

**No news for Kerr-Schild fields**

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Algebraically special fields with no gravitational radiation are described. Kerr-Schild fields, which include as a concrete case the Kinnersley photon rocket, form an important subclass of them.

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**I. INTRODUCTION**

In 1951 Vaidya [1] generalized the Schwarzschild vacuum solution to a spherically symmetric "shining star" solution, emitting pure radiation. In 1969 Kinnersley [2] further generalized this solution to a photon rocket—a particle of arbitrary acceleration emitting an anisotropic jet of radiation. He noted that it belonged to the Kerr-Schild (KS) class of metrics, studied till then in vacuum [3] and in the presence of electromagnetic fields [4]. Later it was shown that the photon rocket is also a type *D* pure radiation solution from the Robinson-Trautman (RT) class [algebraically special expanding solutions without a twist (rotation)] [5]. In 1994 Bonnor [6] found that axially symmetric photon rockets do not emit gravitational radiation, which is strange for an accelerating particle. A suggestion was made, based on the linearized theory, that axial symmetry was responsible for this phenomenon [7]. However, an axially symmetric RT solution of type II was studied as an anisotropic perturbation of the photon rocket, still with no sign of gravitational radiation [8]. Another explanation was given [9], based on the well-known vanishing of the Einstein pseudo-energy-momentum tensor for KS fields [10]. It was extended to the Einstein-Maxwell field of a charged particle, undergoing arbitrarily accelerated motion [11].

The fact that the photon rocket is a member of the RT class was emphasized in [12,13]. In the latter reference the Bondi-Sachs [14,15] formalism was used and applied to axially symmetric RT pure radiation solutions. It was shown that the Kinnersley solution is the only one which does not radiate gravitationally. This put the study of photon rockets on a firm basis, since different pseudotensors for KS metrics lead to different results [16]. Later the news function was calculated for any RT metric and it was found once more that the Kinnersley solution is the only one to emit just photon radiation [17,18]. Rotating pure radiation KS fields (radiating Kerr metric) were given in [19,20] and it was demonstrated [20] that only pure radiation contributes to the mass loss.

Finally, the Bondi mass and the news function were recently found for twisting algebraically special metrics in a paper [21], based on previous research of Tafel and

coauthors [22,23], where the RT class was discussed as a special case.

Meanwhile, all pure radiation KS fields with axial symmetry have been found explicitly [24]. Stephani [5,25] proposed a more compact description of KS fields and rederived the results of Herlt, giving also an example of a nonaxially symmetric solution. This approach has been developed further by the present author [26–29].

In this paper we use the same approach to study fields with vanishing news function and consequently, with no gravitational radiation. The KS metrics form a large class of such solutions but do not exhaust all of them. The intersection between KS and RT fields is represented by the Kinnersley photon rocket, which explains why more general RT solutions always radiate gravitationally.

In Sec. II the notation for twisting algebraically special metrics is fixed and the expressions for the Bondi mass and the news function are given according to [21]. In Sec. III the condition for no news is discussed and different classes of metrics that satisfy it are given.

**II. THE BONDI MASS AND NEWS FUNCTION**

Pure radiation (aligned case) and vacuum algebraically special fields satisfy the Einstein equations

$$R_{\mu\nu} = \Phi k_{\mu} k_{\nu}, \quad (1)$$

where  $k^{\mu}$  is the multiple null eigenvector of the Weyl tensor and  $\Phi \geq 0$ , vanishing for vacuum solutions. In some coordinates  $u, r, \xi, \bar{\xi}$  the interval reads [5]

$$ds^2 = 2\omega(dr + Wd\xi + \bar{W}d\bar{\xi} + H\omega) - 2\frac{r^2 + \Sigma^2}{P^2}d\xi d\bar{\xi}, \quad (2)$$

where

$$\omega = du + Ld\xi + \bar{L}d\bar{\xi}, \quad \partial = \partial_{\xi} - L\partial_u, \quad (3)$$

$$W = -(r + i\Sigma)L_{,u} + i\partial\Sigma, \quad \Sigma = \frac{i}{2}P^2(\partial\bar{L} - \bar{\partial}L), \quad (4)$$

$$H = -r(\ln P)_{,u} - \frac{mr + M\Sigma}{r^2 + \Sigma^2} + P^2\text{Re}[\partial(\bar{\partial}\ln P - \bar{L}_{,u})]. \quad (5)$$

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The functions  $P$ ,  $m$ ,  $M$  are real and  $L$  is a complex function of  $u$ ,  $\xi$ ,  $\bar{\xi}$ . The Einstein equations read [5], Eqs. (30.40)–(30.42)

$$\partial(m + iM) = 3(m + iM)L_{,u}, \quad (6)$$

$$M = P^3 \text{Im} \partial \bar{\partial} \bar{\partial} V, \quad (7)$$

$$P^3 [P^{-3}(m + iM)]_{,u} = P^4 (\partial + 2\partial \ln P - 2L_{,u}) \partial I - \frac{\eta^2}{2}, \quad (8)$$

where  $P \equiv V_{,u}$  and

$$\Phi = \eta^2 \rho \bar{\rho}, \quad \rho^{-1} = -(r + i\Sigma), \quad (9)$$

$$I = P^{-1} (\bar{\partial} \bar{\partial} V)_{,u} = \bar{\partial} (\bar{\partial} \ln P - \bar{L}_{,u}) + (\bar{\partial} \ln P - \bar{L}_{,u})^2. \quad (10)$$

