PHYSICAL REVIEW D **71,** 046003 (2005)

Separation of spontaneous chiral symmetry breaking and confinement via the AdS/CFT correspondence

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(Received 7 January 2005; published 15 February 2005)

We analyze, in the framework of AdS/CFT correspondence, the gauge theory phase structure that is supposed to be dual to the recently found nonsupersymmetric dilatonic deformations to $AdS_5 \times S^5$ in type IIB string theory. Analyzing the probe D7-brane dynamics in the backgrounds of our interest, which corresponds to the fundamental $\mathcal{N} = 2$ hypermultiplet, we show that the chiral bi-fermion condensation responsible for spontaneous chiral symmetry breaking is not logically related to the phenomenon of confinement.

DOI: 10.1103/PhysRevD.71.046003 PACS numbers: 11.25.Tq, 11.30.Qc, 11.30.Rd, 12.38.Aw

I. INTRODUCTION

Quantum chromodynamics (QCD) is believed to describe hadrons in the universe. While much of its perturbative dynamics is by now fairly well understood, it is still hard to analyze, in convincing ways, many nonperturbative phenomena that are relevant in the low energy regime. Among these are the confinement of quarks and the spontaneous breaking of their (approximate) chiral symmetry (SχSB). Although these two aspects of QCD have their same origin in strongly interacting dynamics, there has not been found any logical connection between the two phenomena. In this paper, we provide, in our belief, one convincing example showing the logical separation between confinement and SχSB. Our analysis seems to suggest that spontaneous chiral symmetry breaking of massless quarks may happen without any need of a confining potential between them.

Our analysis is based on the proposal of AdS/CFT correspondence, in which Type IIB string theory on $AdS_5 \times S^5$ background is equivalent to the $\mathcal{N} = 4$ SYM theory on the boundary of AdS_5 [1]. The duality between the two descriptions is supposed to hold even at the level of Hilbert spaces of their corresponding quantum theory; (semiclassical) deformations of $AdS_5 \times S^5$ which vanish asymptotically at the boundary correspond to some quantum states in the dual gauge theory [2,3]. Depending on the deformations in the bulk that we are considering, these states share several interesting properties with the usual vacuum states of realistic gauge theories, such as homogeneity over space and nonvanishing gluon condensation, etc. Henceforth, studying confinement and SχSB on these states may give us an important laboratory for unraveling the relation between the two phenomena.

The deformed backgrounds of our interest are a family of dilatonic deformations of $AdS_5 \times S^5$ that were found in Ref. [4]. A nice fact about these solutions is the existence of a single adjustable parameter, k/μ, which enables us to scan a range of corresponding quantum states. In the gauge theory side, this parameter represents the ratio of the gluon condensation to the energy density of the quantum states. The analysis in Ref. [4] showed that, for $k/\mu < -12$, the potential between (heavy) quark/antiquark pair is confining, while states of $k/\mu > -12$ were argued to exhibit Coulomb-like behavior. However, in Sec. III, we perform a more careful study for the cases of $k/\mu > -12$ to find the screening phase instead for them. We also look at the response to magnetically charged objects and get an interesting phase structure.

To study SχSB on these background states, a small number of light quarks/antiquarks are introduced *à la* Karch and Katz in Sec. IV; probe D7-branes [5]. They are N_f, $\mathcal{N} = 2$ hypermultiplets in fundamental representation of the SU(N) gauge group, and their effect to $\mathcal{N} = 4$ SU(N) SYM dynamics may be neglected in the $N \gg N_f$ limit via quenched approximation. The D7 probe for studying SχSB was analyzed first in [6], and its use also for hadron physics [7–11] is by now a well-established method in the literature (see [12–16] for other setups of introducing flavors). From a careful numerical work, we seem to find convincing evidence that SχSB persists in the region of our parameter space in which the confinement no longer exists. Therefore, on the basis of validity of the AdS/CFT correspondence, it is clear that some states in large N $\mathcal{N} = 4$ SYM theory, which have nonvanishing gluon condensation, serve as a rare ground for logical separation between SχSB and confinement. We summarize and conclude in Sec. V.

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II. BULK SOLUTIONS: DILATONIC DEFORMATION IN $AdS_5 \times S^5$

In Ref. [4], a family of nonsupersymmetric solutions of type IIB supergravity with asymptotic $AdS_5 \times S^5$ geometry was found by turning on generic dilaton deformation to the maximally supersymmetric $AdS_5 \times S^5$ background.[1] (For an earlier example, see [18,19].) Analytic solutions are available only for the cases in the Poincaré patch, which preserves the $\mathbf{R} \times \mathrm{ISO}(3) \times \mathrm{SO}(6)$ subgroup of the full $\mathrm{SO}(2,4) \times \mathrm{SO}(6)$ symmetry of $AdS_5 \times S^5$. Explicitly, these solutions are

$$
\begin{aligned}
ds^2 &= (y-b)^{(1-a)/4}(y+b)^{(1+a)/4}\Big[-\Big(\frac{y-b}{y+b}\Big)dt^2 \\
&\quad + \frac{dy^2}{16(y-b)^{(5-a)/4}(y+b)^{(5+a)/4}} + d\vec{x}^2\Big] + d\Omega_5^2, \\
\phi &= \phi_0 + \frac{k}{8b}\log\Big(\frac{y-b}{y+b}\Big), \qquad F_5 = Q(\omega_5 + *\omega_5),
\end{aligned}
\tag{2.1}
$$

where the metric is in the Einstein frame and we let the AdS radius be unity for simplicity. Here Q is the constant that counts the number N of D3-branes, $d\Omega_5^2$ and ω_5 are the metric and the volume form of unit five sphere, respectively. Clearly, the S^5 part of the original $AdS_5 \times S^5$ is intact and the SO(6) R symmetry of the $\mathcal{N} = 4$ SYM theory is unbroken at this level. The parameters a and b are defined in terms of two quantities, k and μ:

$$
a \equiv \Big(1 + \frac{k^2}{6\mu^2}\Big)^{-(1/2)}, \qquad b \equiv \frac{\mu}{2}\Big(1 + \frac{k^2}{6\mu^2}\Big)^{1/2}. \tag{2.2}
$$

The solutions have a timelike naked singularity at $y = b$. Up to overall scaling, these solutions are parametrized by essentially a single variable k/μ. They can be thought of as describing some quantum states in the bulk AdS_5 spacetime, because their deformations to the maximally supersymmetric $AdS_5 \times S^5$ solution decay sufficiently fast as we approach the boundary. According to AdS/CFT correspondence, we therefore interpret them as the dual geometries of some quantum states of the $\mathcal{N} = 4$ SYM gauge theory living on the boundary $R^{1,3}$.

