PHYSICAL REVIEW D **71,** 043504 (2005)

# *K*-chameleon and the coincidence problem

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(Received 8 December 2004; published 8 February 2005)

In this paper we present a hybrid model of *k*-essence and chameleon, named as *k*-chameleon. In this model, due to the chameleon mechanism, the directly strong coupling between the *k*-chameleon field and matters (cold dark matters and baryons) is allowed. In the radiation-dominated epoch, the interaction between the *k*-chameleon field and background matters can be neglected; the behavior of the *k*-chameleon therefore is the same as that of the ordinary *k*-essence. After the onset of matter domination, the strong coupling between the *k*-chameleon and matters dramatically changes the result of the ordinary *k*-essence. We find that during the matter-dominated epoch, only two kinds of attractors may exist: one is the familiar **K** attractor and the other is a completely *new*, dubbed **C** attractor. Once the Universe is attracted into the **C** attractor, the fraction energy densities of the *k*-chameleon $\Omega_\phi$ and dust matter $\Omega_m$ are fixed and comparable, and the Universe will undergo a power-law accelerated expansion. One can adjust the model so that the **K** attractor does not appear. Thus, the *k*-chameleon model provides a natural solution to the cosmological coincidence problem.

DOI: 10.1103/PhysRevD.71.043504 PACS numbers: 98.80.–k, 98.80.Cq

## I. INTRODUCTION

There are now a lot of cosmological observations, such as SNe Ia [1–3], WMAP [4], SDSS [5], etc. All suggest that the Universe is spatially flat and consists of approximately 70% dark energy with negative pressure, 30% dust matter (cold dark matter plus baryon), and negligible radiation, and that the Universe is undergoing an accelerated expansion. To understand the nature of the dark energy remains as one of the biggest challenges to theorists and cosmologists [6]. The simplest candidate of the dark energy is a tiny positive cosmological constant. However, it is difficult to understand why the cosmological constant is about 120 orders of magnitude smaller than its natural expectation, namely, the Planck energy density. This is the so-called cosmological constant problem. Another puzzle of the dark energy is the cosmological coincidence problem, i.e., why are the dark energy density and the dust matter energy density comparable *now* and why does the Universe begin the accelerated expansion just only *recently*?

In order to have an interpretation to the accelerated expansion of the Universe, many alternatives to the cosmological constant have been proposed. One of the interesting scenarios is the so-called quintessence model [7]. The quintessence is a slowly varying scalar field with a canonical kinetic energy term. With the evolution of the Universe, the scalar field slowly rolls down its potential. A class of tracker solutions of quintessence [8,9] is found in order to solve the cosmological coincidence problem. As shown, however, the quintessence model still needs some fine-tuning in order for the quintessence component to overtake the matter density at the present epoch (for example, see Refs. [9,10]). Motivated by the *k*-inflation [11], in which a scalar field with noncanonical kinetic energy terms acts as the inflaton, the so-called *k*-essence [10,12–14] is introduced to the coincidence problem. In this well-known model, by the help of nonlinear kinetic energy terms, a dynamical solution to the cosmological coincidence problem without fine-tuning is possible [10,12]. In fact, *k*-essence is based on the idea of a dynamical attractor solution which makes it act as a cosmological constant only at the onset of matter domination. Consequently, *k*-essence overtakes the matter energy density and makes the Universe start with accelerated expansion just recently. It is worth noting that to achieve a later-time acceleration attractor, one needs to design the Lagrangian of the model so that $r^2(y_d) > 1$ in order to avoid the dust attractor [10]. After all, the quintessence and *k*-essence fields are very light scalar fields. Such light fields may mediate a long-range force and therefore are subject to tight constraints from the searches of the fifth force [15] and the tests of the equivalence principle (EP) [16].

On the other hand, the coupling between scalar field and matters has been studied for some years (for example, see Refs. [17–23]). Recently, a novel scenario named chameleon [24–28] has been proposed (see also [29]). In this scenario, the scalar field can be directly coupled to matters (cold dark matters or baryons) with gravitational strength, in harmony with general expects from string theory, while this strong coupling can escape from the local tests of EP

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violations and fifth force searches. The basic idea of the chameleon scenario is that the scalar field acquires a mass which depends on the ambient matter density (so the name chameleon). While nearly massless in the cosmos where the matter density is tiny, the chameleon mass is of the order of an inverse millimeter on the Earth where the matter density is high, which is sufficient to evade the tightest constraints from the tests of EP violations and fifth force searches.

It is interesting to wonder what will happen when the $k$-essence is strongly coupled to matters through the chameleon mechanism. May the virtues of $k$-essence and chameleon join together and the shortcomings be avoided? The answer is yes. In this paper, we will combine the $k$-essence with the chameleon and present a so-called $k$-chameleon model, which of course is a hybrid of the $k$-essence and chameleon. Through the chameleon mechanism, the directly strong coupling between $k$-chameleon and matters (cold dark matters and baryons) is allowed. We study the cosmological evolution of $k$-chameleon and find that the $k$-chameleon model can provide a natural solution to the cosmological coincidence problem.

In the $k$-chameleon model, during the radiation-dominated epoch, the interaction between the $k$-chameleon field and ambient matters can be negligible. Therefore the behavior of the $k$-chameleon is the exact same as that of the ordinary $k$-essence without the interaction between the scalar field and background matters. As a result, three kinds of attractors, namely **R**, **K**, and **S** (following the notations of the $k$-essence model [10,12]) may exist. The radiation tracker, i.e., the **R** attractor, has the largest basin of attraction on the whole phase plane so that most initial conditions join onto it and then makes this scenario become insensitive to initial conditions. However, after the onset of matter domination, the strong coupling between the $k$-chameleon and matters dramatically changes the result for the ordinary $k$-essence. In the matter-dominated epoch, the **D** and **S** attractors (which may exist in the ordinary $k$-essence model) are physically forbidden due to the strong coupling between $k$-chameleon and matters. Note that unlike the ordinary $k$-essence model, the disappearance of **D** and **S** attractors naturally occurs in the $k$-chameleon model, and need not any artificial design of the Lagrangian. Actually, during the matter-dominated epoch, only two kinds of attractors may exist: one is the familiar **K** attractor and the other is a completely *new* one named the **C** attractor. The new attractor **C** has some desirable features which may provide a promising solution to the cosmological coincidence problem.

Once the Universe is attracted into the **C** attractor, the fraction energy densities of the $k$-chameleon $\Omega_\phi$ and the matters $\Omega_m$ are fixed and they are comparable. Further, many parameters, such as the parameter of the equation of state of the $k$-chameleon field $w_\phi$ and its kinetic energy term $X$, are also fixed. And the Universe will undergo a power-law accelerated expansion forever. In this sense the $k$-chameleon model gives a natural solution to the cosmological coincidence problem. On the other hand, note that if the kinetic energy term $X$ of the $k$-chameleon is fixed at a somewhat small value (equivalently $y \equiv 1/\sqrt{X}$ is large), the $k$-chameleon can be treated as a canonical chameleon approximately. Therefore, we cannot detect it from the tests of EP violation and fifth force searches on the Earth and in the solar system today, although it is strongly coupled to background matters.

This paper is organized as follows: In Sec. II, a brief review of the chameleon mechanism is given. In Sec. III, we present our $k$-chameleon model and illustrate how the directly strong coupling between $k$-chameleon and matters (cold dark matters and baryons) is allowed while it cannot be detected from the tests of EP violation and fifth force searches on the Earth and in the solar system. In Sec. IV, the cosmological evolution of the $k$-chameleon is studied and the result shows that the $k$-chameleon model may provide a promising solution to the cosmological coincidence problem. A brief conclusion will be given in Sec. V.

We use the units $\hbar = c = 1$ throughout this paper. $M_{\rm pl} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass. We adopt the metric convention as $(+\,-\,-\,-)$.

