Hadronic structure aspects of $K^+ \to \pi^- + l_1^+ + l_2^+$ decays

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As is known from previous studies the lepton number violating decays $K^+ \to \pi^- + l_1^+ + l_2^+$ have good prospects to probe new physics beyond the standard model and provide valuable information on neutrino masses and mixing. We analyze these processes with an emphasis on their hadronic structure aspects applying the relativistic constituent quark model. We conclude that the previously ignored contribution associated with the t-channel Majorana neutrino exchange is comparable with the s-channel one in a wide range of neutrino masses. We also estimate model independent absolute upper bounds on the neutrino contributions to these decays.

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I. INTRODUCTION

Discovery of small but finite neutrino masses and large neutrino flavor mixing has clearly shown the limitations of the standard model (SM) of electroweak interactions and pointed to the physics beyond its framework. The smallness of neutrino masses is commonly considered as a strong indication in favor of the celebrated seesaw picture [1] with its characteristic attributes: high-energy scale of lepton number violation (LNV) associated with new physics as well as very light and very heavy Majorana neutrino mass eigenstates (for a recent review see, for instance, Ref. [2]). This supports the long-standing belief that, contrary to the SM, the lepton number is not conserving and neutrinos are massive Majorana particles. If this is true, the LNV processes, forbidden in the SM, are allowed at small rates and some of them can be observed experimentally. Therefore, theoretical studies and experimental searches for LNV processes are attracting growing interest as the way to probe new physics beyond the SM and to study the properties of neutrinos.

Various LNV processes have been discussed in the literature in this respect (for review see [3,4]). In principle, they can probe Majorana neutrino contribution and provide information on the so-called effective Majorana mass matrices $\langle m_\nu \rangle_{\alpha\beta}$ and $\langle M_N^{-1} \rangle_{\alpha\beta}$ of light and heavy Majorana neutrinos. These quantities under certain assumptions are related to the entries of the Majorana neutrino mass matrix $M^{(\nu)}_{\alpha\beta}$. Currently the most sensitive experiments intended to probe LNV physics beyond the SM, in particular, Majorana neutrino contribution, are those searching for nuclear neutrinoless double beta ($0\nu\beta\beta$) decay [5–7]. Because of the lepton flavor structure of this process its experimental searches are sensitive to a specific flavor set of the LNV parameters. For the Majorana neutrino contribution to this process they are $\langle m_\nu \rangle_{ee}$ and $\langle M_N^{-1} \rangle_{ee}$ entries of the effective Majorana neutrino mass matrices. In order to probe the LNV parameters with another lepton flavor composition one needs to study other LNV processes.

In the present paper we study LNV $K^+ \to \pi^- l_1^+ l_2^+$ decays. Currently the best experimental upper bounds on the branching ratios of these processes are [8]

$$
\begin{aligned}
\mathcal{R}_{\mu\mu} &= \frac{\Gamma(\mathrm{K}^+ \to \mu^+\mu^+\pi^-)}{\Gamma(\mathrm{K}^+ \to \text{all})} \le 3.0 \times 10^{-9},\\
\mathcal{R}_{ee} &= \frac{\Gamma(K^+ \to \pi^- e^+ e^+)}{\Gamma(\mathrm{K}^+ \to \text{all})} \le 6.4 \times 10^{-10}, \qquad (1)\\
\mathcal{R}_{\mu e} &= \frac{\Gamma(K^+ \to \pi^- \mu^+ e^+)}{\Gamma(\mathrm{K}^+ \to \text{all})} \le 5.0 \times 10^{-10}.
\end{aligned}
$$

These processes may receive various contributions from the LNV physics beyond the SM (see, for instance, Ref. [9]), including the Majorana neutrino exchange. We concentrate on the latter case.

Assume the neutrino mass spectrum consists of light ν_k and heavy N_k neutrinos with the masses much smaller $m_{\nu(k)} \ll m_K$ and much larger $M_{N(k)} \gg m_k$ than the K-meson mass $m_K = 494$ MeV respectively. Then the light and heavy neutrino contributions to the amplitude of $K^+ \to \pi^- l_1^+ l_2^+$ decay are proportional to the effective masses $\langle m_\nu \rangle_{l_1 l_2}$ and $\langle M_N^{-1} \rangle_{l_1 l_2}$ with $l_i = e, \mu$. The estimates of these quantities (see, for instance, [3,10]) from the neutrino observations lead to the very small branching ratios of these decays in comparison with the current experimental sensitivities (1) that their experimental observation looks unrealistic even in a distant future. The exception occurs if there exists Majorana neutrino ν_h with the mass in the "resonant" region. For the $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ decay this is the region of 245 MeV $\le m_{\nu_h} \le$ 388 MeV. In this case the ν_h contribution is resonantly enhanced and may result in an observable effect [10].

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In the SM extensions with Majorana neutrinos there are two lowest order diagrams, shown in Fig. 1, which contribute to the $K^+ \to \pi^- l_1^+ l_2^+$ decays. These diagrams were first considered for $K^+ \to \mu^+ \mu^+ \pi^-$ decay long ago in Refs. [11,12] and more recently in Ref. [13].

The contribution from the factorizable s-channel diagram in Fig. 1(a), dominant for the neutrinos with the masses in the resonant region, can be calculated without referring to any hadronic structure model. On the contrary, the t-channel diagram in Fig. 1(b) requires a detailed hadronic structure calculation. Studying neutrino contributions to $K^+ \to \pi^- l_1^+ l_2^+$ decays outside the resonant region one should take into account both diagrams. This implies the analysis based on a certain model of hadronic structure.