An element of the standard AdS/CFT dictionary gives us important information about these quantum states in the gauge theory. In terms of the coordinate r defined by $r^2 = \sqrt{\frac{b}{2}}e^s$ and $y = b\cosh(2s)$, the bulk metric goes to the standard $AdS_5 \times S^5$ metric for large r, and r becomes the usual radial coordinate of asymptotic AdS_5. The dilaton field then asymptotes to

$$
\phi = \phi_0 + \frac{k}{8b}\log\Big(\frac{y-b}{y+b}\Big) \sim \phi_0 - \frac{k}{4}\frac{1}{r^4}, \tag{2.3}
$$

which implies that the corresponding quantum states in the gauge theory have a nonvanishing expectation value of $\mathcal{L}_{\mathrm{CFT}} \sim [1/(2g_{YM}^2)]\mathrm{tr}F^2$:

$$
\langle \mathcal{L}_{\mathrm{CFT}} \rangle = \frac{k}{4}. \tag{2.4}
$$

The ADM energy density of these states was calculated to be proportional to μ.

III. PHASES OF DUAL $\mathcal{N} = 4$ SYM STATES: CONFINEMENT VS SCREENING

The family of supergravity backgrounds in the previous section with varying dilaton profile are supposed to describe some quantum states of $\mathcal{N} = 4$ SYM theory on $R^{1,3}$. According to AdS/CFT dictionary, these states are characterized by expectation values of $\mathrm{tr}F^2$ and the Hamiltonian density. Roughly, we have seen that

$$
\begin{aligned}
k &\sim \frac{1}{2g_{YM}^2}\langle \mathrm{tr}(F^2)\rangle = \frac{1}{2g_{YM}^2}\langle \mathrm{tr}(\vec{E}^2 - \vec{B}^2)\rangle, \\
\mu &\sim \frac{1}{2g_{YM}^2}\langle \mathrm{tr}(\vec{E}^2 + \vec{B}^2)\rangle = \mathcal{E},
\end{aligned}
\tag{3.1}
$$

where we denote the energy density by $\mathcal{E}$. Though these states are quantum states of the superconformal $\mathcal{N} = 4$ SYM theory, they have certain properties that mimic those of interesting vacuum states of more realistic gauge theories; they are homogeneous over spatial R^3 and have nonvanishing gluon condensation. The latter property has long been suspected of being one of the crucial characteristics of QCD vacuum [20]. It is thus a meaningful endeavor to study quantum structure of these states and talk about their "phases." One has to bear in mind that the strength of the gluon condensate here characterizes the macroscopic states of the $\mathcal{N} = 4$ SYM theory and works as a tunable parameter.

One of the key aspects of a given phase of gauge theory is how it reacts to external charges. In the screening phase, external charges are compensated by conducting currents, and subsequently screened within some characteristic length scale. Equivalently, the gauge boson gets massive and does not propagate beyond its mass scale. On the other hand, the confining phase does not break gauge symmetry and charge conservation. Instead, electric flux is confined to a narrow string, resulting in a linear potential between two charges. One of the most profound observations in gauge theory is that magnetic screening due to condensation of a magnetically charged object leads to electric confinement and vice versa. However, there is a caveat here; that is, if there is also a condensation of electrically charged object at the same time, electric confinement will

[1]It is also possible to have solutions with the axion field turned on, but these solutions are readily obtained from the current ones by SL(2, Z) action. See also Ref. [17] for the nonsingular class of dilatonic deformation in AdS_5.

be ruined. In this case, the most plausible expectation is that both electric and magnetic charges are screened.

In this section, we analyze the response of our states of $\mathcal{N} = 4$ SYM theory to various types of external charges, and find some aspects of interesting phases that we discussed in the above. In the spirit of AdS/CFT correspondence, external charges are described by stretched strings in the supergravity background to the boundary [21,22]. The Wilson line expectation value is obtained from the effective world-sheet dynamics of the stretched strings in the supergravity background. For electric charges, the world-sheet dynamics is dictated by F1 Nambu-Goto action, while magnetic or dyonic cases will be described by D1–Dirac-Born-Infeld(DBI) action with/without world-sheet gauge flux turned on.

A. Electric confinement/screening transition

Electric confinement in the above dilatonic backgrounds has been shown to occur in Ref. [4] for $k/\mu < -12$. A Nambu-Goto string stretched between a heavy quark/antiquark pair through the bulk corresponds to the Wilson loop in the dual gauge theory side [21,22]. In the large AdS radius limit, the string behaves classically and one may get the interaction potential via classical analysis of the Nambu-Goto string dynamics. Here we would like to analyze more general cases including magnetic charges as well.

To deal with the general (p, q) string, let us begin with the Dirac-Born-Infeld action,

$$S = -\frac{1}{2\pi\alpha'} \times \int d\tau d\sigma e^{-\phi}\sqrt{-\det(g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + 2\pi\alpha' F_{ab})}, \tag{3.2}$$

where $g_{\mu\nu}$ is the string frame metric which is related to the Einstein frame metric by

$$g_{\mu\nu} = e^{\phi/2} g^E_{\mu\nu}. \tag{3.3}$$

Denoting

$$M = -\det(g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu), \tag{3.4}$$

the Lagrangian density may be written as

$$\mathcal{L} = -\frac{1}{2\pi\alpha' e^\phi}\sqrt{M - (2\pi\alpha' E)^2}, \tag{3.5}$$

where $E = F_{01}$. Let us introduce the displacement D by

$$D = \frac{\partial \mathcal{L}}{\partial E} = \frac{2\pi\alpha' E}{e^\phi \sqrt{M - (2\pi\alpha' E)^2}}. \tag{3.6}$$

