

Hyperquarks and generation number

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In a model in which quarks and leptons are built up from two spin- $\frac{1}{2}$ preons as fundamental entities, a new class of fermionic bound states (hyperquarks) arises. It turns out that these hyperquarks are necessary to fulfill the 't Hooft anomaly constraint, which then links the number of fermionic generations to the number of colors and hypercolors.

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I. INTRODUCTION

The fact that leptons and quarks form families of identical structure and the large number of *ad hoc* parameters of the standard model have led to the conjecture that these particles are composites of more fundamental entities. Among the various approaches to quark and lepton substructure, the Harari-Shupe model, which has only two elementary spin- $\frac{1}{2}$ building blocks stands out because of its simplicity. For the construction of leptons and quarks, one of these (T -preon) carries the fundamental electric charge of ($\frac{e}{3}$), the other one (V -preon) is neutral [1,2].

The preon model should give answers to some fundamental questions, which remain unanswered within the standard model, for example:

- (i) How can the exact equality of the electric charge of proton and positron be explained?
- (ii) Is there a connection between fractional charges and colored fermions?
- (iii) Why are there exactly three identical fermionic families (generations)?

While answers to the first two questions have been suggested, there has not been much progress in answering the third one. The purpose of this paper is to propose an answer to the question why there are three generations of quarks and leptons.

With respect to preon dynamics not much is known except that ordinary quantum chromodynamic (QCD) forces alone are insufficient to bind preons into both colored quarks and colorless leptons. For preon binding a new superstrong interaction (hypercolor force) at the TeV range [3] was suggested. Thus, preons carry both hypercolor and color charges. Quarks and leptons are assumed to be hypercolor singlets. The corresponding gauge group has an analogous structure as the SU(3) color group of QCD and is called quantum hyperchromodynamics [3,4]. The model involves three unbroken local gauge symmetries: hypercolor, color, and electromagnetism, and the total gauge group is $SU(3)_H \times SU(3)_C \times U(1)_Q$.

The small radius of the quarks and leptons, $r < 10^{-19}$ m, corresponds to a preon kinetic energy of $T > 1$ TeV. Most of this kinetic energy must be compensated by a nearly equal potential energy of the superstrong forces, in order to explain the near masslessness of leptons and quarks compared to the effective preon mass. The chiral symmetry at the preon level must not be spontaneously broken at the energy scale of the bound states [5]. Otherwise, the bound state masses would be of the same order of magnitude as the effective preon mass (> 1 TeV). For example, in QCD, where chiral symmetry is spontaneously broken, bound state masses are of the same order of magnitude as the constituent quark masses.

In order to prevent chiral symmetry to be spontaneously broken, a chiral anomaly cancellation condition, the 't Hooft anomaly constraint, must be satisfied [5]. The Harari-Shupe model with only hypercolor singlets does not satisfy this condition. In this paper we introduce a new class of bound states (hyperquarks) which reconciles the Harari-Shupe model with the 't Hooft anomaly constraint. Furthermore, we suggest a simple argument which links the number of fermion families to the number of colors and hypercolors.

II. PREON QUANTUM NUMBERS

In the Harari-Shupe model all quarks and leptons can be built from just two spin- $\frac{1}{2}$ fermions (preons). According to Harari and Seiberg [3] the two types of preons belong to the following representations of the underlying exact gauge group $SU(3)_H \times SU(3)_C \times U(1)_Q$: $T: (3, 3)_{1/3}$ and $V: (3, \bar{3})_0$, where the first (second) argument is the dimension of the representation in hypercolor (color) space and the subscript denotes the electric charge. Although there are two degenerate types of preons (T and V) there is no global SU(2) isospin symmetry on the preon level because the charged and neutral preon belong to different representations in color space. To be consistent with the parity assignment for the standard model fermions, the intrinsic parity of the T and V preons must be different. The preon quantum numbers are summarized in Table I.

Preons are characterized by new quantum numbers [3]. These are the preon number \mathcal{P} and Y number, which are combinations of the number of T -preons $n(T)$ and the

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TABLE I. The color (C), hypercolor (H), electric charge (Q), preon number \mathcal{P} , Y number, and intrinsic parity (Π) of preons and antipreons (see also [6]).

preon	H	C	Q	\mathcal{P}	Y	Π
T	3	3	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	-1
V	3	$\bar{3}$	0	$+\frac{1}{3}$	$-\frac{1}{3}$	+1
\bar{V}	$\bar{3}$	3	0	$-\frac{1}{3}$	$+\frac{1}{3}$	-1
\bar{T}	$\bar{3}$	$\bar{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	+1

number of V -preons $n(V)$:

$$\mathcal{P} = \frac{1}{3}[n(T) + n(V)], \quad (2.1)$$

$$Y = \frac{1}{3}[n(T) - n(V)].$$

The Y number is also related to the baryon number (B) and lepton number (L) of the standard model as $Y = (B - L)$. The antipreon numbers $n(\bar{T})$ and $n(\bar{V})$ are defined as $n(\bar{T}) = -n(T)$ and $n(\bar{V}) = -n(V)$.

