}{r_d\rho^2}\right\} + \frac{\eta\xi}{c}\left\{\frac{3}{64\rho^7}[16m^2\rho^2 + 25(\mathbf{m}\mathbf{r}_0)^2 - 50(\mathbf{n}\mathbf{r}_0)(\mathbf{m}\mathbf{n})(\mathbf{m}\mathbf{r}_0) - 26\rho^2(\mathbf{m}\mathbf{n})^2 + 25r_0^2(\mathbf{m}\mathbf{n})^2]\right.$$
$$\times\left[arctg\left(\frac{ct_0 + (\mathbf{n}\mathbf{r}_0)}{\rho}\right) - arctg\left(\frac{(\mathbf{n}\mathbf{r}_0)}{\rho}\right)\right] + \frac{m^2\{(3r_d^2 + 2\rho^2)r_0^4 - (3r_0^2 + 2\rho^2)r_d^4\}(\mathbf{n}\mathbf{r}_0)}{4r_d^4 r_0^4\rho^4} + \frac{(\mathbf{m}\mathbf{n})^2}{64r_0^4 r_d^8\rho^6}$$
$$\times[[ct_0 + (\mathbf{n}\mathbf{r}_0)](75r_0^2r_d^6 + 50r_0^2r_d^4\rho^2 - 52r_d^4\rho^4 - 78r_d^6\rho^2 + 40r_0^2r_d^2\rho^4 + 144r_0^2\rho^6)r_0^4$$
$$+ (12r_d^6\rho^4 + 28r_0^2r_d^6\rho^2 - 144r_0^4\rho^6 - 75r_0^4r_d^6)r_d^2(\mathbf{n}\mathbf{r}_0)] + \frac{(\mathbf{m}\mathbf{r}_0)^2}{64r_d^8\rho^6}[(75r_d^6 + 50r_d^4\rho^2 + 40r_d^2\rho^4 + 144\rho^6)[ct_0 + (\mathbf{n}\mathbf{r}_0)]]$$
$$- \frac{(\mathbf{m}\mathbf{r}_0)(\mathbf{m}\mathbf{n})[ct_0(\mathbf{n}\mathbf{r}_0) + r_0^2]}{32r_d^8\rho^6}[75r_d^6 + 50r_d^4\rho^2 + 40r_d^2\rho^4 + 144\rho^6] + \frac{(\mathbf{m}\mathbf{r}_0)(\mathbf{m}\mathbf{n})}{32r_0^6r_d^6\rho^6}[75r_0^6r_d^4(r_d^2 + \rho^2) + r_0^6\rho^4(50r_d^2 + 104\rho^2)$$
$$\left. - 25r_0^4r_d^6\rho^2 - 10r_d^6\rho^4(r_0^2 - 4\rho^2)] - \frac{(\mathbf{m}\mathbf{r}_0)^2(\mathbf{n}\mathbf{r}_0)}{64r_0^8\rho^6}[75r_0^6 + 50r_0^4\rho^2 + 40r_0^2\rho^4 + 144\rho^6]\right\}. \quad (24)$$

Here and further the terms, which become equal to zero at $r_d \to \infty$ and at any value $r_0 < \infty$, were omitted.

The constants of integration $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ for the electromagnetic signal emitted at $t = 0$ in the point $\mathbf{r} = \mathbf{r}_0$ and propagating along the ray, which passes through the point $\mathbf{r} = \mathbf{r}_d$, should have the form, which is presented in the Appendix B.

If we use expressions (22)–(24), it is easy to find the time of nonlinear electrodynamic lag Δt of electromagnetic signal carried by the first normal wave, in comparison with the electromagnetic signal carried by the second normal wave:

$$\Delta t = \frac{(\eta_1 - \eta_2)\xi}{c}\left\{\frac{3}{64\rho^7}\left[16m^2\rho^2 + 25(\mathbf{m}\mathbf{r}_0)^2 - 50(\mathbf{n}\mathbf{r}_0)(\mathbf{m}\mathbf{n})(\mathbf{m}\mathbf{r}_0) - 26\rho^2(\mathbf{m}\mathbf{n})^2 + 25r_0^2(\mathbf{m}\mathbf{n})^2\right]\right.$$
$$\times\left[arctg\left(\frac{ct_0 + (\mathbf{n}\mathbf{r}_0)}{\rho}\right) - arctg\left(\frac{(\mathbf{n}\mathbf{r}_0)}{\rho}\right)\right] + \frac{m^2\{(3r_d^2 + 2\rho^2)r_0^4 - (3r_0^2 + 2\rho^2)r_d^4\}(\mathbf{n}\mathbf{r}_0)}{4r_d^4r_0^4\rho^4} + \frac{(\mathbf{m}\mathbf{n})^2}{64r_0^4r_d^8\rho^6}$$
$$\times[[ct_0 + (\mathbf{n}\mathbf{r}_0)](75r_0^2r_d^6 + 50r_0^2r_d^4\rho^2 - 52r_d^4\rho^4 + 40r_0^2r_d^2\rho^4 + 144r_0^2\rho^6 - 78r_d^6\rho^2)r_0^4$$
$$+ (12r_d^6\rho^4 + 28r_0^2r_d^6\rho^2 - 144r_0^4\rho^6 - 75r_0^4r_d^6)r_d^2(\mathbf{n}\mathbf{r}_0)] + \left[\frac{(\mathbf{m}\mathbf{r}_0)^2}{64r_d^8\rho^6}[ct_0 + (\mathbf{n}\mathbf{r}_0)] - \frac{(\mathbf{m}\mathbf{r}_0)(\mathbf{m}\mathbf{n})}{32r_d^8\rho^6}[ct_0(\mathbf{n}\mathbf{r}_0) + r_0^2]\right]$$
$$\times[75r_d^6 + 50r_d^4\rho^2 + 40r_d^2\rho^4 + 144\rho^6] + \frac{(\mathbf{m}\mathbf{r}_0)(\mathbf{m}\mathbf{n})}{32r_0^6r_d^6\rho^6}[75r_0^6r_d^4(r_d^2 + \rho^2) + r_0^6\rho^4(50r_d^2 + 104\rho^2)$$
$$\left. - 25r_0^4r_d^6\rho^2 - 10r_d^6\rho^4(r_0^2 - 4\rho^2)] - \frac{(\mathbf{m}\mathbf{r}_0)^2(\mathbf{n}\mathbf{r}_0)}{64r_0^8\rho^6}[75r_0^6 + 50r_0^4\rho^2 + 40r_0^2\rho^4 + 144\rho^6]\right\}. \quad (25)$$

