Electron-electron interaction in a Maxwell-Chern-Simons model with a purely spacelike Lorentz-violating background

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One considers a planar Maxwell-Chern-Simons electrodynamics in the presence of a purely spacelike Lorentz-violating background. Once the Dirac sector is properly introduced and coupled to the scalar and the gauge fields, the electron-electron interaction is evaluated as the Fourier transform of the Möller scattering amplitude (derived in the nonrelativistic limit). The associated Fourier integrations can not be exactly carried out, but the interaction potential is obtained as a first order solution in v^2/s^2. It is then observed that the scalar potential presents a logarithmic attractive (repulsive) behavior near (far from) the origin. Concerning the gauge potential, it is composed of the pure MCS interaction corrected by background contributions, also responsible for its anisotropic character. It is also verified that such corrections may turn the gauge potential attractive for some parameter values. Such attractiveness remains even in the presence of the centrifugal barrier and gauge invariant $A \cdot A$ term, which constitutes a necessary condition for yielding electron-electron pairing.

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I. INTRODUCTION

In the last few years, Lorentz-violating theories have been the focus of great interest and investigation [1–7], [10–12]. Despite the intensive activity proposing and discussing the consequences of a Lorentz-violating electrodynamics, some experimental data and theoretical considerations indicate stringent limits on the parameters responsible for such a breaking [2,3]. These evidences put the Lorentz-violation as a negligible effect in a factual $(1 + 3)$-dimensional electrodynamics, which raises the question about the feasibility of observing this effect in a lower dimension system or in another environment distinct from the usual high-energy domain in which this matter has been generally regarded so far.

Condensed Matter Systems (CMS) are low-energy systems sometimes endowed with spatial anisotropy which might constitute a nice environment to study Lorentz-violation and to observe correlated effects. Indeed, although Lorentz covariance is not defined in CMS, Galileo covariance holds as a genuine symmetry in such systems (a least for the cases endowed with isotropy). Keeping in mind that a CMS may be addressed as the low-energy limit of a relativistic model, there follows a straightforward correspondence between the breakdown of Lorentz and Galileo symmetries, in the sense that a Galileo's symmetry violating CMS may have as counterpart a relativistic system endowed with the breaking of Lorentz covariance. Considering the validity of this correspondence, it turns out that an anisotropic CMS could be addressed as the low-energy limit of a relativistic model in the presence of a spacelike Lorentz-violating background.

The attainment of an attractive electron-electron (e^-e^-) potential in the context of a planar model incorporating Lorentz-violation is a point that could set up a first connection between such theoretical models and condensed matter physics. Theoretical planar models able to provide attractive e^-e^- interaction potentials may constitute a suitable framework to deal with electronic pairing, a fundamental characteristic of superconducting systems. Historically, the Maxwell-Chern-Simons theories [11] were addressed in the beginning of 1990s as a theoretical alternative to accomplish this objective, without success. Actually, it is known that the MCS-Proca models [12] may better provide an attractive interaction due to the action of the scalar intermediation played by the Higgs sector. Another well defined feature of a planar superconductor concerns the symmetry of the order parameter (standing for the Cooper pair), which is described in terms of a spatially anisotropic d-wave. Certainly, a field theory model able to accounting for a spatially anisotropic electronic pairing must first provide an anisotropic e^-e^- interaction.

The investigation of the e^-e^- interaction can be suitably considered in the context of a Lorentz-violating planar framework. In fact, in a very recent paper [8], the low-energy Möller interaction potential was carried out for the case of a planar electrodynamics [4] incorporating a purely timelike background. With the inclusion of the Dirac sector, the low-energy Möller scattering amplitude (adopted as the appropriate tool to analyze the nonrelativistic electron-electron interaction) was carried out. The interaction potential, obtained from the evaluation of exact Fourier transforms, revealed to be composed of a scalar and a gauge contributions. It has been shown that the scalar potential exhibits a logarithmic attractive repulsive/behavior near/far from the origin, while the gauge potential is composed of the Maxwell-Chern-Simons (MCS) usual interaction [11] corrected by background-depending terms. These terms provide attractiveness (for some parameter values) even in the presence of the centrifugal barrier and the A^2 − gauge invariant term stemming from the Pauli equation, stating the possibility of achieving electron-electron pairing.

Having as main motivation the encouraging outcomes achieved in Ref. [8], in this work one aims at evaluating the electron-electron potential in the context of a Lorentz-violating planar electrodynamics endowed with a purely spacelike background, $v^{\mu} = (0, v)$. As this kind of background fixes a 2-direction in space, it naturally leads to an anisotropic behavior, a consequence of the directional dependence of the solutions in relation to the fixed background ($\mathbf{v}$). By determining such $e^{-}e^{-}$ potential, one can investigate two expected properties concerning the $e^{-}e^{-}$ interaction: attractiveness and anisotropy, which are relevant due its possible connection with superconducting systems. The procedure adopted here is the same one developed in Refs. [8,12]. One starts from the planar Lagrangian defined in Ref. [8], in which the Dirac sector has been already included. One first carries out the $e^{-}e^{-}$ Möller scattering amplitude, whose Fourier transform leads to the interaction potential (according to the Born approximation). The involved Fourier integrations are not exactly solvable, but algebraic solutions are obtained once the approximation $s^2 \gg \mathrm{v}^2$ is considered. The total interaction comprises the scalar and the gauge potentials, since the $e^{-}e^{-}$ interaction is both mediated by the massless scalar and the massive gauge fields. The scalar potential maintains the logarithmic behavior (asymptotically repulsive and attractive near the origin) of the purely timelike case, being different only in the presence of anisotropy. With respect to the gauge potential, it is given by a lengthy expression composed of the pure MCS interaction and many background-depending terms which imply manifest anisotropy. It is also possible to show that these corrections are able to turn this potential attractive for some values of the relevant parameters, which behavior remains even in the presence of the centrifugal barrier (l/mr^2) and the $A \cdot A$ gauge invariant term. Moreover, the total interaction (scalar and gauge potentials) may always be attractive with a suitable tuning of the coupling parameters. This outcome, which is a necessary condition to bring about the formation of electron-electron pairs, puts in evidence that this theoretical framework may be useful to describe electronic pairing in low-energy systems as far as the nonrelativistic approximation is valid. However, it is important to point out that such kind of procedure does not set up a theoretical model for addressing general condensed matter properties.