## II. A BRIEF REVIEW OF THE CHAMELEON MECHANISM

Following Refs. [24–26], consider a canonical chameleon scalar field $\phi$ governed by the action

$$S = \int d^4x\sqrt{-g}\left[-\frac{M_{\rm pl}^2}{2}\mathcal{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + \int d^4x\mathcal{L}_m(\psi_m^{(i)}, g_{\mu\nu}^{(i)}), \tag{1}$$

where $g$ is the determinant of the metric $g_{\mu\nu}$, $\mathcal{R}$ is the Ricci scalar, and $\psi_m^{(i)}$ are various matter fields labeled by $i$. In the chameleon mechanism, the scalar field $\phi$ is supposed to directly interact with matters through a conformal coupling. In other words, each matter field $\psi_m^{(i)}$ couples to a metric $g_{\mu\nu}^{(i)}$ which is related to the Einstein-frame metric $g_{\mu\nu}$ by the rescaling

$$g_{\mu\nu}^{(i)} = e^{2\beta_i\phi/M_{\rm pl}}g_{\mu\nu}, \tag{2}$$

where $\beta_i$ are dimensionless constants. Moreover, the different $\psi_m^{(i)}$ fields are assumed not to interact with each other for simplicity. From the action Eq. (1), the equation of motion for $\phi$ is

$$\nabla^2\phi = -V_{,\phi} - \sum_i \frac{\beta_i}{M_{\rm pl}}e^{4\beta_i\phi/M_{\rm pl}}g_{(i)}^{\mu\nu}T_{\mu\nu}^{(i)}, \tag{3}$$

where

$$\nabla^2\phi \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = \frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\phi]$$

and $T^{(i)}_{\mu\nu} = (2/\sqrt{-g^{(i)}})\delta\mathcal{L}_m/\delta g^{\mu\nu}_{(i)}$ is the stress-energy tensor density for the $i$th form of matter. $V_{,\phi}$ denotes the derivative of $V$ with respect to $\phi$. For nonrelativistic dust-like matter, $g^{\mu\nu}_{(i)}T^{(i)}_{\mu\nu} = \tilde{\rho}_i$, where $\tilde{\rho}_i$ is the energy density. Defined in this way, however, $\tilde{\rho}_i$ is not conserved in the Einstein frame. Instead, it is more convenient to define a matter density $\rho_{mi} \equiv \tilde{\rho}_i e^{3\beta_i\phi/M_{\rm pl}}$ which is independent of $\phi$ and is conserved in the Einstein frame. Thus, Eq. (3) can be recast as

$$\nabla^2\phi = -V_{,\phi} - \sum_i \frac{\beta_i}{M_{\rm pl}}\rho_{mi}e^{\beta_i\phi/M_{\rm pl}} = -V^{\rm eff}_{,\phi}. \tag{4}$$

Note that the dynamics of $\phi$ is not governed solely by $V(\phi)$, but rather by an effective potential

$$V_{\rm eff}(\phi) = V(\phi) + \sum_i \rho_{mi}e^{\beta_i\phi/M_{\rm pl}}, \tag{5}$$

which depends explicitly on the matter density $\rho_{mi}$. The key ingredient to achieve a successful chameleon model is that the effective potential $V_{\rm eff}(\phi)$ has a minimum even when $V(\phi)$ is monotonic. In fact, if $V(\phi)$ is monotonically decreasing and $\beta_i > 0$ or, equivalently, $V(\phi)$ is monotonically increasing and $\beta_i < 0$, the effective potential $V_{\rm eff}(\phi)$ has a minimum $\phi_{\rm min}$ satisfying

$$V^{\rm eff}_{,\phi}(\phi_{\rm min}) = V_{,\phi}(\phi_{\rm min}) + \sum_i \frac{\beta_i}{M_{\rm pl}}\rho_{mi}e^{\beta_i\phi_{\rm min}/M_{\rm pl}} = 0. \tag{6}$$

Meanwhile, the mass of small fluctuations about the minimum $\phi_{\rm min}$ is

$$m^2_{\rm eff} \equiv V^{\rm eff}_{,\phi\phi}(\phi_{\rm min}) = V_{,\phi\phi}(\phi_{\rm min}) + \sum_i \frac{\beta_i^2}{M^2_{\rm pl}}\rho_{mi}e^{\beta_i\phi_{\rm min}/M_{\rm pl}}. \tag{7}$$

In other words, the originally massless scalar field acquires a mass which depends on the local matter density. The denser the environment, the more massive the chameleon is. Actually, while the coupling constants $\beta_i$ can be of order unity as the natural expectations from string theory, it is still possible for the mass of the chameleon, i.e., $m_{\rm eff}$, to be sufficiently large on the Earth to evade current constraints on EP violation and fifth force. On the other hand, through the so-called "thin-shell" effect, the chameleon-mediated force between two large objects, such as the Earth and the sun, is much suppressed, which thereby ensures that solar system tests of gravity are satisfied. For more details, see the original papers [24–27]. The quantum stability analysis of the chameleon model is presented in Ref. [28].

It is worth noting that, in most existing canonical chameleon models, the potentials $V(\phi)$ are assumed to be of the runaway form, namely, it is monotonically decreasing and satisfies

$$\lim_{\phi\to\infty} V = 0, \qquad \lim_{\phi\to\infty}\frac{V_{,\phi}}{V} = 0, \qquad \lim_{\phi\to\infty}\frac{V_{,\phi\phi}}{V_{,\phi}} = 0\cdots$$

as well as

$$\lim_{\phi\to 0} V = \infty, \qquad \lim_{\phi\to 0}\frac{V_{,\phi}}{V} = \infty, \qquad \lim_{\phi\to 0}\frac{V_{,\phi\phi}}{V_{,\phi}} = \infty\cdots.$$

Actually, the fiducial potentials are chosen to be

$$V(\phi) = M^4\left(\frac{M}{\phi}\right)^n \quad \text{and} \quad V(\phi) = M^4\exp(M^n/\phi^n) \tag{8}$$

in Refs. [24,26] and Ref. [25], respectively. However, for the potentials given in Eq. (8), to achieve a successful chameleon model, the mass scale $M$ has to satisfy

$$M \lesssim 10^{-3}\ \text{eV}, \tag{9}$$

which is about 30 orders of magnitude smaller than its natural expectation, namely, the Planck mass. Fortunately, Ref. [27] shows that a chameleon model with a nonrunaway form potential

$$V(\phi) = \frac{1}{2}m^2_\phi\phi^2 + \frac{\xi}{4!}\phi^4 \tag{10}$$

can be a successful example even when the parameter $\xi$ is of order unity. In Sec. III C of the present paper, a completely *new* canonical chameleon with nonrunaway potential and without any fine-tuning like in Eq. (9) will be presented. This new canonical chameleon model is another successful example.

## III. *K*-CHAMELEON MODEL

So far, all chameleon models existing in the literature are of the form of quintessencelike, namely, the kinetic energy term of the scalar field is a canonical one. As is well known, nonlinear kinetic energy terms naturally appear in many models unifying gravity with other particle forces, including supergravity and superstring theory. For many years, the contributions of these higher order terms have been ignored for the reasons of simplicity. The example of $k$-essence [10,12–14] demonstrates that the effects of nonlinear dynamics can be dramatic. Here, motivated by $k$-essence, we put the chameleon and $k$-essence together and present a $k$-chameleon model. Namely, we consider a scalar field with nonlinear kinetic terms and the scalar field is strongly coupled to matters.

### A. Setup

Our starting point is the action

$$S = \int d^4x\sqrt{-g}\left[-\frac{M_{\rm pl}^2}{2}\mathcal{R} + p(\phi, X)\right] + \int d^4x\mathcal{L}_m(\psi_m^{(i)}, g_{\mu\nu}^{(i)}), \tag{11}$$

where $g$ is the determinant of the metric $g_{\mu\nu}$, $\mathcal{R}$ is the Ricci scalar, and $\psi_m^{(i)}$ are various matter fields labeled by $i$. The scalar field $\phi$ interacts directly with matters through a conformal coupling. Explicitly, each matter field $\psi_m^{(i)}$ couples to a metric $g_{\mu\nu}^{(i)}$ which is related to the Einstein-frame metric $g_{\mu\nu}$ by the rescaling

$$g_{\mu\nu}^{(i)} = A_i^2(\phi)g_{\mu\nu}, \tag{12}$$

where $A_i(\phi)$ is a function of the $k$-chameleon field $\phi$. Note that Eqs. (11) and (12) are of the general form arising from string theory, supergravity, and Brans-Dicke theory. Moreover, the different fields $\psi_m^{(i)}$ are assumed not to interact with each other for simplicity. The kinetic energy term is defined by

$$X \equiv \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{13}$$

Note that if one takes $p(\phi, X) = X - V(\phi)$ and $A_i(\phi) = \exp(\beta_i\phi/M_{\rm pl})$, this case then reduces to the canonical chameleon model presented in Sec. II.