It is necessary to note that if we substitute the relations (14) and (15) in this expression by components of vector **m** components, it can be also used for neutron star rotating quasi-stationary.

VII. ANALYSIS OF NONLINEAR ELECTRODYNAMIC LAG EFFECT

As it follows from formula (25), the value of nonlinear electrodynamic lag of electromagnetic signals depends on difference of post-Maxwellian parameters $\eta_1 - \eta_2$. Thus, the value Δt can be different in various models of the nonlinear electrodynamics of vacuum. In particular, the Born-Infeld electrodynamics predicts that $\Delta t = 0$, while according to the Heisenberg-Euler electrodynamics $\Delta t \neq 0$. It allows with the help of experimental data to distinguish the nonlinear theories with different value of $\eta_1 - \eta_2$.

Besides, it follows from expression (25) that Δt value depends significantly on the relative positions of the neutron star with strong magnetic field (gamma ray pulsar or magnetar), the source and the detector of X- and gamma rays. A few typical cases can be noted: the source of emission is close to the neutron star, the source of emission is located at certain (not very large) distance from the neutron star, and the source of emission is placed at a very long distance from the neutron star.

The first case is realized when the source of emission is near the pulsar (or magnetar) magnetic poles. It is the so called polar cap model [22]. The second case is when the emission occurs in the so called outer gap, the part of pulsar magnetosphere along the null charge surface $(\mathbf{\Omega B}) = 0$, where the corotation charge changes sign [23]. The most advanced models were developed by considering so called Deutsch field gamma ray pulsar [24,25]. The third case can take place when the emission from the distant object, for example the active galactic nuclei, propagates in the vicinity of a neutron star.

Thus, we will discuss the obtained formula for the three typical cases.

A. The source of X- and gamma rays is located on the pulsar (or magnetar) magnetic poles

This case of emission generation in polar caps is quite typical for relatively young pulsars with very high magnetic field ($B \geq 10^{13}$ G) like the well-known pulsar in the Crab nebula. The pulsating emission component of such pulsars is in phased in the different energy bands - from radio to gamma emission.

It is assumed that hard electromagnetic emission is generated mainly by the relativistic electrons moving along the magnetic field lines (so called curvature radiation). As it is well-known, the magnetic bremsstrahlung (and curvature radiation as well) beam width is inversely proportional to the electron's Lorentz-factor Γ. Thus, for the ultra-relativistic electrons ($\Gamma > 2$) the beam will be rather narrow and oriented along the tangent to the field line.

At the same time, since X-ray and gamma-quanta of relatively small energy (about dozens—hundreds keV) besides the curvature mechanisms associated with sub-relativistic and relativistic ($\Gamma \geq 1$) electrons will also be produced due to Compton back-scattering of thermal photons, the directional diagram (beam) in the range of X-ray and soft gamma ray radiation can be relatively wide. Thus we can consider the isotropically emitted local areas near the magnetic poles (at the distances no more than 0.5–1.0 stellar radius) as the sources of such photons [22].

The maximum intensity detected in the pulsed component is determined by the projection of the emitting surface on the plane normal to the observation line, i.e., it will depend on the angle θ between the line from the source to observer and the magnetic dipole line as $\cos\theta$.

The pulsed component of the emission of a rotating neutron star with a strong magnetic field can be polarized to a certain degree. For example, the mean pulse profile of the Crab pulsar has twin peaks in different bands. In the optical range polarization degree of both peaks is at the level of $\sim$10% and the polarization vector rotates as the pulsation phase changes [26]. The point is that similar behavior may be also typical for the range of X-rays.

However, although during the OSO (Orbital Solar Observatory) missions the polarization of synchrotron emission of the Crab nebula was detected in the range 10–20 keV at the level of 15% [27], the attempts to measure polarization of the pulsed component of X-rays, i.e., emission of the pulsar itself, on the OSO-8 satellite gave only the upper limits, which for each peak are: $\sim$20% and $\sim$30% at the energies of 2.4–2.8 keV and $\sim$60% and $\sim$50% at the energies of 4.8–5.6 keV [28]. Thus, in further considerations we will assume that pulsed emission of gamma pulsars or magnetars is not polarized if the nonlinear electrodynamic effects is not taken into account.

In the case of emission from the pulsar polar cap region we will assume that its size is small and the emitting area itself is point-like. The relative amplitude of modulation and the mean pulsed profile time structure are determined by the angle φ_m between the pulsar rotation axis and the magnetic dipole axis as well as the angle φ_0 between the pulsar rotation axis and the direction from the pulsar to the observer (detector). It is clear that the maximum of intensity will be observed if the rotation axis, dipole magnetic axis and the line from the pulsar to observer lay in the same plane.

