Relating leptogenesis to low energy flavor violating observables in models with spontaneous CP violation

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In the minimal left-right symmetric model, there are only two intrinsic CP violating phases to account for all CP violation in both the quark and lepton sectors, if CP is broken spontaneously by the complex phases in the VEV's of the scalar fields. In addition, the left- and right-handed Majorana mass terms for the neutrinos are proportional to each other due to the parity in the model. This is thus a very constrained framework, making the existence of correlations among the CP violation in leptogenesis, neutrino oscillation and neutrinoless double beta decay possible. In these models, CP violation in the leptonic sector and CP violation in the quark sector are also related. We find, however, that such connection is rather weak due to the large hierarchy in the bi-doublet VEV required by a realistic quark sector.

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I. INTRODUCTION

The evidence of nonzero neutrino masses opens up the possibility that the leptonic CP violation might be responsible, through leptogenesis, for the observed asymmetry between matter and antimatter in the Universe [1]. It is generally difficult, however, to make a connection between leptogenesis and CP-violating processes at low energies due to the presence of extra phases and mixing angles in the right-handed neutrino sector as in models [2]. (For realistic neutrino mass models based on SUSY GUTs see, for example, [3].) Recently attempts have been made to induce *spontaneous CP violation* (SCPV) from a single source. In one such attempt SM is extended by a singlet scalar field which develops a complex VEV which breaks CP symmetry [4]. Another attempt assumes that there is one complex VEV of the field which breaks the $B-L$ symmetry in SO(10) [5]. In these models there is no compelling reason why all other VEVs have to be real. Here we focus on the minimal left-right symmetric model. In this model SCPV could be due to two intrinsic CP violating phases associated with VEVs of two scalar fields which account for all CP-violating processes observed in Nature; these *exhaust* sources of CP-violation. As the left-handed (LH) and right-handed (RH) Majorana mass matrices are identical up to an overall mass scale, in this model there exist relations between low energy processes, such as neutrino oscillations, neutrinoless double beta decay and lepton flavor violating charged lepton decay, and leptogenesis which occurs at very high energy. Also, it is possible to relate CP-violation in the lepton sector with that in the quark sector. In this paper we explicitly display such relations in two realistic models.

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The paper is organized as follows: In Sec. II, we define the minimal left-right symmetric model and review the formulation of leptogenesis and low energy LFV processes, including CP violation in neutrino oscillation and neutrinoless double beta decay; we then propose in Sec. III a new model which gives rise to bi-large leptonic mixing patterns due to an interplay of both type-I and type-II seesaw terms; we also extract the connections between various LFV processes in this model and a flavor ansatz proposed earlier; Section IV concludes this paper.

II. THE MINIMAL LEFT-RIGHT SYMMETRIC MODEL

The minimal left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model [6,7] is the minimal extension of the SM. It has the following particle content: The left- and right-handed matter fields transform as doublets of $SU(2)_L$ and $SU(2)_R$, respectively,

$$Q_{i,L} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,L} \sim (1/2, 0, 1/3), \quad Q_{i,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,R} \sim (0, 1/2, 1/3) \tag{1}$$

$$L_{i,L} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,L} \sim (1/2, 0, -1), \quad L_{i,R} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,R} \sim (0, 1/2, -1). \tag{2}$$

The minimal Higgs sector that breaks the left-right symmetry to the SM gauge group contains an $SU(2)$ bi-doublet Higgs and two $SU(2)$ triplet Higgses,

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (1/2, 1/2, 0) \tag{3}$$

$$\Delta_L = \begin{pmatrix} \Delta_L^+/\sqrt{2} & \Delta_L^{++} \\ \Delta_L^0 & -\Delta_L^+/\sqrt{2} \end{pmatrix} \sim (1, 0, +2) \tag{4}$$

$$\Delta_R = \begin{pmatrix} \Delta_R^+/\sqrt{2} & \Delta_R^{++} \\ \Delta_R^0 & -\Delta_R^+/\sqrt{2} \end{pmatrix} \sim (0, 1, +2). \tag{5}$$