There is a connection between the \mathcal{P} and Y numbers, and the electric charge Q of the preons

$$Q = \frac{1}{2}(\mathcal{P} + Y), \quad (2.2)$$

which can be readily verified from Table I. This generalized Gell-Mann-Nishijima relation does not only hold for the preons but for all bound states (leptons, quarks, mesons, baryons) [1].

Furthermore, there is an interesting connection between these quantum numbers and the color (hypercolor) gauge symmetries [7]. We generalize the definitions of \mathcal{P} and Y in Eq. (2.1) for a theory with N_H and N_C (hyper)color charges of the preons as follows:

$$\mathcal{P} = \frac{1}{N_H}[n(T) + n(V)], \quad Y = \frac{1}{N_C}[n(T) - n(V)]. \quad (2.3)$$

After inserting these relations into Eq. (2.2) we obtain for any multipreon state of charge Q

$$Q = \frac{1}{2}(\mathcal{P} + Y) = Q(T)n(T) + Q(V)n(V), \quad (2.4)$$

where

$$Q(T) = \frac{1}{2}\left(\frac{1}{N_H} + \frac{1}{N_C}\right), \quad Q(V) = \frac{1}{2}\left(\frac{1}{N_H} - \frac{1}{N_C}\right). \quad (2.5)$$

Therefore, the equality of the number of colors and hypercolors leads to electric neutrality of the V -preon

$$Q(V) = 0. \quad (2.6)$$

From the preceding equations follows that electric charge

TABLE II. Allowed three-preon bound states representing leptons, quarks, and hyperquarks and their quantum numbers (see also [6]). Formally, the hyperquarks are obtained from the corresponding quarks by interchanging: $V \leftrightarrow \bar{V}$.

fermionic group	preon content	bound state	\mathcal{P}	Y	Q	Π
leptons	(VVV)	$(\nu_e, \nu_\mu, \nu_\tau)$	+1	-1	0	+1
	$(\bar{T}\bar{T}\bar{T})$	(e^-, μ^-, τ^-)	-1	-1	-1	+1
quarks	(TTV)	(u, c, t)	+1	$+\frac{1}{3}$	$+\frac{2}{3}$	+1
	$(\bar{T}\bar{V}\bar{V})$	(d, s, b)	-1	$+\frac{1}{3}$	$-\frac{1}{3}$	+1
hyperquarks	$(TT\bar{V})$	$(\tilde{u}, \tilde{c}, \tilde{t})$	$+\frac{1}{3}$	+1	$+\frac{2}{3}$	-1
	$(\bar{T}V\bar{V})$	$(\tilde{d}, \tilde{s}, \tilde{b})$	$+\frac{1}{3}$	-1	$-\frac{1}{3}$	+1

is quantized because the \mathcal{P} and Y numbers occur only as integer multiples of $1/3$.

III. FERMIONIC BOUND STATES

When constructing the three-preon bound states, a new class of states arises in addition to the usual leptons and quarks. We call these states hyperquarks [8]. Hyperquarks have fractional electric charges similar to the quarks but carry hypercolor. We make the following assumptions concerning possible bound states:

- (i) fermionic bound states consist of three preons,
- (ii) the bound states are singlets either in hypercolor (quarks), or color (hyperquarks), or both (leptons); only preons carry both hypercolor and color.

We assume that these are the only allowed combinations.¹ They are shown in Table II. Our assumptions are less restrictive than those of Harari and Seiberg [6]. These authors suggest that in a ‘‘natural’’ theory, nearly massless fermionic bound states must be hypercolor singlets.

It is evident from Table II that integer values of \mathcal{P} correspond to hypercolor singlets (leptons and quarks) whereas integer values of Y correspond to color singlets (leptons and hyperquarks). Fractional values of the quantum numbers \mathcal{P} and Y correspond to open hypercolor or open color charges. Only colored or hypercolored objects can have fractional electric charges.

Furthermore, one can see from the preon content of the quarks and leptons that particles and antiparticles enter in a symmetrical way. This leads to a different interpretation of the apparent matter-antimatter asymmetry in the Universe. In terms of the standard model with elementary leptons and

¹All other combinations have both color and hypercolor different from zero. These combinations contain preons and antipreons of the same type, e.g., $(T\bar{T}V)$, which concerning its quantum numbers are identical to a single preon.

quarks there is an asymmetry between matter and antimatter. On the other hand, at the preon level there is complete symmetry between matter and antimatter. For example, atomic hydrogen, which makes up most of the visible Universe, contains an equal number of preons and anti-preons. We hasten to add that this does not explain the observed asymmetry on the bound state level, the origin of which must be looked for in CP -violating interactions.