At $\varphi_m + \varphi_0 < 90^o$, i.e. when only one polar cap is observable, the time profile of pulsation have one-peak structure. The peak width will be smaller and the relative amplitude of modulation will be higher with increasing angle φ_m. When $\varphi_m + \varphi_0 = 90^o$ the modulation becomes 100%, if $\varphi_m = \varphi_0$, i.e., $\varphi_m = \varphi_0 = 45^o$ the peak width will be equal to half of the pulsation period.

At $\varphi_m + \varphi_0 > 90^o$, as the pulsar rotates, one of the polar caps will be shadowed by the opaque neutron star and while the other cap will appear from the shadow, thus the corresponding time profile of pulsation will be two-peaked. It is clear that the peak widths should not be equal, and the peak corresponding to the polar cap, which is less shadowed, will be wider.

For the radius-vector of point-like emitted area we can write: $\mathbf{r}_0 = \chi R_s \mathbf{m}/m$, where $\chi = +1$ for the source on the neutron star's north magnetic pole, and $\chi = -1$ for the source on the neutron star's south magnetic pole. Let us take into account that the distance from the Earth to the nearest gamma pulsars or magnetars is more than about 1 kps, and $r_0 = R_s \sim 10^2$ km. Thus, in the expression (25) we can take $r_d \to \infty$. As the result, it transforms to:

$$\Delta t = \frac{(\eta_2 - \eta_1)\xi m^2}{64 c R_s^5 [1-(\mathbf{ne})^2]^2}\Big\{\chi(\mathbf{ne})[229 - 260(\mathbf{ne})^2 + 76(\mathbf{ne})^4] + \frac{3[41 - 26(\mathbf{ne})^2]}{\sqrt{[1-(\mathbf{ne})^2]}}\Big[arctg\Big(\frac{\chi(\mathbf{ne})}{\sqrt{1-(\mathbf{ne})^2}} - \frac{\pi}{2}\Big)\Big]\Big\}, \quad (26)$$

where the notation $\mathbf{e} = \mathbf{m}/m$ has been introduced.

Since we assume that the neutron star is opaque, the product $\chi(\mathbf{ne})$ in (26) should be non-negative because at $\chi(\mathbf{ne}) < 0$ the electromagnetic emission propagating along the rays from the source to detector will fall on the neutron star and will not reach the observer.

Analysis of expression (26) shows that the lag time interval Δt has a maximum

$$\Delta t_{\max} = \frac{123\pi(\eta_1 - \eta_2)\xi m^2}{128 c R_s^5}, \qquad (27)$$

if the emission propagates from the source along the rays normal to the neutron star vector $\mathbf{m}$ and becomes equal to zero if the emission propagates along the vector $\chi\mathbf{m}$.

In the case of a rotating neutron star the lag time interval Δt will be a function of time. If we denote the spherical coordinates of the X- and gamma ray detector as θ_d and φ_d, then due to expressions (14) and (15) we obtain:

$$\begin{aligned}(\mathbf{ne}) = {} & \sin\theta_d\{\sin\Theta\sin(\Omega_2 t' + \psi_0 - \varphi_d)\cos\alpha_0 \\ & + \sin\alpha_0[\cos(\Omega_2 t' + \psi_0 - \varphi_d)\cos(\Omega_1 t' + \Phi_0 + \beta_0) \\ & - \cos\Theta\sin(\Omega_2 t' + \psi_0 - \varphi_d)\sin(\Omega_1 t' + \Phi_0 + \beta_0)]\} \\ & + \cos\theta_d\{\cos\alpha_0\cos\Theta + \sin\Theta\sin\alpha_0 \\ & \times\cos(\Omega_1 t' + \Phi_0 + \beta_0)\}.\end{aligned}$$

where t' is the retarded time.

Thus, the detected value of the lag time Δt will be modulated with frequencies Ω_1, Ω_2 and their combinations. The modulation depth and frequency spectrum are fore-determined by mutual orientation of vectors $\mathbf{n}, \boldsymbol{\Omega}_1$ and $\mathbf{m}$.

It is necessary to note that the obtained condition when the lag time falls to zero corresponds to the peak maximum of the mean pulsation profile, while the maximum lag time corresponds to the intensity minimum of the pulsation profile. Indeed, in the assumption of the isotropically emitting point-like region near the polar cap, most of the photons will be emitted along the magnetic dipole line and the maximum of the detected emission intensity will be observed if the angle between the magnetic dipole axis and the line from the source to the detector is minimal, while the minimum of the intensity—when the line from the source to the detector is furthest from the dipole axis. Hence, the maximum lag time should be observed at the rise and fall of the peak on the pulsation time profile and at peak maximum the lag should be equal to zero.

B. The pulsar or magnetar outer gap as the source of X- and gamma rays

In the case of the outer gap model, unlike the polar cap model the emitting region is not so small, thus it can not be regarded as point-like. The outer gap is the area between the last open field line and the null charge surface located, as usual at distances of about $10R_s$ from the neutron star. We will approximate its size also as about $10R_s$ [22].

It is assumed that gamma emission of such relatively old pulsars like Vela pulsar generates mainly in the outer gap. It is typical for such pulsars that the mean pulsation profiles are out of phase in the different energy ranges. As in the case of the polar cap model, it is supposed that the high-energy photons are generated due to the curvature mechanism or by inverse Compton scattering of mainly infrared photons (for old pulsars) [22].