D is conserved and $\partial_\sigma D = 0$; one may obtain an equivalent description of the system by the Legendre transformation,

$$\mathcal{L}' = -DE + \mathcal{L} = -\frac{1}{2\pi\alpha' e^\phi}\sqrt{M(1 + e^{2\phi}D^2)} \tag{3.7}$$

by eliminating E using (3.6). The displacement D counts the number of fundamental strings immersed and it is quantized to take an integer value, which we denote as p. Using the Einstein frame metric, the above Lagrangian density may be written as

$$\mathcal{L} = -\frac{1}{2\pi\alpha'} \times \int d\sigma \sqrt{-\det(g^E_{\mu\nu}\partial_a X^\mu \partial_b X^\nu)}\sqrt{p^2 e^\phi + q^2 e^{-\phi}}, \tag{3.8}$$

where we also introduced an integer q counting the number of D strings. The derivation of the (p, q) string action here is only for $q = 1$, but we generalize it for an arbitrary q. From the above action for the (p, q) string, the S duality of the IIB string theory is manifest. Namely, the above is invariant under the transformation,

$$g'_{E\mu\nu} = g_{E\mu\nu}, \qquad \phi' = -\phi, \qquad p \leftrightarrow q. \tag{3.9}$$

Note that in our dilaton-deformed solutions, the S duality corresponds to simply changing $k \to -k$ and $\phi_0 \to -\phi_0$. From this S duality transformation, it is clear that magnetic charges are confined for $k/\mu > 12$, as electrically charged quarks are confined if $k/\mu < -12$.

To see the details of the interaction and the phase structure, let us assume that the (p, q) string is static and choose the gauge $\tau = t$ and $\sigma = y$. We shall consider the case where the (p, q) string trajectory is independent of x_2 and x_3. The (p, q) string Lagrangian then becomes

$$\mathcal{L} = -\sqrt{\lambda}\int dy \sqrt{A(y)[B(y) + C(y)(dx/dy)^2]}, \tag{3.10}$$

where $\lambda = g^2_{YM}N$ is the t'Hooft coupling and

$$A(y) = (y-b)^{(1+3a)/4}(y+b)^{(1-3a)/4}\left[p^2 e^{\phi_0}\left(\frac{y-b}{y+b}\right)^{k/8b} + q^2 e^{-\phi_0}\left(\frac{y-b}{y+b}\right)^{-(k/8b)}\right],$$
$$B(y) = \frac{1}{16(y-b)(y+b)},$$
$$C(y) = (y-b)^{(1-a)/4}(y+b)^{(1+a)/4}. \tag{3.11}$$

The computation showing heavy quark confinement follows closely the one in Ref. [4]. The equation of motion,

$$\frac{d}{dy}\left(\frac{\sqrt{A}C dx/dy}{\sqrt{B + C(dx/dy)^2}}\right) = 0, \tag{3.12}$$

may be integrated once, and one gets

$$\frac{\sqrt{A}C dx/dy}{\sqrt{B + C(dx/dy)^2}} = \pm q^{-2}, \tag{3.13}$$

with an integration constant q^2. To understand the dynamical implication, we rewrite (3.13) in the form

$$\left(\frac{dy}{dx}\right)^2 + V(y) = 0, \tag{3.14}$$

with the potential

$$V(y) = \frac{C}{B}(1 - q^4AC). \tag{3.15}$$

This can be viewed as a particle moving in one dimension under the potential V, regarding the coordinate x as the "time."

The confinement occurs when the "particle" spends an arbitrarily large time when it approaches the turning point denoted by y_0. At the turning point, one has $dy/dx = 0$ and thus $V(y_0) = 0$, which implies that

$$q_0^4A(y_0)C(y_0) = 1, \tag{3.16}$$

for an appropriate choice of the integration constant $q = q_0$. The condition of spending arbitrarily large time is fulfilled if $V'(y_0) = 0$. This leads to

$$(AC)'|_{y=y_0} = 0, \tag{3.17}$$

where the condition $V(y_0) = 0$ is used.

Let us first consider the case of a (1, 0) string connecting the electrically charged quark/antiquark pair. In this case, the latter condition is solved by

$$y_0 = -ab - k/4. \tag{3.18}$$

For the existence of the solution in the range $y \in (b, \infty)$, one has to impose

$$y_0 - b = -ab - k/4 - b \equiv 2b\beta > 0, \tag{3.19}$$

which is equivalent to

$$\frac{k}{\mu} < -12. \tag{3.20}$$

Then $V(y_0) = 0$ is satisfied by choosing the integration constant q as

$$q_0^4 = \frac{1}{2b}\beta^\beta(1+\beta)^{-(1+\beta)}e^{-\phi_0}. \tag{3.21}$$

For small q, the separation between the quark/antiquark pair is of the order of q according to the IR/UV relation. The energy scale here is much higher than that of the confinement. Thus the quark/antiquark potential for sufficiently small separation is of Coulomb type as expected.

When $\beta > 0$ and q approaches q_0 from below, the string spends more and more time near the turning point $y \sim y_0$. The separation between the quark and antiquark becomes larger and larger as one sends q to q_0 from below, because the time spent near the turning point increases more and more.

In the limit $q \to q_0$, we can compute the tension of the string and the energy scale of confinement. The energy of the string is given by

$$E_s = \sqrt{\lambda}\int dy\sqrt{A[B + C(dx/dy)^2]} = \sqrt{\lambda}\int dx\sqrt{q^4A^2C^2}, \tag{3.22}$$

where we have used the equation of motion. The integral in fact diverges and one may regulate it by subtracting the self-energy of quark and antiquark.

Since $q_0^4A(y_0)C(y_0) = 1$ and the string stays near the turning point for most of the time, we find from (3.22) the tension of the confining string to be

$$\begin{aligned} T_{\rm QCD} &= \sqrt{\lambda}\sqrt{A(y_0)C(y_0)} = \sqrt{\lambda}q_0^{-2} \\ &= \sqrt{\lambda\mu}\frac{(1+\beta)^{(1+\beta)/2}}{\sqrt{a}\beta^{\beta/2}}e^{\phi_0/2}. \end{aligned} \tag{3.23}$$

This sets the scale of confinement. Our result agrees with the previously calculated one in the $\mu \to 0$ limit [23].

B. Screening

In the analysis of Ref. [4], the region of $k/\mu > -12$ corresponding to $\beta < 0$ was not carefully analyzed because the paper was mainly concerned only about the existence of confinement phenomena. We would like to show that this region corresponds, in fact, to the screening phase. In this region the potential V always has a turning point beyond which the singularity is located. This feature of inaccessibility to the singularity is true for all values of the integration constant q^2. One may ask the following. May an infinitely large separated quark/antiquark pair be connected through this string solution, by adjusting the integration constant q? The answer turns out to be no. Namely, there is an upper limit on the separation length between the quark and antiquark pair in the above solutions of string configuration.