Varying the action Eq. (11) with respect to $\phi$ yields the equation of motion for the $k$-chameleon field $\phi$

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}p_{,X}g^{\mu\nu}\partial_\nu\phi] = p_{,\phi} - \sum_i\alpha_i(\phi)A_i^4(\phi)g_{(i)}^{\mu\nu}T_{\mu\nu}^{(i)}, \tag{14}$$

where $p_{,X}$ denotes the derivative of $p$ with respect to $X$, and

$$T_{\mu\nu}^{(i)} = \frac{2}{\sqrt{-g^{(i)}}}\frac{\delta\mathcal{L}_m}{\delta g_{(i)}^{\mu\nu}} \tag{15}$$

is the stress-energy tensor density for the $i$th form of matter, and

$$\alpha_i(\phi) \equiv \frac{\partial\ln A_i(\phi)}{\partial\phi}. \tag{16}$$

For the $i$th nonrelativistic dustlike matter, the energy density in the Einstein frame

$$\rho_i = T_\mu^\mu = g^{\mu\nu}\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}} = A_i^4(\phi)\tilde{\rho}_i, \tag{17}$$

where

$$\tilde{\rho}_i = g_{(i)}^{\mu\nu}T_{\mu\nu}^{(i)} = g_{(i)}^{\mu\nu}\frac{2}{\sqrt{-g^{(i)}}}\frac{\delta\mathcal{L}_m}{\delta g_{(i)}^{\mu\nu}} \tag{18}$$

is the energy density in the matter frame. However, $\tilde{\rho}_i$ is not conserved in the Einstein frame. Instead, it is more convenient to define matter density

$$\rho_{mi} \equiv A_i^3(\phi)\tilde{\rho}_i \tag{19}$$

which is independent of $\phi$ and conserved in the Einstein frame. Then Eq. (14) can be rewritten as

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}p_{,X}g^{\mu\nu}\partial_\nu\phi] = p_{,\phi} - \sum_i\alpha_i(\phi)\rho_i. \tag{20}$$

In addition, from the action Eq. (11), we have the pressure and energy density of the $k$-chameleon field $\phi$ [11]

$$p_\phi = p(\phi, X), \qquad \rho_\phi = 2Xp_{,X} - p, \tag{21}$$

respectively. Consider a flat Friedmann-Robertson-Walker universe, whose metric is

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2, \tag{22}$$

where $a$ is the scale factor. If the scalar field $\phi$ is spatially homogeneous, one then has

$$X = \frac{1}{2}\dot{\phi}^2, \tag{23}$$

where a dot denotes the derivative with respect to the cosmic time $t$. Furthermore, by using Eq. (21), the equation of motion for the $k$-chameleon field $\phi$, namely, Eq. (20), can be recast as

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\sum_i\alpha_i(\phi)\rho_i\dot{\phi}, \tag{24}$$

where $H = \dot{a}/a$ is the Hubble parameter. From the total energy conservation equation

$$\dot{\rho}_{\rm tot} + 3H(\rho_{\rm tot} + p_{\rm tot}) = 0, \tag{25}$$

where the total pressure and energy density are

$$p_{\rm tot} = p_\phi + p_r, \quad \text{and} \quad \rho_{\rm tot} = \rho_\phi + \sum_i\rho_i + \rho_r, \tag{26}$$

respectively, we have

$$\dot{\rho}_i + 3H\rho_i = \alpha_i(\phi)\rho_i\dot{\phi}, \tag{27}$$

and

$$\dot{\rho}_r + 4H\rho_r = 0, \tag{28}$$

where $p_r = \rho_r/3$ and $\rho_r$ are the pressure and energy density of radiation, respectively. Note that there is no coupling between the scalar field $\phi$ and radiation because the trace of the stress-energy tensor of radiation vanishes. Finally we write down the Friedmann equation

$$3H^2M_{\rm pl}^2 = \rho_{\rm tot} = \rho_\phi + \rho_m + \rho_r, \tag{29}$$

where $\rho_m = \sum_i \rho_i$ is the sum of the energy density of all matter components.

### B. Master equations of the $k$-chameleon

In this paper, following $k$-essence [10,12], we only consider a factorizable Lagrangian of the form

$$p(\phi, X) = K(\phi)D(X), \tag{30}$$

where we assume $K(\phi) > 0$. From Eq. (21), we have

$$\begin{aligned} p_\phi &= p(\phi, X) = K(\phi)D(X), \\ \rho_\phi &= 2Xp_{,X} - p = K(\phi)[2XD_{,X}(X) - D(X)] \\ &\equiv K(\phi)E(X). \end{aligned} \tag{31}$$

The parameter of the equation of state is

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{D}{E} = \frac{D}{2XD_{,X} - D}. \tag{32}$$

According to the definition in [11], the sound speed is

$$C_s^2 = \frac{p_{\phi,X}}{\rho_{\phi,X}} = \frac{D_{,X}}{E_{,X}}. \tag{33}$$

Substituting Eqs. (23) and (31) into Eq. (24), we obtain

$$\begin{aligned} \frac{dX}{dN} = -\frac{E}{E_{,X}}\Bigg[ & 3(1 + w_\phi) + \sigma\frac{K_{,\phi}}{K}\frac{\sqrt{2X}}{H} \\ & + \sigma\sum_i \alpha_i(\phi)\rho_i \frac{\sqrt{2X}}{HKE}\Bigg], \end{aligned} \tag{34}$$

where $N \equiv \ln a$ is the so-called $e$-folding time, and $\sigma$ is the sign of $\dot\phi$.

It is convenient to reexpress $D$ as $D = g(y)/y$ and to view it as a function of the new variable $y \equiv 1/\sqrt{X}$. Equations (30)–(33) then become

$$p_\phi = \frac{Kg}{y}, \qquad \rho_\phi = -Kg', \tag{35}$$

$$w_\phi = -\frac{g}{g'y}, \qquad C_s^2 = \frac{g - g'y}{g''y^2}, \tag{36}$$

where a prime denotes the derivative with respect to $y$. Taking into account Eqs. (29), (35), and (36), one can recast Eq. (34) in terms of the new variable as

$$\begin{aligned} \frac{dy}{dN} = \frac{3}{2}\frac{w_\phi(y) - 1}{s'(y)}\Bigg[ & s(y) + \sigma\frac{K_{,\phi}}{2K^{3/2}}\sqrt{\frac{\rho_\phi}{\rho_{\rm tot}}} \\ & + \sigma\sum_i \frac{\alpha_i(\phi)}{2K^{1/2}}\frac{\rho_i}{\sqrt{\rho_\phi\rho_{\rm tot}}}\Bigg], \end{aligned} \tag{37}$$

where

$$s(y) = \left(-\frac{3g'}{8M_{\rm pl}^2}\right)^{1/2} y(1 + w_\phi) = \sqrt{\frac{3}{8M_{\rm pl}^2}}\frac{g - g'y}{\sqrt{-g'}}. \tag{38}$$

Note that the requirements of positivity of the energy density, $\rho_\phi > 0$, and stability of the $k$-chameleon background, $C_s^2 > 0$, imply

$$-g' > 0, \qquad g'' > 0. \tag{39}$$

These conditions indicate that $g$ should be a decreasing convex function of $y = 1/\sqrt{X}$. A sample of the function $g(y)$ is plotted in Fig. 1.