These processes in the case of high-energy gamma-quanta give the emission propagating mainly along the direction of parental electron motion. However, in the range of X-rays and soft gamma rays (dozens - hundreds keV) the beam is, probably, quite isotropic. We can neglect the emission polarization, which is not caused by the nonlinear electrodynamic effects.

Thus, in the case of emission generation in the outer gap we can examine the emitting area, for which $r_0 \sim 10R_s$, the radius-vector φ_0, θ_0 coordinates do not coincide in the general case with the vector $\mathbf{m}$ coordinates φ_m, θ_m. However, because of the null charged surface $(\boldsymbol{\Omega}\mathbf{B}) = 0$, we can assume that $\varphi > 45^o$. Besides, we have $(\mathbf{nr}_0) = r_0\cos(\varphi_0 - \varphi)$, $\rho \sim r_0\sin(\varphi_0 - \varphi)$ (if $\varphi_0 > \varphi$) and $\rho \sim 2r_0\sin(\varphi - \varphi_0$ (if $\varphi_0 < \varphi$). It is also assumed that the detector is located infinitely far. In this case expression (25) gives:

$$\begin{aligned}\Delta t = {} & \frac{(\eta_1 - \eta_2)\xi}{64 c\rho^7}\Big\{3[16m^2\rho^2 - 50(\mathbf{mn})(\mathbf{nr}_0)(\mathbf{mr}_0) - 26\rho^2(\mathbf{mn})^2 + 25r_0^2(\mathbf{mn})^2 + 25(\mathbf{mr}_0)^2] \\ & \times\left[\frac{\pi}{2} - arctg\left(\frac{(\mathbf{nr}_0)}{\rho}\right)\right] + \frac{\rho}{r_0^8}[(\mathbf{mn})^2(\mathbf{nr}_0)r_0^4(12\rho^4 + 28\rho^2 r_0^2 - 75r_0^4) - 16m^2\rho^2 r_0^4(\mathbf{nr}_0)(2\rho^2 + 3r_0^2) \\ & - (\mathbf{nr}_0)(\mathbf{mr}_0)^2(144\rho^6 + 40\rho^4 r_0^2 + 50\rho^2 r_0^4 + 75r_0^6) + 10(\mathbf{mn})(\mathbf{mr}_0)r_0^2(8\rho^6 - 2\rho^4 r_0^2 - 5\rho^2 r_0^4 + 15r_0^6)]\Big\}.\end{aligned}$$

C. The X-ray and gamma ray source is located far away from the pulsar or magnetar

In this case the source of emission is a distant object like the active galactic nuclei (blasar, quasar or Seyfert galaxy), emitting X-rays or gamma rays. Since such objects can be considered as located practically infinitely far, both from the observer and from the pulsar, we can assume that their emission is a flat wave, i.e., the geometric optics approximation can be used. Since at the present time, there is no information about polarization of high-energy emission from extra-galactic objects, we will consider their emission not polarized.

To study this case, it is convenient to orientate the coordinate system axes in such a way that the source, detector and pulsar lay in the one of coordinate planes, for example, YOZ, and the axis Z is parallel to the line connecting the emission source and the detector. Let us also suppose that the source and the detector lay on different sides of the XOY plane, otherwise the effect of nonlinear electrodynamic lag will be negligibly small. Thus, we will obtain:

$$\mathbf{r}_0 = \left\{0, \rho, -\sqrt{r_0^2 - \rho^2}\right\}, \quad \mathbf{n} = \{0, 0, 1\}, \quad (\mathbf{mn}) = -m_z,$$

$$(\mathbf{nr}_0) = -\sqrt{r_0^2 - \rho^2}, \quad (\mathbf{mr}_0) = m_y\rho - m_z\sqrt{r_0^2 - \rho^2},$$

$$\mathbf{r}_d = \left\{0, \rho, \sqrt{r_d^2 - \rho^2}\right\}, \quad ct_0 = \sqrt{r_d^2 - \rho^2} - \sqrt{r_0^2 - \rho^2}.$$

Substituting these relations in expression (25) and tending r_0 and r_d to infinity, we obtain:

$$\Delta t = \frac{3\pi(\eta_1 - \eta_2)\xi[16m^2 + 25m_y^2 - m_z^2]}{64c\rho^5}.$$

The maximum value of Δt

$$\Delta t = \frac{123\pi(\eta_1 - \eta_2)\xi m^2}{64c\rho^5}$$

in this case will be obtained, if the vector $\mathbf{m}$, is directed along the Y axis, and the minimum value

$$\Delta t = \frac{45\pi(\eta_1 - \eta_2)\xi m^2}{64c\rho^5},$$

if the vector $\mathbf{m}$ is directed along the z axis.

The neutron star rotation also leads in this case to the modulation of the value Δt with the frequencies Ω_1, Ω_2 and their combinations.

VIII. DISCUSSION

Let us discuss the possibility of observing the nonlinear electrodynamic effect of X-ray and gamma-ray birefringence in strong magnetic field of pulsars and magnetars.

As it follows from the observations, magnetars possess a very high magnetic field $B \sim 10^{16}$ G and can reveal themselves as anomalous X-ray pulsars and also as sources of recurrent gamma ray bursts, the so-called, soft gamma ray repeaters [29,30].