In what follows we focus on the hadronic structure aspects of $K^+ \to \pi^- l_1^+ l_2^+$ decay. One of the main motivations of our study is the controversial situation existing in the literature on this subject. In Ref. [12] the contribution of the t-channel diagram in Fig. 1(b) has been evaluated in the Bethe-Salpeter approach and is argued to be negligible compared to the s-channel diagram in Fig. 1(a) for any value of neutrino mass. In our mind this result is not supported by any physical reason and appears to be an artifact of this approach. In this situation it is worthwhile to carry out an independent analysis of the t-channel contribution within an alternative approach to hadronic structure calculations.

Our analysis is based on the relativistic constituent quark model [14] which was successful in the description of various meson decay processes. As will be demonstrated, we disagree with the above mentioned conclusion of Ref. [12] and predict that the t-channel neutrino contribution to $K^+ \to \pi^- l_1^+ l_2^+$ decays is comparable with the s-channel one for all the values of neutrino masses outside the resonant region.

The following comment is in order. In view of the fact mentioned before that neutrinos with the masses outside the resonant region give an experimentally undetectable contribution to $K^+ \to \pi^- l_1^+ l_2^+$ decays, the significance of our results mainly consists of establishing a reliable framework for hadronic structure calculations in the analysis of these and similar exotic decays rather than for extraction of neutrino parameters. On the other hand, one may note that our results, obtained for the simplest neutrino exchange mechanism of $K^+ \to \pi^- l_1^+ l_2^+$ decays, can be straightforwardly extended to some other mechanisms offered by the physics beyond the SM, which could lead to *a priori* much larger rates and provide valuable information on the LNV physics.

II. MAJORANA NEUTRINO CONTRIBUTION TO $K^+ \to \pi^- l_1^+ l_2^+$ DECAYS

We consider the SM extension with massive Majorana neutrinos. In this case the weak interaction effective Lagrangian has the standard form. For the studied processes the relevant terms are

$$\mathcal{L}_{\rm int}^{\rm weak} = \frac{G_F}{\sqrt{2}}[V_{ud}\bar{d}O^\alpha u + V_{us}\bar{s}O^\alpha u] \cdot \bar{\nu}_k U_{nk}^* O_\alpha l_n + \text{h.c.} \tag{2}$$

with $O^\alpha = \gamma^\alpha(1-\gamma^5)$. The unitary neutrino mixing matrix U_{ij} relates $\nu_i' = U_{ik}\nu_k$ weak ν_i' and Majorana neutrino mass eigenstates ν_k with the masses m_{ν_k}. The fields l_n denote charged leptons e, μ, and τ.

The lowest order diagrams describing Majorana neutrino contribution to $K^+(p) \to \pi^-(p') + l_1^+(q_1) + l_2^+(q_2)$ decays are shown in Fig. 1. The corresponding matrix elements we write down in the form

$$\begin{aligned} M(l_1^+ l_2^+) &= G_F^2 V_{us} V_{ud} \sum_k U_{l_1 k} U_{l_2 k} m_{\nu_k} [H^{\mu_1\mu_2}(q_1, q_2; m_{\nu_k}) \\ &\quad \cdot L_{\mu_1\mu_2}(q_1, q_2) - (q_1 \leftrightarrow q_2)] \\ &= M(l_1^+ l_2^+)_s + M(l_1^+ l_2^+)_t. \end{aligned} \tag{3}$$

Here the terms $M(l_1^+ l_2^+)_s$ and $M(l_1^+ l_2^+)_t$ denote the contributions of the s- and t-channel diagrams in Fig. 1(a) and 1(b) respectively. The lepton and hadron tensors are defined as

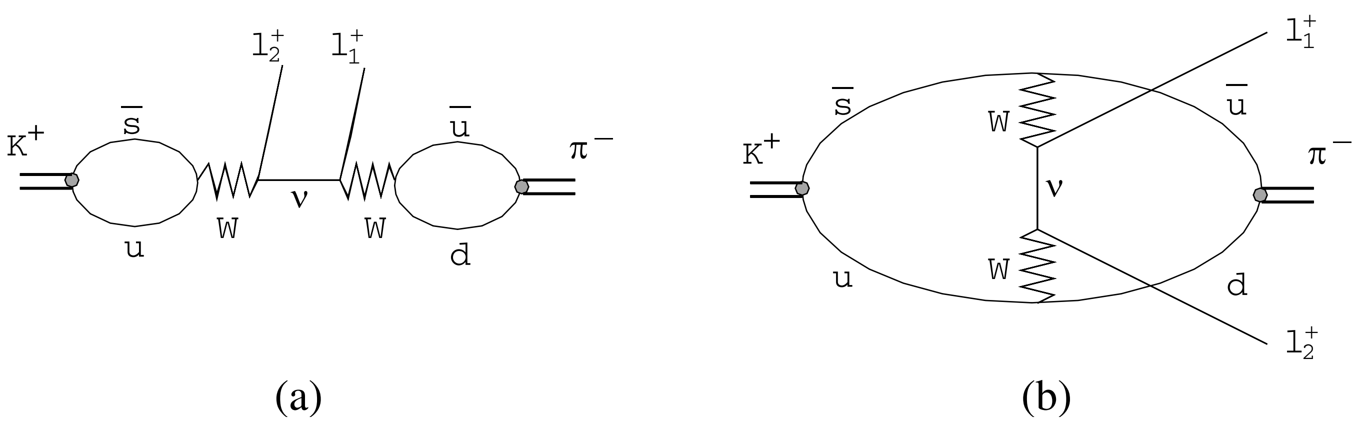

FIG. 1. The lowest order diagrams contributing to $K^+ \to \pi^- l_1^+ l_2^+$ decays.