The $SU(2)_R$ symmetry is broken by the VEV of the triplet Δ_R,

$$<\Delta_R> = \begin{pmatrix} 0 & 0 \\ v_R e^{i\alpha_R} & 0 \end{pmatrix} \tag{6}$$

and the electroweak symmetry is broken by the VEV of the bi-doublet,

$$<\Phi> = \begin{pmatrix} \kappa e^{i\alpha_\kappa} & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix}. \tag{7}$$

To get realistic SM gauge boson masses, the VEV's of the bi-doublet Higgs must satisfy $v^2 \equiv |\kappa|^2 + |\kappa'|^2 \simeq 2m_w^2/g^2 \simeq (174\ \text{GeV})^2$. Generally, a nonvanishing VEV for the $SU(2)_L$ triplet Higgs is induced, and it is suppressed by the heavy $SU(2)_R$ breaking scale similar to the see-saw mechanism for the neutrinos,

$$<\Delta_L> = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix}, \qquad v_L v_R = \beta|\kappa|^2, \tag{8}$$

where the parameter β is a function of the order $\mathcal{O}(1)$ coupling constants in the scalar potential and v_R, v_L, κ and κ' are positive real numbers in the above equations. Because of this see-saw suppression, for a $SU(2)_R$ breaking scale as high as 10^{15} GeV which is required by the smallness of the neutrino masses, the induced $SU(2)_L$ triplet VEV is well below the upper bound set by the electroweak precision constraints [8]. The scalar potential that gives rise to the vacuum alignment described can be found in Ref. [9,10].

The Yukawa sector of the model is given by $\mathcal{L}_{\text{Yuk}} = \mathcal{L}_q + \mathcal{L}_\ell$ where $\mathcal{L}_q$ and $\mathcal{L}_\ell$ are the Yukawa interactions in the quark and lepton sectors, respectively. The Lagrangian for quark Yukawa interactions is given by,

$$-\mathcal{L}_q = \overline{Q}_{i,R}(F_{ij}\Phi + G_{ij}\tilde{\Phi})Q_{j,L} + h.c. \tag{9}$$

where $\tilde{\Phi} \equiv \tau_2\Phi^*\tau_2$. In general, F_{ij} and G_{ij} are Hermitian to preseve left-right symmetry. Because of our assumption of SCPV with complex vacuum expectation values, the matrices F_{ij} and G_{ij} are real. The Yukawa interactions responsible for generating the lepton masses are summarized in the following Lagrangian, $\mathcal{L}_\ell$,

$$-\mathcal{L}_\ell = \overline{L}_{i,R}(P_{ij}\Phi + R_{ij}\tilde{\Phi})L_{j,L} + if_{ij}(L_{i,L}^T C\tau_2\Delta_L L_{j,L} + L_{i,R}^T C\tau_2\Delta_R L_{j,R}) + h.c., \tag{10}$$

where C is the Dirac charge conjugation operator, and the matrices P_{ij}, R_{ij} and f_{ij} are real due to the assumption of SCPV. Note that the Majorana mass terms $L_{i,L}^T\Delta_L L_{j,L}$ and $L_{i,R}^T\Delta_R L_{j,R}$ have identical coupling because the Lagrangian must be invariant under interchanging $L \leftrightarrow R$. The complete Lagrangian of the model is invariant under the unitary transformation, under which the matter fields transform as

$$\psi_L \to U_L\psi_L, \qquad \psi_R \to U_R\psi_R \tag{11}$$

where $\psi_{L,R}$ are left-handed (right-handed) fermions, and the scalar fields transform according to

$$\Phi \to U_R\Phi U_L^\dagger, \qquad \Delta_L \to U_L^*\Delta_L U_L^\dagger, \qquad \Delta_R \to U_R^*\Delta_R U_R^\dagger \tag{12}$$

with the unitary transformations U_L and U_R being

$$U_L = \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{-i\gamma_L} \end{pmatrix}, \qquad U_R = \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{-i\gamma_R} \end{pmatrix}. \tag{13}$$