The anomalous X-ray pulsars are observed, mainly, in the energy range from few keV to 10–20 keV photons. While the soft gamma ray repeaters have thermal spectra with effective temperature $kT \sim 20$–30 keV, i.e., can be observed up to about 100–200 keV. Thus, to study the polarization properties of these objects in hard emission, the traditional techniques, such as the Thomson (Compton) scattering or the Bragg reflection can be used, as well as new methods based on the photoelectron tracing.

Besides the magnetars some gamma ray pulsars possess the magnetic fields, which are high enough to manifest nonlinear electrodynamic effects. Among them the pulsars, with magnetic induction exceeding the character quantum-electrodynamic value $B_q = 4.4 \cdot 10^{13}$ G, are of most interest. Only 6 such gamma ray pulsars are known. For four of them, quite reliable data on the magnetic field is available [31]: B1509-58 has the field of $B \sim 1.5 \cdot 10^{14}$ G, the magnetic fields of Crab, Vela and B1706-44 were estimated at the level of $B \sim 5 \cdot 10^{13}$ G. For two pulsars (B1046-58, J0218+4232) estimations are still not reliable, but are similar to B_q.

Using expression (27), we estimate on the order of magnitude of the time of nonlinear-electrodynamic lag in the case of magnetars and gamma pulsars. Let us consider a magnetar with a surface magnetic field of $B \sim 10^{16}$ G. If we assume that the magnetar's radius is $R_s = 10$ km, we find that the area with $B < B_q$, where the results of our calculations are valid, originates from the impact parameters $\rho = 100$ km. Then, according to the Heisenberg-Euler nonlinear electrodynamics the value Δt can reach $\Delta t = 10^{-7}$ s.

In the case of gamma pulsars the surface magnetic field is less than typical value of a magnetar . However, they may be of a larger size. Since, though the value Δt will be less than of magnetars, for some pulsars it can reach $\Delta t = 10^{-7}$ s. Because of the large distances the X-ray and gamma ray fluxes from pulsars and magnetars are very low in the vicinity of the Earth. Hence, it is possible to detect this effect only using instruments installed onboard satellites, and the measurements should be made outside the Earth's radiation belts.

The polarization measurement technique based on the Thomson (or the Compton for hard X-rays) scattering and pair production effect (for high-energy gamma rays) can be used to detect the effect of nonlinear electrodynamic lag of the signals with mutually orthogonal polarization.

In the 2–20 keV range the telescopes with conical foil mirrors can be used for X-ray photons detection. The effective focusing area in such instruments can reach several square meters. This allows us to detect the considered nonlinear electrodynamic effect in the Crab pulsar emission over a time of about a hundreds seconds. In the case of observations of objects with luminosity of about 1 mCrab, the exposure time should be about 10^5 s, i.e., 24 hours, which is also quite acceptable.

For the range 20–100 keV we obtain than an instrument with geometrical area 10^3sm^2 provides the detection of effect in Crab pulsar emission over a hundreds kiloseconds. To detect the effect in emission from sources with luminosity of about 1 mCrab the geometrical area $\sim 10^4$ sm^2 and exposure times of about a year will be necessary. In the range 0.1–1.0 MeV an instrument with geometrical area 10^4 sm^2 will detect the effect in the Crab pulsar emission over about a day, while detecting the effect in the emission of source with luminosity of about 1 mCrab several years of continuous observations will be necessary.

No reasonable estimates of the instrument geometrical area and exposure time made for the range 0.05–1.0 GeV permit to detect effect even in the Crab pulsar emission.

Considering, in general, the different techniques of nonlinear electrodynamic birefringence observation we can conclude that the polarization measurements employing micro-well gas proportional counters [32] placed in the focal plane of a large conic foil mirror telescope are the most appropriate. However, such a technique is only effective for observations in soft X-rays. This range does not cover the emission from all types of possible objects with very high magnetic field, such as soft gamma ray repeaters and gamma pulsars, which are more contrast in hard X-rays and gamma rays. Besides, the screening of emitted photons by matter surrounding the neutron star is more effective for soft X-rays. Such absorption could not be completely excluded, thus it is unlikely that the polarization of soft X-rays will not be changed during their propagation in the neutron star vicinity.

From this point of view, observations in hard X-rays or gamma rays are preferable. The complexity and cost of a soft X-ray polarimeter and installation on a special spacecraft like the orbital X-ray observatory, are also significant factors, which can not be disregarded. Due to these circumstances experiments, in which Thompson or Compton polarimeters could be used, seems more realistic.

The exposure time and the background level are defined mainly by the type of spacecraft, and the exposure time is exactly the ‘‘unlimited’’ resource, which allows to improve the sensitivity of measurements. The large exposure time in the polarization measurements of the search for nonlinear electrodynamic effects in the pulsar and magnetar emission assumes the necessity of very long (during months or even years) constant spacecraft orientation, which makes it possible to observe the chosen source during all the time of the experiment duration.

Thus the analysis presented above indicates on the instrumental possibility of nonlinear electrodynamic lag measurements, although the realization of experiment is a very complicated technical problem.

In conclusion we should note that all obtained formulas were verified by computer algebra.

ACKNOWLEDGMENTS

This work was partially supported by Presidential Grant of Russian Federation No. 1450.2003.2 and by Russian Foundation for Basic Research under grant No. 04-02-16604.