Since we are attempting to study the cosmological evolution of the $k$-chameleon and trying to find out its attractors, we impose the condition that the coefficients of the last two terms in Eq. (37) to be constants for simplicity. Thus, $K(\phi)$, $\alpha_i(\phi)$, and $A_i(\phi)$ should be the form

$$K(\phi) = \frac{M^2}{\phi^2}, \qquad \alpha_i(\phi) = \frac{\beta_i}{\phi}, \qquad A_i(\phi) = \left(\frac{\phi}{M}\right)^{\beta_i}, \tag{40}$$

where $M$ is a constant mass scale and $\beta_i$ are dimensionless positive constants. Furthermore, although $\beta_i$ may be different for different matter species, we take a same value $\beta$ for all $\beta_i$ for simplicity [$\alpha(\phi)$, $A(\phi)$ for all $\alpha_i(\phi)$, $A_i(\phi)$ accordingly]. In fact, it is straightforward to generalize it to the generic case with different $\beta_i$. In addition, without loss of generality, we restrict ourselves to the most interesting case of positive $\dot\phi$, namely $\sigma = +1$. Under these simplifications, Eq. (37) becomes

$$\frac{dy}{dN} = \frac{3}{2}\frac{w_\phi(y) - 1}{r'(y)}\left[r(y) - \sqrt{\Omega_\phi} + \frac{\beta}{2}\frac{\Omega_m}{\sqrt{\Omega_\phi}}\right], \tag{41}$$

where $r(y) \equiv Ms(y)$ is a dimensionless function of $y$. It is worth noting that if $M = \sqrt{3}M_{\rm pl}$, $r(y)$ then is completely the same as that of the $k$-essence case [10,12]. The fraction energy densities $\Omega_\phi \equiv \rho_\phi/\rho_{\rm tot}$ and $\Omega_m \equiv \rho_m/\rho_{\rm tot}$, where $\rho_m = \sum_i \rho_i$ is the sum of the energy density of all matter components. At the same time, from Eqs. (24)–(28), we reach

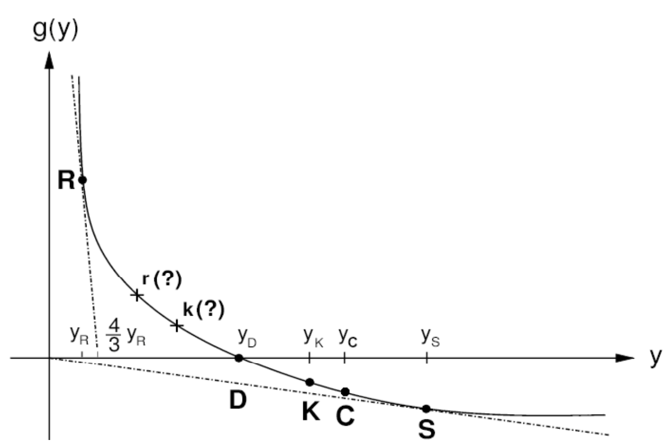


FIG. 1. A sample of function $g(y)$, reproduced from [10]. Here the new attractor **C** appears between $y_d < y < y_s$.

$$\frac{d\Omega_\phi}{dN} = -\frac{\alpha}{H}\Omega_m\dot{\phi} + 3\Omega_\phi(w_{\rm tot} - w_\phi), \tag{42}$$

$$\frac{d\Omega_m}{dN} = \frac{\alpha}{H}\Omega_m\dot{\phi} + 3\Omega_m w_{\rm tot}, \tag{43}$$

where

$$w_{\rm tot} \equiv \frac{p_{\rm tot}}{\rho_{\rm tot}} = w_\phi\Omega_\phi + \frac{1}{3}\Omega_r, \tag{44}$$

and $\Omega_r \equiv \rho_r/\rho_{\rm tot}$ is the fraction energy density of radiation.

Thus we have obtained the master equations governing the whole system. Compared to the ordinary $k$-essence model [10,12], we find that some new terms which describe the coupling between the $k$-chameleon and matters enter these equations. In addition, let us mention that in our $k$-chameleon model, the coupling function $A(\phi)$ is of a form of power law [see Eq. (40)], while in the ordinary chameleon models [24–28] or other coupled models of scalar field to matters in the literature [18,19], the coupling function is of either the exponential type [18,24–28] or linear type [19].

### C. Coupling constant $\beta$ and mass scale $M$

In this subsection we will discuss the constraints on the coupling constant $\beta$ and the mass scale $M$ imposed by the fifth force experiment [15] and the tests of the EP [16], in order to obtain a successful chameleon mechanism.

Note that all the searches of the fifth force and EP violation have been performed only at the present epoch on the Earth or in the solar system. This point is very important. As we will see, in the $k$-chameleon model, the kinetic energy term $X$ of the $k$-chameleon field could be fixed at a somewhat small value (equivalently $y \equiv 1/\sqrt{X}$ is large) at the current accelerated expansion epoch. In this case, the nonlinear kinetic energy terms can be neglected and the $k$-chameleon can be treated as a canonical chameleon approximately.

In this case, $D(X)$ in the Lagrangian $p(\phi, X) = K(\phi)D(X)$ can be approximated to

$$D(X) \simeq c_1X - c_2, \tag{45}$$

where $c_1$ is a dimensionless positive constant and $c_2$ is a positive constant with dimension of energy density. To change the Lagrangian to a canonical form, we make a redefinition of the field variable. Introducing a new scalar field

$$\phi_{\rm new} = -\sqrt{c_1}M\ln\frac{\phi}{M} \tag{46}$$

satisfying

$$X_{\rm new} = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi_{\rm new}\partial_\nu\phi_{\rm new} = c_1K(\phi)X,$$

where $K(\phi) = M^2/\phi^2$ has been considered. Accordingly, the conformal coupling $A(\phi) = (\phi/M)^\beta$ is transformed to

$$A(\phi_{\rm new}) = \exp\left[-\frac{\beta\phi_{\rm new}}{\sqrt{c_1}M}\right], \tag{47}$$

and the Lagrangian becomes the canonical one,

$$p(\phi_{\rm new}, X_{\rm new}) = X_{\rm new} - V(\phi_{\rm new}), \tag{48}$$

where the potential

$$V(\phi_{\rm new}) = c_2\exp\left[\frac{2\phi_{\rm new}}{\sqrt{c_1}M}\right]. \tag{49}$$

The equation of motion Eq. (20) then becomes

$$\nabla^2\phi_{\rm new} = -V^{\rm eff}_{,\phi_{\rm new}}(\phi_{\rm new}), \tag{50}$$

where the effective potential

$$V_{\rm eff}(\phi_{\rm new}) = V(\phi_{\rm new}) + A(\phi_{\rm new})\rho^{\rm new}_{mt}, \tag{51}$$

and the matter energy density

$$\rho^{\rm new}_{mt} = A^3(\phi_{\rm new})\sum_i\tilde{\rho}_i, \tag{52}$$

which is independent of $\phi_{\rm new}$ and is conserved in the Einstein frame, and $\tilde{\rho}_i$ is defined in Eq. (18). Obviously, because $V^{\rm eff}_{,\phi_{\rm new}\phi_{\rm new}}(\phi_{\rm new})$ is always larger than zero, the effective potential has a minimum at

$$\phi^{\rm min}_{\rm new} = \frac{\sqrt{c_1}M}{2+\beta}\ln\frac{\beta\rho^{\rm new}_{mt}}{2c_2} \tag{53}$$

satisfying $V^{\rm eff}_{,\phi_{\rm new}}(\phi^{\rm min}_{\rm new}) = 0$. Thus, the mass of small fluctuations about the minimum $\phi^{\rm min}_{\rm new}$ is

$$m^2_{\rm new} \equiv V^{\rm eff}_{,\phi_{\rm new}\phi_{\rm new}}(\phi^{\rm min}_{\rm new}). \tag{54}$$

After some algebra, we get

$$m^{-1}_{\rm new} = \left(\frac{c_1}{\beta^2 + 2\beta}\right)^{1/2}\left(\frac{M}{M_{\rm pl}}\right)\left(\frac{\beta M^4_{\rm pl}}{2c_2}\right)^{\beta/(4+2\beta)} \times \left(\frac{\rho^{\rm new}_{mt}}{M^4_{\rm pl}}\right)^{-1/(2+\beta)}M^{-1}_{\rm pl}. \tag{55}$$

It is easy to see that, if one takes the natural expectations of the constants as

$$c_1 \sim \beta \sim \mathcal{O}(1), \qquad c_2 \sim \mathcal{O}(M^4_{\rm pl}), \quad \text{and} \quad M \sim \mathcal{O}(M_{\rm pl}), \tag{56}$$

$m^{-1}_{\rm new}$ is indeed a small quantity for any reasonable matter density. For instance, the atmosphere has mean density $\rho_{\rm atm} \sim 10^{-3}$ g/cm$^3$. Substituting into Eq. (55) and assuming Eq. (56), we find

$$m^{-1}_{\rm atm} \sim \mathcal{O}(1\ {\rm mm}), \tag{57}$$

which is sufficient [24,27] to evade current constraints on EP violation and fifth force. Note that the field redefinition

Eq. (46) can be taken as $\phi_{\rm new} = \sqrt{c_1}M\ln(\phi/M)$ and the final result obtained above is still valid. In addition, let us stress here that the potential Eq. (49) is different from those in Refs. [24–27]. Therefore it is interesting to further study this model with more details and other aspects such as the thin-shell effect. We will present these details in a separate paper [30].

