

## Can there be neutrino oscillations in a gamma-ray burst fireball?

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The central engine which powers the gamma-ray burst fireball produces neutrinos in the energy range of about 5–20 MeV. Fractions of these neutrinos may propagate through the fireball which is far away from the central engine. We have studied the propagation of these neutrinos through the fireball which is contaminated by baryons and have shown that resonant conversion of neutrinos is possible for the oscillations of  $\nu_e \leftrightarrow \nu_{\mu,\tau}$ ,  $\nu_e \leftrightarrow \nu_s$ , and  $\bar{\nu}_{\mu,\tau} \leftrightarrow \bar{\nu}_s$  if the neutrino mass square difference and mixing angle are in the atmospheric and/or Liquid Scintillator Neutrino Detector range. On the other hand, it is probably difficult for neutrinos to have resonant oscillation if the neutrino parameters are in the solar neutrino range. From the resonance condition, we have estimated the fireball temperature and the baryon load in it.

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Gamma-ray bursts (GRBs) are short, nonthermal bursts of low energy ( $\sim 100$  KeV–1 MeV) photons and release about  $10^{51}$ – $10^{53}$  ergs in a few seconds making them the most luminous object in the universe. A class of models called the *fireball model* seems to explain the temporal structure of the bursts and the nonthermal nature of their spectra [1–5]. The sudden release of a copious amount of  $\gamma$  rays into a compact region with a size  $c\delta t \sim 100$ – $1000$  K m [1] creates an opaque  $\gamma - e^\pm$  fireball due to the process  $\gamma + \gamma \rightarrow e^+ + e^-$ . The average optical depth of this process [6] is  $\tau_{\gamma\gamma} \approx 10^{13}$ . This optical depth is very large and, even if there are no pairs to begin with, they will form very rapidly and will Compton scatter lower energy photons. Because of the huge optical depth, photons cannot escape freely. In the fireball the  $\gamma$  and  $e^\pm$  pairs will thermalize with a temperature of about 3–10 MeV. The fireball expands relativistically with a Lorentz factor  $\Gamma \sim 100$ – $1000$  under its own pressure and cools adiabatically due to the expansion. The radiation emerges freely to the intergalactic medium (ISM), when the optical depth is  $\tau_{\gamma\gamma} \approx 1$ .

In addition to  $\gamma$ ,  $e^\pm$  pairs, fireballs may also contain some baryons, both from the progenitor and the surrounding medium. These baryons can be either free or in the form of nuclei. If the fireball temperature is high enough (more than 0.7 MeV), then it will be mostly in the form of neutrons and protons. Derishev *et al.* [7,8] argued that roughly equal numbers of neutrons and protons should be present in the fireball. The electrons associated with the matter (baryons) can increase the opacity, hence delaying the process of emission of radiation, and the baryons can be accelerated along with the fireball and convert part of the radiation energy into bulk kinetic energy. But irrespective of it, the baryonic load has to be very small; otherwise the expansion of the fireball will be Newtonian, which is inconsistent with the present observations. Why the baryonic loading is so low in the fireball is still an open question to be answered [1]. The neutrino oscillation

may overcome the baryon loading problem [9,10]. The evolution of the pure fireball (with no baryons) has been studied in Refs. [11,12]. In the expanding fireball, protons are accelerated in shocks and collide with the photons to produce charged pions, which give rise to ultra-high-energy neutrinos [13]. These neutrinos can be detected by  $km^2$  detectors. Observation of these neutrinos will be able to study the oscillating neutrino flavors in the largest possible baseline, test the equivalence principle and many other neutrino properties [14].

The hidden central engine which powers the fireball is still unknown, but observation suggests that it must be compact. The prime candidates are merger of neutron star with neutron star, black hole-neutron star binaries, and hypernova/collapsar models involving a massive stellar progenitor [1–4,15]. The recent observations of GRB030329 strongly favor the collapsar model [16]. In all these models, the gravitational energy is released mostly in the form of  $\nu\bar{\nu}$ , gravitational radiation, and a small fraction ( $\sim 10^{-3}$ ) is responsible to power GRB. Neutrinos of energy about 5–20 MeV are generated due to the stellar collapse or merger event that trigger the burst. Also due to nucleonic bremsstrahlung  $NN \rightarrow NN\nu\bar{\nu}$  and  $e^+e^- \rightarrow \nu\bar{\nu}$  processes muon and tau types of neutrinos can be produced [17] during the merger process and its flux will be very small. Fractions of these neutrinos may propagate through the fireball which is far away from the central engine. Also within the fireball, because of the weak interaction process  $p + e^- \rightarrow n + \nu_e$ , MeV neutrinos can be generated and propagate through it. The fireball plasma being in an extreme condition may affect the propagation of MeV neutrinos through it.