This paper is organized as follows. In Sec. II, one briefly presents the structure of reduced planar model (derived in Ref. [4]), here adopted as stating point. This model is supplemented by the Dirac field. In Sec. III, one presents the spinors which fulfill the two-dimensional Dirac equation, used to evaluate the Möller scattering amplitude associated with the Yukawa and the gauge intermediations. The corresponding interaction potentials are carried out, and the results are discussed. In Sec. IV, one concludes with the final remarks and prospects.

II. THE PLANAR LORENTZ-VIOLATING MODEL

The starting point is the planar Lagrangian obtained from the dimensional reduction of the Carroll-Field-Jackiw (CFJ) electrodynamics [4], which consists in a Maxwell-Chern-Simons electrodynamics coupled to a massless Klein-Gordon field (φ) and to a fixed 3-background vector (v^{μ}) through a Lorentz-violating Chern-Simons-like mixed term, derived from the dimensional reduction of the Carroll-Field-Jackiw model [5]. One then considers the additional presence of a fermion field (ψ):

$$\mathcal{L}_{1+2} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{s}{2}\epsilon_{\mu\nu\kappa}A^{\mu}\partial^{\nu}A^{\kappa} - \frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi + \varphi\epsilon_{\mu\nu\kappa}v^{\mu}\partial^{\nu}A^{\kappa} - \frac{1}{2\alpha}(\partial_{\mu}A^{\mu})^2 + \overline{\psi}(i\not{D} - m_e)\psi - y\varphi(\overline{\psi}\psi). \tag{1}$$

Here, the covariant derivative, $\not{D}\psi \equiv (\not{\partial} + ie_3\not{A})\psi$, states the minimal coupling, whereas the term $\varphi(\overline{\psi}\psi)$ reflects the Yukawa coupling between the scalar and fermion fields, with y being the constant that measures the strength of the electron-phonon coupling. This latter term usually appears in some field theories endowed with a four-fermion interaction $(\overline{\psi}\psi)^2$ which, by the action of a Hubbard-Stratonovich transformation [6], is turned into a typical electron-phonon coupling, present in BCS-like Lagrangians. The mass dimensions of the fields and parameters are the following: $[\varphi] = [A^{\mu}] = 1/2, [\psi] = 1, [s] = [v^{\mu}] = 1, [e_3] = [y] = 1/2$. One then notes that the coupling constants, e_3, y, both exhibit $[\mathrm{mass}]^{1/2}$ dimension, a usual result in (1 + 2) dimensions. In Eq. (1), the first two terms correspond to the Maxwell-Chern-Simons sector, whereas the term $(\partial_{\mu}A^{\mu})^2$ is responsible only for the gauge-fixing in this field model. Finally, one points out that the knowledge of the propagators[1] evaluated in Ref. [4] is essential to the calculations carried out in this work.

[1]The gauge propagator is given by: $\langle A^{\mu}(k)A^{\nu}(k)\rangle = i\{-\frac{1}{k^2-s^2}\theta^{\mu\nu} - \frac{\alpha(k^2-s^2)\boxtimes(k)+s^2(v.k)^2}{k^2(k^2-s^2)\boxtimes(k)}\omega^{\mu\nu} - \frac{s}{k^2(k^2-s^2)}S^{\mu\nu} + \frac{s^2}{(k^2-s^2)\boxtimes(k)}\Lambda^{\mu\nu} - \frac{1}{(k^2-s^2)\boxtimes(k)}T^{\mu}T^{\nu} + \frac{s}{(k^2-s^2)\boxtimes(k)}[Q^{\mu\nu} - Q^{\nu\mu}] + \frac{is^2(v.k)}{k^2(k^2-s^2)\boxtimes(k)}[\Sigma^{\mu\nu} + \Sigma^{\nu\mu}] - \frac{is(v.k)}{k^2(k^2-s^2)\boxtimes(k)}[\Phi^{\mu\nu} - \Phi^{\nu\mu}]\}$, whereas the scalar propagator is: $\langle\varphi\varphi\rangle = \frac{i}{\boxtimes(k)}[k^2 - s^2]$, where: $\boxtimes(k) = [k^4 - (s^2 - v\cdot v)k^2 - (v\cdot k)^2]$. The involved 2-rank tensors are defined as follows: $\theta_{\mu\nu} = \eta_{\mu\nu} - \omega_{\mu\nu}$, $\omega_{\mu\nu} = \partial_{\mu}\partial_{\nu}/\square$, $S_{\mu\nu} = \varepsilon_{\mu\kappa\nu}\partial^{\kappa}$, $Q_{\mu\nu} = v_{\mu}T_{\nu}, T_{\nu} = S_{\mu\nu}v^{\mu}$, $\Lambda_{\mu\nu} = v_{\mu}v_{\nu}$, $\Sigma_{\mu\nu} = v_{\mu}\partial_{\nu}$, $\Phi_{\mu\nu} = T_{\mu}\partial_{\nu}$.