$$L_{\mu_1\mu_2}(q_1, q_2) = l_1^{\lambda_1,+}(q_1)C\gamma_{\mu_1}\gamma_{\mu_2}(1-\gamma_5)l_2^{\lambda_2,+}(q_2), \quad (4)$$

$$H^{\mu_1\mu_2}(q_1, q_2; m_\nu) = H_s^{\mu_1\mu_2}(q_1, q_2; m_\nu) + H_t^{\mu_1\mu_2}(q_1, q_2; m_\nu). \quad (5)$$

Here λ_1, λ_2 are the polarizations of the charged leptons and C is the charge conjugation matrix. The hadron matrix elements $H_s^{\mu_1\mu_2}$ and $H_t^{\mu_1\mu_2}$ correspond to the contributions of the s- and t-channel diagrams in Fig. 1(a) and 1(b) respectively.

The contribution of the factorizable s-channel diagram in Fig. 1(a) can be calculated in a straightforward way without referring to any hadronic structure model with the following result

$$H_s^{\mu_1\mu_2}(q_1, q_2; m_\nu) = p^{\mu_1}p'^{\mu_2}\frac{f_\pi f_K}{m_\nu^2 - (p-q_1)^2}, \quad (6)$$

where the pion f_π and K-meson f_K leptonic decay constants completely parametrize the hadronic structure of this contribution. Their experimental values are $f_\pi = 131$ MeV and $f_K = 161$ MeV. The t-channel diagram in Fig. 1(b) is much more involved and requires calculations on the basis of certain models of hadronic structure. In the following sections we apply for this purpose the relativistic constituent quark model [14].

Let us note that for the case of neutrino mass spectrum consisting of very light, $m_\nu \ll m_K$, and very heavy, $m_\nu \equiv M_N \gg m_K$, neutrinos ($m_K = 493.677$ MeV is the K-meson mass) both s- and t-channel matrix elements in Eq. (3) are reduced to the form

$$M(l_1^+ l_2^+)_{s,t} = \frac{\langle m_\nu\rangle_{l_1l_2}}{m_K}\mathcal{A}_{s,t}^{(\nu)} + \langle M_N^{-1}\rangle_{l_1l_2} m_K \mathcal{A}_{s,t}^{(N)}, \quad (7)$$

with the contributions proportional to the effective Majorana masses of the light ν and heavy N neutrinos defined as

$$\begin{aligned}\langle m_\nu\rangle_{l_1l_2} &= \sum_{k=light} U_{l_1k}U_{l_2k}m_{\nu_k},\\ \langle M_N^{-1}\rangle_{l_1l_2} &= \sum_{k=\mathrm{heavy}} U_{l_1k}U_{l_2k}M_{N_k}^{-1}.\end{aligned} \quad (8)$$

In this limiting case the coefficients $\mathcal{A}_{s,t}^{(\nu)}$ and $\mathcal{A}_{s,t}^{(N)}$ are independent of neutrino masses and mixing. As follows from Eq. (6) the coefficients $\mathcal{A}_s^{(\nu)}$, $\mathcal{A}_s^{(N)}$ do not depend on the hadronic structure model and their values can be easily calculated (see, for instance, Refs. [3,10]). We will show in Sec. IV that $\mathcal{A}_t^{(N)}$ is also hadronic model independent. Thus, the only coefficient in Eq. (7) which requires hadronic model based calculation is $\mathcal{A}_t^{(\nu)}$.

III. FORMALISM OF HADRONIC STRUCTURE CALCULATIONS

Here we present the details of the relativistic constituent quark model [14] which we apply to the calculation of $K^+ \to l_1^+ l_2^+ \pi^-$ decay rates. The model is based on the effective interaction Lagrangian describing the couplings between hadrons and their constituent quarks. The coupling of a meson $H(q_1\bar{q}_2)$ to its constituent quarks q_1 and $\bar{q}_2$ is given by the Lagrangian

$$\mathcal{L}_{\mathrm{int}}^{\mathrm{Str}}(x) = g_H H(x)\int dx_1 \int dx_2 F_H(x, x_1, x_2) \times \bar{q}(x_1)\Gamma_H\lambda_H q(x_2) + \mathrm{h.c.} \quad (9)$$

Here, λ_H and Γ_H are the flavor SU(3) Gell-Mann matrix and certain combination of Dirac γ-matrices corresponding to the flavor and spin quantum numbers of the meson field $H(x)$. The function F_H is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the meson. The translational invariance requires the vertex function F_H to fulfil the identity

$$F_H(x+a, x_1+a, x_2+a) = F_H(x, x_1, x_2) \quad \text{for any 4-vector } a_\mu. \quad (10)$$

We use for this function the following form

$$F_H(x, x_1, x_2) = \delta(x - c_1x_1 - c_2x_2)\Phi_H[(x_1-x_2)^2], \quad (11)$$

where Φ_H is the correlation function of the two constituent quarks with the masses m_1, m_2. Here we introduced the notation $c_i = m_i/(m_1+m_2)$. The form of the vertex function in Eq. (11) implies factorization of its dependence on the center-of-mass coordinate $x = [m_1/(m_1+m_2)]x_1 + [m_2/(m_1+m_2)]x_2$ of the constituent quarks.