In a heat bath, the dispersion relation which governs the propagation of the particle gets modified and this can have a drastic effect on the particle propagation in the heat bath. The neutrino propagation in a thermal bath has been studied extensively [18–21]. In general, the propagating neutrino will experience an effective potential due to the

particles in the heat bath. Particularly the neutrino propagation in the early universe hot plasma, as well as in the supernova medium, has profound implications in the respective physics. For example, the neutrino oscillation in the early universe hot plasma may change the relative abundances of  $\nu_e$  and  $\bar{\nu}_e$  and affect the primordial nucleosynthesis of light elements. In the supernova case, the neutrino oscillation in the dense and compact environment can affect the cooling mechanism.

The electron-type neutrinos have charge current as well as neutral current interactions, but the muon and tau types will experience only the neutral current interactions. To the leading order, the effective potential experience by the neutrinos is proportional to the difference of the particle-antiparticle number densities. So if the system under consideration has an equal number of particles and antiparticles, the leading order contribution vanishes. On the other hand, the next-to-leading order contribution (i.e., the term proportional to  $1/M^4$ , where  $M$  is the vector boson mass) is proportional to the sum of the particle and antiparticle number densities, and this is the leading contribution to the effective potential. The early universe hot plasma has an equal number of particles and antiparticles, hence the leading contribution to the neutrino effective potential will be proportional to  $1/M^4$ . A similar situation can arise for neutrinos propagating in the GRB fireball if one considers a mostly photon-lepton fireball [1,3,4]. Here in the present work we consider a fireball containing mostly photon-lepton with little baryon contamination, which mimics the early universe hot plasma, and we study the neutrino propagation within it.

In a relativistic and nondegenerate  $e^\pm$ , proton and neutron plasma, the effective potential experience by  $\nu_e$  is [19,20]

$$V_{\nu_e} \simeq \sqrt{2}G_F N_\gamma \left[ \mathcal{L}_e - \left( \frac{7\xi(4)}{\xi(3)} \right)^2 \frac{T^2}{M_W^2} \right], \quad (1)$$

and by a muon or tau neutrino it is given by

$$V_{\nu_{\mu,\tau}} \simeq \sqrt{2}G_F N_\gamma \mathcal{L}_{\mu,\tau}, \quad (2)$$

where

$$\mathcal{L}_e = \left( \frac{1}{2} + 2\sin^2\theta_W \right) L_e + \left( \frac{1}{2} - 2\sin^2\theta_W \right) L_p - \frac{L_n}{2}, \quad (3)$$

and

$$\mathcal{L}_{\mu,\tau} = \left( -\frac{1}{2} + 2\sin^2\theta_W \right) (L_e - L_p) - \frac{L_n}{2}. \quad (4)$$

The asymmetry of the particle  $a$  is defined as

$$L_a = \frac{(N_a - \bar{N}_a)}{N_\gamma}, \quad (5)$$

where  $N_\gamma = \frac{2}{\pi^2} \xi(3) T^3$  is the number density of photons.

For the antineutrinos, the effective potential will be given by changing  $L_a \rightarrow -L_a$ . For a pure  $\gamma$  and  $e^\pm$  fireball, we have  $L_p = L_n = 0$ .

We consider the neutrino oscillation processes:  $\nu_e \leftrightarrow \nu_{\mu,\tau}$ ,  $\nu_e \leftrightarrow \nu_s$ , and the antineutrino processes. The effective potential difference for  $\nu_e \leftrightarrow \nu_{\mu,\tau}$  and  $\bar{\nu}_e \leftrightarrow \bar{\nu}_{\mu,\tau}$  processes is given by

$$V \simeq 4.02 \times 10^{-12} T_{\text{MeV}}^3 [\pm L_e - 6.14 \times 10^{-9} T_{\text{MeV}}^2] \text{ MeV}, \quad (6)$$

where  $\pm$  corresponds to  $\nu$  and  $\bar{\nu}$ , respectively, and for  $\nu_e \leftrightarrow \nu_s$  oscillation the effective potential difference is given by Eq. (1). For a pure  $\gamma$ ,  $e^\pm$  fireball ( $CP$  symmetric),  $L_e = 0$  and only the higher order term will contribute. The conversion probability for the above processes for a constant  $V$  is given by

