

Angular distribution asymmetry in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay in the two-Higgs-doublet model with large $\tan\beta$

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We study possible scalar type interactions in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay. One finds that an angular distribution asymmetry, $\mathcal{A}(s)$, can be induced from the interference between the scalar part and vector part amplitudes in this decay. Our analysis shows that, in the two-Higgs-doublet model of type II with large $\tan\beta$, the charged Higgs contribution could make $\mathcal{A}(s)$ up to 4×10^{-3} without conflict with present experimental constraints. Thus in the future precise experiments in τ -charm factories, this angular distribution asymmetry may be an interesting observable either to be helpful in searching for the signal of the charged Higgs boson or to impose the significant bound on it.

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The τ lepton is the only known lepton massive enough to decay into hadrons. Its semi-leptonic decay into a τ neutrino and a hadron system provides an ideal tool for testing the standard model (SM), in both the electroweak and the strong sectors [1–3]. With the increased experimental sensitivities achieved already or in the future, some interesting limits on possible new physics contributions to the τ decay amplitudes may also be expected [2,4]. The main purpose of the present paper is to explore this possibility in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay. In particular, within the two-Higgs-doublet model (2HDM) we will analyze an angular distribution asymmetry induced from this transition, and it is interesting that this asymmetry may be very useful either to search for the signal of the charged Higgs boson or to impose the significant bound on it. Meanwhile, as a byproduct of our calculation, we will show below that the problem with the pion form factor [5,6] could not be solved by including the charged Higgs contribution.

The final two-pseudoscalar mesons in the decay $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ could have spin-parity number $J^P = 0^+$ or 1^- . Conservation of the vector current (CVC) however forbids the production of 0^+ nonstrange states, thus the decay is expected to be dominated by the low-lying vector resonance contributions at low energies [7,8]. It is known that, in the limit of exact isospin symmetry, CVC relates properties of the $\pi^- \pi^0$ system produced in τ decay to those of the $\pi^+ \pi^-$ produced in the reaction $e^+ e^- \rightarrow \pi^+ \pi^-$, which implies the CVC relation

$$\frac{d\Gamma(\tau \rightarrow \pi^- \pi^0 \nu_\tau)}{ds} = \frac{3\Gamma_e^{(0)} \cos^2 \theta_C}{2\pi\alpha^2 m_\tau^2} s \left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left(1 + \frac{2s}{m_\tau^2}\right) \sigma_{e^+ e^- \rightarrow \pi^+ \pi^-}(s) \quad (1)$$

with

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$$\Gamma_e^{(0)} = \frac{G_F^2 m_\tau^5}{192\pi^3}, \quad s = (p_{\pi^-} + p_{\pi^0})^2,$$

G_F is the Fermi coupling constant, and θ_C is the Cabibbo angle. By comparison of $e^+ e^- \rightarrow \pi^+ \pi^-$ data and the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ data, one finds that the CVC relation (1) works very well for the low s , except in the higher s region [5,7,9]. It is thought that some of the discrepancy between them may be understood by including the isospin symmetry breaking effects [10]. However, it has been pointed out by A. Höcker [5], corrections due to the isospin violation from the SU(2)-breaking sources including the mass and width differences of the charged and neutral $\rho(770)$ mesons, can improve the agreement between τ and $e^+ e^-$ data in the ρ peak region, while these cannot correct the discrepancy in the tails, as shown in Fig. 1 of Ref. [5].