IV. 't HOOFT ANOMALY CONSTRAINT AND GENERATION NUMBER

A necessary condition for chiral symmetry conservation needed to explain the near masslessness of fermionic bound states, is the matching of anomalies at the elementary and composite levels, known as the 't Hooft anomaly matching condition [5]. The anomaly condition involves the difference D_i^{abc}

$$D_i^{abc} = d_{i(L)}^{abc} - d_{i(R)}^{abc} \quad (4.1)$$

of structure constants d_i^{abc} of the left-handed (L) and right-handed (R) sector of the fermions (f_i) with

$$d_i^{abc} = \text{Tr}(\lambda_a \{\lambda_b, \lambda_c\})_i, \quad (4.2)$$

where λ_a are the generators of the corresponding symmetry.

The 't Hooft anomaly matching condition is usually formulated as [9]:

$$\sum_i D_i^{abc}(\text{bound states}) = \sum_j D_j^{abc}(\text{preons}). \quad (4.3)$$

Here, the summation on the right-hand side extends over the two types of preons and on the left-hand side over the three types of bound state doublets: leptons, quarks, and hyperquarks (see Table II).

The following considerations are separately valid for each chiral sector and lead to a special case of the 't Hooft anomaly condition, namely, the equality of the sum of bound state and preon charges. As can be seen from Table I and Table II, adding the preon number of the bound states within each doublet leads to a cancellation for both the lepton and quark doublets, whereas the total preon number of the hyperquark doublet and of the fundamental preon doublet are equal.

Similarly, we obtain for the electric charge Q from Tables I and II

$$[Q(\tilde{u}) + Q(\tilde{d})]N_H N_G = [Q(T) + Q(V)]N_H N_C. \quad (4.4)$$

From Eq. (4.4) we conclude that the hyperquarks must occur with a multiplicity of N_G (generation number) in order to balance the number of colors on the preon side, i.e., $N_C = N_G$. For the electric charge Q of the leptons and quarks we get

$$Q(\nu) + Q(e^-) + [Q(u) + Q(d)]N_C = 0, \quad (4.5)$$

which is the usual anomaly freedom condition of the electroweak theory. It is thus explained by the preon content of these fermions. Adding Eq. (4.4) and Eq. (4.5) we obtain

$$\begin{aligned} [Q(\nu) + Q(e^-) + [Q(u) + Q(d)]N_C + [Q(\tilde{u}) + Q(\tilde{d})]N_H]N_G \\ = [Q(T) + Q(V)]N_H N_C. \end{aligned} \quad (4.6)$$

This can be written in compact notation as

$$\sum_i Q_i(\text{bound states}) = \sum_j Q_j(\text{preons}). \quad (4.7)$$

Note that this equality only holds if the multiplicity of the color degrees of freedom on the preon side is balanced by the generation number N_G on the bound state side.

The preceding argument can be repeated by replacing the hyperquarks by quarks in Eq. (4.4) and quarks by hyperquarks in Eq. (4.5). We then obtain $N_G = N_H$ from which follows

$$N_G = N_C = N_H. \quad (4.8)$$

V. SUMMARY AND OUTLOOK

A composite model approach of quarks and leptons gives answers to some fundamental questions, which remain unanswered within the standard model. For example, the exact equality of the electric charge of proton and positron can now be explained by the preon and antipreon content of these particles. Furthermore, the asymmetry between matter and antimatter is now seen as a property of the bound states and is no longer related to an asymmetry in the realm of fundamental particles where this symmetry is restored. Moreover, the connection between fractional electric charge and colored or hypercolored fermionic bound states is to some extent explained in the preon model.

We have suggested that the 't Hooft anomaly cancellation constraint, which is a necessary condition to explain the near masslessness (compared to the preon scale) of quarks and leptons can be fulfilled if a new class of preon bound states, called hyperquarks, is introduced. We have also suggested that the anomaly cancellation condition then leads to a connection between the number of color and hypercolor degrees of freedom and the generation number. This restricts the number of fermionic bound states to exactly three generations.

At what energy scale can one expect the hyperquarks to appear? Because hyperquarks have not yet been observed with present accelerators, a lower limit for the hyperquark mass is $m_{hq} > 1$ TeV. On the other hand, above 10^{16} GeV, i.e., the energy scale of proton decay, quarks transform into leptons due to preon exchange processes, i.e., explicit preon degrees of freedom become important. Because the 't Hooft anomaly condition requires that the preon bound states be massless compared to the preon scale, the hyper-

quark masses are presumably closer to the lower experimental limit of 1 TeV.

We hope to discuss bosonic preon bound states in a future publication.

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