Lifetime differences in heavy mesons with time independent measurements

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Heavy meson pairs produced in the decays of heavy quarkonium resonances at e^+e^- machines (beauty and tau-charm factories) have the useful property that the two mesons are in the CP-correlated states. We show that this leads to time-independent correlations allowing the extraction of the lifetime difference $\Delta\Gamma_D$ and other mixing parameters. In particular, for the decay $\psi(3770) \to D^0\bar{D}^0$ the correlation of a flavor specific decay of one D with a CP-specific decay of the other D is linearly sensitive to the D^0 lifetime difference. The utility of this method is considered at CLEO-c as well as future threshold charm factories. We include the impact of possible CP-violating effects and present the complete results for time-integrated CP-entangled decay rates with CP violation taken into account. We comment on the utility of using this method to extract the lifetime difference of neutral B mesons at future high luminosity B factories.

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I. INTRODUCTION

Neutral flavored mesons such as D^0, B^0, B_s, and K^0 are unique in that $\Delta Q = 2$ transitions can drive meson anti-meson mixing. This unique feature of these mesons makes them ideal tests of the standard model (SM). Indeed in the B^0 and K^0 mesons CP-conserving and CP-violating mixing has been observed and can be explained in terms of the the SM. No definitive signs of mixing have been seen in the D^0 at a level of $\sim 10^{-2}$ again consistent with the SM prediction that mixing is small in this case.

The study of mixing in the D^0 case is thus particularly interesting since it could constitute a signal of new physics. Charm factories operating at the $\psi(3770)$ resonance, such as CLEO-c, provide an ideal area for the study of charm transitions and mixing effects because of the large "clean" sample of D meson decays and the fact that the $D^0\overline{D}^0$ pair is produced in a quantum mechanically entangled state. This coherence can be exploited by observing the correlation between different decay modes of each of the mesons in the $D^0\overline{D}^0$ pair.

Our goal in this paper is to focus mainly on the time-independent methods to determine lifetime difference $y = \Delta\Gamma_D/2\Gamma$ of charmed mesons at electron-positron colliders where the $D^0\bar{D}^0$ can be produced in an entangled state. In particular we will consider methods which are *time independent* and therefore may be applied in experiments where the time history is difficult to obtain, such as the experiments at threshold charm machines such as CLEO-c and BES-III. Results, which are *linear* rather then *quadratic* [1], in y are of particular interest because y is generically small for B_d^0 and D^0 mesons. Our discussion naturally generalizes to the other oscillating hadron species (i.e. $H = D^0$, B^0, and B_s) at electron-positron colliders where a $H\overline{H}^0$ pair also can be produced in an entangled initial state. In addition, our methods can be used to obtain information about other mixing parameters and CP violation.

The basic formalism for the oscillation of neutral mesons is well known, if H^0 is such a meson, mixing will lead to mass eigenstates which are mixtures of H and $\overline{H}^0$:

$$|H_{\frac{1}{2}}\rangle = p|H^0\rangle \pm q|\bar{H}^0\rangle, \tag{1}$$

where the complex parameters p and q are obtained from diagonalizing the $H^0 - \overline{H}^0$ mass matrix [2]. It is conventional to parameterize the mass and width splittings between these eigenstates by

$$x \equiv \frac{m_2 - m_1}{\Gamma}, \qquad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}, \tag{2}$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are the masses and widths of $H_{1,2}$ and the mean width and mass are $\Gamma = (\Gamma_1 + \Gamma_2)/2$ and $m = (m_1 + m_2)/2$. Note that y is constructed from the decays of H into physical states so it should be dominated by the SM contributions, unless new physics significantly modifies $\Delta Q = 1$ interactions. For x the intermediate virtual states could easily receive contributions from new physics which are not obvious in H decays.

Let us now consider the expectations concerning the values of x and y for the various neutral flavored pseudoscalars. In the case of D^0, such studies may be carried out at tau-charm factories running on the $\psi(3770)$ resonance. In the SM it has been argued that $y \sim x \sim O(1\%)$ [3]. The expectation is therefore that the studies of lifetime differences is within the reach of current and proposed charm threshold experiments.

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In the case of B_d^0, $x = 0.730 \pm 0.029$ [4] and y is small, of the order of 0.3% [5]. It is possible that at future high luminosity B factories there would be enough statistics to measure y. In addition, if e^+e^- machines are run at the $\Upsilon(5s)$ resonance, these methods could be used to investigate y in the B_s. In the B_s system, it has been estimated that y may be particularly large (5%-15%) [6] and indeed similar methods have been previously discussed in [7].

In the cases of B_s and D^0, it is thought that CP violation from the interference between decay and mixing amplitudes is small, while in B^0 large CP-violating effects are known to exist [8,9]. In our methods, effects of CP violation from the interference between decay and mixing amplitudes will impact the signal, generally reducing it, so we will derive the formalism in a framework which includes CP violation. In a way, CP-violating effects play a role of "background" in our study, reducing the "signal" which is a CP-conserving lifetime difference y.