More recently, the author of Ref. [6] proposed that the scalar contribution through the interference with the vector part could be up to the percent level at $s \approx 1 \text{ GeV}^2$, which may solve the above discrepancy. However, it will be shown below this scalar type contribution has been overestimated in Ref. [6]; but there is another interesting observable, the angular distribution asymmetry, induced by the scalar contribution in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay. More

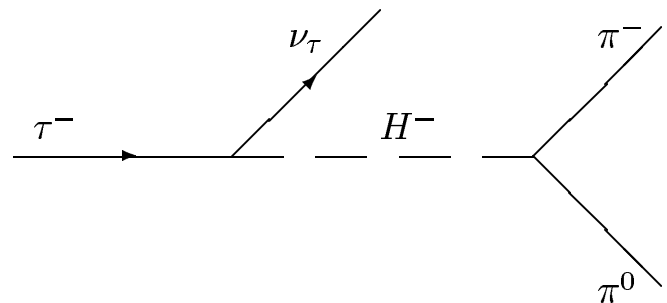


FIG. 1. Tree-level Feynman diagram for the charged Higgs contribution to $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay.

interestingly, this asymmetry may be enhanced in the 2HDM with large $\tan\beta$.

The general scalar and pseudoscalar type interactions in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay have been investigated by the authors of Ref. [11], and the general invariant amplitude for this decay, by assuming only left-handed neutrinos, can be parametrized as

$$\mathcal{M} = G_F \cos\theta_C [F_V(p_{\pi^-} - p_{\pi^0})_\mu \bar{u}(p_{\nu_\tau}) \gamma^\mu (1 - \gamma_5) u(p_\tau) + F_S m_\tau \bar{u}(p_{\nu_\tau}) (1 + \gamma_5) u(p_\tau)], \quad (2)$$

where F_V is the vector form factor, and F_S the scalar one. It is straightforward to get the differential decay rate

$$\begin{aligned} \frac{d\Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau)}{ds} &= \frac{\cos^2\theta_C \Gamma_e^{(0)}}{2m_\tau^2} \lambda^{1/2}(1, m_{\pi^0}^2/s, m_{\pi^-}^2/s) \left(1 - \frac{s}{m_\tau^2}\right) \\ &\times \left\{ |F_V|^2 \left[\lambda(1, m_{\pi^0}^2/s, m_{\pi^-}^2/s) \left(1 + \frac{2s}{m_\tau^2}\right) + \frac{3(m_{\pi^-}^2 - m_{\pi^0}^2)^2}{s^2} \right] + 3|F_S|^2 - 6\text{Re}(F_V F_S^*) \frac{m_{\pi^-}^2 - m_{\pi^0}^2}{s} \right\}, \end{aligned} \quad (3)$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$. Since F_S is very small, in general one cannot expect the term proportional to $|F_S|^2$ would give a significant contribution to $d\Gamma/ds$, which should be below the percent level. It is easy to see that, in the limit of the exact isospin symmetry $m_{\pi^-} = m_{\pi^0}$, the interference term between the scalar and vector amplitudes in Eq. (3) will vanish. Using the experimental value, $m_{\pi^-} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV [12], we have

$$\frac{m_{\pi^-}^2 - m_{\pi^0}^2}{s} \sim 10^{-3} \quad (4)$$

for $s \simeq 1$ GeV². Therefore, contributions to $d\Gamma/ds$ from the scalar interaction cannot be expected to reach the percent level in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay, which thus disagrees with the conclusion obtained in Ref. [6] [our following analysis in the 2HDM can explicitly lead to this conclusion by using Eqs. (3), (9), and (13) and the limits (14)]. On the other hand, one can expect another interesting observable induced from the interference between the scalar and vector interactions in this decay. It is seen that, in the limit of $m_{\pi^-} = m_{\pi^0}$, the differential decay rate in terms of s and θ , the angle between the three-momentum of π^- and the three-momentum of τ^- in the $\pi^- \pi^0$ rest frame, can be written as

$$\begin{aligned} \frac{d^2\Gamma}{dsd\cos\theta} &= \frac{3\Gamma_e^{(0)} \cos^2\theta_C}{4m_\tau^2} \sqrt{1 - \frac{4m_\pi^2}{s}} \left(1 - \frac{s}{m_\tau^2}\right)^2 \\ &\times \left\{ |F_S|^2 + |F_V|^2 \left(1 - \frac{4m_\pi^2}{s}\right) \right. \\ &\times \left[\frac{s}{m_\tau^2} + \left(1 - \frac{s}{m_\tau^2}\right) \cos^2\theta \right] \\ &\left. + 2\text{Re}(F_V F_S^*) \sqrt{1 - \frac{4m_\pi^2}{s}} \cos\theta \right\}, \end{aligned} \quad (5)$$