$$\mathcal{P}(t) = \frac{\Delta^2 \sin^2 2\theta}{\omega^2} \sin^2 \left( \frac{\omega t}{2} \right), \quad (7)$$

where

$$\omega = \sqrt{(V - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}, \quad (8)$$

where  $\Delta = \Delta m^2 / 2E_\nu$ ,  $V$  is the potential difference,  $E_\nu$  is the neutrino energy, and  $\theta$  is the neutrino mixing angle. The oscillation length is given by

$$L_{\text{osc}} = \frac{L_v}{\sqrt{\cos^2 2\theta \left( 1 - \frac{V}{\Delta \cos 2\theta} \right)^2 + \sin^2 2\theta}}, \quad (9)$$

where  $L_v = 2\pi/\Delta$  is the vacuum oscillation length which can be recovered for  $V = 0$ . For resonance to occur we should have, from Eq. (8),

$$V = \Delta \cos 2\theta. \quad (10)$$

We have thus far assumed that lepton asymmetry in the fireball does not vary with distance. But in reality, the lepton asymmetry changes with distance. So in this case we will consider the adiabaticity of the resonant conversion. The adiabaticity condition at the resonance [22] is given by

$$\kappa_r \equiv 2.0 \times 10^{-3} \left( \frac{\Delta \tilde{m}^2 \sin 2\theta}{E_{\text{MeV}}} \right)^2 \frac{l_{\text{cm}}}{T_{\text{MeV}}^3} \left( \frac{dL_e}{dx} \right)^{-1} \geq 1, \quad (11)$$

where  $l_{\text{cm}}$  is some length scale expressed in centimeters,  $x$  is a dimensionless variable,  $\Delta \tilde{m}^2$  is in units of  $\text{eV}^2$ , and  $T_{\text{MeV}}$  and  $E_{\text{MeV}}$  are expressed in units of MeV. The above condition depends on the neutrino parameters and the change in the  $L_e$  as one goes away from the center of the fireball.

As already stated in the introduction, baryon loading is still an outstanding problem. It is believed that the contamination is very low ( $10^{-8} M_\odot - 10^{-5} M_\odot$ ) [1,2], so that the fireball can have an ultrarelativistic expansion. For simplicity, we consider a charge neutral spherical fireball

( $L_e = L_p$ ) of initial radius  $R$  with an equal number of protons and neutrons in it. Then the baryon load in the fireball is

$$M_b \sim 2.23 \times 10^{-4} R_7^3 T_{\text{MeV}}^3 L_e M_\odot, \quad (12)$$

where  $R_7$  is in units of  $10^7$  cm. As stated above, the baryon contamination is of order  $10^{-8} M_\odot$  to  $10^{-5} M_\odot$ , which corresponds to  $L_e \sim 4.47 \times 10^{-5} R_7^{-3} T_{\text{MeV}}^{-3}$  to  $L_e \sim 4.47 \times 10^{-2} R_7^{-3} T_{\text{MeV}}^{-3}$ , respectively. The thermalized fireball has a temperature  $T \sim (L/4\pi\sigma R^2)^{1/4} \sim 3\text{--}10$  MeV, with  $L$  the luminosity and  $\sigma$  the Stephen-Boltzmann constant.

The processes (active to active oscillations) do not depend on the baryon asymmetry in the fireball, simply because the neutral current contribution to the potential of all the active neutrinos is the same, and for an active to active oscillation this contribution cancels out, leaving only the dependence on  $L_e$ . But the active-sterile oscillation depends on both the lepton and the baryon asymmetry. In Eq. (6),  $V > 0$  for the neutrino process if

$$L_e > 6.14 \times 10^{-9} T_{\text{MeV}}^2, \quad (13)$$

and depending on the fireball parameters and the neutrino properties the resonance condition can be satisfied. On the other hand, the antineutrino process will never satisfy the resonance condition because the potential is always negative. The resonance condition of Eq. (10) can be written as

$$L_e T_{\text{MeV}}^3 = 0.124 \frac{\Delta \tilde{m}^2 \cos 2\theta}{E_{\text{MeV}}}, \quad (14)$$

and the resonance length is given by

$$L_{\text{res}} \simeq 248 \frac{E_{\text{MeV}}}{\Delta \tilde{m}^2 \sin 2\theta} \text{ cm}. \quad (15)$$

Putting the value of  $L_e$  from Eq. (14) in Eq. (13), the constraint on the fireball temperature is

$$T_{\text{MeV}}^5 < 0.2 \times 10^8 \frac{\Delta \tilde{m}^2 \cos 2\theta}{E_{\text{MeV}}}. \quad (16)$$

Thus, the fireball temperature derived in Eq. (16) is the one required in order to have resonant conversion of the neutrinos. As we have already discussed, during the stellar collapse or merger events that trigger the burst, neutrinos of energy about 5–20 MeV are copiously produced and some of these neutrinos will propagate through the fireball. Apart from this, due to inverse beta decay, MeV neutrinos can also be generated within the fireball. So here we will take  $E_{\text{MeV}} = 5$  and 20 for our calculation to estimate the fireball parameters.