The paper will proceed as follows: In Sec. II we will discuss the formalism which applies in the case where CP violation is negligible, as well as the application to $D^0\overline{D}^0$ production at a charm factory assuming there is no CP violation in D^0 oscillation or decay. In Sec. III we will generalize the formalism to the case where CP violation is present and consider the application to B and D mesons. In Sec. IV we will give our conclusions. In the appendix we give the time-integrated correlated decay rates for H^0H^0 decaying to various combinations of final states where indirect CP violation is present. These formulas generalize and expand the results presented in [1].

II. FORMALISM IF *CP* IS CONSERVED: CHARMED MESONS

At present, the information about the $D^0 - \overline{D}^0$ mixing parameters x and y comes from the time-dependent analyses that roughly can be divided into two categories. All of them involve *time-dependent* studies of $D^0 - \overline{D}^0$ oscillations.

First, more traditional analyses study time dependence of $D \to f$ decays, where f is the final state that can be used to tag the flavor of the decayed meson. The most popular is the nonleptonic doubly Cabibbo suppressed (DCS) decay $D \to K^+\pi^-$. Time-dependent studies allow one to separate the doubly Cabibbo suppressed decay from the mixing contribution,

$$\Gamma[D^0(t) \to K^+\pi^-] = e^{-\Gamma t}|A_{K^-\pi^+}|^2 \times \left[R + \sqrt{R}R_m(y'\cos\phi - x'\sin\phi)\Gamma t + \frac{R_m^2}{4}(y^2 + x^2)(\Gamma t)^2\right], \quad (3)$$

where R and δ parameterize the magnitude and the phase of the ratio of doubly Cabibbo suppressed and Cabibbo favored (CF) decay amplitudes, $\sqrt{R}e^{i\delta} = A_{\rm DCS}/A_{\rm CF}$. Furthermore, $R_m = |p/q|$ and $\phi = \arg(p/q)$. The parameters x' and y' are related to x and y by

$$x' = x\cos\delta + y\sin\delta, \qquad y' = y\cos\delta - x\sin\delta. \quad (4)$$

Since x and y are small, the best constraint comes from the linear terms in t that also are linear in x and y. Using this method, direct extraction of x and y is not possible from Eq. (3) due to unknown relative strong phase δ (see [10] for extensive discussion). This phase, however, can be measured independently at CLEO-c [11]. The corresponding formula also can be written for $\overline{D}^0$ decay with $x' \to -x'$ and $R_m \to R_m^{-1}$ [12].

Another method to measure D^0 mixing is to compare the lifetimes extracted from the analysis of D decays into the CP-even and CP-odd final states. This study is similarly sensitive to a linear function of y, via

$$\frac{\tau(D \to K^-\pi^+)}{\tau(D \to K^+K^-)} - 1 = y\cos\phi + x\sin\phi\left[\frac{1 - R_m^2}{2}\right]. \quad (5)$$

This method, however, only can be applied in the situation where the D mesons are moving relatively fast, so that the time development of a D (or lifetime) can be studied. This method, as well as the previous one, is not applicable in the experiments where D's are produced at threshold.

Time-dependent or time-integrated studies of the semileptonic transitions are sensitive to the quadratic form $x^2 + y^2$ and at the moment are not competitive with the analyses discussed above.

A separate class of mixing studies involve investigations of the correlated decays $\Gamma[\psi_C \to (H \to K^\pm\pi^\mp) \times (H \to K^\pm\pi^\mp)]$ where DCS contribution cancels out [1]. This method, however, also is sensitive only to the quadratic form $x^2 + y^2$.

The essential point of our method is best illustrated in the simple cases where CP violation may be neglected. In such cases, $p = q$, so mass eigenstates become eigenstates of CP, which we denote:

$$|H_\pm\rangle = \frac{1}{\sqrt{2}}[|H^0\rangle \pm |\bar{H}^0\rangle]. \quad (6)$$

It follows then that these CP eigenstates $|H_\pm\rangle$ do not evolve with time.

At threshold e^+e^- machines such as *BABAR* or CLEO-c we can take advantage of this fact that the $H^0\bar{H}^0$ production is through resonances of specific charge conjugation. The $H^0\bar{H}^0$ will therefore be in an entangled state with the same quantum numbers as the parent resonance. In particular, since both mesons are pseudoscalars, charge conjugation reads $C = (-1)^L$, if the production resonance has angular momentum L. This implies that the quantum mechanical state at the time of $H\overline{H}^0$ production is

$$\Psi_L \equiv |H^0\overline{H}^0\rangle_L = \frac{1}{\sqrt{2}}\{|H^0(k_1)\overline{H}^0(k_2)\rangle + C|H^0(k_2)\overline{H}^0(k_1)\rangle\}, \quad (7)$$

where k_1 and k_2 are the momenta of the mesons. Rewriting this in terms of the CP basis we arrive at

$$\Psi_{C=+1} = \frac{1}{\sqrt{2}}\{|H_+(k_1)H_+(k_2)\rangle - |H_-(k_1)H_-(k_2)\rangle\},$$
$$\Psi_{C=-1} = \frac{1}{\sqrt{2}}\{|H_-(k_1)H_+(k_2)\rangle + |H_+(k_1)H_-(k_2)\rangle\}. \quad (8)$$