The upshot of this subsection is that we illustrate briefly how the directly strong coupling between the $k$-chameleon and matters (cold dark matters and baryons) is allowed while we cannot detect it from the tests of EP violations and fifth force searches on the Earth or in the solar system today. The coupling constant $\beta$ need not be tuned to an extremely small value and can be of order unity, in harmony with general expectations from string theory. The mass scale $M$ can be of order $M_{\rm pl}$. In other words, it need not be tuned like Eq. (9) mentioned above as in Refs. [24–26].

## IV. COSMOLOGICAL EVOLUTION OF *K*-CHAMELEON AND THE COSMOLOGICAL COINCIDENCE PROBLEM

In this section, we will study the cosmological evolution of the $k$-chameleon. Just like the cases of quintessence [8,9] and $k$-essence [10,12–14], the key point is to find out its attractors during the evolution. During the radiation- and matter-dominated epochs, there are several sets of possible attractors in the $k$-chameleon model. Note that, as illustrated in Sec. III C, the mass scale $M$ can be of order $M_{\rm pl}$. Thus, for simplicity, we take $M = \sqrt{3}M_{\rm pl}$ from now on.

### A. Radiation-dominated epoch

In this epoch, the fraction energy density of matter $\Omega_m \simeq 0$. One can see that all terms describing the coupling between the $k$-chameleon and matters can be neglected, and the master equations governing the system, namely, Eqs. (41)–(44), reduce to corresponding ones for the ordinary $k$-essence case [10,12]. Thus, the cosmological evolution of the $k$-chameleon in this epoch is completely the same as that of ordinary $k$-essence. A detailed study was presented in Ref. [10] for this case. Therefore we will not repeat here and only mention some key points and make some remarks.

#### *1. Attractors*

During the radiation-dominated epoch, three kinds of attractors, **R** (radiation), **K** ($k$-field), and **S** (de Sitter) attractors (following notations of the $k$-essence model [10,12]) may exist. The location of the **R** attractor, i.e., $y_r = \text{const}$, is determined by

$$y_r g'(y_r) = -3g(y_r), \tag{58}$$

which corresponds to $y_r < y_d$ to ensure $g > 0$ (positive pressure), where $y_d$ is the location of the function $g(y)$ across the $y$ coordinate axis, i.e., $g(y_d) = 0$ (see Fig. 1 of the present paper or Fig. 1 of [10]). The fraction energy density of the $k$-chameleon is given by

$$\Omega_\phi^{(r)} = r^2(y_r) = -2g'(y_r)y_r^2, \tag{59}$$

and the **R** attractor exists only if $r^2(y_r) < 1$. The parameter of the equation of state $w_\phi(y_r) = 1/3$, and the $k$-chameleon mimics the radiation. Note that $\Omega_\phi^{(r)} = r^2(y_r)$ has to be in the range 1%–10% in order to satisfy the constraints from the big bang nucleosynthesis (BBN) [31,32].

The location $y_k$ of the second attractor **K** is determined by

$$\Omega_\phi^{(k)} = r^2(y_k) = 1, \tag{60}$$

which implies that the $k$-chameleon dominates over other components. The parameter of the equation of state

$$w_\phi(y_k) = -1 + \frac{2\sqrt{2}}{3}\frac{1}{\sqrt{-g_k' y_k^2}} = \text{const}, \tag{61}$$

and the scale factor is

$$a \propto t^{2/3[1+w_\phi(y_k)]} = t^{\sqrt{-g_k' y_k^2/2}}. \tag{62}$$

If $\sqrt{-g_k' y_k^2/2} > 1$, the solution describes a power-law inflation. Physically, this condition is equivalent to $w_{\rm tot}(y_k) = w_\phi(y_k) < -1/3$.

The third attractor **S** is defined by $w_\phi(y_s) = -1$ and $\Omega_\phi^{(s)} = r^2(y_s) \simeq 0$. From Eq. (36), we have

$$g(y_s) = g_s' y_s, \qquad C_s^2 = 0. \tag{63}$$

Geometrically, **S** is a fixed point on the curve $g(y)$, which makes the tangent of $g$ at $y_s$ pass through the origin (see Fig. 1). It is clear that $y_s$ always exists for the convex decreasing function $g(y)$ unless $y_s \to \infty$.

The combination of cosmologically relevant attractors during the radiation-dominated epoch can be one of three types:

(i) **R**, **S**, and *no* other attractors at $y_r < y < y_s$;

(ii) **R**, **S**, and **K** plus possibly other attractors at $y < y_d$;

(iii) **R**, **S** (*no* **K** attractor), and at least one additional attractor **r** (?) or **k** (?) (following notations of $k$-essence).

The phase diagrams for these three cases have been drawn in Figs. 2–4 of Ref. [10], respectively. Obviously, one can see from these phase diagrams that the **R** attractor has the largest basin of attraction on the whole phase plane for all three cases.

#### *2. Remarks*

The stability analysis of the attractors was done in Ref. [10]. With a closer look, we find that the stability analysis is valid only for the **R** and **D** (dust) attractors (which may appear during the matter-dominated epoch in the ordinary $k$-essence model). Actually, because the **K** and **S** are not determined by $w_\phi = w_m$ [following notations of Ref. [10], $w_m$ denotes the state parameter of background matter (radiation or dust)], Eq. (24) of Ref. [10] is not valid for them. In fact, Eq. (24) of Ref. [10] should be changed to $d\delta\Omega_\phi/dN = 3w_\phi(y_k)\delta\Omega_\phi$ and $d\delta\Omega_\phi/dN = 3\delta\Omega_\phi$ for **K** and **S**, respectively. Therefore, the stability condition for the **K** attractor is $\sqrt{-g_k' y_k^2/2} > 2/3$ while **S** is always *unstable*. This point can be seen clearly in the phase diagrams, i.e., Figs. 2–6 of Ref. [10]: the phase flow can get near to **S**, but never reaches it, and the phase flow is then forced to leave it.

The second remark is about the basin of attraction of the **R** attractor. In Ref. [14], the statement "**R** attractor has the largest basin of attraction on the whole phase plane in the radiation-dominated epoch" was criticized by numerically analyzing two concrete models. In these two models the function $D(X)$ in the Lagrangian has the following forms:

$$D(X) = -2.01 + 2\sqrt{1+X} + 3 \times 10^{-17}X^3 - 10^{-24}X^4,$$

and

$$D(X) = -2.05 + 2\sqrt{1 + f(X)},$$

where

$$f(X) = X - 10^{-8}X^2 + 10^{-12}X^3 - 10^{-16}X^4 + 10^{-20}X^5 - 10^{-24}X^6/2^6,$$

which are first given in Refs. [10,12], respectively. (Note that in Refs. [10,12,14], the unit $3M_{\rm pl}^2 = 1$ was used.) However, we notice that the conclusion "**R** attractor has the largest basin of attraction on the whole phase plane in the radiation-dominated epoch" is based on another non-analyzable Lagrangian, i.e., Eq. (42) of Ref. [10],

$$g(y) = g_{\rm glue}(y)\left(1 - \frac{y}{s^2 y_d}\right),$$

where $g(y)$ is a function parametrized by five parameters: $y_r$, $g_r'$, $y_d$, $g_d'$, and $s^2 y_d$ (see Sec. V of Ref. [10] for more detail). By choosing suitable parameters, as shown in Secs. VA and VB of Ref. [10], that the **R** attractor has indeed the largest basin of attraction on the whole phase plane in the radiation-dominated epoch is possible.

The third remark we would like to stress is about the implication of $y_s$ mentioned above. In the left side of $y_s$, i.e., $y < y_s$, one has $w_\phi > -1$ and $C_s^2 > 0$ while $g'' > 0$. Because the $g''$ is a continuous function, and the sound speed $C_s^2$ cannot diverge at $y_s$, which implies that $g'' \neq 0$ at $y_s$. Therefore $g'' > 0$ should still hold in the right side of $y_s$, i.e., $y > y_s$. However, in the right side of $y_s$, $g - g'y < 0$, one has $w_\phi < -1$ and $C_s^2 < 0$. This is physically forbidden since the $k$-chameleon becomes unstable. As a result, the phase flow cannot pass across $y_s$. Any physically reasonable $y$ must be less than $y_s$, which can be seen clearly from Figs. 2–6 of Ref. [10] as well. Actually, a general discussion on this point has been made in Ref. [33], which shows that a dynamical transition from the states with $w_\phi > -1$ to those with $w_\phi < -1$ or vice versa is physically impossible. One can see that in fact, the condition $y < y_s$ is equivalent to $w_\phi > -1$ physically.