Under these unitary transformations, the VEV's transform as

$$\kappa \to \kappa e^{-i(\gamma_L-\gamma_R)}, \quad \kappa' \to \kappa' e^{i(\gamma_L-\gamma_R)}, \quad v_L \to v_L e^{-2i\gamma_L}, \quad v_R \to v_R e^{-2i\gamma_R}. \tag{14}$$

Thus by redefining the phases of matter fields with the choice of $\gamma_R = \alpha_R/2$ and $\gamma_L = \alpha_\kappa + \alpha_R/2$ in the unitary matrices U_L and U_R, we can rotate away two of the complex phases in the VEV's of the scalar fields and are left with only two genuine CP violating phases, $\alpha_{\kappa'}$ and α_L,

$$<\Phi> = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix}, \quad <\Delta_L> = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix}, \quad <\Delta_R> = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \tag{15}$$

The quark Yukawa interaction $\mathcal{L}_q$ gives rise to quark masses after the bi-doublet acquires VEV's

$$M_u = \kappa F_{ij} + \kappa' e^{-i\alpha_{\kappa'}} G_{ij}, \quad M_d = \kappa' e^{i\alpha_{\kappa'}} F_{ij} + \kappa G_{ij}. \tag{16}$$

Thus the relative phase in the two VEV's in the SU(2) bi-doublet, $\alpha_{\kappa'}$, gives rise to the CP violating phase in the CKM matrix. To obtain realistic quark masses and CKM matrix elements, it has been shown that the VEV's of the bi-doublet have to satisfy $\kappa/\kappa' \simeq m_t/m_b \gg 1$ [11]. When the triplets and the bi-doublet acquire VEV's, we obtain the following mass terms for the leptons

$$M_e = \kappa' e^{i\alpha_{\kappa'}} P_{ij} + \kappa R_{ij}, \quad M_\nu^{\text{Dirac}} = \kappa P_{ij} + \kappa' e^{-i\alpha_{\kappa'}} R_{ij} \tag{17}$$

$$M_\nu^{RR} = v_R f_{ij}, \quad M_\nu^{LL} = v_L e^{i\alpha_L} f_{ij}. \tag{18}$$

The effective neutrino mass matrix, M_ν^{eff}, which arises from the Type-II see-saw mechanism [12], is thus given by

$$M_\nu^{\text{eff}} = M_\nu^{II} - M_\nu^{I} \tag{19}$$

$$M_\nu^I = (M_\nu^{\rm Dirac})^T (M_\nu^{RR})^{-1} (M_\nu^{\rm Dirac}) = (\kappa P + \kappa' e^{-i\alpha_{\kappa'}} R)^T (v_R f)^{-1} (\kappa P + \kappa' e^{-i\alpha_{\kappa'}} R) \quad (20)$$

$$M_\nu^{II} = v_L e^{i\alpha_L} f. \quad (21)$$

Assuming the charged lepton mass matrix is diagonal, the Yukawa couplings R_{ij} can be determined by the charged lepton masses,

$$R = \begin{pmatrix} \mathcal{O}(m_e/m_\tau) & 0 & 0 \\ 0 & \mathcal{O}(m_\mu/m_\tau) & 0 \\ 0 & 0 & \mathcal{O}(1) \end{pmatrix}. \quad (22)$$

In the limit $\kappa \gg \kappa'$, the conventional type-I see-saw term [12,13] is dominated by the term proportional to κ,

$$M_\nu^I = (M_\nu^{\rm Dirac})^T (M_\nu^{RR})^{-1} (M_\nu^{\rm Dirac}) \simeq \frac{\kappa^2}{v_R} P^T f^{-1} P = \frac{v_L}{\beta} P^T f^{-1} P. \quad (23)$$

