Lorentz-breaking effects in scalar-tensor theories of gravity

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In this work, we study the effects of breaking Lorentz symmetry in scalar-tensor theories of gravity taking torsion into account. We show that a space-time with torsion interacting with a Maxwell field by means of a Chern-Simons-like term is able to explain the optical activity in synchrotron radiation emitted by cosmological distant radio sources. Without specifying the source of the dilaton-gravity, we study the dilaton-solution. We analyze the physical implications of this result in the Jordan-Fierz frame. We also analyze the effects of the Lorentz-breaking in the cosmic string formation process. We obtain the solution corresponding to a cosmic string in the presence of torsion by keeping track of the effects of the Chern-Simons coupling and calculate the charge induced on this cosmic string in this framework. We also show that the resulting charged cosmic string yields important effects concerning the background radiation. The optical activity in this case is also worked out and discussed.

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I. INTRODUCTION

The idea of the possible existence of extra dimensions of space-time as proposed in the Kaluza-Klein theory [1] has inspired the formulation of scalar-tensor theories of gravity. In these theories, the gravitational interaction is mediated by one or several long-range fields, in addition to the usual tensor field of the theory of general relativity and represents the simplest and most natural generalization of general relativity [2–4]. The relevance of these scalar-tensor theories resides in the fact that in the very early Universe, or in the presence of strong gravitational fields, both the scalar and tensor aspects of gravity have to be taken into account. Nowadays, these effects are small but, to some extent, they can be observed.

In addition to the scalar and tensor fields, we shall consider another one which may have an important role: the torsion field [5]. This could influence such physical phenomena as, for example, neutrino oscillations [6], and may have been an important element in the early Universe, when quantum effects of gravity were drastically important.

In this paper, we study the cosmic string configuration in the context of scalar-tensor theories of gravity [7] with torsion [8–10] when a Maxwell-Chern-Simons term is present and analyze their role in the geometric and topological features of the cosmic string solution. It is worthy mentioning that cosmic strings with torsion and Brans-Dicke type models have also been discussed in Refs. [11–13], respectively. A cosmic string is called a topological defect and corresponds to a regular, classical solution to a gauge field theory which arises when a symmetry of the theory is spontaneously broken. In particular, in the framework of cosmology, it may be generatedBezerra during phase transitions in the early Universe [14,15]. The grand unified theory (GUT) defects carry a large energy density and, hence, are of interest in cosmology, as potential sources to explain the most energetic events in the Universe, like the cosmological gamma-ray bursts (GRBs) [16,17], ultra high energy cosmic rays (UHECRs) and very high energy neutrinos [18]. More recently, stimulated by a suggestion of Ref. [17], it has been shown in Refs. [19–21] that cusps in cosmic string loops can emit gravitational wave (GW) bursts whose amplitude, depending on the parameters of the model (in particular, the string tension and the number of cusps per loops) can be interesting for GW detection.

Over the past years, there has been a considerable interest in theories with the Lorentz and *CPT* violations[22–25]. These theories may be implemented by a Chern-Simons type model in four dimensions. More recently, Jackiw [26] and Jackiw and Pi [27] have opened up a very interesting stream of investigation with Lorentz violation being extended to the realm of general relativity. In these works, a Chern-Simons action is proposed for gravity in terms of the Cotton tensor and an external background vector, and Lorentz violation takes place at the level of the gravitational degrees of freedom.

In three dimensions, the Chern-Simons models have attracted considerable attention due to the fact that the Maxwell-Chern-Simons-Higgs (MCSH) theory in a three-dimensional Minkowski space-time [28] has some similarities with the theory of high-T_C superconductivity. At large distances, the Chern-Simons term dominates over the Maxwell term and so it is reasonable to consider the simplest Abelian Chern-Simons-Higgs model, from which it was shown [29] that there exists a vortex solution to the three-dimensional Abelian Chern-Simons-Higgs model,

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and an electrically charged vortex solution with a Chern-Simons term [30]. Motivated by these reasons, in our work we study the possibility of building a cosmic string solution in the presence of a four-dimensional Chern-Simons type term [31]; in this dimension it presents a vector coupling, which can be identified with the dilaton gradient in a scalar-tensor theory of gravity.

In a previous paper [32], only the effect of the interaction between the cosmic string–dilaton-solution in the background was considered; now, we analyze another aspect associated with the charge induced in the core of the string. An interesting application of a cosmic string with Chern-Simons torsion coupling is to analyze the possible existence of a preferred direction in the sky. This subject has already been discussed in the context of theories of gravity [22–24,33–38] and observational cosmology [39]. The idea is that the electromagnetic radiation traveling through the intergalactic medium interacts with its components and, if this radiation is initially plane-polarized, the plane of polarization will rotate. Thus, if the Faraday rotation is taken into account, there is a residual rotation that does not vanish. Such a phenomenon, if it exists, would imply the violation of the Lorentz invariance [33], with remarkable consequences for fundamental physics [40].