Let us study the resonance condition for solar, atmospheric, and the Liquid Scintillator Neutrino Detector (LSND) reactor neutrinos, where we know approximately the neutrino mass square differences and the neutrino mixing angles from the recent experimental results. These can constrain the fireball parameters.

The recent analysis of the salt phase data of SNO [23] combined with the KamLAND [24] reactor antineutrino results gives  $6 \times 10^{-5} \text{ eV}^2 < \Delta m^2 < 10^{-4} \text{ eV}^2$  and  $0.8 < \sin 2\theta < 0.98$  with a confidence level of 99%. The best fit point has  $\Delta m^2 \sim 7.1 \times 10^{-5} \text{ eV}^2$  and  $\sin 2\theta \sim 0.83$ . Using the best fit point in Eq. (14), we obtain  $L_e \simeq 0.5 \times 10^{-5} T_{\text{MeV}}^{-3} E_{\text{MeV}}^{-1}$ . The condition in Eq. (16) gives  $T_{\text{MeV}} < 2.8$  and  $< 2.1$ , respectively, for  $E_{\text{MeV}} = 5$  and 20. Similarly, the resonance length  $L_{\text{res}} \sim 211$  and 845 km are obtained, respectively, for  $E_{\text{MeV}} = 5$  and 20. Using the resonance value of  $L_e$  in Eq. (12), we obtain  $M_b \sim 1.15 \times 10^{-9} R_7^3 M_\odot E_{\text{MeV}}^{-1}$  which is independent of the fireball temperature. For  $E_{\text{MeV}} = 5$  and 20, we obtain  $M_b \sim 0.23 \times 10^{-9} R_7^3 M_\odot$  and  $M_b \sim 0.58 \times 10^{-10} R_7^3 M_\odot$ , respectively. One can see that, in this case, the temperature of the fireball is less compared to the lower limit of 3 MeV. Also the baryon load in the fireball is less, which can be improved by increasing  $R_7$ . By considering the solar neutrino mixing angle and the  $\Delta m^2$ , for neutrino oscillation in the fireball, the fireball temperature has to be less than 3 MeV. If the  $L_e$  vary with distance, then at the resonance  $\kappa_r \sim 0.8 \times 10^{-14} l_{\text{cm}} (L'_e)^{-1}$ , which implies  $L'_e = dL_e/dx$  has to be very small to satisfy the condition in Eq. (11).

The Super-Kamiokande Collaboration recently reported the atmospheric neutrino oscillation parameters [25]  $1.9 \times 10^{-3} \text{ eV}^2 < \Delta m^2 < 3.0 \times 10^{-3} \text{ eV}^2$  and  $0.9 \leq \sin^2 2\theta \leq 1.0$  with a 90% confidence level. Taking  $\Delta m^2 \sim 2.5 \times 10^{-3} \text{ eV}^2$  and  $\sin^2 2\theta \sim 0.9$ , we obtain  $L_e \simeq 0.98 \times 10^{-4} T_{\text{MeV}}^{-3} E_{\text{MeV}}^{-1}$  and this implies  $T_{\text{MeV}} < 5$  and  $< 3.8$ , respectively, for  $E_{\text{MeV}} = 5$  and 20. So these temperatures come within the range of the fireball as discussed earlier. The resonance lengths for  $E_{\text{MeV}} = 5$  and 20 are, respectively, 5.2 and 21 km. Also, for the above ranges of neutrino energy, the baryon load in the fireball is  $M_b \sim 0.44 \times 10^{-8} R_7^3 M_\odot$  and  $M_b \sim 0.11 \times 10^{-8} R_7^3 M_\odot$ , respectively. Thus, before coming out of the fireball, the neutrinos can have many resonant oscillations. If the lepton asymmetry varies with distance, then  $l_{\text{cm}}/L'_e \geq 10^{12}$  to have resonant conversion.