The interaction Lagrangian for the particular case of charged kaon and pion takes the form

$$\begin{aligned}\mathcal{L}_{\mathrm{int}}^{\pi,K}(x) = {}& ig_K K^+(x)\int dx_1\int dx_2 F_K(x, x_1, x_2)\bar{u}(x_1)\\ &\times \gamma^5 s(x_2) + ig_\pi \pi^+(x)\int dx_1 \int dx_2\\ &\times F_\pi(x, x_1, x_2)\bar{u}(x_1)\gamma^5 d(x_2) + \mathrm{h.c.}\end{aligned} \quad (12)$$

The coupling constants g_H in Eqs. (9) and (12) are determined by the so-called *compositeness condition* which requires the renormalization constant of an elementary meson field $H(x)$ to vanish

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2}\tilde{\Pi}'_H(M_H^2) = 0, \quad (13)$$

where $\tilde{\Pi}'_H$ is the derivative of the meson self-energy function. In the case of pseudoscalar mesons we have

$$\tilde{\Pi}'_P(p^2) = \frac{1}{2p^2} p^\alpha \frac{d}{dp^\alpha} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_P^2(-k^2) \times \mathrm{tr}[\gamma^5 S_1(\not k + c_1 \not p)\gamma^5 S_2(\not k - c_2 \not p)], \tag{14}$$

where $\tilde{\Phi}_P(-k^2)$ is the Fourier-transform of the correlation function $\Phi_P[(x_1 - x_2)^2]$ and $S_i(\not k)$ is the quark propagator. We use the free fermion propagators for the valence quarks

$$S_i(\not k) = \frac{1}{m_i - \not k} \tag{15}$$

with an effective constituent quark mass m_i. In order to avoid the appearance of the imaginary parts in the physical amplitudes we require

$$M_P < m_1 + m_2 \tag{16}$$

for the meson mass M_P.

Finally we specify the correlation function Φ_H in Eqs. (11) and (14) characterizing finite size of hadrons. Any choice for its Fourier-transform $\tilde{\Phi}_H$ is appropriate as long as it falls off sufficiently fast in the ultraviolet region of the Euclidean space to render Feynman diagrams ultraviolet finite. We adopt the Gaussian form for this function

$$\tilde{\Phi}_H(k_E^2) \doteq \exp(-k_E^2/\Lambda_H^2), \tag{17}$$

where k_E is Euclidean momentum. The hadronic size parameters Λ_H and the constituent quark masses $m_{u,d,s}$ are determined by fitting to the experimental data for the leptonic decay constants f_H of mesons H. The model expressions for the leptonic decay constants f_P of pseudoscalar mesons are derived from the Lagrangian (12) and take the form

$$\mathcal{F}_P(p^2)p^\mu = \frac{3g_P}{4\pi^2} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_P(-k^2) \times \mathrm{tr}[O^\mu S_1(\not k + c_1 \not p)\gamma^5 S_2(\not k - c_2 \not p)] \tag{18}$$

with the definition $f_P \doteq \mathcal{F}_P(M_P^2)$. The best fit to the experimental values of the decay constants $f_\pi = 131$ MeV and $f_K = 161$ MeV is obtained with

$$m_{u(d)} = 0.235\ \mathrm{GeV}, \qquad m_s = 0.333\ \mathrm{GeV}, \qquad \Lambda_\pi = 1.0\ \mathrm{GeV}, \qquad \Lambda_K = 1.6\ \mathrm{GeV}. \tag{19}$$

This completes the definition of the model which we apply to the analysis of $K^+ \to l_1^+ l_2^+ \pi^-$ decays.

IV. $K^+ \to l_1^+ l_2^+ \pi^-$ HADRONIC MATRIX ELEMENTS AND DECAY RATES

Now let us turn to the calculation of the hadronic matrix elements of $K^+ \to l_1^+ l_2^+ \pi^-$ decays within the above presented approach. The Lagrangian describing these processes consists of the three terms

$$\mathcal{L}_{K-\mathrm{dec}} = \mathcal{L}_{\mathrm{int}}^{\mathrm{weak}} + \mathcal{L}_{\mathrm{int}}^{\pi} + \mathcal{L}_{\mathrm{int}}^{K}, \tag{20}$$

where the first term is the weak interaction Lagrangian (2) while the second and the third terms determine π and K meson interactions with quarks defined in Eq. (12). In the lowest order this Lagrangian generates the contributions corresponding to the diagrams in Fig. 1. In what follows we concentrate on the contribution of the t-channel diagram in Fig. 1(b). The expression for the corresponding hadronic matrix element introduced in Eqs. (3) and (5) takes the form

$$H_t^{\mu_1\mu_2}(q_1, q_2; m_\nu) = -3g_\pi g_K \int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i} \tilde{\Phi}_K(-k_1^2)\tilde{\Phi}_\pi(-k_2^2)\mathrm{tr}[\gamma^5 S_s(k_1 - c_2 p)O^{\mu_1} S_u(k_2 - p'/2)\gamma^5 S_d(k_2 + p'/2) \times O^{\mu_2} S_u(k_1 + c_1 p)] \frac{1}{m_\nu^2 - (k_1 - k_2 + q_{12})^2}, \tag{21}$$

where $q_{12} = c_1 q_1 - c_2 q_2 + (1/2 - c_2)p'$ with $c_1 = m_u/(m_u + m_s)$ and $c_2 = m_s/(m_u + m_s)$. The minus sign comes from the one fermion loop.