For the sake of our discussions, we consider that a plane-polarized electromagnetic radiation has the polarization plane rotated when it is traveling in the presence of the gravitational field generated by a screwed cosmic string in scalar-tensor theories of gravity. The motivation to consider such a background with torsion was already discussed in [41,42]. On the other hand, the assumption that gravity may be intermediated by a scalar field (or, more generally, by many scalar fields), in addition to the usual tensor field, has been considerably reassessed over the recent years. It has been argued that gravity may be described by a scalar-tensorial gravitational field, at least at sufficiently high energy scales.

This paper is organized as follows: In Sec. II, we present some aspects of scalar-tensor theories. In Sec. III, we introduce the Chern-Simons coupling to the free field theory and study the cosmic string solution with Chern-Simons effects. In Sec. IV, we discuss the possibility that this approach can be in accordance with the experiments where an optical activity appears. Finally, in Sec. V, we present our concluding remarks. An appendix follows where we collect some technical aspects that may be helpful for the understanding of Sec. III.

II. SCALAR-TENSOR THEORIES OF GRAVITY WITH TORSION

In this section, we set forth some results concerning the scalar-tensor theories of gravity with torsion. Let us consider that, in this case, gravity is represented by an action in the Jordan-Fierz frame [43,44], which is given by

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-\tilde{g}}\left[\tilde{\phi}\tilde{R} - \frac{\omega(\tilde{\phi})}{\tilde{\phi}}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}\right] + I_m, \tag{2.1}$$

where the matter action I_m in Eq. (2.1) is related to the dilaton-matter coupling. The function ω in a general scalar-tensor theory has a $\tilde{\phi}$ dependence, but in the specific case of the Brans-Dicke theory it is a constant. The scalar curvature $\tilde{R}$, appearing in Eq. (2.1) can be written as

$$\tilde{R} = \tilde{R}(\{\}) + \delta\frac{\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}}{\tilde{\phi}^2}, \tag{2.2}$$

where $\tilde{R}(\{\})$ is the Riemann scalar curvature in the Jordan-Fierz frame and δ is the torsion coupling constant [44]. It is worthy to stress that in the scalar curvature $\tilde{R}$, the scalar function $\tilde{\phi}$ (the dilaton field) can act as a source of the torsion field. Therefore, in the absence of string spin, the torsion field may be generated by the gradient of this scalar field [43]. In this case, the torsion can be propagated with the scalar field, and it can be written as

$$S_{\mu\nu}{}^{\lambda} = (\delta^\lambda_\mu\partial_\nu\tilde{\phi} - \delta^\lambda_\nu\partial_\mu\tilde{\phi})/2\tilde{\phi}. \tag{2.3}$$

The most general affine connection $\Gamma^\alpha_{\lambda\nu}$ in this theory has a contribution arising from the contortion tensor $K^\alpha_{\lambda\nu}$, as given below:

$$\Gamma_{\lambda\nu}{}^{\alpha} = \{^{\alpha}_{\lambda\nu}\} + K_{\lambda\nu}{}^{\alpha}, \tag{2.4}$$

where the quantity $\{^{\alpha}_{\lambda\nu}\}$ is the Christoffel symbol computed from the metric tensor $g_{\mu\nu}$, and the contortion tensor, $K^\alpha_{\lambda\nu}$, can be written in terms of the torsion field as

$$K_{\lambda\nu}{}^{\alpha} = -\frac{1}{2}(S^\alpha_\lambda + S^\alpha_\nu - S_{\lambda\nu}{}^{\alpha}). \tag{2.5}$$

Now, let us introduce a Maxwell-Chern-Simons coupling and analyze its consequences. Thus, we will consider the action for the matter in (2.1), which we will indicate by $I_{\rm MCS}$, as given by

$$I_{\rm MCS} = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{3!}\lambda\varepsilon^{\mu\nu\alpha\beta}\tilde{F}_{\mu\nu}A_\alpha\tilde{S}_\beta\right], \tag{2.6}$$

where we couple the electromagnetic dual field $\tilde{F}_{\mu\nu}$ and the vector potential A_α to the torsion vector $\tilde{S}_\beta = -\frac{\lambda}{3}S_\beta$, which is responsible for the appearance of the preferred cosmic direction, as suggested by observations [38]. The Chern-Simons term in the action above yields a Lorentz symmetry breaking if S_β (or, equivalently, $\tilde{S}_\beta$) is taken as a fixed background vector. Lorentz breaking takes place in the active point of view, as discussed in Ref. [45] and as we shall discuss in the next paragraph. Moreover, as pointed out in [45], if $\tilde{S}_\mu$ is a constant vector or a gradient of a scalar, U(1) gauge invariance is not violated, up to a surface term. However, the situation here is not the same as the gauge symmetry breaking realized by the torsion as proposed in Ref. [46]. The parameter λ is the coupling constant of the theory, whose expected value will be estimated later on using current astronomical data sets. In what follows, we shall investigate the role played by the Chern-Simons term in the scalar-tensor screwed cosmic string background.