Thirdly, let us discuss the implication of the LSND and KARMEN results [26] on the neutrino oscillation in the GRB fireball. The combined analysis of LSND and KARMEN 2 results gives  $0.45 \text{ eV}^2 < \Delta m^2 < 1 \text{ eV}^2$  and  $2 \times 10^{-3} < \sin^2 2\theta < 7 \times 10^{-3}$  with a confidence level of 90%. We consider  $\Delta m^2 \sim 0.5 \text{ eV}^2$  and  $\sin 2\theta \sim 0.07$  to estimate the fireball parameters. This gives  $T_{\text{MeV}} < 18$  and  $< 14$ , respectively, for  $E_{\text{MeV}} = 5$  and 20, and for these two values of  $E_{\text{MeV}}$  the baryon load is, respectively,  $M_b \sim 0.3 \times 10^{-5} R_7^3 M_\odot$  and  $M_b \sim 0.7 \times 10^{-6} R_7^3 M_\odot$ . The resonance lengths for these two neutrino energies are, respectively, 0.4 and 1.4 km. So this shows that the propagating neutrinos have to oscillate several times before they come out of the fireball. If the lepton asymmetry has a profile, then in this case  $l_{\text{cm}}/L'_e \geq 10^{10}$  to satisfy the resonant oscillation.

If the neutrino oscillation parameters are in the atmospheric and/or LSND range, the oscillation length at resonance within the fireball will vary from a few meters to  $\sim 21$  km. As the resonance length is small compared to the size of the fireball, there will be many resonant oscillations for  $\nu_e \leftrightarrow \nu_{\mu,\tau}$  before they emerge out of the fireball. So the average conversion probability in this case is  $\mathcal{P}(t) \sim 0.5$ , where we have considered the fact that  $L_e$  does not vary with distance. But if the oscillation parameters are in the solar neutrino range, it is probably difficult to have neutrino oscillation. This is due to the fact that the fireball temperature in this case is less (i.e.,  $< 3$  MeV).

For the oscillation  $\nu_e \leftrightarrow \nu_s$ , with charge neutral fireball and  $L_p = L_n$ , the effective potential is given by

$$V \simeq 4.02 \times 10^{-12} T_{\text{MeV}}^3 \left[ \frac{L_e}{2} - 6.14 \times 10^{-9} T_{\text{MeV}}^2 \right] \text{ MeV.} \quad (17)$$

The resonance condition can also be satisfied for the above process for  $V > 0$ . Both processes  $\nu_e \leftrightarrow \nu_{\mu,\tau}$  and  $\nu_e \leftrightarrow \nu_s$  are equally probable if we consider the atmospheric neutrino oscillation parameter or the LSND one, with average probability  $\sim 0.5$ . Finally, let us consider the process  $\bar{\nu}_{\mu,\tau} \leftrightarrow \bar{\nu}_s$ , for which

$$V \simeq 2.0 \times 10^{-12} T_{\text{MeV}}^3 L_e \text{ MeV.} \quad (18)$$

Here the higher order contribution to the neutrino potential is absent and, due to this, there is no constraint on the baryon loading of the fireball. Only the charge neutrality of

the fireball is sufficient enough for the resonant oscillation to take place.

From the collapsar or merger model of SN with SN and/or from the collision of SN with a black hole, lots of neutrinos will be produced and fractions of these neutrinos will propagate through the fireball. Here we have studied the propagation of these neutrinos through the GRB fireball by assuming the later to be spherical with a radius  $R$ , charge neutral, and  $L_p = L_n$ . Also, we have assumed that  $L_e > 6.14 \times 10^{-9} T_{\text{MeV}}^2$  so that the potential difference will be positive for neutrinos and there can be resonant oscillation. By using the known neutrino mass square difference and mixing angle from the solar, atmospheric, and reactor experiments, we estimate the fireball temperature, the baryon load, and the lepton asymmetry in it. Our result shows that, if the neutrino oscillation parameters are in the solar neutrino range, there will probably be very few or no oscillations take place. On the other hand, if the neutrino oscillation parameters are in the atmospheric and/or LSND range as discussed above, there can be many resonant oscillations for  $\nu_e \leftrightarrow \nu_{\mu,\tau}$ ,  $\nu_e \leftrightarrow \nu_s$ , and  $\bar{\nu}_{\mu,\tau} \leftrightarrow \bar{\nu}_s$  before they emerge out of the fireball, and about 50% of these neutrinos will resonantly convert. These MeV neutrino signals will be similar to the one from supernovae, for example, SN1987A. Unfortunately with the present generation neutrino telescopes these  $\nu_s$  cannot be detected due to their cosmological distance, and the fluxes are extremely negligible compared to the galactic supernovae.

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