III. THE MOLLER SCATTERING AMPLITUDE AND THE INTERACTION POTENTIAL

The two-particle interaction potential is given by the Fourier transform of the two-particle scattering amplitude in the low-energy limit (Born approximation). In the case of the nonrelativistic Möller scattering, one should consider only the t-channel (direct scattering) [13] even for indistinguishable electrons, since in this limit one recovers the classical notion of trajectory. From Eq. (1), there follow the Feynman rules for the interaction vertices: $V_{\psi\varphi\psi} = iy$; $V_{\psi A \psi} = ie_3\gamma^{\mu}$, so that the e^-e^- scattering amplitudes are written as:

$$-i\mathcal{M}_{\text{scalar}} = \overline{u}(p_1')(iy)u(p_1)[\langle\varphi\varphi\rangle]\overline{u}(p_2')(iy)u(p_2), \quad (2)$$

$$-i\mathcal{M}_{A} = \overline{u}(p_1')(ie_3\gamma^{\mu})u(p_1)[\langle A_{\mu}A_{\nu}\rangle]\overline{u}(p_2')(ie_3\gamma^{\nu})u(p_2), \quad (3)$$

with $\langle\varphi\varphi\rangle$ and $\langle A_{\mu}A_{\nu}\rangle$ being the scalar and photon propagators. Expressions (2) and (3) represent the scattering amplitudes for electrons of equal polarization mediated by the scalar and gauge particles, respectively. The spinors $u(p)$ stand for the positive-energy solution of the Dirac equation $(\not{p} - m)u(p) = 0$. The $\gamma-$ matrices, in turn, satisfy the $so(1,2)$ algebra, $[\gamma^{\mu}, \gamma^{\nu}] = 2i\epsilon^{\mu\nu\alpha}\gamma_{\alpha}$, and correspond to the (1+2)-dimensional representation of the Dirac matrices, that is, the Pauli ones: $\gamma^{\mu} = (\sigma_z, -i\sigma_x, i\sigma_y)$. Taking into account these definitions, one obtains the spinors,

$$u(p) = \frac{1}{\sqrt{N}}\begin{bmatrix} E+m \\ -ip_x - p_y \end{bmatrix}, \qquad \overline{u}(p) = \frac{1}{\sqrt{N}}[E+m \quad -ip_x + p_y], \quad (4)$$

which fulfill the normalization condition $\overline{u}(p)u(p) = 1$ whenever the constant $N = 2m(E+m)$ is adopted. The Möller scattering should be easily analyzed in the center of mass frame, where the momenta of the incoming and outgoing electrons are read at the form: $P_1^{\mu} = (E, p, 0)$, $P_2^{\mu} = (E, -p, 0)$, $P_1'^{\mu} = (E, p\cos\theta, p\sin\theta)$, $P_2'^{\mu} = (E, -p\cos\theta, -p\sin\theta)$, whereas θ is the scattering angle (in the CM frame). The 3-current components, $j^{\mu}(p) = \overline{u}(p')\gamma^{\mu}u(p)$, and the transfer 3-momentum arising from this convention are explicitly written in Ref. [8].

A. The scalar potential

Starting from the expression of the scalar propagator $\langle\varphi\varphi\rangle$ (see 1), considering the transfer momentum, $k^{\mu} = (0, \mathbf{k})$, and a purely spacelike background, $v^{\mu} = (0, \mathbf{v})$, the following scattering amplitude arises from Eq. (2):

$$\mathcal{M}_{\text{scalar}} = -y^2\frac{[\mathbf{k}^2 + s^2]}{\mathbf{k}^2[\mathbf{k}^2 + s^2 + \mathrm{v}^2\sin^2\alpha]}, \quad (5)$$

where α is the angle defined by the vectors $\mathbf{v}$ and $\mathbf{k}$. Taking into account the Born approximation, the potential associated with the Yukawa interaction reads as,

$$V_{\text{scalar}}(r) = -\frac{y^2}{(2\pi)^2}\int e^{ik.r}\frac{[\mathbf{k}^2 + s^2]}{\mathbf{k}^2[\mathbf{k}^2 + s^2 + \mathrm{v}^2\sin^2\alpha]}d^2k. \quad (6)$$

Such Fourier integration can not be exactly computed. However, this integration may be solved in the regime in which $s^2 \gg \mathrm{v}^2$. As far as this condition holds, the following expansion,

$$\frac{[\mathbf{k}^2 + s^2]}{\mathbf{k}^2[\mathbf{k}^2 + s^2 + \mathrm{v}^2\sin^2\alpha]} \simeq \frac{1}{\mathbf{k}^2} - \frac{\mathrm{v}^2\sin^2\alpha}{\mathbf{k}^2[\mathbf{k}^2 + s^2]}, \quad (7)$$

is valid as a first order approximation (in v^2/s^2). In order to solve Eq. (6), two other angles are of interest: φ and β - defined by the relations $\cos\varphi = \mathbf{r}\cdot\mathbf{k}/rk$, $\cos\beta = \mathbf{r}\cdot\mathbf{v}/r\mathrm{v}$, respectively. While the background vector, $\mathbf{v}$, sets up a fixed direction in space, the coordinate vector, $\mathbf{r}$, defines the position where the potentials are to be measured; so, β is the (fixed) angle that indicates the directional dependence of the fields in relation to the background direction. Being confined into the plane, these angles satisfy a simple relation: $\alpha = \varphi - \beta$, whose consideration leads to $\sin^2\alpha = c_2 + c_1\cos^2\varphi + c_3\sin 2\varphi$, with $c_1 = (1 - 2\cos^2\beta)$, $c_2 = \cos^2\beta$, $c_3 = -(\sin 2\beta)/2$. This expression allows the evaluation of the angular integration (on the variable φ) contained in Eq. (6), given below:

$$\int_0^{2\pi} e^{ikr\cos\varphi}\sin^2\alpha d\varphi = 2\pi\left[(c_1 + c_2)J_0(kr) - \frac{c_1}{kr}J_1(kr)\right]. \quad (8)$$

Taking into account these preliminary results, one shall now proceed with the integration on the $\mathbf{k}$-variable, obtaining the following scalar interaction potential:

$$V_{\text{scalar}}(r) = \frac{y^2}{(2\pi)}\left\{\left[1 - \frac{\mathrm{v}^2}{s^2}\right]\ln r - \frac{\mathrm{v}^2\sin^2\beta}{s^2}K_0(sr) - \frac{\mathrm{v}^2\cos 2\beta}{s^4}\frac{1}{r^2}[1 - srK_1(sr)]\right\}. \quad (9)$$