Let us explain in more detail in which sense Lorentz symmetry is violated. Conventional Lorentz transformations, as we adopt them in the theory of special relativity, are implemented as coordinate changes and we usually refer to them as observer Lorentz transformations. However, we can also consider the so-called particle Lorentz transformations, which consist in applying boosts or rotations on particles and localized fields, but never on the background fields, contrary to the observer Lorentz transformations, which act also on background fields.

Distinguishing between observer and particle Lorentz transformations is crucial for the kind of model we are considering here, where the Chern-Simons term described in the action of Eq. (2.6) is to be regarded as arising from a constant background field S^{μ}, which is seen as a global feature of the model and is not related to localized experimental conditions, as it is the case of the electromagnetic field A^{μ}, which is a perturbation that propagates in a space-time dominated by S^{μ}. So, in applying particle Lorentz transformations, the Chern-Simons term of Eq. (2.6) does not display Lorentz invariance, since S^{μ} is not acted upon by any Λ-matrix belonging to a Lorentz group, so that the Λ's acting on $\tilde{F}^{\mu\nu}$ and A^{α} do not combine to produce the $\det\Lambda = 1$-factor that would appear if S^{μ} were boosted, as it happens for the class of observer Lorentz transformations. This confirms that Lorentz covariance breaks down.

Now, using Eq. (2.3) the torsion vector (in the Jordan-Fierz frame) is defined as

$$\tilde{S}_{\mu} = \frac{3}{2}\partial_{\mu}\ln\tilde{\phi}. \tag{2.7}$$

We should reinforce here that we are actually associating ϕ to the torsion vector, and not to the Weyl covector, because our connection is metric (nonmetricity is not considered in the space-time we adopt). We refer the reader to Refs. [12,47] where the degeneracy between the torsion vector and the Weyl covector is discussed in connection with metric-affine models of gravity.

Although the action proposed in Eq. (2.1) shows explicitly this scalar-tensor gravity feature, for technical reasons, we will adopt the Einstein (conformal) frame in which the kinematic terms of the scalar and tensor fields do not mix. In this frame, the action can be written as

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[R - 2g^{\mu\nu}\xi\partial_{\mu}\phi\partial_{\nu}\phi] + I_{\mathrm{MCS}}, \tag{2.8}$$

where I_{MCS} is the action of Maxwell-Chern-Simons given by (2.6) with $\tilde{S}_{\mu}$, interchanged by S_{μ} which is given by

$$S_{\mu} = \frac{1}{2}\alpha(\phi)\partial_{\beta}\phi, \tag{2.9}$$

where $g_{\mu\nu}$ is written in the Einstein frame, $R(\{\})$ is the curvature scalar without torsion and ξ is the parameter that includes the torsion contribution, δ, so defined that

$$\xi = 1 - 2\delta\alpha^2. \tag{2.10}$$

This more convenient formulation of the theory in terms of the gravitational field variables, $g_{\mu\nu}$ and ϕ, is obtained by means of the conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu}, \tag{2.11}$$

and by a redefinition of the quantity

$$G\Omega^2(\phi) = \tilde{\phi}^{-1}.$$

This transformation puts into evidence that any gravitational phenomena will be affected by the variation of the gravitational *constant*, G, in the scalar-tensor gravity, a feature that is exhibited through the definition of a new parameter,

$$\alpha^2(\phi) \equiv \left(\frac{\partial\ln\Omega(\phi)}{\partial\phi}\right)^2 = [2\omega(\tilde{\phi}) + 3]^{-1},$$

which can be interpreted as the field-dependent coupling strength between matter and the scalar field. In order to make our calculations as general as possible, we will not fix the factors $\Omega(\phi)$ and $\alpha(\phi)$, leaving them as arbitrary functions of the scalar field.

In this context, the field equation of the electromagnetic field becomes

$$\partial_{\mu}F^{\mu\nu} = 2\lambda\alpha(\phi)^{*}F^{\mu\nu}\partial_{\mu}\phi, \tag{2.12}$$

where $*F$ stands for the Hodge dual. For some purposes, it is more interesting to write down these equations of motion in terms of the electric and magnetic fields. Then, we consider the electric field E^i and magnetic field B^i, defined as usual by:

$$E^i = F^{0i}, \qquad B^i = -\epsilon^{ijk}F_{jk}. \tag{2.13}$$

In what follows, let us consider a spatially flat isotropic Friedmann-Robertson-Walker (FRW) background,

$$ds^2 = a^2(\eta)(-d\eta^2 + dx^2 + dy^2 + dz^2), \tag{2.14}$$

where η is the conformal time coordinate, defined by $d\eta = dt/a(t)$, $a(t)$ being the cosmological scale factor. Then, we have the following equations

$$\begin{aligned}\vec{\nabla}\cdot\tilde{\mathbf{E}} &= 2\lambda\alpha(\phi)(\vec{\nabla}\phi)\cdot\tilde{\mathbf{B}},\\ \partial_{\eta}\tilde{\mathbf{E}} - \vec{\nabla}\times\tilde{\mathbf{B}} &= 2\lambda\alpha(\phi)[\partial_{\eta}\phi\tilde{\mathbf{B}} - \vec{\nabla}\phi\times\tilde{\mathbf{E}}],\end{aligned} \tag{2.15}$$