APPENDIX A:THE CONSTANTS OF INTEGRATION FOR THE SIGNALS EMITTED FROM POINT $\mathbf{r} = \mathbf{r_0}$ AT THE TIME $t = 0$.

$$\begin{aligned}
\mathbf{r}_2 =& \frac{1}{\rho^4}\{[r_0^2(\mathbf{nr}_0) - 2\rho^2(\mathbf{nr}_0) - r_0^3 + \rho^2 r_0 - 2r_0^3(\mathbf{v}_1\mathbf{r}_0) + 2r_0\rho^2(\mathbf{v}_1\mathbf{r}_0)]\mathbf{n} + +(\mathbf{nr}_0)[2r_0(\mathbf{v}_1\mathbf{r}_0) + r_0 - (\mathbf{nr}_0)]\mathbf{r}_0 - r_0(\mathbf{nr}_0)\rho^2\mathbf{v}_1\} \\
&- \frac{15[(r_0^2 - 2\rho^2)\mathbf{n} - (\mathbf{nr}_0)\mathbf{r}_0]}{16\rho^3} arctg\left(\frac{(\mathbf{nr}_0)}{\rho}\right), \\
\mathbf{r}_3 =& \frac{3}{64\rho^9}\{2\mathbf{m}\rho^2[(25r_0^2 - 26\rho^2)(\mathbf{nm}) - 25(\mathbf{nr}_0)(\mathbf{r}_0\mathbf{m})] + 5\mathbf{r}_0[16m^2\rho^2(\mathbf{nr}_0) + 60\rho^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) \\
&+ 35(\mathbf{nr}_0)(\mathbf{r}_0\mathbf{m})^2 + 35r_0^2(\mathbf{nm})^2(\mathbf{nr}_0) - 26\rho^2(\mathbf{nr}_0)(\mathbf{nm})^2 - 70r_0^2(\mathbf{nm})(\mathbf{r}_0\mathbf{m})] \\
&- \mathbf{n}[80m^2\rho^2 r_0^2 + 300\rho^2(\mathbf{nr}_0)(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) - 96m^2\rho^4 - 350r_0^2(\mathbf{nr}_0)(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) + 104\rho^4(\mathbf{nm})^2 - 280\rho^2 r_0^2(\mathbf{nm})^2 \\
&- 200\rho^2(\mathbf{r}_0\mathbf{m})^2 + 175r_0^4(\mathbf{nm})^2 + 175r_0^2(\mathbf{r}_0\mathbf{m})^2]]\} arctg\left(\frac{(\mathbf{nr}_0)}{\rho}\right) - \frac{1}{64\rho^6 r_0^8}\{2\mathbf{m}\rho^2 r_0^2[40\rho^4(\mathbf{r}_0\mathbf{m}) + 12\rho^2 r_0^2(\mathbf{nm})(\mathbf{nr}_0) \\
&+ 28r_0^4(\mathbf{nm})(\mathbf{nr}_0) - 10\rho^2 r_0^2(\mathbf{r}_0\mathbf{m}) - 25r_0^4(\mathbf{mr}_0)] + \mathbf{r}_0[32m^2\rho^4 r_0^4 + 80m^2\rho^2 r_0^6 - 40\rho^2 r_0^4(\mathbf{nr}_0)(\mathbf{nm})(\mathbf{r}_0\mathbf{m}) \\
&- 200r_0^6(\mathbf{nr}_0)(\mathrm{nm})(\mathbf{mr}_0) - 144\rho^6(\mathbf{mr}_0)^2 - 12\rho^4 r_0^4(\mathbf{nm})^2 + 40\rho^4 r_0^2(\mathbf{mr}_0)^2 - 60\rho^2 r_0^6(\mathbf{nm})^2 + 70\rho^2 r_0^4(\mathbf{mr}_0)^2 \\
&+ 25r_0^8(\mathbf{nm})^2 + 175r_0^6(\mathbf{mr}_0)^2] - 2\mathbf{n}r_0^2[32m^2\rho^4 r_0^2(\mathbf{nr}_0) + 64m^2\rho^2 r_0^4(\mathbf{nr}_0) + 40\rho^4(\mathbf{nr}_0)(\mathbf{mr}_0)^2 - 16\rho^2 r_0^4(\mathbf{nm})^2(\mathbf{nr}_0) \\
&+ 60\rho^2 r_0^2(\mathbf{nr}_0)(\mathbf{mr}_0)^2 + 50r_0^6(\mathbf{nm})^2(\mathbf{nr}_0) + 125r_0^4(\mathbf{nr}_0)(\mathbf{mr}_0)^2 + 20\rho^4 r_0^2(\mathbf{nm})(\mathbf{mr}_0) \\
&+ 80\rho^2 r_0^4(\mathbf{nm})(\mathbf{mr}_0) - 175r_0^6(\mathbf{nm})(\mathbf{mr}_0)]\}. \qquad \text{(A1)}
\end{aligned}$$

APPENDIX B: THE CONSTANTS OF INTEGRATION FOR THE SIGNALS PASSING THROUGH POINT $\mathbf{r} = \mathbf{r}_d$.

$$\mathbf{v}_1 = -\frac{(ct_0 - r_d + r_0)}{\rho^2 ct_0}[\mathbf{r}_0 - (\mathbf{n}\mathbf{r}_0)\mathbf{n}],$$

$$\mathbf{v}_2 = \frac{15[\mathbf{r}_0 - (\mathbf{n}\mathbf{r}_0)\mathbf{n}]}{16\rho^3 ct_0}\left\{[ct_0 + (\mathbf{n}\mathbf{r}_0)]arctg\left(\frac{ct_0 + (\mathbf{n}\mathbf{r}_0)}{\rho}\right) - (\mathbf{n}\mathbf{r}_0)arctg\left(\frac{(\mathbf{n}\mathbf{r}_0)}{\rho}\right)\right\}$$
$$- \frac{[ct_0 r_0 - (\mathbf{n}\mathbf{r}_0)(r_d - r_0)](r_d - r_0)}{c^2 t_0^2 \rho^4}\mathbf{r}_0 + \frac{[r_d r_0 - (\mathbf{n}\mathbf{r}_0)^2][ct_0(\mathbf{n}\mathbf{r}_0) - r_d r_0 + r_0^2]}{c^2 t_0^2 \rho^4}\mathbf{n},$$