Near the origin, $r \rightarrow 0$, the modified Bessel functions behave as $K_0(r) \rightarrow -\ln r$, $K_1(sr) \rightarrow 1/sr + sr\ln r/2$, apart from constant terms. In such a way, the potential V_{scalar} goes like:

$$\lim_{r\rightarrow 0}V_{\text{scalar}}(r) = \frac{y^2}{(2\pi)}\left[1 - \frac{\mathrm{v}^2}{2s^2}(1 + \sin^2\beta)\right]\ln r. \quad (10)$$

Far from the origin, $r \rightarrow \infty$, the Bessel functions decay exponentially whereas the logarithmic function increases. In this limit, one has:

$$\lim_{r\rightarrow\infty}V_{\text{scalar}}(r) = \frac{y^2}{(2\pi)}\left[1 - \frac{\mathrm{v}^2}{2s^2}\right]\ln r. \quad (11)$$

Remarking the condition ($s^2 \gg \mathrm{v}^2$), under which this solution was derived, the scalar potential turns out always attractive near the origin and repulsive asymptotically, exhibiting a logarithmic behavior corrected by background-depending terms near and far from the origin. This logarithmic asymptotic behavior reflects the absence of screening concerning the scalar intermediation, which is ascribed to the presence of a masslesslike term, $1/k^2$, in the body of the scattering amplitude. In comparing the solution attained here with the scalar potential derived for the case of a purely timelike background, given in Ref. [8], it is instructive to point out that both possess a similar logarithmic behavior in the limits $r \to 0$ and $r \to \infty$. The difference lies mainly in the directional dependence on the $\beta -$ angle, responsible for the anisotropy, absent in the purely timelike case.

B. The gauge potential

Although the propagator of the gauge sector is composed by 11 terms, only six of them will contribute to the scattering amplitude, namely $\theta^{\mu\nu}, S^{\mu\nu}, \Lambda^{\mu\nu}, T^{\mu}T^{\nu}, Q^{\mu\nu}, Q^{\nu\mu}$. This is a consequence of the current-conservation law ($k_\mu J^\mu = 0$) The first two terms yield, in the nonrelativistic limit, the Maxwell-Chern-Simons (MCS) scattering amplitude, already carried out in Refs. [11]. The other four terms lead to background-depending scattering amplitudes. In order to obtain the total scattering amplitude mediated by the gauge field, one must previously evaluate the following current-current amplitude terms,

$$j^\mu(p_1)(S_{\mu\nu})j^\nu(p_2) = j^{(0)}(p_1)S_{0i}j^{(i)}(p_2) + j^{(i)}(p_1)S_{i0}j^{(0)}(p_2), \tag{12}$$

$$j^\mu(p_1)(T_\mu T_\nu)j^\nu(p_2) = j^{(0)}(p_1)[(\vec{\mathrm{v}}\cdot\vec{\mathrm{v}})(\vec{k}\cdot\vec{k}) - (\vec{\mathrm{v}}\cdot\vec{k})^2]j^{(0)}(p_2), \tag{13}$$

$$j^\mu(p_1)(\Lambda_{\mu\nu})j^\nu(p_2) = j^{(i)}(p_1)[v_i v_j]j^{(j)}(p_2), \tag{14}$$

$$j^\mu(p_1)(Q_{\mu\nu} - Q_{\nu\mu})j^\nu(p_2) = [j^{(i)}(p_1)v_i j^0(p_2) - j^{(l)}(p_2)v_l j^0(p_1)](\vec{\mathrm{v}}\times\vec{k}), \tag{15}$$

which carried out in the nonrelativistic limit, with $v^\mu = (0, 0, \mathrm{v})$ and $k^\mu = (0, \mathbf{k})$, lead to:

$$j^\mu(p_1)(S_{\mu\nu})j^\nu(p_2) = \mathbf{k}^2/m - (2i/m)\mathbf{k}\times\mathbf{p}, \quad j^\mu(p_1)(T_\mu T_\nu)j^\nu(p_2) = [\mathrm{v}^2\mathbf{k}^2\sin^2\alpha], \tag{16}$$

$$j^\mu(p_1)(\Lambda_{\mu\nu})j^\nu(p_2) = -\frac{\mathrm{v}^2\mathbf{k}^2}{4m^2}e^{i\theta}, \quad j^\mu(p_1)(Q_{\mu\nu} - Q_{\nu\mu})j^\nu(p_2) = \frac{\mathrm{v}^2\mathbf{k}^2}{4m^2}[1 - e^{i\theta}], \tag{17}$$

where the vector, $\mathbf{p} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$, is defined in terms of the 2-momenta ($\mathbf{p}_1, \mathbf{p}_2$) of the incoming electrons, and θ is the scattering angle in the CM frame. The total scattering amplitude associated with the gauge sector is obviously given by:

$$\mathcal{M}_{\text{gauge}} = \mathcal{M}_{MCS} + \mathcal{M}_\Lambda + \mathcal{M}_{TT} + \mathcal{M}_{QQ}. \tag{18}$$

Here, the term M_{MCS} stands for the Maxwell-Chern-Simons scattering amplitude (contributed by the terms $\theta^{\mu\nu}, S^{\mu\nu}$of the gauge propagator), whereas the other three amplitude terms explicitly depend on the background. They all are displayed as below:

$$\mathcal{M}_{MCS} = e_3^2\left\{\left(1 - \frac{s}{m}\right)\frac{1}{\mathbf{k}^2 + s^2} - \frac{2s}{m}\frac{i\mathbf{k}\times\mathbf{p}}{\mathbf{k}^2(\mathbf{k}^2 + s^2)}\right\},$$
$$\mathcal{M}_\Lambda = \frac{e_3^2 s^2 \mathrm{v}^2}{4m^2}\frac{\mathbf{k}^2}{[\mathbf{k}^2 + s^2]\boxtimes(k)}e^{i\theta}, \tag{19}$$