where $\tilde{\mathbf{E}} = a^2\mathbf{E}$ and $\tilde{\mathbf{B}} = a^2\mathbf{B}$. The equation of the motion for the dilaton field in this background is given by

$$(\partial_\eta^2 - \nabla^2)\phi + \frac{2}{a}\partial_\eta a \partial_\eta \phi = 0, \tag{2.16}$$

where ϕ is taken to be an arbitrary function of the space and time coordinates. Assuming that the solution for ϕ has the form

$$\phi(\eta, \vec{x}) = \phi_0(\eta)\cos(\vec{k}\cdot\vec{x}) \tag{2.17}$$

and substituting this into Eq. (2.16), we get the following equation

$$\partial_\eta^2\phi + \frac{2}{a}\partial_\eta a \partial_\eta \phi + k^2\phi = 0. \tag{2.18}$$

We can notice that, as a consequence of this result, the overall homogeneity of the Universe over long distance scales is not disturbed by the inclusion of a spatial part in ϕ. Now, we study the modification introduced by this background in the Poynting vector. From the field Eqs. (2.15), we derive the wave equations for the electric and magnetic fields, which take the form that follows:

$$(\partial_\eta^2 - \nabla^2)\tilde{\mathbf{B}} = -2\nabla(\nabla\phi\tilde{\mathbf{E}}) + 2\nabla(\dot{\phi}\,\tilde{\mathbf{B}}), \tag{2.19}$$

$$(\partial_\eta^2 - \nabla^2)\tilde{\mathbf{E}} = 2\nabla(\dot{\phi}\,\tilde{\mathbf{E}} - 2\nabla\phi\dot{\tilde{\mathbf{E}}}) + \nabla(\nabla\cdot\tilde{\mathbf{E}}) - 2\ddot{\phi}\,\tilde{\mathbf{B}}. \tag{2.20}$$

It is easy to see that these equations reduce to the usual Maxwell equations whenever $\phi = 0$ or constant. From the equations given in (2.15), we find that

$$\nabla\cdot\mathbf{S} + \frac{\partial U}{\partial t} + \tilde{\mathbf{E}}\cdot\tilde{\mathbf{J}} = 0, \tag{2.21}$$

where $\mathbf{S} = (\tilde{\mathbf{E}}\times\tilde{\mathbf{B}})$ is the Poynting vector and $U = \frac{1}{2}\times(\tilde{\mathbf{E}}^2 + \tilde{\mathbf{B}}^2)$ is the electromagnetic energy density. The presence of a term of the form $\tilde{\mathbf{E}}\cdot\tilde{\mathbf{J}}$ indicates a sort of dissipation effect. It can be interpreted as the analogue of the Ohm's law, where the current is proportional to $\tilde{\mathbf{B}}$ and given by $\tilde{\mathbf{J}} = 2\lambda\alpha(\phi)\dot{\phi}\,\tilde{\mathbf{B}}$. This sort of current induced by $\tilde{\mathbf{B}}$ is a feature of models with a Maxwell-Chern-Simons term.

III. SCREWED COSMIC STRING MODEL WITH CHERN-SIMONS COUPLING

In this section, let us investigate the solution that corresponds to a cosmic string when the Chern-Simons coupling is included. We analyze the vortex regime of the fields. The action for screwed cosmic string $I_m(\tilde{g}_{\mu\nu}, \Psi)$ in an Abelian Higgs model can be written as

$$I_m = I_{\rm SCS} + I_{\rm MCSH}, \tag{3.1}$$

where $I_{\rm SCS}$ is the action associated with a screwed cosmic string and $I_{\rm MCSH}$ is the Maxwell-Chern-Simons-Higgs action. First, let us consider $I_{\rm SCS}$, which can be written as

$$I_{\rm SCS} = \int d^4x\sqrt{\tilde{g}}\left[-\tfrac{1}{2}D_\mu\Phi(D^\mu\Phi)^* - \tfrac{1}{4}H_{\mu\nu}H^{\mu\nu} - V(|\Phi|)\right], \tag{3.2}$$

where $D_\mu\Phi = (\partial_\mu + iX_\mu)\Phi$ is the covariant derivative. The field strength $H_{\mu\nu}$ is defined in the standard fashion, namely, $H_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$, with X_μ being the gauge field. The action given by Eq. (3.2) has a U(1) symmetry associated with the Φ-field and it is broken by the vacuum, giving rise to vortices of the Nielsen-Olesen type [48]:

$$\Phi = \varphi(r)e^{i\theta}, \qquad X_\theta = \frac{1}{q}[P(r) - 1], \qquad X_t = X_t(r) \quad \text{time-like}. \tag{3.3}$$

The boundary conditions for the fields $\varphi(r)$ and $P(r)$ are the same as those for ordinary cosmic strings, namely,

$$\varphi(r) = \eta, \quad r\to\infty, \qquad P(r) = 0, \quad r\to\infty, \qquad \varphi(r) = 0, \quad r = 0, \qquad P(r) = 1, \quad r = 0. \tag{3.4}$$

The configuration for the other component compatible with the cosmic string stability is given by the following boundary conditions:

$$X_t(r) = 0, \qquad r\to\infty, \qquad X_t(r) = b, \qquad r = 0. \tag{3.5}$$

It is worthy to draw the attention to the fact that these components are important to study the behavior of the charges and currents.