Consequently, the connection between CP violation in the quark sector and that in the lepton sector, which is made through the phase $\alpha_{\kappa'}$, appears only at the subleading order, $\mathcal{O}(\kappa'/\kappa)$, thus making this connection rather weak. It has also been shown in Ref. [10] that in order to avoid flavor changing neutral current, the phase $\alpha_{\kappa'}$ has to be close to zero. In this case, leptonic CP violation is not constrained by $\alpha_{\kappa'}$, and thus it can be large (due to nonzero α_L). We will neglect these subleading order terms in this paper. In this case there is thus only one phase, α_L, that is responsible for all leptonic CP violation. As the charged lepton mass matrix is diagonal, the leptonic mixing matrix, the so-called Maki-Nakagawa-Sakata (MNS) matrix, is obtained by diagonalizing the effective neutrino mass matrix $M_\nu^{\rm diag} = U_{\rm MNS}^\dagger M_{\rm eff}^\nu U_{\rm MNS}^* = {\rm diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$, where $m_{\nu_{1,2,3}}$ are real and positive, and it can be parameterized as the product of a CKM-like mixing matrix, which has three mixing angles and one CP violating phase, with a diagonal phase matrix,

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_\ell} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_\ell} & s_{23}c_{13}e^{i\delta_\ell} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_\ell} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_\ell} & c_{23}c_{13}e^{i\delta_\ell} \end{pmatrix} \times \begin{pmatrix} 1 & & \\ & e^{i\alpha_{21}/2} & \\ & & e^{i\alpha_{31}/2} \end{pmatrix}. \quad (24)$$

The Dirac phase, δ_ℓ, as well as the two Majorana phases, α_{21} and α_{31}, are given in terms of a single phase, α_L, in this model. The relations among these three leptonic CP violating phases thus ensue. The analytic relations between α_L and the three leptonic CP phases are very complicated, and are not explicitly expressed. The effects of these three CP phases appear in the following processes.

A. *CP* Violation in Neutrino Oscillation

The Dirac CP violating phase, δ_ℓ, affects neutrino oscillation. The transition probability of the flavor eigenstate ν_α into ν_β ($\alpha, \beta = e, \mu, \tau$) reads,

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} Re(U_{\alpha i}U_{\beta j}U_{\alpha j}^* U_{\beta i}^*) \times \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) + 2\sum_{i>j} J_{CP} \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \quad (25)$$

where the CP violation is governed by the leptonic Jarlskog invariant, J_{CP}, which can be expressed model-independently in terms of the effective neutrino mass matrices as [14]

$$J_{CP} = -\frac{{\rm Im}(H_{12}H_{23}H_{31})}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}, \qquad H \equiv M_\nu^{\rm eff} M_\nu^{\rm eff\dagger}, \quad (26)$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ ($i, j = 1, 2, 3$) with m_i being the mass eigenvalues of the effective neutrino mass matrix, $M_\nu^{\rm eff}$.

B. Neutrinoless Double Beta Decays

Neutrinoless double beta ($0\nu\beta\beta$) decay is, on the other hand, only sensitive to the two Majorana phases, α_{21} and α_{31}. Their dependence in the $0\nu\beta\beta$ matrix element, $\langle m_{ee}\rangle$, is

$$|\langle m_{ee}\rangle|^2 = m_1^2|U_{e1}|^4 + m_2^2|U_{e2}|^4 + m_3^2|U_{e3}|^4 + 2m_1m_2|U_{e1}|^2|U_{e2}|^2\cos\alpha_{21} + 2m_1m_3|U_{e1}|^2|U_{e3}|^2\cos\alpha_{31} + 2m_2m_3|U_{e2}|^2|U_{e3}|^2\cos(\alpha_{31} - \alpha_{21}), \quad (27)$$

where U_{ei} ($i = 1, 2, 3$) are the matrix elements in the first row of the MNS matrix.