### B. Matter-dominated epoch

In this epoch, the fraction energy density of radiation $\Omega_r \simeq 0$, and $\Omega_m + \Omega_\phi = 1$ since the Universe is spatially flat, as indicated by many astronomical observations [4,5]. In this case, the terms due to the strong coupling between $k$-chameleon and matters cannot be neglected in the master equations (41)–(44). As will be seen shortly, due to the appearance of the coupling, the $k$-chameleon model will have a big difference from the ordinary $k$-essence model. In this subsection, we will find out all the possible attractor solutions and then study their stability.

#### *1. Attractors*

At first, we would like to point out that the **D** attractor (the scalar field tracks the dust $[w_\phi(y_d) = 0]$ during the matter-dominated epoch) and the **S** attractor, which may exist in the ordinary $k$-essence model [10,12], are physically forbidden in our $k$-chameleon model, due to the existence of strong coupling between the chameleon and dust matters (cold dark matter and baryons). To see this point, let us note that in the cases of $\Omega_m \simeq 1$, $\Omega_\phi \simeq 0$, and $w_{\rm tot} \simeq 0$ for the **S** attractor and $w_{\rm tot} = w_\phi = 0$, but $\Omega_m \neq 0$ or 1 for the **D** attractor, the third term in Eq. (41) is extremely large (for the case $\Omega_\phi \to 0$) while the first terms in Eqs. (42) and (43) cannot be neglected. Thus, *no* solutions with $y = y_{\rm attractor} = $ const satisfying $dy/dN = 0$ and $d\Omega_\phi/dN = 0$, $d\Omega_m/dN = 0$ exist. Therefore the attractors **D** and **S** will no longer appear in the $k$-chameleon model.

However, we find that the $k$-chameleon dominated attractor **K** with $\Omega_\phi^{(k)} = 1$ can still occur in the $k$-chameleon model. Its characteristics are described by Eqs. (60)–(62). A remarkable feature we would like to stress here is that this solution describes a power-law inflation provided $\sqrt{-g_k' y_k^2/2} > 1$. As we will see below, if this attractor is stable, this condition for accelerated expansion can be satisfied automatically.

Except for the **K** attractor, we find that there is a new attractor solution in the $k$-chameleon model, which arises due to the strong coupling between the $k$-chameleon field and the dust matter. The new attractor is dubbed as **C** attractor (the letter **C** stands for "Chameleon"), and has some very interesting features. Let us describe the **C** attractor in some detail.

Considering $\rho_\phi = -Kg'$ and $K(\phi) = M^2/\phi^2$ [see Eqs. (35) and (40)] and setting $M = \sqrt{3}M_{\rm pl}$, from the Friedmann equation (29) one has

$$H = \frac{\sqrt{-g'}}{\sqrt{\Omega_\phi}}\frac{1}{\phi}. \tag{64}$$

Note from the relation $\alpha = \beta/\phi$, we have

$$\frac{\alpha}{H} = \frac{\beta}{\sqrt{-g'}}\sqrt{\Omega_\phi}, \tag{65}$$

which is independent of $\phi$. Therefore it is possible to find an attractor solution **C** with $y = y_c =$ const and fixed $\Omega_\phi^{(c)}$, $\Omega_m^{(c)} = 1 - \Omega_\phi^{(c)}$ and satisfying the master equations (41)–(44),

$$r(y_c) = \frac{2+\beta}{2}\sqrt{\Omega_\phi^{(c)}} - \frac{\beta}{2}\frac{1}{\sqrt{\Omega_\phi^{(c)}}}, \tag{66}$$

and

$$\left(\frac{\alpha}{H}\right)_c \dot{\phi}_c + 3w_\phi(y_c)\Omega_\phi^{(c)} = 0. \tag{67}$$

Substituting Eqs. (23), (36), and (65) into Eq. (67), we get

$$\sqrt{\Omega_\phi^{(c)}} = -\frac{\sqrt{2}\beta}{3g_c}\sqrt{-g_c'}, \tag{68}$$

which is determined only by the coupling constant $\beta$ and the function $g(y)$ at $y = y_c$. Substituting $\Omega_\phi^{(c)}$ into Eq. (66), we have

$$r(y_c) = \frac{9g_c^2 + 2\beta(2+\beta)g_c'}{6\sqrt{2}g_c\sqrt{-g_c'}}. \tag{69}$$

Comparing with $r(y_c) = Ms(y_c) = \sqrt{3}M_{\rm pl}s(y_c)$ and using Eq. (38), we obtain

$$g(y_c) = g_c = -\frac{2\beta(2+\beta)}{9y_c}. \tag{70}$$

Geometrically, this means that the **C** attractor locates at the intersection of curve $g(y)$ and the hyperbola $h(y) \equiv -2\beta(2+\beta)/(9y)$ in the plot of $g(y)$ versus $y$. Because the asymptotes of the hyperbola $h(y)$ are the two coordinate axes, and $g(y) < 0$ when $y > y_d$, this intersection always exists in the regime $y_c > y_d$ so that the $k$-chameleon contributes a negative pressure. On the other hand, note that the curve $g(y)$ is monotonically decreasing while the hyperbola $h(y)$ is monotonically increasing in the regime of $y_d < y < y_s$, therefore there is only one intersection. In other words, the **C** attractor always exists and given a function $g(y)$, there is only one **C** attractor.

Next let us have a look at the other physical features of the **C** attractor. From Eqs. (68) and (70), the fraction energy density of the $k$-chameleon is

$$\Omega_\phi^{(c)} = \frac{9y_c^2(-g_c')}{2(2+\beta)^2}. \tag{71}$$

The usual restriction on $\Omega_\phi^{(c)}$ is $0 < \Omega_\phi^{(c)} < 1$. However, as mentioned at the end of Sec. IVA2, the condition $y_c < y_s$ has to be imposed. In that case, one has $g_c - g_c'y_c > 0$ while $g_c < 0$, $g_c' < 0$, and $w_\phi(y_c) > -1$. By using Eqs. (68) and (70), we find that the bounds of $\Omega_\phi^{(c)}$ should be

$$\frac{\beta}{2+\beta} < \Omega_\phi^{(c)} < 1. \tag{72}$$

Note that $\beta \sim \mathcal{O}(1)$, this result is quite interesting. For instance, the lower bounds of $\Omega_\phi^{(c)}$ are $1/3$, $1/2$, $3/5$, and $2/3$ for $\beta = 1, 2, 3$, and 4, respectively. Therefore, in the **C** attractor, it is not strange that the fraction energy densities of the $k$-chameleon and dust matters are comparable. In addition, from Eqs. (36), (70), and (71), one has

$$w_\phi(y_c) = -\frac{\beta}{2+\beta}\frac{1}{\Omega_\phi^{(c)}}. \tag{73}$$

Thus, the requirement $w_\phi(y_c) > -1$ leads to the same lower bound to $\Omega_\phi^{(c)}$ as given in Eq. (72). Furthermore, from Eqs. (44) and (73), we find

$$w_{\rm tot}(y_c) = w_\phi(y_c)\Omega_\phi^{(c)} = -\frac{\beta}{2+\beta}. \tag{74}$$

From the Einstein equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm pl}^2}\rho_{tot}(1 + 3w_{\rm tot}), \tag{75}$$

the Universe undergoes an accelerated expansion provided $\beta > 1$, which is equivalent to $w_{\rm tot}(y_c) < -1/3$. We can look at this from another angle. Because $\dot{\phi}_c = \sqrt{2X_c} = \sqrt{2}/y_c$ is constant, one has $\phi \propto t$. Then, from Eq. (64), $H \propto t^{-1}$ and the scale factor $a \propto t^\lambda$. If $\lambda > 1$, the Universe undergoes a power-law inflation. Let us find out the explicit expression of $\lambda$ for the **C** attractor. From Eq. (25), one has

$$\rho_{\rm tot}(y_c) \propto a^{-3[1+w_{\rm tot}(y_c)]} = a^{-6/(\beta+2)}. \tag{76}$$

Since $\Omega_\phi^{(c)}$ and $\Omega_m^{(c)} = 1 - \Omega_\phi^{(c)}$ are both fixed, $\rho_\phi(y_c)$ and $\rho_m(y_c)$ decrease in the same manner as $\rho_{\rm tot}(y_c) \propto a^{-6/(\beta+2)}$. Substituting $a \propto t^\lambda$ and Eq. (76) into Eq. (75), we get $\lambda = (\beta + 2)/3$ by comparing the power of $t$. In fact, from Eqs. (64) and (71) one can find the same result

$$a \propto t^{(\beta+2)/3} \tag{77}$$

by using $\phi = \dot{\phi}_c t$. In short, if $\beta > 1$, the **C** attractor solution describes a power-law inflation.