We note that the characteristic energy scale of $K^+ \to l_1^+ l_2^+ \pi^-$ is set by m_K. Therefore for the neutrino masses $m_\nu \gg m_K$, the neutrino propagators in the matrix elements of these processes can be substituted by the constant

$$\frac{1}{m_\nu^2 - k^2} \to \frac{1}{m_\nu^2}.$$

Thus the direct dependence on the final lepton momenta q_1 and q_2 drops out from the invariant matrix elements in Eqs. (6) and (21). Using the Fierz identity

$$\mathrm{tr}(T_1 O^\mu T_2 O_\mu) = -\mathrm{tr}(T_1 O^\mu)\mathrm{tr}(T_2 O_\mu) \tag{22}$$

in Eq. (21) and recalling the definition of the weak decay constants f_π and f_K in Eq. (18), one finds that

$$H_t^{\mu_1\mu_2}(q_1, q_2; m_\nu) = \frac{1}{3} H_s^{\mu_1\mu_2}(q_1, q_2; m_\nu) \quad \text{for } m_\nu \gg m_K. \tag{23}$$

Thus, in this limit the t-channel contribution can be evaluated in a hadronic model independent way as well as the s-channel contribution.

In the case of arbitrary finite neutrino masses, after straightforward but quite tiresome calculations, explained in the Appendix, we end up with the following expression for the t-channel hadronic matrix element

$$H_t^{\mu_1\mu_2}(q_1,q_2;m_\nu) = H_t^g(s_1,s_2;m_\nu)g^{\mu_1\mu_2} + H_t^{p'p}(s_1,s_2;m_\nu)(p^{\mu_1}p'^{\mu_2} + p'^{\mu_1}p^{\mu_2} + i\varepsilon^{\rho_1\rho_2\mu_1\mu_2}p_{\rho_1}p'_{\rho_2}) + H_t^{p'q}(s_1,s_2;m_\nu)(p'^{\mu_1}q_{12}^{\mu_2} + q_{12}^{\mu_1}p'^{\mu_2} - i\varepsilon^{\rho_1\rho_2\mu_1\mu_2}p'_{\rho_1}q_{12\rho_2}). \quad (24)$$

An approximate analytic representation for the structure functions $H_t^k(s_1,s_2;m_\nu)$ is given in (A4). We define the kinematical variables as

$$s_1 = (q_1+q_2)^2 = (p-p')^2,$$
$$s_2 = (q_2+p')^2 = (p-q_1)^2,$$
$$s_3 = (p'+q_1)^2 = (p-q_2)^2,$$

where $p^2 = m_K^2$, $p'^2 = m_\pi^2$, $q_i^2 = m_{l_i}^2$, and $s_1+s_2+s_3 = m_K^2+m_\pi^2+m_{l_1}^2+m_{l_2}^2$.

With these definitions the $K^+ \to l_1^+ l_2^+ \pi^-$ decay rate can be written in the form

$$\Gamma(K^+ \to l_1^+ l_2^+ \pi^-) = \frac{G_H^4 V_{us}^2 V_{ud}^2}{256\pi^3 m_K^3}\sum_{k,n}\alpha_k^{l_1l_2}(\alpha_n^{l_1l_2})^* \times \int_{(m_{l_1}+m_\pi)^2}^{(m_K-m_{l_2})^2} ds_3 \int_{s_2^-}^{s_2^+} ds_2 \mathcal{F}(s_2,s_3)_{kn}, \quad (25)$$

where $\alpha_k^{l_1l_2} = U_{l_1k}U_{l_2k}m_{\nu_k}$ and

$$s_2^\pm = m_{l_1}^2 + m_K^2 - \frac{1}{2s_3}[(s_3+m_K^2-m_{l_2}^2)(s_3+m_{l_1}^2-m_\pi^2) \mp \lambda^{1/2}(s_3,m_K^2,m_{l_2}^2)\lambda^{1/2}(s_3,m_{l_1}^2,m_\pi^2)], \quad (26)$$

The integrand in Eq. (25) is

$$\mathcal{F}(s_2,s_3)_{kn} = 2[H^{\mu_1\mu_2}(q_1,q_2;m_{\nu_k}) + H^{\mu_2\mu_1}(q_2,q_1;m_{\nu_k})][H^{\dagger\nu_1\nu_2}(q_1,q_2;m_{\nu_n}) + H^{\dagger\nu_2\nu_1}(q_2,q_1;m_{\nu_n})] \times q_1^{\rho_1}q_2^{\rho_2}\mathrm{tr}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\rho_2}\gamma_{\nu_2}\gamma_{\nu_1}\gamma_{\rho_1}(1-\gamma^5). \quad (27)$$

An explicit form of the function $\mathcal{F}(s_2,s_3)_{kn}$, which we do not show here for its complexity, is derived by the substitution of the expression for $H^{\mu_1\mu_2}$ from Eqs. (5), (6), and (24). Then we carry out the twofold integration in Eq. (25) numerically. The results of these calculations we discuss in the next section.

V. NUMERICAL RESULTS

One of the main purposes of our study is to examine the relative contribution of the t-channel diagram, Fig. 1(b), to $K^+ \to l_1^+ l_2^+ \pi^-$ decays. We are doing this in terms of the decay rates of these processes by comparing their values obtained in the case when both s- and t-channel diagrams are taken into account, Γ_{s+t}, with the case when the t-channel diagram is switched off, Γ_s. For the sake of simplicity we analyze the contribution of only one neutrino mass eigenstate ν_h with an arbitrary mass m_{ν_h}. Varying m_{ν_h} in a wide range of its values we assume ν_h to be an additional mass eigenstate to the three ordinary very light neutrinos. This additional neutrino state may appear in models with sterile neutrino species (see, for instance, Refs. [10,15]) and may *a priori* have an arbitrary mass.