and the phase space is given by

$$4m_\pi^2 \leq s \leq m_\tau^2, \quad -1 \leq \cos\theta \leq 1.$$

Note that the interference term between F_V and F_S in Eq. (5) is proportional to $\cos\theta$, and will vanish after integrating over θ in the full phase space, which is consistent with Eq. (3) in the isospin limit. However, this term can lead to an angular distribution asymmetry, which is defined as

$$\mathcal{A}(s) = \frac{\int_0^1 \left(\frac{d^2\Gamma}{dsd\cos\theta}\right) d\cos\theta - \int_{-1}^0 \left(\frac{d^2\Gamma}{dsd\cos\theta}\right) d\cos\theta}{\int_0^1 \left(\frac{d^2\Gamma}{dsd\cos\theta}\right) d\cos\theta + \int_{-1}^0 \left(\frac{d^2\Gamma}{dsd\cos\theta}\right) d\cos\theta}. \quad (6)$$

Thus together with Eq. (5), we have

$$\begin{aligned} \mathcal{A}(s) &= \frac{3\Gamma_e^{(0)} \cos^2\theta_C}{2m_\tau^2} \left(1 - \frac{4m_\pi^2}{s}\right) \left(1 - \frac{s}{m_\tau^2}\right)^2 \\ &\times \text{Re}(F_V F_S^*) \left(\frac{d\Gamma}{ds}\right)^{-1}, \end{aligned} \quad (7)$$

where

$$\begin{aligned} \frac{d\Gamma}{ds} &= \frac{\Gamma_e^{(0)} \cos^2\theta_C}{2m_\tau^2} \sqrt{1 - \frac{4m_\pi^2}{s}} \left(1 - \frac{s}{m_\tau^2}\right)^2 \\ &\times \left[|F_V|^2 \left(1 + \frac{2s}{m_\tau^2}\right) \left(1 - \frac{4m_\pi^2}{s}\right) + 3|F_S|^2 \right]. \end{aligned} \quad (8)$$

It is known that, in the low energy region $\sqrt{s} \leq 1$ GeV, F_V can be well described by the $\rho(770)$ meson dominance [7,8], which reads

$$F_V = \frac{m_\rho^2}{m_\rho^2 - s - im_\rho \Gamma_\rho(s)} \quad (9)$$

with

$$\begin{aligned} \Gamma_\rho(s) &= \frac{m_\rho s}{96\pi f_\pi^2} \left\{ \left(1 - \frac{4m_\pi^2}{s}\right)^{3/2} \theta(s - 4m_\pi^2) \right. \\ &\left. + \frac{1}{2} \left(1 - \frac{4m_K^2}{s}\right)^{3/2} \theta(s - 4m_K^2) \right\}. \end{aligned} \quad (10)$$

In order to get a significant asymmetry $\mathcal{A}(s)$ in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay, a sizable contribution to F_S must be gen-

erated. Theoretically, F_S in the SM is an isospin symmetry breaking effect [13]. Experimentally there is no $(\pi\pi)$ scalar resonance observed so far in this low energy region of this decay [6] (future precise experiments are expected but not available yet so far). Large mass scalar particles in the high energy region may give contributions to F_S , however, which in general will be strongly suppressed by the inverse of their large mass squared. Therefore it seems difficult to observe the scalar effects in this decay in the low energy region both from $d\Gamma/ds$ and from $\mathcal{A}(s)$. This situation may be changed in the 2HDM however, in which a significant charged Higgs contribution to F_S could be expected with the large value of $\tan\beta$.