In summary, in the $k$-chameleon model there may exist two attractor solutions, **K** and **C**, and *no* **D** and **S** attractor solutions in the matter-dominated epoch. If $r^2(y) < 1$ for any $y < y_s$, the **K** attractor cannot exist. Thus, the combination of cosmologically relevant attractors during the matter-dominated epoch can be one of two types:

(i) Only **C** and *no* **K**;
(ii) **C** and **K**.

#### *2. Stability analysis of the attractors*

As mentioned in the beginning of Sec. IVA2, the stability analysis of the attractors **K** and **C** ought to be treated separately. We study the behavior of small deviations from the **K** and **C** attractor solutions one by one.

(i) **K** attractor: In this case, $\Omega_\phi^{(k)} = 1$, $\Omega_m^{(k)} = 1 - \Omega_\phi^{(k)} = 0$. Substituting $y(N) = y_k + \delta y$ and $\Omega_\phi(N) = \Omega_\phi^{(k)} + \delta\Omega_\phi$ into Eqs. (41)–(43) and linearizing these equations, we get

$$\frac{d\delta y}{dN} = \frac{3}{2}\frac{w_\phi(y_k) - 1}{r_k'}\left(r_k'\delta y - \frac{1}{2}\delta\Omega_\phi\right),$$
$$\frac{d\delta\Omega_\phi}{dN} = \left[3w_\phi(y_k) + \frac{\sqrt{2}\beta}{\sqrt{-g_k' y_k^2}}\right]\delta\Omega_\phi. \tag{78}$$

Considering Eq. (36), one has $w_\phi(y_k) < 0$ since $-g_k' > 0$ and $g_k < 0$. Thus, the solutions of $\delta y$ and $\delta\Omega_\phi$ decay only if

$$3w_\phi(y_k) + \frac{\sqrt{2}\beta}{\sqrt{-g_k' y_k^2}} < 0. \tag{79}$$

Substituting Eq. (61) into it, the stability condition for the **K** attractor becomes

$$\sqrt{-g_k' y_k^2/2} > \frac{2+\beta}{3}. \tag{80}$$

Note that, if $\beta > 1$ (the same requirement for the **C** attractor describes power-law inflation), Eq. (80) becomes $\sqrt{-g_k' y_k^2/2} > 1$ which ensures the **K** attractor describes power-law inflation too.

(ii) **C** attractor: In this case, $\Omega_m^{(c)} = 1 - \Omega_\phi^{(c)}$. Substituting $y(N) = y_c + \delta y$ and $\Omega_\phi(N) = \Omega_\phi^{(c)} + \delta\Omega_\phi$ into Eqs. (41)–(43) and linearizing these equations, we obtain

$$\frac{d\delta y}{dN} = B_1\delta y + B_2\delta\Omega_\phi, \qquad \frac{d\delta\Omega_\phi}{dN} = B_3\delta y + B_4\delta\Omega_\phi, \tag{81}$$

where

$$B_1 = \frac{3}{2}[w_\phi(y_c) - 1],$$
$$B_2 = \frac{3}{2}\frac{w_\phi(y_c) - 1}{r'(y_c)}\left(-\frac{1}{4\sqrt{\Omega_\phi^{(c)}}}\right)\left(\beta + 2 + \frac{\beta}{\Omega_\phi^{(c)}}\right),$$
$$B_3 = 3\Omega_\phi^{(c)}(\Omega_\phi^{(c)} - 1)w_\phi'(y_c) + \frac{\beta}{\sqrt{2}}\frac{g_c''}{y_c(\sqrt{-g_c'})^3} \times\sqrt{\Omega_\phi^{(c)}}(\Omega_\phi^{(c)} - 1) + \frac{\sqrt{2}\beta}{y_c^2\sqrt{-g_c'}}\sqrt{\Omega_\phi^{(c)}}(1 - \Omega_\phi^{(c)}),$$
$$B_4 = 3w_\phi(y_c)(2\Omega_\phi^{(c)} - 1) + \frac{\beta(3\Omega_\phi^{(c)} - 1)}{y_c\sqrt{2\Omega_\phi^{(c)}(-g_c')}}. \tag{82}$$

From Eq. (81), we have

$$\frac{d^2\delta y}{dN^2} - (B_1 + B_4)\frac{d\delta y}{dN} + (B_1B_4 - B_2B_3)\delta y = 0,$$
$$\frac{d^2\delta\Omega_\phi}{dN^2} - (B_1 + B_4)\frac{d\delta\Omega_\phi}{dN} + (B_1B_4 - B_2B_3)\delta\Omega_\phi = 0. \tag{83}$$

We can see that the solutions of $\delta y$ and $\delta\Omega_\phi$ decay only if

$$B_1 + B_4 < 0 \quad \text{and} \quad B_1B_4 - B_2B_3 > 0. \tag{84}$$

By using Eqs. (36), (38), (70), (71), (73), and (82), and considering $r(y) = Ms(y) = \sqrt{3}M_{\rm pl}s(y)$, we can obtain

$$B_1 = -\frac{4\beta + 2\beta^2 + 9(-g_c')y_c^2}{6(-g_c')y_c^2},$$

$$B_2 = -\frac{(2+\beta)^2[4\beta + 2\beta^2 + 9(-g_c')y_c^2]}{27g_c''(-g_c')y_c^4},$$

$$B_3 = \frac{3[9(-g_c')^2 y_c + \beta(\beta+2)g_c''][2(\beta+2)^2 - 9(-g_c')y_c^2]}{4(2+\beta)^4(-g_c')},$$

$$B_4 = \frac{\beta[2(\beta+2)^2 - 9(-g_c')y_c^2]}{6(2+\beta)(-g_c')y_c^2}.$$

From these quantities, we have

$$B_1 + B_4 = -\frac{3(1+\beta)}{2+\beta}, \tag{85}$$

$$B_1B_4 - B_2B_3 = \frac{[9(-g_c')y_c^2 + 2\beta(\beta+2)][2(\beta+2)^2 - 9(-g_c')y_c^2]}{4(2+\beta)^2 g_c'' y_c^3}. \tag{86}$$

Note that $\beta > 0$, $-g_c' > 0$, $g_c'' > 0$ [see Eq. (39)], and $\Omega_\phi^{(c)} < 1$, which implies $2(\beta+2)^2 - 9(-g_c')y_c^2 > 0$ [see Eq. (71)]. Then it is easy to see that the stability conditions

Eq. (84) for the **C** attractor are satisfied. Thus, we conclude that the **C** attractor is stable.

### C. Cosmological evolution of the *k*-chameleon and the coincidence problem

As mentioned in Sec. IVA, in the radiation-dominated epoch, the behavior of the *k*-chameleon field is the same as that of the ordinary *k*-essence without interaction between the scalar field and background matters [10]. As a result, three kinds of attractors, namely, **R**, **K**, and **S**, may exist. However, the radiation tracker, **R**, has been argued to have the largest basin of attraction on the whole phase plane so that most initial conditions join onto it and then make this scenario become insensitive to initial conditions [10]. Yet the contribution of the *k*-chameleon to the total energy density must not spoil the BBN or not dominate over the matter density at the end of the radiation-dominated epoch. It has been argued that if the contribution of the scalar field energy density in the **R** attractor satisfies [10]

$$\Omega_\phi^{(r)} \simeq 10^{-2}\text{–}10^{-1}, \tag{87}$$

the scalar field (*k*-essence or *k*-chameleon) will not violate the constraint from the BBN.