Thus in the $L = \text{odd}; C = -1$ case, which would apply to the experimentally important $\psi(3770)$ and $\Upsilon(4S)$ resonances, the CP eigenstates of the H mesons are anticorrelated while if $L = \text{even}; C = +1$ the eigenstates are correlated.[1] In either case the CP conservation implies that correlation between the eigenstates is independent of when they decay. In this way, if meson $H(k_1)$ decays to the final state which is also a CP eigenstate, then the CP eigenvalue of the meson $H(k_2)$ is therefore determined: it is the same as $H(k_1)$ for $C = +1$; opposite in the case of $C = -1$.

Using this eigenstate correlation as a tool to investigate CP violation has been suggested by [13].[2] Here we suggest using this time-independent correlation for the experimental investigation of lifetime differences. The idea is fairly straightforward: we look at decays of the form $\psi_C \to (H \to S_\sigma)(H \to Xl\nu)$ where S_σ is a CP eigenstate of eigenvalue $\sigma = \pm 1$.

With $H(k_2)$ in a definite CP state which is known, the semileptonic *width* of this meson should be independent of the CP quantum number since this component of the width is flavor specific and is not effected by the mixing. It follows, however, that the semileptonic *branching ratio* of $H(k_2)$ will be inversely proportional to the total width of that meson. Since we know whether $H(k_2)$ is a H_+ or a H_- from the decay of $H(k_1)$, we can easily determine y in terms of the semileptonic branching ratios of $H_\pm$.

This can be expressed by introducing the ratio

$$R_\sigma^C = \frac{\Gamma[\psi_C \to (H \to S_\sigma)(H \to Xl^\pm \nu)]}{\Gamma[\psi_C \to (H \to S_\sigma)(H \to X)]\mathrm{Br}(H^0 \to Xl^\pm \nu)}, \quad (9)$$

where X in $H \to X$ stands for the inclusive set of all final states. Clearly then, a deviation from $R_\sigma^C = 1$ implies a lifetime difference. In fact, from this experimentally obtained quantity, we extract y by

$$R_\sigma^C = \frac{1}{1 + C\sigma y}, \qquad y = C\sigma\left[\frac{R_\sigma^C - 1}{R_\sigma^L}\right]. \quad (10)$$

An equivalent formulation of Eq. (10) is to define the semileptonic branching ratio for the CP eigenstates:

$$\mathcal{B}_\pm^\ell = \mathrm{Br}(H_\pm \to Xl^+\nu) + \mathrm{Br}(H_\pm \to Xl^-\bar{\nu}). \quad (11)$$

Thus if both $\mathcal{B}_+^\ell$ and $\mathcal{B}_-^\ell$ are measured we also can determine y as

$$y = \frac{1}{4}\left(\frac{\mathcal{B}_+^\ell(D)}{\mathcal{B}_-^\ell(D)} - \frac{\mathcal{B}_-^\ell(D)}{\mathcal{B}_+^\ell(D)}\right). \quad (12)$$

This formula provides a simple way of measuring the lifetime difference y in charm threshold experiment.

Let us consider the application of our method in the case of the production of $D^0\overline{D}^0$ mesons at an electron-positron collider. In the context of the standard model, CP violation is expected to be small in D^0 hence the above formalism so the assumption that CP is conserved should apply directly to this case. In the next section we will discuss the generalization to the case where CP violation cannot be neglected.

The experiment can be preformed for instance at the tau-charm factory CLEO-c at CESR (Cornell Electron Storage Ring) or at the future experiments at BES-III. In this case the energy of the electron-positron collisions is tuned to the $\psi(3770)$ resonance which therefore decays to the $C = -1$ $D\overline{D}^0$ state. There are numerous candidates for CP eigenstate S_σ appearing in Eq. (10) which have branching ratios in the few percent range, for instance $K_S\pi^0$ (1.05%); $K_S\omega$ (1.05%); $K_S\eta'$ (0.85%); $\pi^+\pi^-$ (0.15%). In addition, the modes $K^{*0}\pi^0$ and $K^{*0}\rho^0$ may be used provided the K^{*0} itself decays to a CP eigenstate, $K_{S,L}\pi^0$ and one can separate the main amplitude from cross channel processes.

The statistical uncertainty in y determined by Eq. (10) is given by

$$\Delta y = (2N_0\mathcal{A}^\ell\mathcal{B}^\ell\mathcal{A}^\sigma\mathcal{B}^\sigma)^{-(1/2)}, \quad (13)$$

where N_0 is the initial number of ψ's, $\mathcal{A}^\sigma$ and $\mathcal{A}^\ell$ are the acceptances for the CP eigenstate modes and the semileptonic modes, respectively, while $\mathcal{B}^\sigma$ and $\mathcal{B}^\ell$ are the branching ratios for those modes. In general, of course, we can combine the statistics for a number of modes so, as an example, if we assume that $\mathcal{B}^\ell = 12\%$, $\mathcal{B}^\sigma = 2\%$, with $\mathcal{A}^\ell\mathcal{A}^\sigma = 0.1$ then $N_0 = 10^8$ gives $\Delta y = 0.5\%$.