Equation (8) is, in fact, a definition of  $\eta^2$ , the actual field equations being (6) and (7), which are the same as in the vacuum case. Equations (7) and (8) give also

$$P^3 (P^{-3} m - \text{Re} \partial \bar{\partial} \bar{\partial} V)_{,u} = -P^2 (\partial \bar{\partial} V)_{,u} (\bar{\partial} \bar{\partial} V)_{,u} - \frac{\eta^2}{2}, \quad (11)$$

where  $M$  is excluded and this condition may replace Eq. (8).

In [21] it was shown that  $\bar{I}$  is the Bondi news function, which enters the tensor  $n_{ab}$  and its derivative

$$n_{ab} dx^a dx^b = -2P_s^{-1} (\partial \bar{\partial} V d\xi^2 + \bar{\partial} \bar{\partial} V d\bar{\xi}^2), \quad (12)$$

$$(n_{ab})_{,u} dx^a dx^b = -2\bar{I} d\xi^2 - 2I d\bar{\xi}^2, \quad (13)$$

while the Bondi mass is given by

$$M_B = m \hat{P}^{-3} - \frac{1}{2} \hat{P}^{-1} (\sigma^0 \bar{\sigma}^0)_{,u} + P_s^4 \text{Re} [\hat{P}^{-1} I_{,u} (\partial \hat{u})^2 - 2\partial I \partial \hat{u}], \quad (14)$$

where

$$\hat{P} \equiv P P_s^{-1}, \quad P_s = 1 + \frac{1}{2} \xi \bar{\xi}, \quad (15)$$

$$\sigma^0 = P_s \partial \bar{\partial} V, \quad (\sigma^0)_{,u} = P_s P \bar{I}, \quad (16)$$

and  $\hat{u}$  is an approximate Bondi coordinate. Then Eq. (11) becomes the energy loss formula in the Bondi-Sachs approach.

### III. FIELDS WITH NO NEWS

The condition for vanishing news is obviously  $I = 0$ . Equation (10) gives in this case

$$\partial \bar{\partial} V = F(\xi, \bar{\xi}), \quad (17)$$

where  $F$  is an arbitrary complex function. All, but the first term in the expression (14) for the Bondi mass, vanish

$$M_B = \frac{m P_s^3}{P^3}. \quad (18)$$

Equation (8) simplifies and its real part gives for the mass loss

$$(M_B)_{,u} = -\frac{\eta^2 P_s^3}{2P^3} = -\frac{\eta^2 M_B}{2m}. \quad (19)$$

There is no gravitational radiation; the Bondi mass decreases due to the emission of photons and when  $\eta = 0$ , it stays constant.

A large class of fields, satisfying the no news condition (17) are the Kerr-Schild fields, which can be defined by [25]

$$\partial \bar{\partial} V = 0. \quad (20)$$

In this case the tensor  $n_{ab}$  vanishes identically. One can choose a gauge with  $m = 1$ ,  $M = 0$ ,  $L = ih\xi$ , where  $h$  is real. Equation (19) becomes

$$(\ln V_{,u})_{,u} = \frac{\eta^2}{6}, \quad (21)$$

which is, in fact, Eq. (32.79) from [5]. One can integrate it twice to find  $V$  as a function of  $\eta$ . In the vacuum case  $V$  must be linear in  $u$  and the reality condition then uniquely fixes  $h$ . These solutions have been given already in [3] and include the Kerr black hole. In the pure radiation case the general solution is not known. Herlt [5,24] has found the general axisymmetric solution. This includes the radiating Kerr metric [20]. An example of a nonaxisymmetric solution was given in [25]. In all these metrics  $\eta = \eta(u)$ , so presumably it is more natural to base the classification of KS fields on the form of  $\eta$  and not on the symmetries of the metric.

For RT solutions  $L = 0$ ,  $M = 0$  and the condition  $I = 0$  becomes

$$P_{\xi\xi} = 0, \quad P_{\bar{\xi}\bar{\xi}} = 0. \quad (22)$$

Together they give the nonaxisymmetric solution

$$P = \alpha(u) \xi \bar{\xi} + \beta(u) \xi + \bar{\beta}(u) \bar{\xi} + \delta(u), \quad (23)$$

where  $\alpha$ ,  $\delta$  are real. In general,  $\alpha$ ,  $\beta$ ,  $\delta$  may be interpreted in terms of the acceleration of a particle moving along a spacelike, timelike, or null world line [5], Sec. 28.3. When  $K = 2(\alpha\delta - \beta\bar{\beta})$  is positive we get the Kinnersley photon rocket. The case with no  $u$  dependence represents the Vaidya shining star [1]. Consequently, the only RT solution with no gravitational radiation is the photon rocket, which belongs also to the KS class. This confirms the previous results [6,9,13,17,18].