In the minimal version of the SM only one Higgs doublet is required, and a single physical neutral Higgs boson is left over after spontaneous electroweak symmetry breaking. The 2HDM is the simplest extension of the SM with one extra Higgs doublet, which contains three neutral and two charged Higgs bosons. To the purpose of the present discussion, we shall work in the context of the 2HDM of type II [14], where two-Higgs scalar doublets (H_u and H_d) are coupled separately to the right-handed up-type quarks, and the right-handed down-type quarks and the charged leptons. On the other hand, the 2HDM of type II is particularly interesting being the Higgs sector of the minimal supersymmetric standard model (MSSM) [15]. In this case, flavor-changing neutral current amplitudes are naturally absent at the tree-level [16], however, it is possible to accommodate large down-type Yukawa couplings, provided that the ratio $v_u/v_d = \tan\beta$, where $v_{u(d)}$ is the vacuum expectation value of the Higgs doublet $H_{u(d)}$, is large. Phenomenologically, there has been considerable interest in the large $\tan\beta$ effects in B decays such as $B \rightarrow \mu^+ \mu^-$ [17] due to the neutral Higgs contributions at the loop level, as well as in the τ leptonic decays $\tau \rightarrow \ell \nu_\ell \nu_\tau$ and semi-leptonic decays $\tau \rightarrow \pi/K \nu_\tau$ due to the charged Higgs contributions starting from the tree-level [18]. The effects of the charged Higgs boson in τ decays have also been studied in Ref. [19].

The tree-level Yukawa interaction in the 2HDM of type II (including the MSSM) can be written as

$$\mathcal{L}_Y = Y_d \bar{d}_R Q_L H_d + Y_u \bar{u}_R Q_L H_u + Y_L \bar{\ell}_R L H_d + \text{H.c.}, \quad (11)$$

where $Y_{u,d,\ell}$ are 3×3 Yukawa couplings matrices. Thus quarks and charged leptons together with W^\pm and Z^0 will get massive after spontaneous symmetry breaking. Different from the SM case, now five physical Higgs particles: two charged ones H^\pm and three neutral ones h^0, H^0, A^0 , will be left over [14]. One can find that the charged Higgs exchange will give the tree-level contribution to $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay (we do not think loop contributions can significantly change our conclusion since we are only interested in the order-of-magnitude estimate in the present calculation), which is

$$\mathcal{L}^{H^\pm} = \frac{G_F}{\sqrt{2}} \cos\theta_c \frac{m_\tau \tan^2\beta}{m_{H^\pm}^2} \bar{\nu}_\tau (1 + \gamma_5) \tau \left[m_d \bar{d} (1 - \gamma_5) u + \frac{m_u}{\tan^2\beta} \bar{d} (1 + \gamma_5) u \right], \quad (12)$$

and the corresponding Feynman diagram has been drawn in Fig. 1. It is obvious that the scalar contribution to F_S from the above \mathcal{L}^{H^\pm} will be strongly suppressed by $1/m_{H^\pm}^2$ for $m_{H^\pm} \sim O(10^2 \text{ GeV})$, which however can be substantially compensated by large $\tan\beta$. In the large $\tan\beta$ limit (so we can neglect the term proportional to m_u), one has

$$F_S = \frac{m_d \tan^2\beta}{m_{H^\pm}^2} \frac{m_\pi^2}{m_u + m_d}. \quad (13)$$

Unfortunately, at present there is no evidence for m_{H^\pm} , experimentally. From Ref. [12], only the lower limit $m_{H^\pm} > 79.3 \text{ GeV}$ is bounded, one can expect the possibility of the significant F_S for $\tan\beta \approx 30 \sim 50$. On the other hand, some measurements have given the bounds on $\tan\beta/m_{H^\pm}$ in the 2HDM of type II, which read [20–22]

$$\begin{aligned} m_{H^\pm} &> 1.28 \tan\beta \text{ GeV} (95\% \text{CL}), \\ \tan\beta/m_{H^\pm} &< 0.40 \text{ GeV}^{-1} (95\% \text{CL}), \\ \tan\beta/m_{H^\pm} &< 0.40 \text{ GeV}^{-1} (90\% \text{CL}). \end{aligned} \quad (14)$$