$$\mathbf{v}_3 = \frac{3}{64 ct_0 \rho^9}\Big\{[5[(\mathbf{m}\mathbf{n})^2(\mathbf{n}\mathbf{r}_0)(35r_0^2 - 26\rho^2) + 60(\mathbf{m}\mathbf{n})(\mathbf{m}\mathbf{r}_0)\rho^2 - 70(\mathbf{m}\mathbf{n})(\mathbf{m}\mathbf{r}_0)r_0^2 + 35(\mathbf{m}\mathbf{r}_0)^2(\mathbf{n}\mathbf{r}_0) + 16m^2(\mathbf{n}\mathbf{r}_0)\rho^2]\mathbf{r}_0$$
$$+ 2\rho^2[25(\mathbf{m}\mathbf{n})r_0^2 - 26(\mathbf{m}\mathbf{n})\rho^2 - 25(\mathbf{m}\mathbf{r}_0)(\mathbf{n}\mathbf{r}_0)]\mathbf{m} - [(\mathbf{m}\mathbf{n})^2(175r_0^4 - 255r_0^2\rho^2 + 78\rho^4)$$
$$- 50(\mathbf{m}\mathbf{n})(\mathbf{m}\mathbf{r}_0)(\mathbf{n}\mathbf{r}_0)(7r_0^2 - 5\rho^2) + 5(35(\mathbf{m}\mathbf{r}_0)^2 + 16m^2\rho^2)(r_0^2 - \rho^2)]\mathbf{n}]\left[arctg\left(\frac{ct_0 + (\mathbf{n}\mathbf{r}_0)}{\rho}\right) - arctg\left(\frac{(\mathbf{n}\mathbf{r}_0)}{\rho}\right)\right]$$
$$+ 5ct_0[[(35r_0^2 - 36\rho^2)(\mathbf{m}\mathbf{n})^2 - 70(\mathbf{m}\mathbf{n})(\mathbf{m}\mathbf{r}_0)(\mathbf{n}\mathbf{r}_0) + 35(\mathbf{m}\mathbf{r}_0)^2 + 16m^2\rho^2]\mathbf{r}_0 + 10\rho^2[(\mathbf{m}\mathbf{n})(\mathbf{n}\mathbf{r}_0) - (\mathbf{m}\mathbf{r}_0)]$$
$$\times \mathbf{m}[(35r_0^2 - 26\rho^2)(\mathbf{m}\mathbf{n})^2(\mathbf{n}\mathbf{r}_0) - 10(\mathbf{m}\mathbf{n})(\mathbf{m}\mathbf{r}_0)(7r_0^2 - 26\rho^2) + (\mathbf{n}\mathbf{r}_0)(35(\mathbf{m}\mathbf{r}_0)^2 + 16m^2\rho^2)]\mathbf{n}]arctg\left(\frac{ct_0 + (\mathbf{n}\mathbf{r}_0)}{\rho}\right)\Big\}$$
$$+ \frac{(\mathbf{m}\mathbf{n})}{32 r_d^8 \rho^6}[(\mathbf{m}\mathbf{n})(\mathbf{n}\mathbf{r}_0) - (\mathbf{m}\mathbf{r}_0)][75r_d^6 + 50r_d^4\rho^2 + 40r_d^2\rho^4 - 144\rho^6]\mathbf{r}_0 - \frac{(r_0^2 - r_d^2)}{ct_0}$$
$$\times \left[\frac{(\mathbf{m}\mathbf{n})^2}{64 r_0^2 r_d^8 \rho^6}(25r_0^2 r_d^6 - 30r_0^2 r_d^4\rho^2 - 60r_d^6\rho^2 + 144r_0^2\rho^6 - 40r_0^2 r_d^2\rho^4 - 12r_d^4\rho^4) - \frac{5(\mathbf{m}\mathbf{n})(\mathbf{m}\mathbf{r}_0)(\mathbf{n}\mathbf{r}_0)}{8r_0^2 r_d^4 \rho^6}(5r_d^2 + \rho^2)\right.$$
$$\left. + \frac{(\mathbf{m}\mathbf{r}_0)^2}{64 r_0^2 r_d^8 \rho^6}(175r_d^6 + 70r_d^4\rho^2 + 40r_d^2\rho^4 - 144\rho^6)\right]\mathbf{r}_0 - \Big\{\frac{1}{32 r_d^8 \rho^6}(75r_0^2 r_d^6 + 50r_0^2 r_d^4\rho^2 + 40r_0^2 r_d^2\rho^4 - 144r_0^2\rho^6)$$
$$\times [r_0^2\mathbf{m}\mathbf{n})^2 - (\mathbf{m}\mathbf{r}_0)(\mathbf{m}\mathbf{n})(\mathbf{n}\mathbf{r}_0)] - \frac{(\mathbf{m}\mathbf{n})^2(\mathbf{n}\mathbf{r}_0)}{64 ct_0 r_d^8 \rho^6}[25r_0^2 r_d^6 - 30r_0^2 r_d^4\rho^2 - 40r_0^2 r_d^2\rho^4 + 144r_0^2\rho^6 - 25r_d^8 + 26r_d^6\rho^2$$
$$+ 52r_d^4\rho^4 - 144r_d^2\rho^6] + \frac{5(\mathbf{m}\mathbf{n})(\mathbf{m}\mathbf{r}_0)}{32 ct_0 r_d^6 \rho^6}[20r_0^2 r_d^4 + 4r_0^2 r_d^2\rho^2 - 20r_d^6 - 15r_d^4\rho^2 - 2r_d^2\rho^4 - 8\rho^6]$$
$$- \frac{(\mathbf{m}\mathbf{r}_0)^2(\mathbf{n}\mathbf{r}_0)}{64 ct_0 r_0^2 r_d^8 \rho^6}[175r_0^2 r_d^6 + 70r_0^2 r_d^4\rho^2 + 40r_0^2 r_d^2\rho^4 - 144r_0^2\rho^6 - 175r_d^8] - \frac{m^2(\mathbf{n}\mathbf{r}_0)(r_0^2 - r_d^2)}{4ct_0 r_0^2 r_d^4 \rho^4}[5r_d^2 + 2\rho^2]\Big\}\mathbf{n}. \quad \text{(B1)}$$