$$\mathcal{M}_{TT} = e_3^2\mathrm{v}^2\frac{\mathbf{k}^2}{[\mathbf{k}^2 + s^2]\boxtimes(k)}\sin^2\alpha,$$
$$\mathcal{M}_{QQ} = -\frac{e_3^2 s\mathrm{v}^2}{2m}\frac{\mathbf{k}^2}{[\mathbf{k}^2 + s^2]\boxtimes(k)}[1 - e^{i\theta}], \tag{20}$$

where the term, $\boxtimes(k) = [\mathbf{k}^2(\mathbf{k}^2 + s^2 + \mathrm{v}^2\sin^2\alpha)]$, is read off from the field propagators evaluated in Ref. [4]. The amplitude $\mathcal{M}_{MCS}$ leads to the well-know Maxwell-Chern-Simons potential (see Refs. [11]),

$$V_{MCS}(r) = \frac{e_3^2}{(2\pi)}\left[\left(1 - \frac{s}{m}\right)K_0(sr) - \frac{2}{ms}(1 - srK_1(sr))\frac{l}{r^2}\right], \tag{21}$$

which presents a purely logarithmic behavior near the origin,

$$V_{MCS}(r) \to -(e^2/2\pi)[1 - s/m - sl/m]\ln r, \tag{22}$$

and a typical $-1/r^2$ behavior in the asymptotic limit. This preliminary MCS result will be corrected by the other background-depending contributions, still to be evaluated. Hence, the remaining task consists in carrying out the Fourier transforms of the three amplitudes above. Starting from the $\mathcal{M}_\Lambda$-amplitude, the corresponding potential is written as follows:

$$V_\Lambda(r) = \frac{1}{(2\pi)^2}\frac{e_3^2 s^2 \mathrm{v}^2}{4m^2}\int_0^\infty \times \int_0^{2\pi}\frac{e^{ikr\cos\varphi}}{[\mathbf{k}^2 + s^2][\mathbf{k}^2 + s^2 + \mathrm{v}^2\sin^2\alpha]}e^{i\theta}kdkd\varphi,$$

As usual, this integral can not be exactly solved, so that the first order expansion (in v^2/s^2),

$$\frac{1}{[\mathbf{k}^2+s^2][\mathbf{k}^2+s^2+\mathrm{v}^2\sin^2\alpha]} \simeq \frac{1}{[\mathbf{k}^2+s^2]^2} - \frac{\mathrm{v}^2\sin^2\alpha}{[\mathbf{k}^2+s^2]^3}, \tag{23}$$

must be adopted. Besides this approximation, an important point concerns the existing relationship between the scattering angle (θ) and the integration angle (φ): $\theta = (2\varphi - \pi)$, which is decisive for the solution of the relevant angular integration, now read as

$$\int_0^{2\pi} e^{ikr\cos\varphi} e^{i\theta} d\varphi = -(2\pi)[J_2(kr)]. \tag{24}$$

Considering it and emphasizing that only the first term on the right hand side of Eq. (23) will provide a first order contribution (in v^2), the following potential expression comes out:

$$V_\Lambda(r) \simeq \frac{e_3^2}{(2\pi)}\frac{\mathrm{v}^2}{4m^2}\left\{-\frac{2}{s^2r^2} + K_0(sr) + \left(\frac{2}{sr}+\frac{sr}{2}\right)K_1(sr)\right\}, \tag{25}$$

where one notes that the directional dependence on the angle β does not appear in this first order result. Moreover, it behaves as $-\ln r$ near the origin and as $-1/r^2$ far from it.

In turn, the interaction potential associated with the $\mathcal{M}_{TT}$ amplitude,

$$V_{TT}(r) = \frac{e_3^2\mathrm{v}^2}{(2\pi)^2}\int_0^\infty \frac{e^{i\vec{k}.\vec{r}}\sin^2\alpha}{[\mathbf{k}^2+s^2][\mathbf{k}^2+s^2+\mathrm{v}^2\sin^2\alpha]}d^2\vec{k}, \tag{26}$$

can not be exactly solved as well, in such a way the first order approximation,

$$\frac{\sin^2\alpha}{[\mathbf{k}^2+s^2][\mathbf{k}^2+s^2+\mathrm{v}^2\sin^2\alpha]} \simeq \frac{\mathrm{v}^2\sin^2\alpha}{[\mathbf{k}^2+s^2]^2}, \tag{27}$$

must be properly considered. The associated angular integration is given by Eq. (8), so that the resulting potential takes on the form:

$$V_{TT}(r) \simeq \frac{e_3^2\mathrm{v}^2}{(2\pi)}\left\{\frac{c_1}{2s^2}K_0(sr) - \frac{c_1}{s^4r^2} + \frac{\sin^2\beta}{2s}rK_1(sr) + \frac{c_1}{s^3r}K_1(sr)\right\}. \tag{28}$$

One can now solve the last Fourier transformation for the scattering amplitude M_{QQ}, written as follows:

$$V_{QQ}(r) = \frac{1}{(2\pi)^2}\frac{e_3^2\mathrm{v}^2 s}{2m} \times \int_0^\infty \frac{e^{i\vec{k}.\vec{r}}(1-e^{i\theta})}{[\mathbf{k}^2+s^2][\mathbf{k}^2+s^2+\mathrm{v}^2\sin^2\alpha]}d^2\vec{k}, \tag{29}$$

which must be rewritten according to the approximation (23) and solved by means of the angular integration (24), so that one achieves the following first order outcome:

$$V_{QQ}(r) \simeq \frac{e_3^2}{(2\pi)}\frac{\mathrm{v}^2}{2m}\left\{-\frac{2}{s^3r^2} + \frac{3}{2}rK_1(sr) + \frac{1}{s}\left[K_0(sr) + \frac{2}{sr}K_1(sr)\right]\right\}. \tag{30}$$

It is worth pointing out that the three potentials, $V_\Lambda, V_{TT}, V_{QQ}$, behave in the same way both near and away from the origin. Indeed, it is easy to show that these potentials go as a constant as $r \to 0$, and as $-1/r^2$ for $r \to \infty$. On the other hand, remarking that the rest mass of the electron represents a large energy threshold against low-energy excitations, one should adopt the following condition $m^2 >> s^2$ as a sensible premise. Thereby, the potential V_{TT} turns out to be proportionally more significant than V_{QQ} and V_Λ; in accordance with the order of magnitude of the multiplicative factors ($\mathrm{v}^2/4m^2, \mathrm{v}^2/2m, \mathrm{v}^2$) which appear in Eqs. (25), (30), and (28), one concludes that V_Λ is the less meaningful one.