We present our results for the particular case of $K^+ \to \mu^+\mu^+\pi^-$ decay in Table I for the total decay rate $\Gamma_{s+t}/|U_{\mu h}|^4$ and for the ratio Γ_{s+t}/Γ_s as functions of neutrino mass m_{ν_h}. For other decays $K^+ \to l_1^+ l_2^+ \pi^-$ the results are similar.

The following comments are in order. The s-channel diagram has the two singular neutrino propagators $1/(m_{\nu_h}^2 - s_{2,3})$. Therefore, for the neutrino mass m_{ν_h} in the resonant intervals

$$(m_\mu + m_\pi) \approx 245\text{ MeV} \le m_{\nu_h} \le (m_K - m_\mu) \approx 388\text{ MeV}, \quad \text{for } K^+ \to \mu^+\mu^+\pi^-,$$
$$(m_e + m_\pi) \approx 140\text{ MeV} \le m_{\nu_h} \le (m_K - m_e) \approx 493\text{ MeV}, \quad \text{for } K^+ \to e^+e^+\pi^-, e^+\mu^+\pi^- \quad (28)$$

one must take into account the total decay width Γ_{ν_h} of ν_h-neutrino substituting $m_{\nu_h} \to m_{\nu_h} - (i/2)\Gamma_{\nu_h}$. The total decay width Γ_{ν_h} receives all the possible contributions from the leptonic and semileptonic charged and neutral current decay modes allowed by the energy-momentum conservation for the Majorana neutrino ν_h with the mass in the resonant intervals (28). For the resonant interval of the $K^+ \to \mu^+\mu^+\pi^-$ decay this quantity has been calculated in Refs. [10,15] as a function of m_{ν_h}. In our analysis we use

TABLE I. The total $K^+ \to \mu^+\mu^+\pi^-$ decay rate Γ_{s+t} and the ratio Γ_{s+t}/Γ_s vs neutrino mass m_{ν_h}.

m_{ν_h} (eV)	$\Gamma_{s+t}/\lvert U_{\mu h}\rvert^4$ (GeV)	$\frac{\Gamma_{s+t}}{\Gamma_s}$	m_{ν_h} (KeV)	$\Gamma_{s+t}/\lvert U_{\mu h}\rvert^4$ (GeV)	$\frac{\Gamma_{s+t}}{\Gamma_s}$	m_{ν_h} (MeV)	$\Gamma_{s+t}/\lvert U_{\mu h}\rvert^4$ (GeV)	$\frac{\Gamma_{s+t}}{\Gamma_s}$	m_{ν_h} (GeV)	$\Gamma_{s+t}/\lvert U_{\mu h}\rvert^4$ (GeV)	$\frac{\Gamma_{s+t}}{\Gamma_s}$
1	$0.13 \cdot 10^{-47}$	0.85	1	$0.13 \cdot 10^{-41}$	0.85	1	$0.13 \cdot 10^{-35}$	0.85	1	$0.26 \cdot 10^{-31}$	1.33
250	$0.80 \cdot 10^{-43}$	0.85	250	$0.80 \cdot 10^{-37}$	0.85	250	$0.12 \cdot 10^{-17}$	1.00	250	$0.44 \cdot 10^{-36}$	1.78
500	$0.32 \cdot 10^{-42}$	0.85	500	$0.32 \cdot 10^{-36}$	0.85	500	$0.20 \cdot 10^{-30}$	1.00	500	$0.11 \cdot 10^{-36}$	1.78
750	$0.71 \cdot 10^{-42}$	0.85	750	$0.71 \cdot 10^{-36}$	0.85	750	$0.41 \cdot 10^{-31}$	1.25	750	$0.49 \cdot 10^{-37}$	1.78

its average value over the resonant intervals (28) which is $\Gamma_{\nu_h} \approx 10^{-14}$ GeV.

Because of the smallness of Γ_{ν_h}, the s-channel diagram in Fig. 1(a) blows up in the resonant intervals and absolutely dominates over the t-channel one. This effect is clearly seen in Table I.

Let us note that the values of the decay rates in the resonant intervals (28), dominated by Γ_s, give model independent theoretical upper limits on the neutrino contributions to the studied processes

$$\Gamma(K^+ \to l_1^+ l_2^+ \pi^-) \le \Gamma_s^{\rm res}(K^+ \to l_1^+ l_2^+ \pi^-). \tag{29}$$

The t-channel contributions introduce negligible corrections to these limits. For the corresponding branching ratios we obtain the following order of magnitude upper limits

$$\mathcal{R}_{\mu\mu}, \mathcal{R}_{ee}, \mathcal{R}_{\mu e} \le 10^{-1}. \tag{30}$$

The derivation of more accurate limits requires a comprehensive evaluation of the total decay width Γ_{ν_h} of the neutrino mass eigenstate ν_h as a function of m_{ν_h} in the resonant intervals (28), which is beyond the scope of the present study. Apparently the limits in Eq. (30) are much larger than the experimental limits in Eq. (1). This allows one to derive stringent limits on the mixing matrix elements $U_{eh}, U_{\mu h}$. In this way an upper limit $|U_{\mu h}|^2 \le 10^{-9}$ has been derived in Ref. [10] from $K^+ \to \mu^+\mu^+\pi^-$ decay.