The new features appear in the matter-dominated epoch. As mentioned in Sec. IV B, in the matter-dominated epoch, the **D** and **S** attractors, which may occur in the ordinary *k*-essence model, are physically forbidden in the *k*-chameleon model, due to the strong coupling between *k*-chameleon and dust matters. Actually, in the *k*-chameleon model, two kinds of attractors may exist: one is the familiar **K** attractor and the other is a completely *new* attractor **C**. The new attractor **C** has some desirable features which can make the fraction energy densities of the *k*-chameleon and dust matters (cold dark matter and baryon) comparable. During the matter-dominated epoch, the relevant attractors can appear in the following two possible sets: (1) only **C** and (2) **C** and **K**. If $\beta > 1$, the Universe will undergo a power-law inflation, regardless whether the *k*-chameleon enters into the **C** or **K** attractor. Suppose that during the radiation-dominated epoch, the Universe enters into the **R** attractor, after the onset of matter domination, it will enter into the **C** attractor with a large possibility since the values of $\Omega_\phi^{(r)}$ and $y_r$ are required to be somewhat small in order to satisfy the BBN constraint. In particular, one can adjust the model so that the **K** attractor does not exist [for instance if $r^2(y) < 1$ for $y < y_s$], thus the Universe has to enter into the unique **C** attractor where the fraction energy densities of the *k*-chameleon $\Omega_\phi$ and the matters $\Omega_m$ are fixed and stay comparable forever. In this way the *k*-chameleon model leads to a natural solution to the cosmological coincidence problem.

The evolution of the *k*-chameleon heavily depends on the function $g(y)$ and other components of the Universe like radiation and dust matter. In Fig. 2 we plot a sketch of possible phase diagrams of the evolution of the *k*-chameleon, where only two attractors **R** and **C** appear during the evolution of the Universe. In this plot, we expect that during the radiation-dominated epoch, for most initial conditions, the *k*-chameleon is attracted to the **R** attractor satisfying the constraint (87). In this epoch, the *k*-chameleon mimics the equation of state of the radiation component of the Universe. With the increase of $\Omega_m$ for the dust matter component, the *k*-chameleon will no longer track the radiation component due to the interaction between the *k*-chameleon and dust matter. During the matter-dominated epoch, the $\Omega_\phi$ will continue to decrease until $y$ reaches to $y_c$, which can be seen from (42) and (43). Beyond $y_c$, $\Omega_\phi$ will increase and $y$ decreases toward $y_c$ and passes through it, which can be seen from (41). When $y$ decreases to some value ($<y_d$), it increases towards $y_c$, again. After several such processes, finally the *k*-chameleon is expected to reach the stable attractor **C**, where the fraction energy densities of dust matter and dark energies are comparable and the Universe undergoes an accelerated expansion.

In order to have a better picture of the *k*-chameleon model, it is helpful to do some numerical analysis. The astronomical observations, such as SNe Ia [1–3], WMAP [4], suggest that $\Omega_\phi \simeq 0.7$, $\Omega_m \simeq 0.3$, and $w_\phi < -0.76$ at 95% C.L. today. If we adopt $\Omega_\phi(y_c) = 0.7$ and impose the constraint $w_\phi(y_c) < -0.75$, we see from Eqs. (72) and (73) that $2.2 < \beta < 4.7$ has to be obeyed. From Eqs. (73) and (74), we find that $w_\phi(y_c) \simeq -0.86$ and $w_{\rm tot}(y_c) = -3/5$ for $\beta = 3$, while $w_\phi(y_c) \simeq -0.95$ and $w_{\rm tot}(y_c) = -2/3$ for $\beta = 4$. In these cases, the Universe undergoes an accelerated expansion as $a \propto t^{5/3}$ and $t^2$ for $\beta = 3$ and 4, respectively.

In the **C** attractor, the kinetic energy term $X_c = 1/y_c^2$ is fixed at a constant value. It is possible to design the function $g(y)$ to get a somewhat large $y_c$. For instance, if $\Omega_\phi(y_c) = 0.7$ and $\beta = 3$, we have from Eq. (71) that

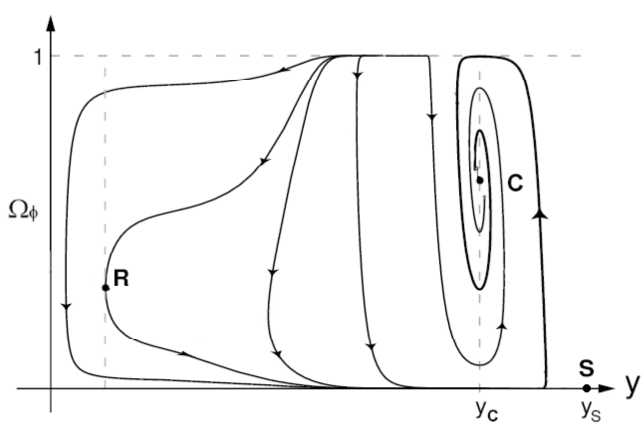


FIG. 2. A sketch of possible phase diagrams for the case where only the **R** and **C** attractors appear.

$y_c \simeq 100$ and $X_c \simeq 10^{-4}$ while $g'_c \simeq -4 \times 10^{-4}$ (the unit $3M_{\rm pl}^2 = 1$ has been used here). Comparing $g'_c \simeq -4 \times 10^{-4}$ with a particular example $g'_d \simeq -5 \times 10^{-3}$ in Ref. [10], one can see that this value is reasonable, since $y_c \simeq 100 \gg y_d = 17$. Therefore, in this example, the $k$-chameleon indeed can be treated as a canonical chameleon approximately. As a result, we cannot detect it from the tests of EP violations and fifth force searches on the Earth or in the solar system today, although it is strongly coupled to ambient matters, as illustrated in Sec. III C.

Note that in order to exclude the **K** attractor, one has to adjust the model so that $r^2(y) < 1$ and decreases monotonically in the region $y_r < y < y_s$ [10]. On the other hand, one requires that the **C** attractor exists in the region $y_d < y < y_s$. One may wonder whether or not these two conditions can be met simultaneously. To see this, let us take an example. We have from Eq. (66) that $r^2(y_c) = [(2+\beta)\Omega_\phi^{(c)} - \beta]^2/(4\Omega_\phi^{(c)})$. If $\beta = 4$ and $\Omega_\phi^{(c)} = 0.7$, one then has $r^2(y_c) \simeq 1.4\%$. In the radiation-dominated epoch, in order to satisfy the constraint from the BBN, $r^2(y_c) = \Omega_\phi^{(r)}$ should be in the region 1%–10%. So we see that those two conditions can be satisfied, if $r^2(y_r) > 1.4\%$ and the decreasing of $r^2(y)$ is sufficiently slow so that $y_c$ and $y_s$ can be somewhat large values. Note that $y_c$ is always less than $y_s$ since $y_s$ locates at $r^2(y_s) = 0$.

## V. CONCLUSION

Recently a chameleon mechanism has been suggested [24–28], in which a scalar field (chameleon) can be strongly coupled to ambient matters, but it still satisfies the constraints from the fifth force and EP violation experiments on the Earth and in the solar system. In this paper we have combined the chameleon mechanism to the $k$-essence model of dark energy and have presented a $k$-chameleon model. During the radiation-dominated epoch, the evolution of the $k$-chameleon is the same as that of the ordinary $k$-essence, and three kinds of attractors, **R**, **K**, and **S**, may appear. One can construct a model where the **R** attractor has the biggest basin of attraction so that for most initial conditions, the Universe will be attracted to the **R** attractor. During the matter-dominated epoch, the **D** and **S** attractors, which may appear in the ordinary $k$-essence model, are forbidden in the $k$-chameleon model, due to the strong coupling between the $k$-chameleon and background matters (cold dark matter and baryons). Except for the familiar **K** attractor, a new attractor, dubbed **C** attractor, exists in the $k$-chameleon model. In the **C** attractor, the fraction energy densities of the chameleon field (dark energy) and the dust matter (cold dark matters and baryons) are fixed and comparable, and the Universe enters into an accelerated expansion phase in the power-law manner if the coupling constant $\beta > 1$. We can adjust the model so that the **K** attractor does not exist, and the Universe then has to enter into the **C** attractor. Thus the $k$-chameleon model provides a natural solution to the cosmological coincidence problem.

## ACKNOWLEDGMENTS

H. W. is grateful to Ding-Fang Zeng, Xun Su, Fei Wang, Fu-Rong Yin, and Wei-Shui Xu for helpful discussions. R. G. C. would like to express his gratitude to the Physics Department, Baylor University, for its hospitality to him during his visit. This work was supported by a grant from the Chinese Academy of Sciences, grants from the NSFC, China (No. 10325525 and No. 90403029), and a grant from the Ministry of Science and Technology of China (No. TG1999075401).

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