III. FORMALISM IF CP IS VIOLATED IN H^0H^0 OSCILLATIONS

In the case where CP violation is present in the $H^0\overline{H}^0$ mixing, it is necessary to consider general time-dependent entangled states of the $H^0\overline{H}^0$ pair. Following the notation of [2], we will denote the wave function $|H(t)\rangle$ at a given moment in time t by a two-element vector:

$$|H(t)\rangle = a|H^0\rangle + \overline{a}|\overline{H}^0\rangle \equiv \begin{pmatrix} a \\ \overline{a} \end{pmatrix}. \quad (14)$$

CPT conservation forces the general mass matrix in the following form

$$\mathcal{M} \equiv \hat{M} + i\hat{\Gamma}/2 = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix}, \quad (15)$$

[1] While $L = \text{even}$ resonances are not directly produced in e^+e^- collisions, quantum mechanically *symmetric* states can be produced in the decays, such as $\psi(4140) \to D\overline{D}\gamma$. In the following, $C = +1$ case refers to this situation.

[2] For other measurements that involve CP correlations to study CP violation in D mesons, see [14].

where $\hat{M}$ and $\hat{\Gamma}$ are Hermitian while A, p, and q are the same general complex numbers introduced in Eq. (1). The effects of CP violation in the system are parameterized by the ratio p/q which can be expressed in terms of the complex parameter ϵ or alternatively in terms of the real parameters R_m and ϕ:

$$\frac{p}{q} \equiv \frac{1+\epsilon}{1-\epsilon} \equiv R_m e^{i\phi}. \tag{16}$$

In the limit of CP conservation in mixing matrix, $R_m = 1$. Even if CP is violated, in the case of heavy neutral mesons, it is expected that $R_m \approx 1$. The phase ϕ, of course, depends on the convention one uses for weak phases that can be traded off against the weak phase in the decay in the usual way. In our discussion it will be useful to assume that we are using a convention where we have absorbed any weak phase from the decay into the mixing.

For an isolated meson, the wave function at time t is related to the wave function at $t = 0$ by

$$|H(t)\rangle = U_t |H(0)\rangle, \tag{17}$$

where the time evolution operator U_t satisfies the equation

$$i\frac{dU_t}{dt} = \mathcal{M} U_t, \qquad U_0 = 1. \tag{18}$$

This Schrödinger-like equation can be solved to yield the result

$$U_t = \begin{pmatrix} g_+(t) & (p/q)g_-(t) \\ (q/p)g_-(t) & g_+(t) \end{pmatrix}. \tag{19}$$

Here, the time dependence of D^0 and $\overline{D}^0$ is driven by

$$\begin{aligned} g_+(t) &= (\cosh(y\tau/2)\cos(x\tau/2) - i\sinh(y\tau/2) \\ &\quad \times \sin(x\tau/2))e^{-\mu\tau/2}, \\ g_-(t) &= (-\sinh(y\tau/2)\cos(x\tau/2) + i\cosh(y\tau/2) \\ &\quad \times \sin(x\tau/2))e^{-\mu\tau/2}, \end{aligned} \tag{20}$$

with $\tau = \Gamma t$ and $\mu = 1 + 2im/\Gamma$.

Let us now consider the time-integrated decay rate for a single H to a final state f. If a and $\overline{a}$ are the amplitudes for H^0 and $\overline{H}^0$ to decay to f, respectively, and $|\psi_0\rangle$ is the initial wave function for the meson, then the time-integrated decay rate is

$$\begin{aligned} 2\Gamma_f(\rho_0) &= (Q+P)\,\mathrm{Tr}[\rho_f\rho_0] + (Q-P)\,\mathrm{Tr}[\upsilon^\dagger\rho_f\rho_0] \\ &\quad - 2\mathrm{Re}[(yQ - ixP)\,\mathrm{Tr}[\rho_f\upsilon\rho_0]], \end{aligned} \tag{21}$$

where

$$\rho_0 = |\psi_0\rangle\langle\psi_0|, \qquad \rho_f = \begin{pmatrix} |a|^2 & a^*\overline{a} \\ \overline{a}^* a & |\overline{a}| \end{pmatrix}, \qquad \upsilon = \begin{pmatrix} 0 & p/q \\ q/p & 0 \end{pmatrix}, \tag{22}$$

and

$$P = 1/(1+x^2), \qquad Q = 1/(1-y^2). \tag{23}$$

Of particular interest is the case where f is a CP eigenstate with $CP = \sigma = \pm 1$. If we assume $R_m = 1$, as would be the case for $B_d \to \psi K_s$ in the standard model, then

$$\frac{1}{2}(\Gamma_f(H^0) + \Gamma_f(\overline{H}^0)) = \frac{1 - \sigma y\cos\phi}{1-y^2}\Gamma_0, \tag{24}$$

$$\frac{1}{2}(\Gamma_f(H^0) - \Gamma_f(\overline{H}^0)) = \sigma\frac{x\sin\phi}{1+x^2}\Gamma_0, \tag{25}$$

where Γ_0 is the decay rate for H to f.