To show this, let us first note that the separation length is given by

$$L = 2\int_{y_\star}^\infty dy\frac{1}{\sqrt{-V}} = 2\int_{y_\star}^\infty dy\frac{\sqrt{B}}{\sqrt{C}\sqrt{q^4AC-1}}, \tag{3.24}$$

where $y_\star$ is the turning point. For small q satisfying the condition $e^{\phi_0}bq^4 \ll 1$, the turning point $y_\star \sim 1/(e^{\phi_0}q^4)$ is much larger than b and, thus, the potential V may well be approximated by

$$V \sim 16y^{5/2}(1 - e^{\phi_0}q^4y). \tag{3.25}$$

Then the separation is approximately given by

$$L \sim \frac{e^{\phi_0/4}q}{2}\int_1^\infty\frac{dt}{t^{5/4}}\frac{1}{\sqrt{t-1}}. \tag{3.26}$$

This is the case of sufficiently small separation and the expression for the separation is essentially the same as the one for the strings in the pure AdS case, because the strings are staying in the near boundary region of the asymptotically AdS space.

To study the upper limit on the separation distance, one should look at the large q behavior of L. When $e^{\phi_0} b q^4 \gg 1$, the contribution of the integral from infinity to $y - b = O(b)$ is of order $1/(e^{\phi_0/2} q^2)$, which is small. The turning point occurs in the regime $y - b \ll b$, and the contribution from near the turning point reads as

$$\delta L = 2 \int_{z_\star} \frac{dz}{4z^{(5-a)/4}(2b)^{(5+a)/4}} \frac{1}{\sqrt{e^{\phi_0} q^4 z^{|\beta|}(2b)^{1+\beta} - 1}}. \tag{3.27}$$

Thus we conclude that

$$\delta L \sim q^{-(3+a)/(2|\beta|)}, \tag{3.28}$$

which is negligible in the large q limit. Obviously, the intermediate region contributes only of order one to the integral. This shows that the separation has a maximum value for some q, which we denote by $L_{\max}$.

What really happens if the separation of the external quarks becomes larger than $L_{\max}$? In this case, the strings follow the trajectory of the trivial solution, $\frac{dx}{dy} = 0$. The strings from the boundary quarks and antiquarks are stretched straight toward the singularity without any change of x coordinate. Very near the singularity corresponding to IR regime of the dual field theory, the strings are joined by changing x coordinate. This situation is depicted in Fig. 1. One may worry about the part of the string very near the singularity. (This part describes the physics of the field lines in the extreme IR regime of the energy scale.) However, one may see that the contribution to the energy of this part is zero at any rate. To see this, let us first note that the energy integral of the configuration is given by

$$E_s = \sqrt{\lambda} \int \sqrt{ABdy^2 + ACdx^2}. \tag{3.29}$$

Since $dy = 0$ for the part of joining two different straight strings,

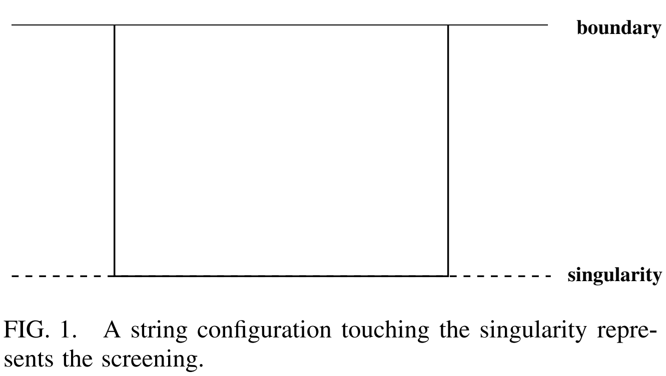

FIG. 1. A string configuration touching the singularity represents the screening.

$$E_{\rm joint} = \sqrt{\lambda} \int \sqrt{AC} dx = \sqrt{\lambda} L \frac{(y-b)^{|\beta|/2}}{(y+b)^{-(1+\beta)/2}} \Bigg|_{y=b} = 0, \tag{3.30}$$

where in the last equality we have used the fact $\beta < 0$. Thus, the boundary condition at the singularity does not matter and the result of vanishing string energy has its own validity despite the singularity.

From the above discussion, the nature of the interaction is clear. The charges are not confined for $k/\mu > -12$. When the separation becomes larger than $L_{\max}$, the quark/antiquark potential diminishes, representing the phenomenon of screening. Therefore we conclude that the regime of $k/\mu > -12$ corresponds to a screening phase.

C. Confinement vs screening of heavy quarks

From the discussion above, one may expect that the system shows the phase transition as one varies the parameter k/μ by adjusting k or μ. At $k/\mu = -12$, the system undergoes the phase transition between a confinement phase and a screening phase for heavy quarks. The appearance of the tension of the electric-flux string in the confining phase may serve as an order parameter. At the critical point of the $k/\mu = -12$ or $\beta \to 0$ limit, the electric-flux string tension takes a finite value of

$$T_{\rm QCD} = 5\sqrt{\lambda\mu} e^{\phi_0/2}. \tag{3.31}$$

Namely, the tension jumps to the finite value at the phase transition. This may be understood as follows. Because of the Gauss law, the total electric flux around charges should remain preserved irrespective of confinement or screening. Then, when quarks are confined by the transition, the electric-flux lines form a linear tube and the finite tension simply comes from the existing energy of the field profile. Thus, the tension should start with a finite value.

D. Doubly screening phase

In this subsection, let us consider the response of D-strings describing the interaction between magnetically charged objects. From the Lagrangian in (3.8), the dynamics of $(1, 0)$ strings for a given k is mapped into $(0, 1)$ strings with $-k$. Thus, without further computation, one may see that magnetically charged quarks are confined when $k/\mu > 12$ and screened otherwise.

The full phase structure is drawn in Fig. 2. Region I with $k/\mu < -12$ describes the phase where the electrically charged quarks are confined. Then the magnetically charged objects should be screened, which is indeed the case as discussed above. Region III with $k/\mu > 12$ corresponds to the phase where magnetic charges are confined while quarks are screened. This corresponds to the S dual of the region I.