C. Leptogenesis

In the left-right symmetric model with the particle content we have, leptogenesis receives contributions both from the decay of the lightest RH neutrino, N_1, as well as from the decay of the $SU(2)_L$ triplet Higgs, Δ_L [15,16]. We consider the $SU(2)_L$ triplet Higgs being heavier than the lightest RH neutrino, $M_{\Delta_L} > M_{R_1}$. For this case, the decay of the lightest RH neutrino dominates. In the SM, the canonical contribution to the lepton number asymmetry from one-loop diagrams mediated by the Higgs doublet and the charged leptons is given by [16],

$$\epsilon^{N_1} = \frac{3}{16\pi}\left(\frac{M_{R_1}}{v^2}\right) \cdot \frac{\mathrm{Im}[\mathcal{M}_D (M_\nu^I)^* \mathcal{M}_D^T]_{11}}{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}}. \tag{28}$$

Now, there is one additional one-loop diagram mediated by the $SU(2)_L$ triplet Higgs. It contributes to the decay amplitude of the right-handed neutrino into a doublet Higgs and a charged lepton, which gives an additional contribution to the lepton number asymmetry [16],

$$\epsilon^{\Delta_L} = \frac{3}{16\pi}\left(\frac{M_{R_1}}{v^2}\right) \cdot \frac{\mathrm{Im}[\mathcal{M}_D (M_\nu^{II})^* \mathcal{M}_D^T]_{11}}{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}}, \tag{29}$$

where $\mathcal{M}_D$ is the neutrino Dirac mass term in the basis where the RH neutrino Majorana mass term is real and diagonal,

$$\mathcal{M}_D = O_R M_D, \quad f^{\mathrm{diag}} = O_R f O_R^T. \tag{30}$$

Because there is no phase present in either $M_D = P\kappa$ or M_ν^I or O_R, the quantity $\mathcal{M}_D (M_\nu^I)^* \mathcal{M}_D^T$ is real, leading to a vanishing ϵ^{N_1}. We have checked explicitly that this statement is true for *any* chosen unitary transformations U_L and U_R defined in Eq. (13). On the other hand, the contribution, ϵ^{Δ_L}, due to the diagram mediated by the $SU(2)_R$ triplet is proportional to $\sin\alpha_L$. So, as long as the phase α_L is nonzero, the predicted value for ϵ^{Δ_L} is finite. A nonvanishing value for ϵ^{N_1} is generated at the subleading order when terms of order $\mathcal{O}(\kappa'/\kappa)$ in M_D are included. At the leading order, leptogenesis is generated solely from the decay mediated by the $SU(2)_L$ triplet Higgs.

III. SPECIFIC MODELS WITH BI-LARGE NEUTRINO MIXING

In this section we consider two models which give bi-large neutrino mixing. (i) Model I assumes hierarchial mass ordering, $m_3 \gg m_{1,2}$, in the neutrino sector. Unlike most previous models in which either type-I or type-II see-saw mass term is supposed to dominate over the other, the bi-large mixing pattern arises in Model I due to an interplay between the type-I and type-II see-saw mass terms. (ii) In Model II we incorporate the flavor ansatz proposed in Ref. [17] into the LR model defined in Sec. II with the assumption of SCPV. In Ref. [17], as the coupling constants are complex, the total number of independent phases is 12. Now, as there is only one phase in our leptonic sector, all these 12 phases either vanish or are related leading to very pronounced correlation among CP violating processes. The main difference between these two models is their predictions for the atmospheric mixing angle; the deviation from maximal mixing is negligibly small in Model I whereas Model II has a sizable deviation. In both models we assume the neutrino Yukawa coupling P_{ij} to be proportional to the up quark mass matrix,

$$P_{ij} = q\begin{pmatrix} \frac{m_u}{m_t} & 0 & 0 \\ 0 & \frac{m_c}{m_t} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{31}$$

where the parameter q is a constant of proportionality. A nonzero value for U_{e3} is predicted in both models.