The total gauge potential, $V_{\rm gauge}(r) = V_{MCS} + V_\Lambda + V_{TT} + V_{QQ}$, is then written as a nontrivial combination of Bessel functions and $1/r^2$ terms, explicitly as:

$$V_{\rm gauge}(r) = \frac{e_3^2}{(2\pi)}\left\{\left[1 - \frac{s}{m} + \mathrm{v}^2\left(\frac{1}{2ms} + \frac{1}{4m^2} - \frac{\cos 2\beta}{2s^2}\right)\right]K_0(sr) - \left[\frac{2l}{ms} + \mathrm{v}^2\left(\frac{1}{ms^3} + \frac{1}{2s^2m^2} - \frac{\cos 2\beta}{s^4}\right)\right]\frac{1}{r^2} + \left[\frac{2l}{mr} + \mathrm{v}^2\left(\frac{1}{s^2m} + \frac{1}{2m^2s} - \frac{\cos 2\beta}{s^3}\right)\frac{1}{r} + \mathrm{v}^2\left(\frac{s}{8m^2} + \frac{\sin^2\beta}{2s} + \frac{3}{4m}\right)r\right]K_1(sr)\right\}. \tag{31}$$

Near the origin, this gauge potential is reduced to a simple expression,

$$V_{\rm gauge}(r) \to -\frac{e_3^2}{(2\pi)}\left[1 - \frac{s}{m} - \frac{sl}{m}\right]\ln r, \tag{32}$$

which corresponds exactly to the limit of the MCS gauge potential, already established in Eq. (22). This is an expected result, once it has been already established that all the potentials $V_\Lambda, V_{TT}, V_{QQ}$ behave as a constant in the limit $r \to 0$. It is still interesting to observe that the gauge potential derived in the case of a purely timelike background (see Ref. [4]) also presents this exact dependence, which shows that all background induced corrections are negligible in close proximity to the origin for both time and spacelike backgrounds. Far from the origin, the Bessel functions decay exponentially, so that the gauge potential is ruled by the $1/r^2$ terms, which remain as dominant. So, one has:

$$V_{\rm gauge}(r) \longrightarrow -\frac{e_3^2}{(2\pi)}\left[\frac{2l}{ms} + {\rm v}^2\left(\frac{1}{2m^2s^2} - \frac{1}{ms^3} - \frac{\cos 2\beta}{s^4}\right)\right]\frac{1}{r^2}, \tag{33}$$

This is also similar to the asymptotic behavior of the pure MCS potential, $-(2l/ms)r^{-2}$, here supplemented by background corrections, which in turn do not modify the $1/r^2$ physical behavior. Such analysis indicates that the gauge potential is always attractive in the limit $r \longrightarrow \infty$, once one relies on the approximation $s^2 \gg {\rm v}^2$. The behavior of this potential near the origin depends on the sign of the coefficient $(1 - s/m - sl/m)$ in the very way as it occurs with the pure MCS potential: it will be attractive for $s > m/(1 + l)$ or repulsive for $s < m/(1 + l)$. Now, regarding the condition, $m >> s$, there follows a repulsive gauge potential at the origin. Since this potential is always attractive far from the origin, there must exist a region in which the potential is negative (a well region) even in the case in which $s < m/(1 + l)$. This general behavior is attested in Fig. 1, which graphic exhibits a simultaneous plot for the gauge potential expression and for the pure MCS potential, given by Eqs. (21) and (31), respectively.

Such illustration confirms the equal behavior near and away from the origin, at the same time it demonstrates that the presence of the background may turn this potential attractive at some region. Yet, this result is not definitive once it is known that one should address carefully the low-energy potential as to avoid a misleading interpretation. As

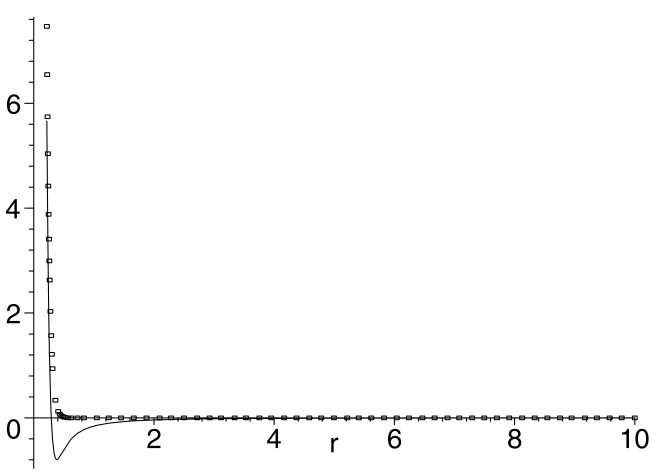

FIG. 1. Plot of the pure MCS potential (box dotted line) × plot of the gauge potential (continuos line) for the following parameter values: $s = 20$, $\mathbf{v} = 5$, $\beta = \pi/2$, $m_e = 5.10^5$.