This easily can be generalized to the case of the entangled initial state which presents itself in the creation of $H^0\overline{H}^0$ pairs from a Ψ_L resonance. As was shown in Eqs. (7) and (8), the states of interest can be decomposed into the coherent sum of products of flavor (or CP) eigenstates. Using Eq. (7) we can write the time-integrated correlated decay rate for $\Psi_L \to (H \to f_1)(H \to f_2)$ is

$$\begin{aligned} \Gamma^{f_1f_2}(\Psi_C) &= \Gamma_{f_1}(H^0)\Gamma_{f_2}(\overline{H}^0) + \Gamma_{f_1}(\overline{H}^0)\Gamma_{f_2}(H^0) \\ &\quad + (-1)^L[\Gamma_{f_1}(\rho_{+-})\Gamma_{f_2}(\rho_{-+}) \\ &\quad + \Gamma_{f_1}(\rho_{-+})\Gamma_{f_2}(\rho_{+-})], \end{aligned} \tag{26}$$

where ρ_{ik} are the matrices

$$\rho_{++} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \rho_{--} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \rho_{+-} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \rho_{-+} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{27}$$

Let us return to the calculation of y through the determination of R^C_σ defined in Eq. (9). In the case $C = +1$ the numerator is $\Gamma^{S_\sigma X_l}(\Psi_1)$ and so Eq. (26) implies

$$\Gamma^{S_\sigma X_l}(\Psi_C) \propto \frac{2 + x^2(R_m^{\pm 2}+1) + y^2(R_m^{\pm 2}-1)}{2(1+x^2)(1-y^2)}, \tag{28}$$

where we assume there is no further CP violation in the decay amplitude, and the $\pm$ signs are for the positively and negatively charged leptons, respectively.

The denominator of Eq. (9) is given by Eq. (26) where f_1 is S_σ and all possible values of f_2 are summed over. In this case the L-dependent terms vanish and the rest simplifies to

$$\Gamma^{f_1X}(\Psi_C) \equiv \sum_{f_2}\Gamma^{f_1f_2}(\Psi_C) = \Gamma_{f_1}(\mathbf{1})/2, \tag{29}$$

where $\mathbf{1}$ is the identity matrix. This is proportional to the average between the time-integrated decay rate of H^0 and $\overline{H}^0$ to the final state f_1. It is easiest to see that in the CP eigenstate basis spanned by the states of Eq. (8). Indeed, in the particular example of $L =$ odd the time-integrated decay rate is

$$\Gamma^{f_1X}(\Psi_1) = \int dt_1 dt_2 \frac{1}{2} \times [\mathrm{Tr}[U_{t2}^\dagger \rho_X U_{t2} \rho_{++}] \mathrm{Tr}[U_{t1}^\dagger \rho_{f1} U_{t1} \rho_{--}] + \mathrm{Tr}[U_{t2}^\dagger \rho_X U_{t2} \rho_{--}] \mathrm{Tr}[U_{t1}^\dagger \rho_{f1} U_{t1} \rho_{++}] - \mathrm{Tr}[U_{t2}^\dagger \rho_X U_{t2} \rho_{+-}] \mathrm{Tr}[U_{t1}^\dagger \rho_{f1} U_{t1} \rho_{-+}] - \mathrm{Tr}[U_{t2}^\dagger \rho_X U_{t2} \rho_{-+}] \mathrm{Tr}[U_{t1}^\dagger \rho_{f1} U_{t1} \rho_{+-}]], \tag{30}$$

where ρ_X and ρ_{f1} are the matrices for $H \to X$ and $H \to f_1$ decay amplitudes, respectively [see Eq. (22)]. It is easy to see that in the *mass eigenstate* basis $U_{t2}^\dagger \rho_X U_{t2}|_{\mathrm{mass}} = \mathrm{diag}(\Gamma_1 e^{-\Gamma_1 t_2}, \Gamma_2 e^{-\Gamma_2 t_2})$. In principle, this needs to be translated back to the *CP* eigenstate basis. However, integration with respect to t_2 yields a unit matrix, which is invariant under the change of basis. This simplifies the Eq. (30) considerably, which, after taking the corresponding traces transforms into

$$\Gamma^{f_1X}(\Psi_1) = \frac{1}{2} \int dt\, \mathrm{Tr}[\rho_{f1} U_t U_t^\dagger]],$$

which, in the limit $R_m = 1$, becomes for the semileptonic final state (a complete expression is available in the appendix)