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[1] D. L. Burke, R. C. Field, G. Horton-Smith, J. E. Spencer, and D. Walz, S. C. Berridge, W. M. Bugg, K. Shmakov, A. W. Weidemann, C. Bula, K. T. McDonald, E. J. Prebys, C. Bamber, S. J. Boege, T. Koffas, T. Kotseroglou, A. C. Melissinos, D. D. Meyerhofer, D. A. Reis, and W. Ragg, Phys. Rev. Lett. **79**, 1626 (1997).

[2] V. I. Denisov, Phys. Rev. D **61**, 036004 (2000).

[3] N. N. Rosanov, Zh. Eksp. Teor. Fiz. **103**, 1996 (1993); [**76**, 991 (1993)].

[4] V. I. Denisov, J. of Optics **2**, 372 (2000).

[5] V. I. Denisov, I. V. Krivchenkov, and N. V. Kravtsov, Phys. Rev. D **69**, 066008 (2004).

[6] V. N. Rodionov, Zh. Eksp. Teor. Fiz. **125**, 453 (2004)

[7] V. I. Denisov and I. P. Denisova, Opt. Spectrosc. **90**, 282 (2001); **90**, 928 (2001).

[8] V. I. Denisov and I. P. Denisova, Teor. Mat. Fiz. **129**, 1421 (2001).

[9] V. I. Denisov, I. P. Denisova, and S. I. Svertilov, Doklady Physics **46**, 705 (2001).

[10] V. G. Kadyshevsky and V. N. Rodionov, Theor. Math. Phys. (Engl. Transl.) **136**, 517 (2003).

[11] S. L. Adler, Ann. Phys. **67**, 599 (1971).

[12] P. A. Vshivtseva, V. I. Denisov, and I. P. Denisova, Doklady Physics **47**, 798 (2002).

[13] V. I. Denisov and S. I. Svertilov, Astron. Astrophys. **399**, L39 (2003).

[14] V. I. Denisov, I. V. Krivtshenkov, and I. P. Denisova, Sov. Phys. JETP **95**, 194 (2002).

[15] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Nauka, Moscow, 1988) and (Pergamon, Oxford, 1975).

[16] V. I. Denisov, Theor. Math. Phys. **132**, 1070 (2002).

[17] C. Will, *Theory and Experiment in Gravitational Physics* (Cambrige University Press, Cambrige, 1981).

[18] W. Heisenberg and H. Euler, Z. Phys. **26**, 714 (1936).

[19] M. Born, Proc. Roy. Soc. A **143**, 410 (1934); M. Born and L. Infeld *ibid.***144**, 425 (1934).

[20] I. P. Denisova and M. Dalal, J. Math. Phys. **38**, 5820 (1997).

[21] I. P. Denisova, *Vvedenie v tenzornoe ischislenie i ego prilozenija (Introduction into tensor calculus and it's applications)* (Moskva: UNTS-DO, Moscow, 2003).

[22] A. K. Harding and A. Muslimov, Astrophys. J. **508**, 328 (1998).

[23] K. S. Cheng, C. Ho, and M. A. Ruderman, Astrophys. J. **300**, 500 (1986).

[24] M. G. Higgins and R. N. Henriksen, Mon. Not. R. Astron. Soc. **292**, 934 (1997).

[25] M. G. Higgins and R. N. Henriksen, Mon. Not. R. Astron. Soc. **295**, 188 (1998).

[26] D. C. Ferguson, W. J. Cocke, and T. Gehrels, Astrophys. J. **190**, 375 (1974).

[27] M. C. Weisskopf, E. H. Silver, H. L. Kestenbaum, K. S. Long, and R. Novick, Astrophys. J. Lett. **220**, L117 (1978).

[28] E. H. Silver, M. C. Weisskopf, H. L. Kestenbaum, K. S. Long, R. Novick, and R. S. Wolf, Astrophys. J. **225**, 221 (1978).

[29] C. Thomson and R. C. Duncan, Mon. Not. R. Astron. Soc. **275**, 255 (1995).

[30] C. Kouveliotou, T. Strohmayer, K. Hurley, *et al.*, Astrophys. J. Lett. **510**, L115 (1999).

[31] D. J. Thomson, Adv. Space Res. **25**, 659 (2000).

[32] J. K. Black, P. Deines-Jones, S. E. Ready, and R. A. Street, Nucl. Instrum. Methods Phys. Res., Sect. A **513**, 639 (2003).