Using the above bounds and from Eq. (13), we find that F_S could be up to 10^{-3} in the 2HDM of type II with large $\tan\beta$. Of course, this small value of F_S could only give the negligible contribution to $d\Gamma/ds$ defined in Eq. (8) or (5), however, it may lead to an interesting angular distribution asymmetry $\mathcal{A}(s)$ in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay defined in Eq. (7). To illustrate the order-of the asymmetry $\mathcal{A}(s)$, we take the most conservative bound listed in Eq. (14), $\tan\beta/m_{H^\pm} = 0.4 \text{ GeV}^{-1}$, and $\mathcal{A}(s)$ in the range of $0.3 \text{ GeV}^2 \leq s \leq 1 \text{ GeV}^2$ has been plotted in Fig. 2 (we are interested in the low energy region, in which the contribution to F_V is almost saturated by the $\rho(770)$ meson). It is seen that this differential asymmetry could be up

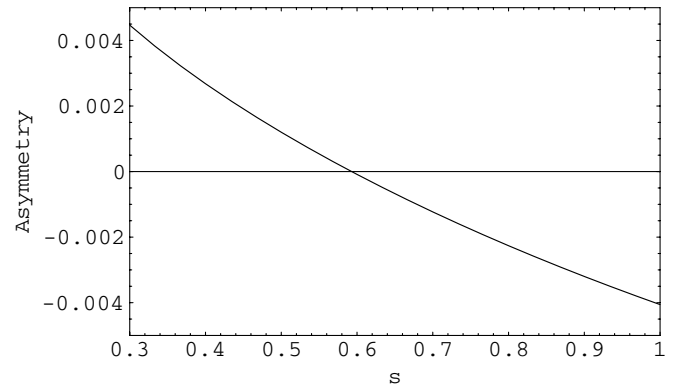


FIG. 2. The differential angular distribution asymmetry $\mathcal{A}(s)$ for $0.3 \text{ GeV}^2 \leq s \leq 1 \text{ GeV}^2$ with $\tan\beta/m_{H^\pm} = 0.4 \text{ GeV}^{-1}$.

to 4×10^{-3} in the 2HDM of type II at large $\tan\beta$ without conflict with the present experimental constraints, which may be detected in the future precise experiments in τ -charm factories. Note that $\mathcal{A}(s)$ defined in Eq. (7) is proportional to $\text{Re}(F_V F_S^*)$, and from Eq. (9),

$$\text{Re}(F_V) = \frac{m_\rho^2(m_\rho^2 - s)}{(m_\rho^2 - s)^2 + m_\rho^2 \Gamma_\rho^2(s)}, \quad (15)$$

which vanishes for $s = m_\rho^2$, thus the sign of the differential asymmetry will be changed (as shown in Fig. 2), and the integrated asymmetry over s is not very significant.

We have analyzed the decay of $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ by considering possible scalar type interactions. We find that the inclusion of scalar type interactions cannot still explain the present discrepancy from the data between $\sigma(e^+ e^- \rightarrow$

$\pi^+ \pi^-)$ and $d\Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau)/ds$ if it really exists, which disagrees with the conclusion obtained in Ref. [6]. However, scalar type interactions can lead to an angular distribution asymmetry in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay. Interestingly, due to the charged Higgs contribution in the 2HDM of type II with large $\tan\beta$, present experimental constraints allows that the differential asymmetry $\mathcal{A}(s)$ could be up to 4×10^{-3} , thus it is expected that the future precise measurements of this asymmetry may either help to search for the signal of the charged Higgs boson or impose the significant bound on it.

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