$$\Gamma^{XX_l}(\Psi_C) \propto \frac{1 + C\cos\phi}{(1 - y^2).} \tag{31}$$

In the limit $R_m = 1$ the ratio of Eqs. (28) and (31) becomes

$$R_\sigma^C = \frac{1}{1 + C\sigma y \cos\phi}. \tag{32}$$

In which case the generalization of Eq. (10) is

$$y\cos\phi = C\sigma \frac{R_\sigma^C - 1}{R_\sigma^C}. \tag{33}$$

So we can regard the measurement of R_σ^C as leading to a determination of $y\cos\phi$. A similar result holds for the nonleptonic final state (such as $D \to K\pi$, with corrections proportional to R). In the case where $R_m \neq 1$ the corresponding expression depends on x as well as y. For instance, for $C = 1$

$$R_\sigma^C = \frac{PQ(1 + R_m x^2 \cosh a_m + R_m y^2 \sinh a_m)}{(Q\cosh^2 a_m - P\sinh^2 a_m - xP\sinh a_m \sin\phi - yQ\cosh a_m \cos\phi)}, \tag{34}$$

where $a_m = \log(R_m) = \log\sqrt{1 + A_m} \approx A_m/2$ [12]. Expanding this to first order in a_m we obtain

$$R_\sigma^C = \frac{1}{1 - \sigma y \cos\phi} + \frac{(x^2 + y^2)(1 - y\cos\phi) + x(1 - y^2)\sin\phi}{(1 - y\cos\phi)^2(1 + x^2)} a_m + O(a_m^2). \tag{35}$$

Thus, we can define $\hat{y}$ by

$$\hat{y}\cos\phi = \sigma \frac{R_\sigma^C - 1}{R_\sigma^C}. \tag{36}$$

If we expand $\hat{y}$ to first order in a_m we obtain

$$\hat{y} = y - a_m \left[\frac{(x^2 + y^2)(1 - y\cos\phi) + (1 - y^2)x\sin\phi)}{(1 + x^2)\cos\phi}\right]. \tag{37}$$

Clearly then, Eq. (33) gives y only if a_m is known to be small. The actual value of a_m can be experimentally obtained from the semileptonic decay asymmetry [4].

In our discussion we now will assume that $R_m \approx 1$ and so the ratio R_σ^L gives us $y\cos\phi$ through Eq. (33). The error in determining y is thus given by the generalization of Eq. (13)

$$\Delta y \cos\phi = (2N_0 \mathcal{A}^\ell \mathcal{B}^\ell \mathcal{A}^\sigma \mathcal{B}^\sigma)^{-(1/2)}. \tag{38}$$

In the case of D^0, the systematics for $\Delta y\cos\phi$ is the same as the systematics for Δy in the *CP* conserving case discussed above.

In the case of B^0, ϕ which is equal to 2β in the standard model has been measured at the *BABAR* and BELLE experiments [8,9]. The average of these two results is currently $\sin 2\beta = 0.736 \pm 0.049$ [4] thus $\cos 2\beta \approx 0.6$. Therefore, if we take $N_0 = 10^8$ and use only ψK_S decay mode with $\psi \to l^+ l^-$ and assume that $\mathcal{A}_\sigma \mathcal{A}_l \approx 1/4$ then $\Delta y \cos 2\beta = 0.06$ corresponding to $\Delta y = 0.1$. Clearly bringing in additional S_σ modes will improve determination of Δy. We also can improve the statistics by using flavor specific decays of the B^0 other than pure leptonic decays. The *BABAR* and Belle experiments have made considerable progress in their ability to accomplish this and obtain an effective value of $\mathcal{A}_l B_l \approx 0.7$. Using this result, the above gives $\Delta y \approx 0.026$.

To produce correlated B_s pairs one needs to run an electron-positron machine at the $\Upsilon(5s)$ resonance. This state can decay into $B_s B_s$, $B_s^* B_s$, and $B_s^* B_s^*$ where the B_s^* decays to $B_s\gamma$. As discussed in [7] if there are zero or two photons in the final state (i.e. the decay was to $B_s B_s$ or $B_s^* B_s^*$) then the $B_s B_s$ is in a $C =$ odd state while if there is one photon in the final state (i.e. $B_s^* B_s$) then the final $B_s B_s$ state is $C =$ even.

The branching ratio to S_σ states in the case of B_s is in principle much larger than in the case of B_0. For instance, the branching ratio for $B_s \to D_s^+ D_s^-$ should be similar to the measured branching ratio for $B^0 \to D^- D_s^+$ which is about 0.8%. Likewise one can estimate the branching ratio of $B_s \to J/\psi \eta^{(\prime)}$ at about 0.15%. In addition analogous states such as $D_s^* D_s^*$ etc. should have branching ratios on the order of 0.1% at least. The acceptance for such states may be lower than for ψK_s so we will assume that $\mathcal{A}_\sigma \mathcal{A}_l \approx 0.1$ with a branching ratio to CP states of about 0.8%. Using these assumptions, if one had a high luminosity $\Upsilon(5s)$ machine that was able to produce 10^8 B_s pairs then $\Delta y \cos\phi = 0.7\%$ which would be the same as Δy if the standard model expectation that $\phi = 0$ was correct. As was shown in [7], precision of about 0.28% can be reached if one includes all final states with quark content of $c\bar{c}s\bar{s}$.