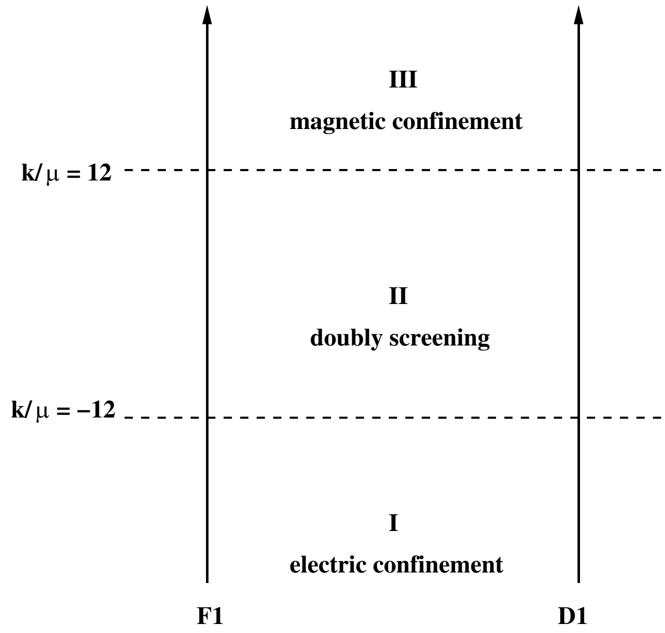

FIG. 2. The full phase diagram.

Region II with $-12 < k/\mu < 12$ describes the phase where both the quarks and magnetic charges are screened, which we call "doubly screening phase." As far as we know, there were no such examples previously where both the electric and magnetic charges are screened. Presumably this phase structure is possible due to the S-duality symmetry of the underlying $\mathcal{N} = 4$ SYM theory.

IV. SPONTANEOUS CHIRAL SYMMETRY BREAKING: D7 PROBE ANALYSIS

A. Generalities

A few N_f D7-branes parallel to a stack of large number N of D3-branes introduces N_f fundamental $\mathcal{N} = 2$ hypermultiplets in the low energy gauge dynamics on D3-branes. For N large enough, $N \gg N_f$, back reaction of the D7-branes to the near horizon limit of supergravity background may be irrelevant, and the low energy gauge dynamics on D3-branes with $\mathcal{N} = 2$ fundamental hypermultiplets is supposed to be dual to $AdS_5 \times S^5$ with probe D7-branes [5]. The open string dynamics on the probe D7-branes corresponds to the dynamics of $\mathcal{N} = 2$ hypermultiplets in the "ambient" $\mathcal{N} = 4$ SYM theory. This is because the gauge theory interpretation of the probe approximation is to take the quenched approximation neglecting effects of hypermultiplets to the dynamics of $\mathcal{N} = 4$ SU(N) SYM with large N. However, it should be noted that these probe $\mathcal{N} = 2$ fundamental hypermultiplets experience full dynamics of $\mathcal{N} = 4$ SYM theory.

In the D-brane picture in flat ten-dimensional spacetime, let the D3-brane world volume span along {0123}, and the D7-brane along {012 345 67} directions. The distance between D3 and D7 in the transverse {89} space gives rise to a mass term in the Lagrangian for hypermultiplets. More specifically, the asymptotic value of $w = \sqrt{x_8^2 + x_9^2}$ for large $\rho^2 = x_4^2 + x_5^2 + x_6^2 + x_7^2$ of the D7 world volume corresponds to the bare mass m_f of the hypermultiplets.[2] For the maximally supersymmetric configuration, w is constant on D7 [for instance, D7 lies at constant $(x_8, x_9) = (w_0, 0)$ while D3 is sitting at the origin]. However, for nonsupersymmetric states such as those we are considering, w is generically a varying function of ρ.

In the supergravity picture, it is not difficult to identify the bare mass for the hypermultiplets in the framework of AdS/CFT correspondence. The maximally supersymmetric supergravity background in the near horizon limit is $AdS_5 \times S^5$ with the string frame metric

$$ds^2 = \frac{1}{f(r)}\left(\sum_{\mu=0}^{3} dx_\mu dx^\mu\right) + f(r)\left(\sum_{i=4}^{9} dx^i dx^i\right) = \frac{r^2}{l^2}\left(\sum_{\mu=0}^{3} dx_\mu dx^\mu\right) + \frac{l^2}{r^2}dr^2 + l^2 d\Omega_5^2, \qquad (4.1)$$

where $r^2 = \sum_{i=4}^{9} x_i^2 = \rho^2 + w^2$ and $f(r) = \frac{4\pi N g_s}{r^2} = \frac{l^2}{r^2}$ is the warping factor. The world-volume profile of the probe D7-brane in this background is simply given by identifying the flat D-brane picture coordinate $\{x^M\}$ ($M = 0, \ldots, 9$) with the coordinate $\{x^M\}$ in (4.1). For example, maximally supersymmetric D7 lying on the plane $(x_8, x_9) = (w_0, 0)$ fills the AdS_5 part of $\{x^\mu, r\}$ for $w_0 \le r < \infty$, in addition to wrapping the S^3 cycle in the S^5. The wrapped S^3 is defined by $x_4^2 + x_5^2 + x_6^2 + x_7^2 = \rho^2 = r^2 - w_0^2$ and it vanishes at $r = w_0$, ensuring a smooth D7 world volume in $AdS_5 \times S^5$. Note that the D7-brane is absent in energy scales below $r = w_0$; this is consistent with the field theory expectation that we should not find any hypermultiplet below its mass scale $m_f = w_0$. Moreover, w_0 is a free parameter representing a family of the D7 profiles; this gives us the freedom of changing the bare mass of hypermultiplets. An especially interesting limit would be the chiral symmetry limit[3] of $m_f = w_0 = 0$.