The potential $V(\varphi, \sigma)$ triggering the spontaneous symmetry breaking can be built in the most general case as

$$V(\varphi) = \frac{\lambda_\varphi}{4}(\varphi^2 - \eta^2)^2, \tag{3.6}$$

where λ_φ is a coupling constant. This potential possesses also all the ingredients which yields the formation of a cosmic string, in analogy with the ordinary cosmic string case, where $X_z = X_t = 0$ and without an external field.

The term $I_{\rm MCSH}$ is given by

$$I_{\rm MCSH} = \int d^4x\sqrt{\tilde{g}}\left[-\frac{\lambda_1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda_2}{2}\varepsilon^{\mu\nu\alpha\beta}H_{\mu\nu}X_\alpha S_\beta + \frac{\lambda_3}{2}\varepsilon^{\mu\nu\alpha\beta}H_{\mu\nu}Y_\alpha S_\beta\right]. \tag{3.7}$$

In this action, we introduced the external field $F_{\mu\nu}$ that will be analyzed in connection with the Chern-Simons coupling. The parameters λ_1, λ_2, and λ_3 will be analyzed with respect to the charge effects.

Let us consider the dilaton-torsion solution in the weak-field approximation [49,50]. This approximation is reasonable because the present dilaton-torsion effects are small if compared with the $g_{\mu\nu}$-effects; in this case, the calculations become relatively simple. For more details on the relations involving the parameters of the metric, the reader should consult the appendix and the references quoted therein.

In this case, the equation for the dilaton can be written as

$$\Box\phi = -\varepsilon T(r), \tag{3.8}$$

where $\varepsilon = 4\pi G^*\alpha_0$ and T is the trace of $T^{\mu}_{(0)\nu}$. Let us assume that the dilaton field, $\phi_{(1)}$ can be written as

$$\phi_{(1)}(t, r, z) = \chi(r) + f(r)\psi(z, t), \tag{3.9}$$

where we are considering that the components which interact with the string are t and z, and therefore, the scalar field $\phi_{(1)}$ depends on the radial coordinate as well as on the coordinates t and z. The function $f(r)$ is required to vanish outside the string core.

Using solution (3.9) in Eq. (3.8), we obtain, up to the first order in ε, the following set of equations

$$\chi'' + \frac{1}{r}\chi' = -T(r). \tag{3.10}$$

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} = \hat{\omega} f \tag{3.11}$$

and

$$\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial z^2} = \hat{\omega}\psi. \tag{3.12}$$

with $\hat{\omega} = k^2 - \omega^2$ being a constant. In the ansatz (3.9), χ corresponds to a field which depends only on the radial coordinate. Note that as T is the source of the pure radial component of the dilaton, it enters only in the Eq. (3.10).

In order to have compatibility with the external solution, $f(r)$ must have the following behavior $\lim_{r\to\infty} f(r) = 0$. Now, let us consider the spacelike dilaton $\hat{\omega} = k^2$. The timelike dilaton also can be considered but the dilaton in this case must be massive [49] and it has different effects. In the spacelike dilaton case, we have that the solutions of Eqs. (3.11) and (3.12) are given, respectively, by the following expressions

$$f(r) = f_I I_0(kr) + f_k K_0(kr), \tag{3.13}$$

$$\psi = \psi_0 \sin(kz). \tag{3.14}$$

and in this situation, by using the procedure of the appendix, we get that

$$\chi = 2G_0\alpha\xi^{-1}[\mu + \tau + (\bar{\epsilon}Q)^2]\ln\frac{r}{r_0}. \tag{3.15}$$

Notice that the solutions (3.13) and (3.14) depend only on the coordinates r and z. For these the solutions, we have the same interpretation already presented in the literature [49].

Now, let us study the cosmic string configuration in the weak-field approximation considering the field equations. In this case, we have analyzed the Chern-Simons effects in the core of the string with $\lambda_1 = \lambda_3 = 0$ in (3.7). Thus, the equation of motion for the gauge field $X_\mu(r)$ is given by

$$\partial_\alpha H^{\alpha\beta} + \lambda_2\epsilon^{\nu\beta k\lambda}H_{\nu k}S_\lambda = j^\beta. \tag{3.16}$$

If we use the screwed cosmic string ansatz, we find the following equations

$$\vec{\nabla}\times\vec{B} - \lambda_2\vec{S}\times\vec{E} = \vec{J} \tag{3.17}$$

and

$$\vec{\nabla}\cdot\vec{E} - \lambda_2\alpha(\phi_0)f(r)\vec{S}\cdot\vec{B} = \rho. \tag{3.18}$$