IV. CONCLUSIONS

In summary, we discussed the possibility of time-independent measurements of lifetime differences in D and B systems. It is important to reiterate that time-dependent measurements are quite difficult at the symmetric e^+e^- threshold machines due to the fact that the pair-produced heavy mesons are almost at rest [15]. The techniques described above will provide a *time-integrated* quantity that is separately sensitive to the lifetime difference y.

This will be particularly useful in the case of D^0 where a charm factory running at the $\psi(3770)$ resonance can yield the measurement with precision of $\Delta y \cos\phi \approx 0.5\%$ which is in the range of some standard model predictions. At a $\Upsilon(5S)$ B factory with a luminosity sufficient to produce 10^8 B_s pairs, a precision of $\Delta y \cos\phi \approx 0.7\%$ should be achievable which is much smaller that the standard model prediction of 5%–15%. Thus, even if only $\sim 10^6$ B_s pairs are produced, precision on the order of the standard model prediction can be obtained. In the case of the $\Upsilon(4S)$ B factory with 10^8 BB pairs, $\Delta y \cos 2\beta \approx 3\%$, which does not probe it to the level of the standard model estimate in this case. Such a measurement will be possible at a perspective high luminosity B factory.

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APPENDIX: CORRELATED DECAYS WITH *CP* VIOLATION

Here we provide the expressions for the time-integrated decay of a correlated $H^0 - \overline{H}^0$ state to various pairs of final states using the formalism discussed in the text.

The final states we consider are

(1) $S_\pm$, a CP eigenstate such as in $D^0 \to K_s \pi^0$ or $B_{d(s)} \to J/\psi K_S(\phi)$.

(2) $L^\pm$, a flavor specific semileptonic decay to a final state containing $\ell^\pm$.

(3) G, a hadronic final state such that both H^0 and $\overline{H}^0$ can decay to it. For example, in charmed mesons, $D^0 \to G^+$ is Cabibbo favored and $D^0 \to G^-$ is doubly Cabibbo suppressed, as in $D^0 \to K^-\pi^+$. This implies that the ratio of DCS to CF decay rates R is small and the results can be expanded in terms of this ratio. Alternatively, the ratio of amplitudes can be of order one in B decay, as in the example of $B_s \to D_s^+ K^-$, so all powers of R must be kept.

(4) X is an inclusive set of all final states.

It is now easy to construct all possible combinations of the above final states. For the case of antisymmetric initial state ($L =$ odd), we have for $\Gamma_{\rm odd}^{f_1, f_2}$

$$\Gamma_{\rm odd}^{S_\sigma L^\pm} = [2 + (1 + R_m^{\pm 2})x^2 - (1 - R_m^{\pm 2})y^2]\left[\frac{\Gamma_0(S_\sigma)\Gamma_0(L^+)}{2(1-y^2)(1+x^2)}\right], \quad \text{(A1)}$$

$$\Gamma_{\rm odd}^{S_+ S_-} = [8R_m^2 + (1 + R_m^4)(x^2 + y^2) + 2R_m^2((1 + 2\cos^2\phi)x^2 + (1 + 2\sin^2\phi)y^2)]\left[\frac{\Gamma_0(S_+)\Gamma_0(S_-)}{2(1-y^2)(1+x^2)}\right], \quad \text{(A2)}$$

$$\Gamma_{\rm odd}^{S_+ S_+} = [(x^2 + y^2)(\cosh^2 a_m - \cos^2\phi)] \times \left[\frac{\Gamma_0^2(S_\sigma)}{(1-y^2)(1+x^2)}\right], \quad \text{(A3)}$$

$$\Gamma_{\rm odd}^{L^\pm L^\pm} = [R_m^{\mp 2}(x^2 + y^2)]\left[\frac{\Gamma_0^2(L^+)}{2(1-y^2)(1+x^2)}\right], \quad \text{(A4)}$$

$$\Gamma_{\rm odd}^{L^\pm L^\mp} = [2 + x^2 - y^2]\left[\frac{\Gamma_0^2(L^+)}{2(1-y^2)(1+x^2)}\right], \quad \text{(A5)}$$

$$\Gamma_{\rm odd}^{S_\sigma X} = [1 + y^2\sinh^2 a_m + x^2\cosh^2 a_m - \sigma(y(1 + x^2)\cosh a_m \cos\phi + x(1 - y^2) \times \sinh a_m \sin\phi)]\left[\frac{2\Gamma_D \Gamma_0(S_\sigma)}{(1-y^2)(1+x^2)}\right], \quad \text{(A6)}$$

$$\Gamma_{\rm odd}^{L^\sigma X} = \left[1 + x^2\cosh^2 a_m + y^2\sinh^2 a_m + \frac{\sigma}{2}(x^2+y^2)\sinh 2a_m\right]\left[\frac{2\Gamma_D\Gamma_0(L^+)}{(1-y^2)(1+x^2)}\right], \quad (A7)$$