discussed in literature (see Hagen and Dobroliubov [11]), in concerning a nonperturbative calculation one must consider not only the centrifugal barrier term (l^2/mr^2), but also the gauge invariant $\mathbf{A}^2$ − term coming from the Pauli equation, $[(\vec{p} - e\vec{A})^2/m_e + e\phi(r) - \frac{\vec{\sigma}.\vec{B}}{m_e}]\Psi(r, \phi) = E\Psi(r, \phi)$. The centrifugal barrier term is generated by the action of the Laplacian operator, $[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}]$, on the total wavefunction $\Psi(r, \phi) = R_{nl}(r)e^{i\phi l}$; on the other hand, the $\mathbf{A}^2$-term is essential to ensure the gauge invariance in the nonrelativistic domain. As this term does not appear in the context of a nonperturbative low-energy evaluation, for the same is associated with two-photon exchange processes (see Hagen and Dobroliubov [11]), it must be suitably added up the low-energy potential in order to assure the gauge invariance. In the presence of these two terms, the pure MCS potential reveals to be really repulsive instead of attractive. Hence, to correctly analyze the low-energy behavior of the gauge potential, it is necessary to add up the centrifugal barrier and the $\mathbf{A} \cdot \mathbf{A}$ term to the gauge potential previously obtained, leading to the following effective potential:

$$V_{\rm eff}(r) = V_{\rm gauge}(r) + \frac{l^2}{m_e r^2} + \left(\frac{e^2}{m_e}\right)\vec{A} \cdot \vec{A} \tag{34}$$

In order to proceed with this analysis, it is necessary to know the expression for the vector potential ($\mathbf{A}$), which was not determined in Ref. [7]. This potential may be obtained solving a system of two coupled differential equations read off from Ref. [7], namely: $\nabla^2(\nabla^2 - s^2)\vec{A} = s\vec{\nabla}^*\rho - s[\nabla(\overrightarrow{\rm v} \times \nabla\varphi)]^*$, $\nabla^2\varphi + (1/s)(\vec{\rm v} \times \vec{\nabla})(\vec{\nabla} \times \vec{A}) = 0$. We proceed decoupling them, yielding the following equation for the vector potential: $[\nabla^2(\nabla^2 - s^2) - (\vec{\rm v}^* \cdot \nabla)(\vec{\rm v}^* \cdot \vec{\nabla})]\vec{A} = s\vec{\nabla}^*\rho$. The solution for this equation (by the usual methods) leads to a first order approximate expression:

$$\begin{aligned}\vec{A}(r) = \frac{e}{(2\pi)}\bigg\{&-\frac{1}{sr}(1 - {\rm v}^2/s^2\sin^2\beta - {\rm v}^2\cos 2\beta/2s^2) \\ &+ (1 - {\rm v}^2/s^2\sin^2\beta + {\rm v}^2\cos 2\beta/2s^2)K_1(sr) \\ &+ \frac{2{\rm v}^2\cos 2\beta}{s^3 r}K_0(sr) - \frac{4{\rm v}^2\cos 2\beta}{s^5 r^3}[1 - rK_1(sr)] \\ &- \frac{{\rm v}^2\sin^2\beta}{2s}rK_0(sr)\bigg\}\hat{r}^*.\end{aligned} \tag{35}$$

One should now compare the gauge potential (31) with the effective potential, given by Eq. (34). In this way, one performs a graphical analysis of these two functions for small and large electron mass, as it is shown in Fig. 2.

For a large mass value ($m_e = 5.10^5$), one observes that the effective potential (continuos line) does not differ from the gauge potential (circle dotted curve), so that both graphics result perfectly overlapped. This fact reveals that the terms $l^2/m_e r^2$, A^2/m_e are not decisive to alter the behavior of the gauge potential in the regime of large

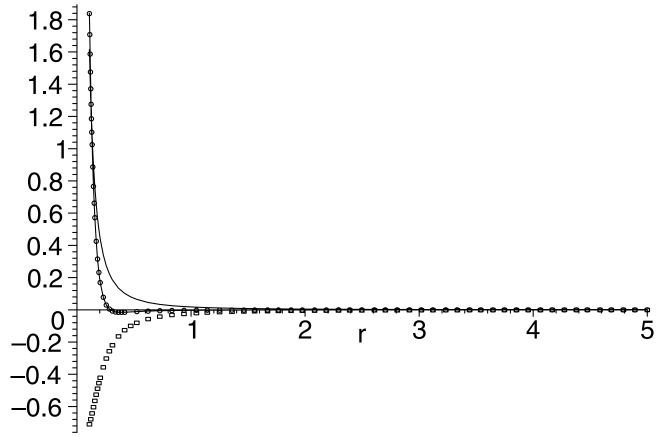

FIG. 2. Plot of the gauge potential (dotted line) × effective potential (continuos line) for two set of parameters with distinct mass value: ($s = 20$, $\upsilon = 5$, $m = 50$, $\beta = \pi/2$, $L = 1$) and ($s = 20$, $\upsilon = 5$, $m = 5.10^5$, $\beta = \pi/2$, $L = 1$).

mass ($m_e/s \simeq 10^5$). On the other hand, for a small mass parameter ($m_e/s \simeq 1$), one notes that the gauge potential (box dotted curve) may differ drastically from the effective potential (continuos single curve). Therefore, in the small mass regime, the low-energy potential has to be replaced by the effective one in order to yield the correct gauge invariant behavior, requirement not necessary in the large mass regime.

Another deserving attention point concerns the influence of the background direction on the solutions. The graphic in Fig. 3 presents three simultaneous plots of the gauge potential for different values of the β angle.

Such an illustration reflects the system anisotropy: depending on the β value, the potential may become totally repulsive or exhibit a region in which it is attractive. The interest in such an effect is related to its possible connection with the anisotropic order parameter of high-T$_c$ superconductors. An interaction potential which intensity varies with a fixed direction indeed leads to an anisotropic wavefunction, which certainly deserves further investigation.