A. Model I

Assuming the matrix f_{ij} have the following hierarchical elements,

$$f_{ij} = \begin{pmatrix} t^2 & t & -t \\ t & 1 & 1 \\ -t & 1 & 1 \end{pmatrix}, \tag{32}$$

with t being a small and positive number. This mass matrix gives rise to bi-large mixing pattern with solar mixing angle given by $\tan\theta_{12}^0 = \frac{1}{2}[\frac{t}{\sqrt{2}} + \sqrt{\frac{t^2}{2} + 4}]$, which is always greater than one (the light side region), and thus inconsistent with the presence of the matter effects observed experimentally. But the contribution to the total effective neutrino mass matrix from the conventional type-I see-saw term,

$$M_\nu^I = \frac{q^2}{2\beta}\begin{pmatrix} 0 & \frac{1}{t}\frac{m_u m_c}{m_t^2} & -\frac{1}{t}\frac{m_u}{m_t} \\ \frac{1}{t}\frac{m_u m_c}{m_t^2} & 0 & \frac{m_c}{m_t} \\ -\frac{1}{t}\frac{m_u}{m_t} & \frac{m_c}{m_t} & 0 \end{pmatrix} v_L, \tag{33}$$

can reduce the solar mixing angle so that it is in the dark side region consistent with the solar experiment. For $t \sim \mathcal{O}(0.1)$, the (23) and (32) elements dominate. In the limit $m_u = 0$, the resulting atmospheric mixing angle is maximal and the mixing angle $\sin\theta_{13} = 0$ leading to a vanishing leptonic Jarlskog invariant, $J_{CP} = 0$. However, when m_u is turned on, $\sin\theta_{13}$ acquires a nonzero value, which is suppressed by (m_u/m_t). It is therefore important to keep all three diagonal elements in the matrix P_{ij} nonzero in our analysis. With the approximation, $m_u{:}m_c{:}m_t \simeq \epsilon^8{:}\epsilon^4{:}1$, where $\epsilon = 0.22$ is the sine of the Cabibbo angle, to the leading order in ϵ the leptonic Jarlskog invariant is given by,

$$J_{CP} \simeq -\frac{2st^2(1-t^2)v_L^6}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}\frac{m_u}{m_t}\sin\alpha_L. \tag{34}$$

In our analyses, we set $m_u/m_t = (0.22)^8$ and $m_c/m_t = (0.22)^4$. As the absolute mass scale of the neutrinos does not depend on the parameters (t, s, α_L), where s is defined as $s = q^2/(2\beta)$, these parameters can be determined using the following neutrino oscillation parameters from experimental data at 1σ as input [18],

$$\frac{\Delta m^2_{\odot}}{\Delta m^2_{\rm atm}} = 0.0263 \sim 0.0447, \quad \sin^2 2\theta_{\rm atm} > 0.9, \quad \tan^2\theta_{\odot} = 0.35 \sim 0.44. \tag{35}$$