These equations are compatible with the asymptotic conditions for the string, i.e., outside it S_z vanishes and S_r does not vanish. Based on discussions concerning the dilaton-solution, the timelike gradient of the dilaton (2.7) is given, in the linearized approximation, by

$$S_\mu = -\alpha(\phi_0)\partial_z\phi_{(1)}\delta^z_\mu = -\alpha(\phi_0)f(r)S_z(kz)\delta^z_\mu, \tag{3.19}$$

where $S_z(kz) = k\psi_0\cos(kz)$. If we use the Gauss law, we find

$$Q = \int d^3xj^0 = \pi\lambda_2\alpha(\phi_0)S_z\int_0^{r_0} f(r)B(r)rdr. \tag{3.20}$$

Another interesting equation is related with the internal electric field, in analogy with the London equation, and is given by

$$J = \int d^3xj^\theta = \pi\lambda_2\alpha(\phi_0)S_z\int_0^{r_0} f(r)Erdr, \tag{3.21}$$

where $E = H_{rt}$ is the electric field inside the string. Therefore, from the previous results, we can get the interesting conclusion that the screwed cosmic string in presence of the Chern-Simons-Higgs coupling is charged.

In what follows, we consider the effects of the Maxwell-Chern-Simons-Higgs coupling on the cosmic string gravitational field. To study the effects presented in the last section, let us consider the vector S_μ as spacelike as in (3.19). The configuration of the external field that interacts with the cosmic string is $Y_t(r)$. In this situation, we have the following equation for the gauge field $X_\mu(r)$,

$$\partial_\alpha H^{\alpha\beta} + \epsilon^{\nu\beta k\lambda}(\lambda_2H_{\nu k} + \lambda_3F_{\nu k})S_\lambda = j^\beta. \tag{3.22}$$

This equation is compatible with the asymptotic conditions of the string, i.e., outside the string S_λ vanishes, because of the $f(r)$ dependence. The equation that is equivalent to Eq. (3.21) and includes the external field is given by

$$J = \int d^3zj^\theta = \alpha(\phi_0)\pi S_z\int f(r)\hat{E}rdr, \tag{3.23}$$

where $\hat{E} = \lambda_2E + \lambda_3E_{\text{ext}}$, with E_{ext} being the external electric field.

In the electric case, where we use the spacelike torsion vector, we have that the external electromagnetic field that interacts with the string is Y_t. The equation of motion for X_μ gives us the same result as in (3.20).

Now we allow the external field Y_μ to interact with this charge. The equation of motion for this, with $\lambda_1 = 1$, is given by

$$\partial_\mu F^{\mu\nu} + \lambda_3 \epsilon^{\nu\rho\alpha\beta} S_\rho H_{\alpha\beta} = 0. \tag{3.24}$$

whose solution is

$$E_{\text{ext}} = \epsilon(z) \frac{Q}{\sqrt{2}\pi r}, \tag{3.25}$$

where we have considered $Q = 2\sqrt{2}\pi^2 \int_0^{r_0} f(r)B(r)rdr$ and $\epsilon(z) = \lambda_2\lambda_3\alpha S_z$.

Up to now we have considered, in the weak-field approximation, the dilaton-solution and the field equations. The Einstein equation solutions in this approximation can be studied in more detail in the appendix. The solution we get is the metric of a superconducting cosmic string as given below:

$$ds^2 = (1 - h_{tt})[-dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)], \tag{3.26}$$

with the component h_{tt} in the Jordan-Fierz frame given by

$$h_{tt} = -4G_0 \bigg\{ (\bar{\epsilon}Q)^2 \ln(\frac{\rho}{r_0}) + \mu - \tau - (\bar{\epsilon}Q)^2 + \frac{\alpha^2 \xi^{-1}}{2} [\mu + \tau + (\bar{\epsilon}Q)^2] \bigg\} \ln\bigg(\frac{\rho}{r_0}\bigg), \tag{3.27}$$

where $\bar{\epsilon} \equiv \lambda \gamma S \alpha(\phi_0)$.

The interesting aspect to stress here is the fact that, in the presence of a fixed torsion vector responsible for the Lorentz violation, we get the possibility of finding out a superconducting cosmic string. If we consider that the Lorentz-breaking effect (that may become important at the Planck scale) influences the cosmic string phase transition, superconducting effects may appear at a very early era.

IV. ANALYSIS OF THE CHERN-SIMONS-LIKE COUPLING IN A SCALAR-TENSOR SCREWED COSMIC STRING BACKGROUND

In this section, we study the cosmic string background in the context of a scalar-tensor theory including torsion with Lorentz breaking induced by the charge inside the string. We examine, in particular, the effect on the polarization of the synchroton radiation coming from cosmological distant sources, and the relation between the electric and magnetic components of the radiation. The radio emission from distant galaxies and quasars present the polarization vectors which are not randomly oriented, as naturally expected. This interesting phenomenon suggests that the space-time between the source and observer may exhibit some optical activity. In this case, we consider that torsion in the action (2.6) is generated by the cosmic string, an important aspect analyzed here, associated to the fact that the cosmic string is charged.