For the nonsupersymmetric backgrounds of our interest, it is possible to identify suitable coordinate $\{x^M\}$ that has a natural interpretation of flat coordinate in the D-brane picture. The string frame metric in this coordinate is[4]

[2]We use the notation $x_i = x^i$ for the spatial coordinates.
[3]By chiral symmetry, we mean a chiral U(1) symmetry which we discuss more in Sec. IV C.
[4]From now on, we set $\phi_0 = 0$ because it plays no special role except the trivial overall scaling.

$$ds^2 = \left(\frac{r^4-1}{r^4+1}\right)^{k/(8b)} \left\{ -(r^2 - r^{-2})^{(1+3a)/2} \times (r^2 + r^{-2})^{(1-3a)/2} dx^0 dx^0 + \frac{1}{r^2}\left(\sum_{i=4}^{9} dx^i dx^i\right) + (r^2 - r^{-2})^{(1-a)/2}(r^2 + r^{-2})^{(1+a)/2} d\vec{x}\cdot d\vec{x} \right\}, \quad (4.2)$$

where $r^2 = (x_4^2 + x_5^2 + x_6^2 + x_7^2) + (x_8^2 + x_9^2) \equiv \rho^2 + w^2$ as before. The above metric is obtained from (2.1) by combining dy^2 and $d\Omega_5^2$ with a change of variable $y = b\cosh(2s)$ and $r^2 = e^s$. The $R^{1,3}$ coordinate $\{x^0, \vec{x}\}$ has also been rescaled appropriately. The singularity is now positioned at $r^2 = \rho^2 + w^2 = 1$. The probe D7 world volume covers $x^0, \ldots, x^7$ and its transverse position is given by $(x_8, x_9) = [w(\rho), 0]$ without loss of generality. Hence, it fills the (approximate) AdS_5 space over $w(0) \le r < \infty$ (equivalently $0 \le \rho < \infty$), and the wrapped S^3 cycle in S^5 shrinks to zero at $r = w(0)$. As we have explained in the previous paragraph, the field theory situation dual to this configuration is a quantum state of $\mathcal{N} = 4$ SYM with $\mathcal{N} = 2$ fundamental hypermultiplet in quenched approximation, whose bare mass is identified with $m_f = w(\infty)$. A family of profiles with varying $w(\infty)$ allows us to tune the bare mass and, in lucky cases, to get the chiral limit.

According to the AdS/CFT proposal, however, the bare mass is not the only information we can extract from $w(\rho)$. Viewing $w(\rho)$ as an effective scalar field in the AdS_5, its asymptotic value at $\rho \to \infty$ couples to some scalar operator in the field theory. We have actually identified this operator; we have seen that $w(\infty)$ couples to the mass operator of the fundamental $\mathcal{N} = 2$ hypermultiplet,

$$\begin{aligned}\delta\mathcal{L}_{\rm SYM} &= w(\infty)\int d^2\theta \tilde{Q}_f Q_f - (\text{H.c.}) \\ &\sim w(\infty)(\tilde{q}_L^f q_L^f + \text{H.c.}) + (\text{bosonic}) \\ &= w(\infty)\bar{q}_D^f q_D^f + (\text{bosonic}),\end{aligned} \quad (4.3)$$

where we have introduced Dirac fermions,

$$q_D^f = \begin{pmatrix} q_L^f \\ i\sigma^2(\tilde{q}_L^f)^* \end{pmatrix}. \quad (4.4)$$

The fermion mass operator is of dimension 3, and the AdS/CFT dictionary tells us that its expectation value is encoded in the coefficient of subleading $\sim\frac{1}{\rho^2}$ behavior of $w(\rho)$ in $\rho \to \infty$. Note that the bosonic piece in the above has a vanishing expectation value in the symmetric phase. As we have the freedom of choosing the bare mass $w(\infty)$, it is possible to see interesting dependence of the bi-fermion mass operator condensate on the bare mass parameter. In the chiral limit, we may discuss about the occurrence of spontaneous chiral symmetry breaking.

B. A subtlety

In this subsection, we show that the expectation value of the fermion mass operator for the hypermultiplet is precisely given by the coefficient of subleading $\frac{1}{\rho^2}$ in $w(\rho)$ as $\rho \to \infty$. This is equivalent to a subtle question of choosing correct field variable, from whose asymptotic behavior we should read off the expectation value of the field theory operator. This is a relevant caveat to care about because $w(\rho)$ has a highly nonstandard form of action functional in AdS_5 derived from the D7 DBI action. The relevant part of the D7-brane DBI action is

$$S_{\rm D7} = \tau_7 \int d^8\xi e^{-\phi}\sqrt{-\det\left(\frac{\partial x^M}{\partial \xi^\mu}\frac{\partial x^N}{\partial \xi^\nu} G_{MN}^{(10)}\right)}, \quad (4.5)$$

where $G^{(10)}$ is the ten-dimensional metric of (4.2), and the dilaton profile is

$$e^{-\phi} = \left(\frac{r^4+1}{r^4-1}\right)^{k/(4b)}. \quad (4.6)$$

Choosing the gauge $\xi^i = x^i$ $(i = 0, \ldots, 7)$, and $(x^8, x^9) = [w(\rho), 0]$, we obtain the effective action for $w(\rho)$,

$$S \sim \int d^4x \int_0^\infty d\rho \rho^3 Z(\rho^2 + w^2)\sqrt{1 + \left(\frac{dw}{d\rho}\right)^2}, \quad (4.7)$$

where $Z(x)$ is a complicated function which goes to unity for large x;

$$Z(x) = \left(1 - \frac{1}{x^4}\right)\left(\frac{x^2-1}{x^2+1}\right)^{k/(4b)}. \quad (4.8)$$

Another fact that will be important for us later is $Z'(x) \sim \frac{1}{x^3}$ for large x.

For a smooth D7 embedding, we need to impose the boundary condition, $\frac{dw}{d\rho}(0) = 0$. In the asymptotic $\rho \to \infty$ region, Z goes to unity and the solution of the equation of motion behaves as

$$w(\rho) \sim m + \frac{C}{\rho^2}. \quad (4.9)$$

Naively, the ρ integration in (4.7) diverges because, for large ρ,

$$\rho^3 Z(\rho^2 + w^2)\sqrt{1 + \left(\frac{dw}{d\rho}\right)^2} \sim \left(\rho^3 - \frac{k}{2b}\frac{1}{\rho}\right) + \left(\frac{k}{2b}(2m^2 + 1) + 2C^2\right) \times \frac{1}{\rho^3} + \cdots, \quad (4.10)$$

and we need a suitable regularization procedure. However, what we are interested in will be variations of the value of (4.7) under changing $w(\infty) = m$ and, for this purpose, it is enough to regularize (4.7) by subtracting the value of it at some fixed reference solution $w_0(\rho)$;