We search the *full* parameter space spanned by (t, s, α_L), by allowing α_L to vary between $[0, 2\pi]$, t to vary between $[0, 1]$ (so that there is normal hierarchy among the light neutrino masses), and s to vary between $[100, 1000]$ (as there are no allowed regions beyond $[100, 1000]$). We slice the (t, s, α_L)-space given above into $(250, 250, 72)$ *equally* spaced points and test whether each of these points satisfied the constraints from the oscillation data given in Eq. (35). The allowed region for (t, s, α_L) which satisfy these data is shown in Fig. 1. The absolute scale for v_L is not essential as it does not affect the qualitative behavior of the correlations, and can be changed by rescaling t and s. The essential parameter that has to be taken into account is r, which differs for each data point (t, s, α_L). We find that for all points in the allowed region given in Fig. 1, by allowing the $SU(2)_L$ triplet VEV $v_L = r \times (0.0265 \text{ eV})$ with $r = (0.713 - 1.16)$, the predicted absolute mass scales of $\Delta m^2_{\odot}$ and $\Delta m^2_{\rm atm}$ individually satisfy the experimental 1σ limits, $\Delta m^2_{\rm atm} = (1.9 \sim 3.0) \times 10^{-3}$ eV^2 and $\Delta m^2_{\odot} = (7.9 \sim 8.5) \times 10^{-5}$eV^2 [18].

Once the parameters (t, s, α_L, r) are determined, there are no more adjustable parameters. Using this data set, we can then predict the (13) element of the MNS matrix, $|U_{e3}|$, the leptonic Jarlskog invariant, J_{CP}, the matrix element for neutrinoless double beta decay, $\langle m_{ee}\rangle$, and the amount of leptogenesis. Figure 2 shows the correlation between the deviation of the atmospheric mixing angle from $\pi/4$ and the predicted value for $|U_{e3}|$, which is in the range of $(0.5 - 3) \times 10^{-3}$. The current experimental upper bound for $|U_{e3}|$ is 0.122 [18], and an improvement on this bound can be achieved in the very long baseline neutrino experi-