Replacing the linearized solution given by Eq. (3.15) into Eq. (2.6), as we did in our previous paper [8], we have

$$S_\mu = -3\alpha \partial_\mu \chi. \tag{4.1}$$

Again, we stress that though S_μ is expressed in a covariant form, Lorentz symmetry is understood to be broken in the sense of active transformations, as already discussed in Sec. II. In this context, the equation for the electromagnetic field becomes

$$\partial_\mu F^{\mu\nu} = 2\lambda\alpha(\phi_0)^* F^{\mu\nu} \partial_\mu \chi. \tag{4.2}$$

For some purposes, it is more interesting to write down these equations of motion in terms of the electric and magnetic fields. Then, we consider the electric field E^i, and magnetic field B^i defined as usual:

$$E^i = F^{0i} B^i = -\epsilon^{ijk} F_{jk} \tag{4.3}$$

Thus, Eq. (4.2) can be written using the linearized solution given by (3.26).

Now, let us consider a FRW background given by (2.14). Therefore, the equations of motion can be written as

$$\begin{aligned} \vec{\nabla} \cdot \tilde{\mathbf{E}}_{\text{ext}} &= 2\lambda\alpha \vec{\nabla}\chi \cdot \tilde{\mathbf{B}}_{\text{ext}}, \\ \partial_\eta \tilde{\mathbf{E}}_{\text{ext}} - \vec{\nabla} \tilde{\mathbf{B}}_{\text{ext}} &= -2\lambda\alpha \vec{\nabla}\chi \tilde{\mathbf{E}}_{\text{ext}} \end{aligned} \tag{4.4}$$

Using the usual procedure [32], we find that the dispersion relation in powers of S_α, to first order, gives us

$$k_\pm = \omega \pm 2\lambda\xi^{-1} G_0[\mu + \tau + (\bar{\epsilon}Q)^2] \hat{s} \cos(\gamma). \tag{4.5}$$

In this case, the parameter Q is the charge in the vortex induced by Lorentz breaking and τ is the tension of the string.

The angle β between the polarization vector and the galaxy's major axis is defined as

$$\langle \beta \rangle = \frac{1}{2} \frac{r}{\Lambda_s} \cos(\vec{k}, \vec{s}), \tag{4.6}$$

where $\langle \beta \rangle$ represents the mean rotation angle after Faraday's rotation is removed, r is the distance to the galaxy, $\vec{k}$ the wave vector of the radiation, and $\vec{s}$ a unit vector.

The rotation of the polarization plane is a consequence of the difference in the propagation speed of the two modes, κ_+, κ_-, the main dynamical quantities computed above. This difference, defined as the angular gradient with respect to the radial (coordinate) distance, is expressed as

$$\frac{1}{2}(\kappa_+ - \kappa_-) = \frac{d\beta}{dr}, \tag{4.7}$$

where β measures the specific entire rotation of the polarization plane, per unit length r, and is given once again by $\beta = \frac{1}{2} \Lambda_s^{-1} r \cos\gamma$. In the case of the screwed cosmic string, the constant Λ_s that encompasses the cosmic distance scale for the optical activity to be observed can be written as a function of the cosmic string energy density μ as

$$\Lambda_s^{-1} = 2\lambda G_0 \alpha^2 \xi^{-1}[\mu + \tau + (\bar{\epsilon}Q)^2]. \tag{4.8}$$

It is illustrative to consider a particular form for the arbitrary function $\alpha(\phi) = 2 \times 10^{-2}$, corresponding to the Brans-Dicke theory. In this work we use the Nodland and Ralston data to $\Lambda_s^{-1} = 10^{-32}$ eV to obtain the estimative of the value of the torsion coupling constant to Chern-Simons theory λ. Using COBE data that $2G_0\alpha(\phi_0)\xi^{-1}[\mu + \tau + (\bar{\epsilon}Q)^2] \sim 10^{-6}$, we find that $\lambda \sim 10^{-26}$ eV.

Indeed, these results are in agreement with the recent measurement of optical polarization of light from quasars and galaxies [35,36,38,51]. An interesting set of potential explanations for this effect has been put forward in Refs. [41,42,52,53]. More recently, an interesting discussion on new constraints on Lorentz symmetry violation has been reported in the work of Ref. [54], where the authors reassess the Lorentz violation parameters for electrons, positrons, and photons from GRB021206 and synchrotron electrons in the Crab nebula. In our work, we are only concerned with photons in a Lorentz-violating background; the consideration of electrons, and positrons is under investigation [21], where we focus on the matter sector (electrons and positrons) of a Lorentz-violating model with an underlying $N = 1$-supersymmetry [55].