As seen from Table I, the ratio Γ_{s+t}/Γ_s is less than one below the resonant region and greater than one above it. This behavior is explained by the fact that the interference of the s- and t-channel diagrams is destructive for m_{ν_h} below the resonant region and constructive above it. One can also notice that the ratio Γ_{s+t}/Γ_s approaches its asymptotic value

$$\frac{\Gamma_{s+t}}{\Gamma_s} = \left(\frac{4}{3}\right)^2 \approx 1.78 \qquad (m_\nu \to \infty) \tag{31}$$

at $m_\nu \approx 10$ GeV. This asymptotic relation follows from Eq. (23) and is independent of the hadronic model.

VI. SUMMARY

We studied the hadronic structure aspects of the lepton number violating $K^+ \to \pi^- l_1^+ l_2^+$ decays within the Relativistic Constituent Quark Model. We considered a particular mechanism of these decays via Majorana neutrino exchange and derived the decay rates as functions of neutrino mass. Our special interest was focused on the relative contribution of the t-channel neutrino exchange diagram in Fig. 1(b). We have shown that it is comparable with the contribution of the s-channel diagram for all values of neutrino mass m_{ν_h} except for the resonant domains (28) where the s-channel diagram Fig. 1(a) absolutely dominates in the decay rates. Outside of this domain the relative contribution of the t-channel diagram varies between ~20% and ~45%. This conclusion contrasts with the previous study of Ref. [12] claiming this contribution to be always negligible in the considered decays. We also pointed out that the values of the decay rates in the resonant regions of neutrino mass represent hadronic model independent theoretical absolute upper bounds for the Majorana neutrino contribution to $K^+ \to \pi^- l_1^+ l_2^+$ decay processes.

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APPENDIX: TECHNICAL DETAILS

Here we present the details of the calculations leading to the expression (24) for the t-channel hadronic matrix element.

We start with the expression in Eq. (21). Using the vertex functions in the form (17) and the α-representation for the denominators of quark propagators

$$\frac{1}{m^2 - K^2} = \int_0^\infty d\alpha e^{-\alpha(m^2-K^2)}$$

one can write down

$$H_t^{\mu_1\mu_2} = 3g_\pi g_K \prod_{i=1}^{5} \int_0^\infty d\alpha_i e^{-\alpha_1 m_s^2 - (\alpha_2+\alpha_3+\alpha_4)m_q^2 - \alpha_5 m_\nu^2 + (\alpha_1 c_2^2 + \alpha_2 c_1^2)p^2 + (\alpha_3+\alpha_4)p'^2/4 + \alpha_5 q_{12}^2} \int \frac{d^4k_1}{(2\pi)^4 i}$$
$$\times \int \frac{d^4k_2}{(2\pi)^4 i} e^{kak+2kr} \mathrm{tr}\{\gamma^5[m_s + \not{k}_1 - c_2\not{p}]O^{\mu_1}[m_u + \not{k}_2 - \not{p}'/2]\gamma^5[m_d + \not{k}_2 + \not{p}'/2]O^{\mu_2}[m_u + \not{k}_1 + c_1\not{p}]\}, \tag{A1}$$

where

$$a = \begin{pmatrix} \mathrm{w_K} + \alpha_1 + \alpha_2 + \alpha_5 & -\alpha_5 \\ -\alpha_5 & \mathrm{w}_\pi + \alpha_3 + \alpha_4 + \alpha_5 \end{pmatrix}$$

with $w_P = 1/\Lambda_P^2$ and

$$r = \begin{pmatrix} (\mathrm{c}_1\alpha_2 - \mathrm{c}_2\alpha_1)\mathrm{p} + \alpha_5 \mathrm{q}_{12} \\ (\alpha_3 - \alpha_4)\mathrm{p}'/2 - \alpha_5 \mathrm{q}_{12} \end{pmatrix}.$$

Then we use the differential representation of the numerator

$$\mathrm{n\,um}(\not k_1, \not k_1) e^{2kr} = \mathrm{num}\Big(\frac{1}{2}\not\partial_1, \frac{1}{2}\not\partial_2\Big) e^{2kr}$$

with $\not\partial_i = \gamma^\alpha \partial/\partial r_i^\alpha$. Integrating out the loop momenta

$$\int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i} e^{kak+2kr} = \frac{1}{2^8\pi^4}\frac{1}{|a|^2} e^{-ra^{-1}r}$$

we arrive at the expression

$$H^{\mu_1\mu_2}(q_1, q_2; m_\nu)_t = \frac{3g_\pi g_K}{2^8\pi^4} \prod_{i=1}^5 \int_0^\infty \frac{d\alpha_i}{|a|^2} e^{-z} \cdot \mathrm{num}^{\mu_1\mu_2}, \tag{A2}$$