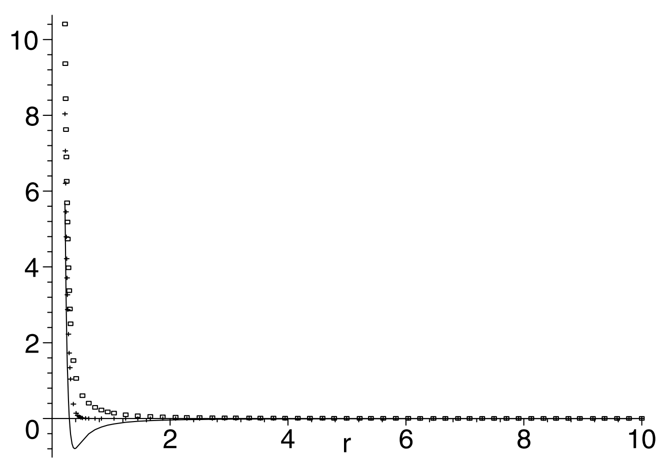

FIG. 3. Plot of the gauge potential for $s = 20$, v$= 5$, $m = 5.10^5$ and $\beta = \pi/2$ (continuos line), $\beta = 3\pi/4$ (box dotted line), $\beta = \pi$ (cross dotted line).

As a final comment, one should remark that the real potential corresponding to the total $e^- e^-$ interaction comprises the gauge and the scalar contributions: $V_{\rm total}(r) = V_{\rm scalar} + V_{\rm gauge}$. The character attractive or repulsive of this total potential arises from the combination of these two expressions for each radial region. Near the origin, for instance, the total interaction goes as:

$$V_{\rm total}(r) \rightarrow \frac{1}{(2\pi)}\left\{-e_3^2\left[1 - \frac{s}{m} - \frac{sl}{m}\right] + y^2\left[1 - \frac{\mathrm{v}^2}{2s^2}(1 + \sin^2\beta)\right]\right\}\ln r. \quad (36)$$

In the regime of large mass, the total interaction will be attractive near the origin whenever the phonic constant y^2 overcomes the 2-dimensional U(1) coupling, e_3^2 (or repulsive for $y^2 < e_3^2$). Far from the origin, the total potential exhibits the very logarithmic behavior stated in Eq. (11). It should be noted that this asymptotic behavior will change solely in the case in which a new mass parameter is introduced in, as it occurs when a spontaneous symmetry breaking takes place. This is mentioned in more detail in the final remarks section. By adjusting the value of the phonic constant y, one can certainly conclude that the total potential may always be negative at some region regardless the character (repulsive or attractive) of the gauge interaction. The relevance of this result is related to the possibility of obtaining $e^- e^-$ bound states in the framework of this particular model.

IV. FINAL REMARKS

In this work, one has considered the Möller scattering in a planar Maxwell-Chern-Simons electrodynamics incorporating a Lorentz-violating purely spacelike background. The interaction potential was evaluated as the Fourier transform of the scattering amplitude (Born approximation) carried out in the nonrelativistic limit, exhibiting two distinct contributions: the scalar (stemming form the Yukawa exchange) and the gauge one (mediated by the MCS-Proca gauge field). The scalar Yukawa interaction turns out to be logarithmically attractive near the origin and repulsive far from it, in much the same way as in the purely timelike case. As for the gauge interaction, it is composed of a pure MCS potential corrected by background-depending contributions, which are able to induce physical interesting modifications despite the smallness of the background against the topological mass ($\mathrm{v}^2/s^2 << 1$). Near and far from the origin, this gauge potential goes like the pure MCS counterpart, so that the alterations only appear at an intermediary radial region. Namely, it is verified that the gauge potential becomes attractive for some parameter values. Such attractiveness remains even in the presence of the centrifugal barrier and gauge invariant $\mathbf{A}\cdot\mathbf{A}$ term. Besides the possibility of having an attractive gauge interaction, it should be mentioned that the total interaction (scalar plus gauge potential) may always result attractive, once a fine tuning of the coupling constant values (y, e_3) is realized. This is a necessary condition for the formation of electron-electron bound states.

The real possibility for obtaining electronic pairing may be checked by means of a quantum-mechanical numerical analysis of the nonrelativistic interaction potential here derived. Such potential should be introduced in the Schrödinger equation, whose numerical solution will provide the corresponding e^-e^- binding energies for each set of the stipulated parameter values. One should remark that the values must be chosen in accordance with the usual scale of low-energy excitations in a condensed matter system. This analysis may be performed for the potentials obtained both in the case of a purely timelike and spacelike background, keeping also in mind the presence of anisotropy observed in the latter case.

One must now comment on the validity of the approximation which has been here adopted. At first sight, the higher order terms (in v^2) are always negligible before the first order ones. Indeed, this is true for terms that decay quickly at large distances. Near the origin, although, it might occur that a high order term (in v^2) comes to increase with r more rapidly, overcoming a first order term, fact which is really associated with its radial dependence in the limit $r \rightarrow 0$. Such a behavior would be observed if a second order term (in v^2) had a more pronounced power in $(1/r)$ than the first order one. This fact was not noted in all second order performed evaluations, which confirms the validity of the approximation adopted as well as the outcomes obtained in this work.

The absence of screening, first observed in Refs. [7,8], is here manifest only in the scalar potential expression by means of the asymptotic logarithmic term, once the gauge sector revealed a much different $(\sim 1/r^2)$ asymptotic behavior. Some usual planar models, in $(1+2)$ dimensions, are known for exhibiting a confining (logarithmic) potential as representation of the gauge interaction; such behavior, however, does not reflect a convenient physical interaction since it increases with distance. To represent a physical interaction, it may be changed to a condensating potential, which may be attained when the model is properly supplemented by a new mass parameter. The consideration of the Higgs mechanism is a suitable tool to provide a Proca mass for the gauge field and to induce an efficient screening for the corresponding field strengths and solutions, bypassing this difficulty. In a recent work [9], it was accomplished the dimensional reduction of an Abelian-Higgs Lorentz-violating model endowed with the CFJ term. The classical solutions for field strengths $(\mathbf{E}, \mathbf{B})$ and four-potential $(A^0, \mathbf{A})$ related to this planar model were analyzed and solved [10], yielding entirely shielded solutions and interesting deviations in comparison with the pure MCS-Proca electrodynamics. This preliminary outcome indicates that the Möller scattering in this framework [14] will also lead to a totally screened interaction potential, with the logarithmic term being replaced by $\mathrm{K}_0, \mathrm{K}_1$ functions.

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