V. CONCLUDING REMARKS

In this work, we show that is possible to build up a cosmic string solution in the presence of a Chern-Simons coupling in the case where the Lorentz-breaking vector is the dilaton gradient. Actually, supersymmetry imposes that, for the Chern-Simons-type Lorentz-breaking to be realized, the background vector that condensates must indeed be the gradient of a scalar field [55]; this is why we adopted the torsion as the dilaton gradient.

There are very important consequences of this fact in the cosmic string solution, as, for example, the existence of a charge induced by the Lorentz breaking in the core of the string. Moreover, birefringence effects may also show up. Both the external and internal solutions are consistent in the case of a timelike cosmic string. In the spacelike string, there are problems concerning the solution of the dilaton equation, as already discussed [49]. In order to solve this problem, it is necessary to introduce a massive scalar field. This case was not analyzed in the present work, but it will be the subject of a future investigation. According to our present results, the background generated by this cosmic string is birefringent and agrees with our previous analysis. The difference between them is that, in the present case, the current has a scalar-tensor parameter which gives a damping in the produced effect; but, even in this situation, this effect could have had interesting consequences at the time in which cosmic strings were probably formed.

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APPENDIX A: DETAILS ON THE WEAK-FIELD APPROXIMATION

In this appendix, we follow the procedure presented in the analysis concerning superconducting cosmic strings [9,50]. Let us study with more detail the weak-field approximation. To do this, we assume that the metric $g_{\mu\nu}$, and scalar field ϕ can be written as

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}, \qquad \Omega(\phi) = \Omega_0 + \Omega_0'\phi_{(1)}, \\ T_{\mu\nu} &= T_{(0)\mu\nu} + T_{(1)\mu\nu}, \qquad \phi = \phi_0 + \phi_{(1)}, \end{aligned} \tag{A1}$$

where $\Omega(\phi_0) = \Omega_0$, ϕ_0 is the constant dilaton value in the absence of the string and $T_{\mu\nu}$ is the energy-momentum tensor.

In order to find the metric, we use the Einstein-Cartan equations in the form $G^{\mu\nu}(\{\}) = 8\pi G_0 T^{\mu\nu}_{(0)}$, where the tensor $T_{(0)\mu\nu}$ (being first order in G) does not contain the torsion contribution due to the fact that we are working in the weak-field approximation and $\tilde{G}_0 = \tilde{\phi}^{-1} \equiv G\Omega_0^2$. Using the energy conservation in the weak-field approximation, we obtain after integration,

$$\int_0^{r_0} r dr (T^{\theta}_{(0)\theta} + T^{r}_{(0)r}) = r_0^2 T^{r}_{(0)r}(r_0) = \frac{1}{2} r_0^2 Y_t'^2(r_0). \tag{A2}$$

Now, let us analyze the Lorentz-breaking effects on the energy-momentum tensor. To do this, we use Eq. (3.25), which gives us

$$Y_t' = \frac{\epsilon Q}{\sqrt{2}\pi r}, \tag{A3}$$

where Q is the charge density in the core of the string. In this case, the energy-momentum tensor of the string source $T_{(0)\mu\nu}$ (in Cartesian coordinates)

$$\nabla^2 h_{\mu\nu} = -16\pi G \langle |(T_{(0)\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T_{(0)}|\rangle. \tag{A4}$$

Then, calculating the expectation value, we get

$$\langle|T_{(0)tt}|\rangle = \mu\delta(x)\delta(y) + \frac{(\bar{\epsilon}Q)^2}{4\pi}\nabla^2\left(ln\frac{r}{r_0}\right)^2,$$
$$\langle|T_{(0)zz}|\rangle = -\tau\delta(x)\delta(y) + \pi\frac{(\bar{\epsilon}Q)^2}{4\pi}\nabla^2\left(ln\frac{r}{r_0}\right)^2,$$
$$\langle|T_{(0)ij}|\rangle = -(\bar{\epsilon}Q)^2\delta_{ij}\delta(x)\delta(y) + \frac{(\bar{\epsilon}Q)^2}{2\pi}\partial_i\partial_j ln(r/r_0), \tag{A5}$$

with $\langle S_z\rangle$ given by

$$<S_z^2> = \psi_0^2\int_{-\pi}^{\pi}\cos^2 u du = \pi\psi_0^2 = S^2 \tag{A6}$$

$\bar{\epsilon}^2 = \lambda^2\gamma^2 S^2\alpha^2$, where $u = kz$ and the energy per unit length μ, and the tension per unit length τ are given, respectively, by

$$\mu = -2\pi\int_0^{r_0} T^t_{(0)t} rdr, \tau = -2\pi\int_0^{r_0} T^z_{(0)z} rdr; \tag{A7}$$

$T^{\mu}_{(0)\mu}$ represents the trace of the energy-momentum tensor without torsion. Actually, these quantities are not conserved in the Einstein frame, but we know [49] that the z-contribution of the dilaton to the metric vanishes, since the relevant quantity is the vacuum value and then the only contribution comes from the induced current. The solution to Eq. (A4) can be found by using the procedure of the current literature. After the junction with the external solution, we find the metric (3.26) that also gives us the dilaton-solution (3.15).

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