where

$$\begin{aligned}
\mathrm{num}^{\mu_1\mu_2} = \mathrm{tr}\Big[&\gamma^5[m_s - (a^{-1}\not r)_1 = c_2\not p]O^{\mu_1}[m_u - (a^{-1}\not r)_2 - \not p'/2]\gamma^5[m_d - (a^{-1}\not r)_2 + \not p'/2]O^{\mu_2}[m_u - (a^{-1}\not r)_1 + c_1\not p] \\
&- \frac{1}{2}a_{21}^{-1}\gamma^5\gamma^\alpha O^{\mu_1}\gamma^\alpha\gamma^5[m_d - (a^{-1}\not r)_2 + \not p'/2]O^{\mu_2}[m_u - (a^{-1}\not r)_1 + c_1\not p] \\
&- \frac{1}{2}a_{21}^{-1}\gamma^5\gamma^\alpha O^{\mu_1}[m_u - (a^{-1}\not r)_2 - \not p'/2]\gamma^5\gamma^\alpha O^{\mu_2}[m_u - (a^{-1}\not r)_1 + c_1\not p] \\
&- \frac{1}{2}a_{11}^{-1}\gamma^5\gamma^\alpha O^{\mu_1}[m_u - (a^{-1}\not r)_2 - \not p'/2]\gamma^5[m_d - (a^{-1}\not r)_2 + \not p'/2]O^{\mu_2}\gamma^\alpha \\
&- \frac{1}{2}a_{22}^{-1}\gamma^5[m_s - (a^{-1}\not r)_1 + c_2\not p]O^{\mu_1}\gamma^\alpha\gamma^5\gamma^\alpha O^{\mu_2}[m_u - (a^{-1}\not r)_1 + c_1\not p] \\
&- \frac{1}{2}a_{12}^{-1}\gamma^5[m_s - (a^{-1}\not r)_1 - c_2\not p]O^{\mu_1}\gamma^\alpha\gamma^5[m_d - (a^{-1}\not r)_2 + \not p'/2]O^{\mu_2}\gamma^\alpha \\
&- \frac{1}{2}a_{12}^{-1}\gamma^5[m_s - (a^{-1}\not r)_1 - c_2\not p]O^{\mu_1}[m_u - (a^{-1}\not r)_2 - \not p'/2]\gamma^5\gamma^\alpha O^{\mu_2}\gamma^\alpha \\
&+ \frac{1}{4}a_{21}^{-1}a_{12}^{-1}\gamma^5\gamma^\beta O^{\mu_1}\gamma^\beta\gamma^5\gamma^\alpha O^{\mu_2}\gamma^\alpha + \frac{1}{4}a_{22}^{-1}a_{11}^{-1}\gamma^5\gamma^\beta O^{\mu_1}\gamma^\alpha\gamma^5\gamma^\alpha O^{\mu_2}\gamma^\beta \\
&+ \frac{1}{4}a_{12}^{-1}a_{21}^{-1}\gamma^5\gamma^\beta O^{\mu_1}\gamma^\alpha\gamma^5\gamma^\beta O^{\mu_2}\gamma^\alpha \Big]
\end{aligned}$$

and

$$z = \alpha_1 m_s^2 + (\alpha_2 + \alpha_3 + \alpha_4)m_q^2 + \alpha_5 m_\nu^2 - (\alpha_1 c_2^2 + \alpha_2 c_1^2)p^2 - (\alpha_3 + \alpha_4)p'^2/4 - \alpha_5 q_{12}^2 + ra^{-1}r. \tag{A3}$$

Thus, we have reduced the two-loop integrations to the 5-fold integrals over α-parameters. We use the FORM code [16] for the calculation of the trace and end up with the ten independent Lorentz structures

$$\begin{aligned}
H^{\mu_1\mu_2}(q_1, q_2; m_\nu)_t = {}& H_t^g(s_1, s_2; m_\nu)g^{\mu_1\mu_2} + H_t^{pp}(s_1, s_2; m_\nu)p^{\mu_1}p^{\mu_2} + H_t^{p'p'}(s_1, s_2; m_\nu)p'^{\mu_1}p'^{\mu_2} + H_t^{qq}(s_1, s_2; m_\nu)q_{12}^{\mu_1}q_{12}^{\mu_2} \\
&+ H_t^{p'p}(s_1, s_2; m_\nu)(p^{\mu_1}p'^{\mu_2} + p'^{\mu_1}p^{\mu_2}) + H_t^{pq}(s_1, s_2; m_\nu)(p^{\mu_1}q_{12}^{\mu_2} + q_{12}^{\mu_1}p^{\mu_2}) \\
&+ H_t^{p'q}(s_1, s_2; m_\nu)(p'^{\mu_1}q_{12}^{\mu_2} + q_{12}^{\mu_1}p'^{\mu_2}) + H_t^{epp'}(s_1, s_2; m_\nu)i\varepsilon^{\rho_1\rho_2\mu_1\mu_2}p_{\rho_1}p'_{\rho_2} \\
&+ H_t^{epq}(s_1, s_2; m_\nu)i\varepsilon^{\rho_1\rho_2\mu_1\mu_2}p_{\rho_1}q_{12\rho_2} + H_t^{ep'q}(s_1, s_2; m_\nu)i\varepsilon^{\rho_1\rho_2\mu_1\mu_2}p'_{\rho_1}q_{12\rho_2}.
\end{aligned}$$

We have shown that

$$H_t^{pp} = H_t^{qq} = H_t^{pq} = H_t^{epq} \equiv 0, \qquad H_t^{epp'} = H_t^{p'p}, \qquad H_t^{ep'q} = -H_t^{p'q}.$$

The function $H_t^{p'p'}$ has been numerically found to be negligibly small. We calculated the three remaining structure functions $H_t^A(s_1, s_2)$ ($A = g, p'p, p'q$) using the FORTRAN code and then approximated them by the functions

$$H_t^A(s_1, s_2; m_\nu) = \frac{H(s_1^{\min}, s_2^{\min})}{1 + b_1x_1 + b_2x_2 + b_{11}x_1^2 + b_{22}x_2^2 + +b_{12}x_1x_2}, \tag{A4}$$

with $x_i = s_i - s_i^{\min}$ $(i = 1, 2)$. For $\mathrm{K}^+ \to \mu^+\mu^+\pi^-$ one has $s_1^{\min} = 4m_\mu^2$, $s_2^{\min} = (m_\pi + m_\mu)^2$. The coefficients

b_1, b_2, b_{11}, b_{22}, b_{12}, and $H(s_1^{\min}, s_2^{\min})$ depend on neutrino mass m_ν. The code for their numerical calculations is available from the authors.

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