Some general classes of twisting vacuum solutions have been found [5,30] under the assumptions that  $L_{,u} - \partial \ln P$  and  $m + iM$  are independent of  $u$ . Among them is the case with no news  $I = 0$  [5], Sec. 29.2.4, which is solved explicitly. Thirty years ago it was shown [31] that these

are the only solutions which are nonradiative in the sense that the Weyl tensor for large  $r$  behaves like  $1/r^3$ . If the further condition  $L_{,u} = 0$  is imposed, the metric becomes independent of  $u$  [5], Sec. 29.2.5. Then  $P$  is given by Eq. (23) with constant  $\alpha, \beta, \delta$  and the expressions for

$m, M, L$  are given by Eq. (29.61) from [5]. The subclass  $m + iM = \text{const}$  contains well-known type  $D$  solutions, such as Kerr and NUT, Kerr and Debney/Demiański four-parameter solution [32,33]. When, in addition,  $M = 0$  we come again to the KS vacuum solutions [5].

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- [1] P. Vaidya, Proc. Indian Acad. Sci. A **33**, 264 (1951).  
 [2] W. Kinnersley, Phys. Rev. **186**, 1335 (1969).  
 [3] R. P. Kerr and A. Schild, Proc. Symp. Appl. Math. **17**, 199 (1965).  
 [4] G. C. Debney, R. P. Kerr, and A. Schild, J. Math. Phys. (N.Y.) **10**, 1842 (1969).  
 [5] H. Stephani, D. Kramer, M. A. H. MacCallum, C. Hoenselaers, and E. Herlt, *Exact Solutions of Einstein's Field Equations* (Cambridge University Press, Cambridge, 2003), 2nd ed.  
 [6] W. B. Bonnor, Classical Quantum Gravity **11**, 2007 (1994).  
 [7] T. Damour, Classical Quantum Gravity **12**, 725 (1995).  
 [8] W. B. Bonnor, Classical Quantum Gravity **13**, 277 (1996).  
 [9] F. H. J. Cornish and B. Micklewright, Classical Quantum Gravity **13**, 2499 (1996).  
 [10] M. Gürses and F. Gürsey, J. Math. Phys. (N.Y.) **16**, 2385 (1975).  
 [11] F. H. J. Cornish and B. Micklewright, Classical Quantum Gravity **13**, 2505 (1996).  
 [12] S. Dain, O. M. Moreschi, and R. J. Gleiser, Classical Quantum Gravity **13**, 1155 (1996).  
 [13] U. von der Gönnna and D. Kramer, Classical Quantum Gravity **15**, 215 (1998).  
 [14] H. Bondi, M. G. J. van der Burg, and A. W. K. Metzner, Proc. R. Soc. London A **269**, 21 (1962).  
 [15] R. K. Sachs, Proc. R. Soc. London A **270**, 103 (1962).  
 [16] F. H. J. Cornish and B. Micklewright, Gen. Relativ. Gravit. **31**, 913 (1999).  
 [17] F. H. J. Cornish and B. Micklewright, Classical Quantum Gravity **16**, 611 (1999).  
 [18] F. H. J. Cornish, Classical Quantum Gravity **17**, 3945 (2000).  
 [19] D. Kramer, in Proceedings of the Tezisy 3rd Soviet Gravitational Conference, Erevan, 1972, p. 321.  
 [20] P. C. Vaidya and L. K. Patel, Phys. Rev. D **7**, 3590 (1973).  
 [21] W. Natorf and J. Tafel, Classical Quantum Gravity **21**, 5397 (2004).  
 [22] J. Tafel and S. Pukas, Classical Quantum Gravity **17**, 1559 (2000).  
 [23] J. Tafel, Classical Quantum Gravity **17**, 4397 (2000).  
 [24] E. Herlt, Gen. Relativ. Gravit. **12**, 1 (1980).  
 [25] H. Stephani, Gen. Relativ. Gravit. **15**, 173 (1983).  
 [26] B. V. Ivanov, J. Math. Phys. (N.Y.) **40**, 5849 (1999).  
 [27] B. V. Ivanov, Phys. Rev. D **60**, 104005 (1999).  
 [28] B. V. Ivanov, Phys. Rev. D **61**, 064011 (2000).  
 [29] B. V. Ivanov, gr-qc/9910072.  
 [30] I. Robinson and J. R. Robinson, Int. J. Theor. Phys. **2**, 231 (1969).  
 [31] D. W. Trim and J. Wainwright, J. Math. Phys. (N.Y.) **15**, 535 (1974).  
 [32] R. P. Kerr and G. Debney, J. Math. Phys. (N.Y.) **11**, 2807 (1970).  
 [33] M. Demiański, Phys. Lett. **42A**, 157 (1972).