$$S_{\rm R} \equiv \int d^4x \int_0^\infty d\rho \rho^3 \left[Z(\rho^2 + w^2)\sqrt{1 + \left(\frac{dw}{d\rho}\right)^2} - Z(\rho^2 + w_0^2)\sqrt{1 + \left(\frac{dw_0}{d\rho}\right)^2} \right], \quad (4.11)$$

which is now convergent due to (4.10). The standard AdS/CFT correspondence is then

$$\exp(iS_{\rm R}[m]) = \left\langle \exp\left(i \int d^4x m \bar{q}_D^f q_D^f \right) \right\rangle, \quad (4.12)$$

where $S_{\rm R}[m]$ is the above regularized action evaluated for the solution of the equation of motion with $w(\infty) = m$. Hence, we have

$$\frac{\delta S_{\rm R}[m]}{\delta m} = \int d^4x \langle \bar{q}_D^f q_D^f \rangle. \quad (4.13)$$

In fact, it is not difficult to calculate the left-hand side of the above relation. Suppose that $w + \delta w$ is the solution of the equation of motion with $(w + \delta w)(\infty) = m + \delta m$ for infinitesimal δm. The variation of $S_{\rm R}[m]$ is

$$\delta S_{\rm R}[m] = \int d^4x \int_0^\infty d\rho \rho^3 \left[2wZ'(\rho^2 + w^2) \times \sqrt{1 + \left(\frac{dw}{d\rho}\right)^2}\delta w + Z(\rho^2 + w^2)\frac{\frac{dw}{d\rho}\frac{d\delta w}{d\rho}}{\sqrt{1 + (\frac{dw}{d\rho})^2}} \right]. \quad (4.14)$$

The convergence of this expression may easily be seen from the property $Z'(x) \sim \frac{1}{x^3}$, and we are allowed to perform integration by part for the second term. The resulting integrand which is proportional to δw vanishes because $w(\rho)$ satisfies the equation of motion, and the surviving surface contribution at $\rho = \infty$ is

$$\int d^4x \lim_{\rho\to\infty} \left[\rho^3 Z(\rho^2 + w^2)\frac{\frac{dw}{d\rho}}{\sqrt{1 + (\frac{dw}{d\rho})^2}}\delta w \right] = \int d^4x(-2C\delta m), \quad (4.15)$$

using $\rho^3 \frac{dw}{d\rho} \sim -2C$ and $\delta w \sim \delta m$ for large ρ. Comparing with (4.13), we thus have

$$\langle \bar{q}_D^f q_D^f \rangle = -2C. \quad (4.16)$$

C. The chiral symmetry $\mathbf{U(1)}_c$

In this subsection, we discuss more about the chiral U(1) symmetry of the gauge theory we are considering. In fact, it is clear in the D-brane picture of the D3-D7 system that there must be a global U(1) symmetry which corresponds to the rotation of D7's position in the transverse (x_8, x_9) plane. [Recall that the D3-branes are aligned along {0123}, while D7-branes are along {01234567}. Let us put N D3-branes at $(x_8, x_9) = (0, 0)$, and N_f D7-branes at $(x_8, x_9) = (w_1, w_2)$.] The distance between D3 and D7 introduces a mass term for $\mathcal{N} = 2$ fundamental hypermultiplets,

$$(w_1 + iw_2)\int d^2\theta \tilde{Q}_f Q^f + \text{H.c.} \sim (w_1 + iw_2)\bar{q}_D^f q_D^f + (\text{bosonic}). \quad (4.17)$$

The rotation $(w_1 + iw_2) \to e^{2i\alpha}(w_1 + iw_2)$ of the D7-brane's position does not change anything on the D3-brane world volume in the D-brane picture, and hence there should exist a compensating chiral rotation which is a global symmetry of the gauge theory. We should also expect the same chiral symmetry in the nonsupersymmetric backgrounds of our interest, because the solutions preserve the SO(6) symmetry of S^5, which includes (x_8, x_9)-plane rotation as a subgroup.

Looking at the superpotential term,

$$\int d^2\theta \tilde{Q}_f Z Q^f + \int d^2\theta \text{tr}(Z[X, Y]), \quad (4.18)$$

where X, Y and Z are adjoint chiral superfields in $\mathcal{N} = 4$ SYM theory ($Z = X_8 + iX_9$), it is easy to realize that this symmetry is a R symmetry. From the D-brane picture, we should assign charges 1 and 0 to Z and X, Y, respectively. Then $d^2\theta$ has charge -1 (or θ_α has charge $\frac{1}{2}$), and this forces us to take charge 0 for $\tilde{Q}_f$ and Q_f. The reason behind the R symmetry is clear; when we rotate D7-branes in the (x^8, x^9) plane, the corresponding ten-dimensional type IIB Killing spinor of the D3-D7 system with eight real components also rotates accordingly.

Note that $\tilde{q}_f$ and q_f both have charge $-\frac{1}{2}$ under this $\text{U}(1)_c$. In terms of the Dirac spinor q_D^f, the charge is $\frac{1}{2}\gamma^5 = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, and a nonvanishing expectation value of $\langle \bar{q}_D^f q_D^f \rangle$ will break this chiral symmetry spontaneously. Although $\text{U}(1)_c$ has a quantum anomaly which is proportional to $-N_f \times C_2(F)$, where $C_2(F)$ is the Casimir invariant of fundamental representation, it is negligible in the large N, t'Hooft limit [24,25]. From the D-brane picture, it comes as a surprise that there is an anomaly for $\text{U}(1)_c$ in the effective field theory on D3, because this is a simple coordinate rotation in the (x^8, x^9) plane. The resolution of the puzzle lies in the fact that D7-brane sources a nontrivial profile of RR-scalar C_0 around it, such that rotation in the (x^8, x^9) plane induces a shift monodromy of C_0 field which is exactly proportional to the number of D7-branes, N_f [13]. The RR-scalar C_0, however, couples to the D3-branes by

$$C_0 \int \text{tr}(F \wedge F). \quad (4.19)$$

Therefore, the shift of the θ parameter due to the field theory anomaly of $U(1)_c$ rotation is precisely canceled by the shift monodromy of the bulk field C_0, and the total anomaly is absent in the whole system. This may well be called an example of anomaly inflow[5] (see also [26] for a related discussion).