$$\Gamma_{\rm odd}^{G^+X} = [x^2 + y^2 + (1+R^2)(2+x^2-y^2)R_m^2 + R^2(x^2+y^2)R_m^4 + 2Rx(1-y^2)\sin(\delta+\phi) \times R_m(1-R_m^2) - 2Ry(1+x^2)\cos(\delta+\phi) \times R_m(1+R_m^2)]\left[\frac{\Gamma(G^+)\Gamma_D}{4(1-y^2)(1+x^2)R_m^2}\right], \quad (A8)$$

$$\Gamma_{\rm odd}^{GS_\sigma} = [R_m^2(1+R^2) + (R^2+R_m^2)(1+R_m^2)x^2 + (R^2-R_m^2)(1-R_m^2)y^2 + 4rR_m(y^2(\cos\phi\sin\phi\sin\delta - \sin^2\phi\cos\delta) + x^2(\cos\phi\sin\phi\sin\delta + \cos^2\phi\cos\delta) + \cos\delta)] \times \left[\frac{2\Gamma(G^+)\Gamma(S_\sigma)}{(1-y^2)(1+x^2)R_m^2}\right], \quad (A9)$$

$$\Gamma_{\rm odd}^{G^\pm L^\pm} = [2R^2 + (R_m^{\mp 2} + R^2)x^2 + (R_m^{\mp 2} - R^2)y^2] \times \left[\frac{\Gamma(G)\Gamma(L^+)}{2(1-y^2)(1+x^2)}\right], \quad (A10)$$

$$\Gamma_{\rm odd}^{G^\pm L^\mp} = [2 + (R^2R_m^{\pm 2} + 1)x^2 + (R^2R_m^{\pm 2} - 1)y^2] \times \left[\frac{\Gamma(G)\Gamma(L^+)}{2(1-y^2)(1+x^2)}\right], \quad (A11)$$

$$\Gamma_{\rm odd}^{G^\pm G^\mp} = [(1+R^4)R_m^2(-2-x^2+y^2) - R^2(x^2+y^2) - R^2R_m^4(x^2+y^2) + 2R^2R_m^2[(2+x^2-y^2) \times \cos 2\delta + (x^2+y^2)\cos 2\phi]] \times \left[\frac{\Gamma^2(G)}{2(y^2-1)(1+x^2)R_m^2}\right], \quad (A12)$$

$$\Gamma_{\rm odd}^{G^\pm G^\pm} = [(x^2+y^2)(R^4R_m^{\pm 2} - 2R^2\cos 2(\delta\pm\phi) + R_m^{\mp 2})] \times \left[\frac{\Gamma^2(G^+)}{2(1-y^2)(1+x^2)}\right]. \quad (A13)$$

In the case of charmed mesons, $R \ll 1$. Neglecting possible CP-violating effects and taking the ratio of Eqs. (A13) and (A12) simultaneously expanding numerator and denominator in R, x, and y we reproduce the well-known result that DCS/CF interference cancels out in the ratio for $L =$ odd [1] and gives the result, identical to the semileptonic final state, $(x^2 + y^2)/2$.

The results for $L =$ even are more cumbersome, so we present only a few of $\Gamma_{\rm even}^{f_1,f_2}$:

$$\Gamma_{\rm even}^{S_\sigma L^\pm} = [(x^2+y^2)(3+(x^2-y^2)+x^2y^2) + R_m^{\pm 2}(2+(1+x^4)x^2 - (1-4x^2-x^4)y^2 + (1-x^2)y^4) - 4R_m^{\pm 1}\sigma((1+x^2)^2 y\cos\phi \mp (1-y^2)^2 x\sin\phi)]\left[\frac{\Gamma_0(S_\sigma)\Gamma_0(L^+)}{2(1-y^2)^2(1+x^2)^2}\right], \quad (A14)$$

$$\Gamma_{\rm even}^{S_+S_-} = [(x^2+y^2)(x^2+(x^2-1)y^2+3) \times (\cosh 2a_m - \cos 2\phi)]\left[\frac{\Gamma_0^2(S_\sigma)}{(1-y^2)^2(1+x^2)^2}\right], \quad (A15)$$

$$\Gamma_{\rm even}^{L^\pm L^\pm} = [R_m^{\mp 2}(x^2+y^2)(x^2+(x^2-1)y^2+3)] \times \left[\frac{\Gamma_0^2(L^+)}{2(1-y^2)^2(1+x^2)^2}\right], \quad (A16)$$

$$\Gamma_{\rm even}^{L^\pm L^\mp} = [x^4 + x^2 - (x^2-1)y^4 + (x^4+4x^2-1)y^2 + 2)] \times \left[\frac{\Gamma_0^2(L^+)}{2(1-y^2)^2(1+x^2)^2}\right]. \quad (A17)$$

Taking the ratios of the decay rates presented above, one easily can generalize the results of [1] to the case of CP nonconservation.

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