D. Separation between SχSB and confinement

The equation of motion for $w(\rho)$ from the effective action (4.7) is somewhat complicated, and does not seem to have any analytic solutions. We have performed numerical analysis for solving the equation of motion, and have identified the asymptotic data, m and C, for each solution. In the previous subsections, we have seen that m corresponds to the bare mass of $\mathcal{N} = 2$ fundamental hypermultiplets, while C is directly proportional to the condensate of the bi-fermion mass operator for the hypermultiplets. Hence, a solution whose asymptotic behavior is characterized by $m = 0$, but $C \neq 0$, signals that chiral symmetry is spontaneously broken in the gauge theory living on the boundary.

The effective action (4.7) for the probe D7-brane has a parameter

$$\frac{k}{b} = 2\left(\frac{k}{\mu}\right)\left(1 + \frac{k^2}{6\mu^2}\right)^{-(1/2)}, \tag{4.20}$$

representing a family of bulk type IIB supergravity backgrounds, which in turn correspond to a family of homogeneous quantum states of $\mathcal{N} = 4$ SYM theory with $\mathcal{N} = 2$ hypermultiplets in AdS/CFT correspondence. In Sec. II, we analyzed "phases" of these states and observed that their phase structure is sensitive to the value of k/μ (equivalently, k/b). Specifically, for $k/\mu < -12$ ($k/b < -4.8$), we have an electric confinement, while for $k/\mu > +12$ ($k/b > +4.8$), magnetically charged objects are confined. An interesting phase seems to happen for $-12 < k/\mu < +12$ ($-4.8 < k/b < +4.8$), in which both electric charges and magnetic charges are screened.

The existence of spontaneous chiral symmetry breaking (SχSB), that is whether there is a solution with $m = 0$ but $C \neq 0$ in the bulk, also depends on the parameter k/μ (or k/b). Our numerical study shows that there is such a solution for $k/\mu < -2.97$ (or $k/b < -3.78$). As an exemplar case, Fig. 3 is describing solutions with varying $w(\infty) = m$ when $k/\mu = -7$ (or $k/b = -4.62$). It is evident from the figure that the value of C does not vanish for the solution with $m = 0$. What happens when $k/\mu > -2.97$ (or $k/b > -3.78$) is that solutions start to meet the singularity at $\rho^2 + w^2 = 1$ as we lower the value of m. We thus cannot extract useful information for these cases.

The above analysis has a profound implication. For $-12 < k/\mu < -2.97$ (or $-4.8 < k/b < -3.78$), the corresponding quantum states of the gauge theory are in the screening phase, while massless fermions of fundamental representation form a nonvanishing bi-fermion condensation. This contradicts a prevailing lore that bi-fermion condensation would require a confining potential between two charges. On the basis of the AdS/CFT correspondence for probe D7-branes, we thus claim to have provided the first example of separation between spontaneous chiral symmetry breaking and confinement.

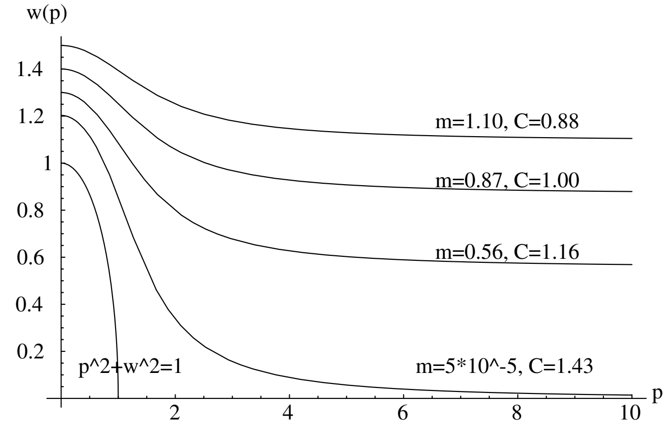

FIG. 3. Numerical solutions for $w(\rho)$ when $k/\mu = -7$ (or $k/b = -4.62$). It is clear that the solution with $m = 0$, but $C \neq 0$, exists. The line $\rho^2 + w^2 = 1$ is the position of the singularity.

V. CONCLUSION

In this work, we have considered dilatonic deformations of AdS geometry that are dual to some quantum states of the $\mathcal{N} = 4$ SYM theory with nonvanishing gluon condensation, k, as well as homogeneous energy density μ. As varying the parameter k/μ, we have identified the phases of these states by studying the interaction between quarks/antiquarks and also between magnetically charged objects. The regime $k/\mu < -12$ is electrically confining, where quarks are confined and magnetic charges are screened. The opposite regime of $k/\mu > 12$ corresponds to the S-dual transformed phase, where magnetic charges are confined. For $-12 < k/\mu < 12$, interestingly both fundamental quarks as well as magnetic charges are , whose phase we call doubly screening phase.

We then introduced the probe D7-branes and studied possible spontaneous chiral symmetry breaking. The $\mathcal{N} = 2$ fundamental hypermultiplet arising from the D3-D7 strings possesses the classical chiral $U(1)_c$, which suffers from quantum anomaly. However, we are working in the large N limit of D3-branes and the effect of the anomaly may be ignored. By studying the D7 moduli dual to the fermion mass operator of the hypermultiplet, we have shown that there is a nonvanishing bi-fermion condensate in the zero-mass limit, leading to the spontaneous breaking of the chiral symmetry. We demonstrated that this happens even within the screening phase with no confinement.

[5]We thank Jaemo Park for a discussion on this.

It is our hope that the conclusions we have drawn from analyzing these states of $\mathcal{N} = 4$ SYM theory reflect some truth of generic confining gauge theories. At least, it seems to suggest that spontaneous chiral symmetry breaking does not necessarily require confinement.

ACKNOWLEDGMENTS

We would like to thank Kimyeong Lee, Jaemo Park, Soo-Jong Rey, and Hyun Seok Yang for helpful discussions. We also thank other participants of the "AdS/CFT and Quantum Chromodynamics" (October 28–30, 2004), Hanyang University, Korea, for inspiring discussions. D. B. is supported in part by KOSEF ABRL R14-2003-012-01002-0 and KOSEF R01-2003-000-10319-0. D. B. also likes to thank the warm hospitality of KIAS where part of this work was done. H.-U. Y. is partly supported by Grant No. R01-2003-000-10391-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

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