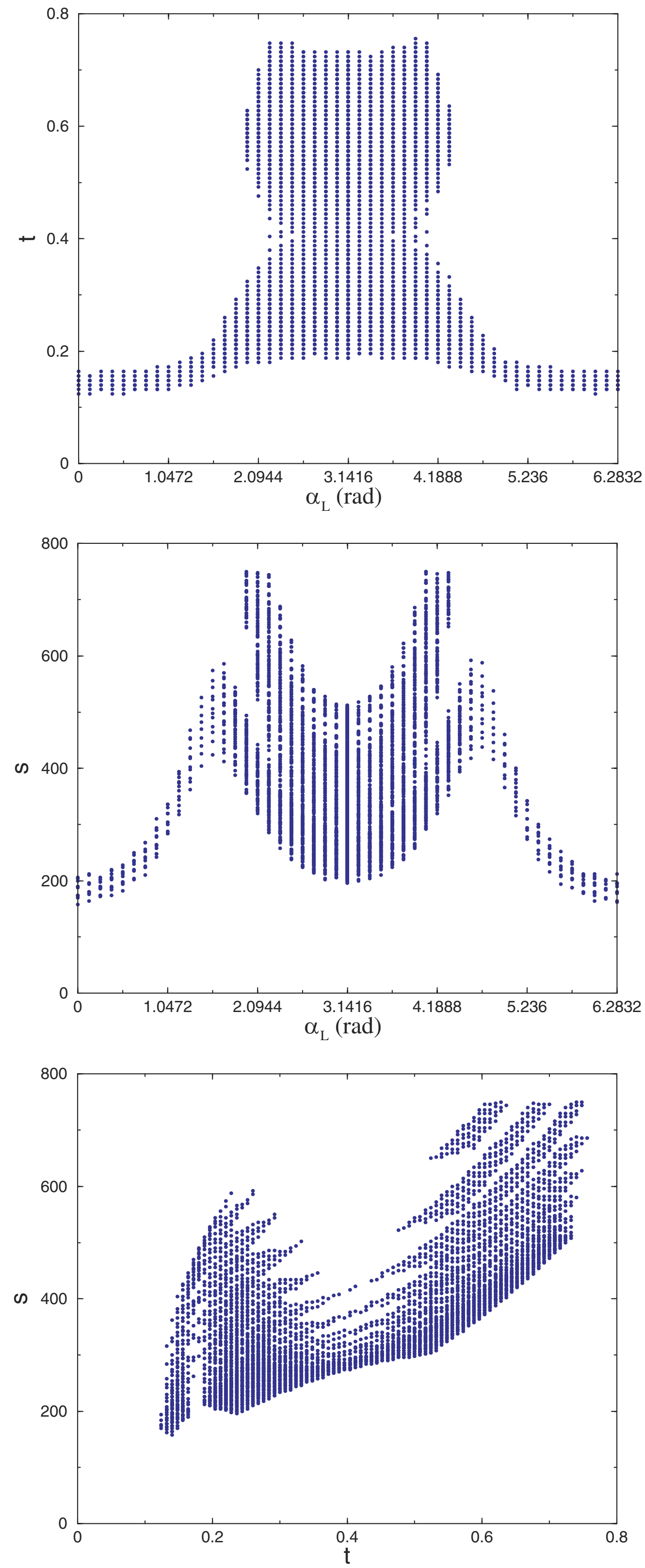

FIG. 1 (color online). Full allowed parameter space on the (α_L, t, s) plane in Model I.

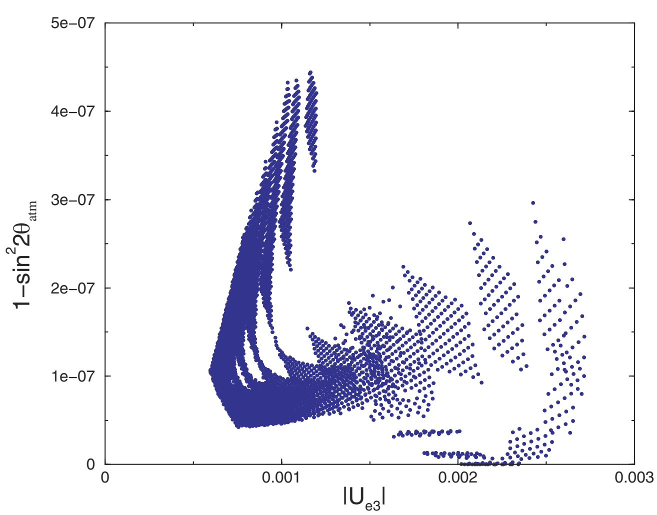

FIG. 2 (color online). The correlation between the leptonic mixing matrix element, $|U_{e3}|$, and the deviation of the atmospheric mixing angle $\sin^2 2\theta_{\rm atm}$ from 1 in Model I.

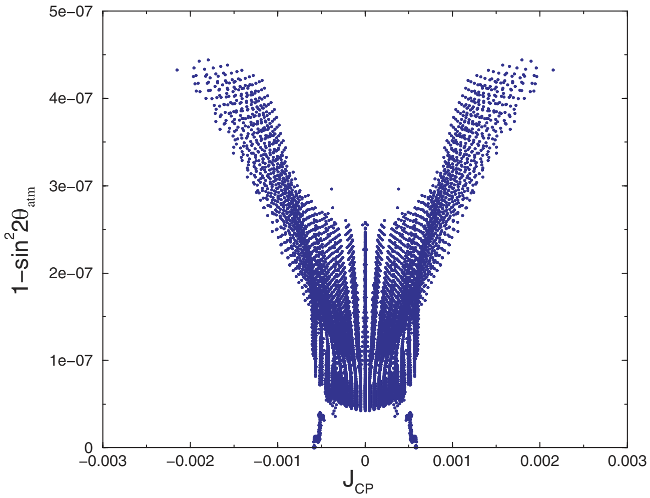

FIG. 3 (color online). Correlation between the leptonic Jarlskog invariant and the deviation of the atmospheric mixing angle $\sin^2 2\theta_{\rm atm}$ from 1 in Model I.

ment [19]. Figure 3 shows the correlation between $1 - \sin^2 2\theta_{\rm atm}$ and the predicted value for J_{CP}, which ranges from 0 to 0.002; in addition, a large value for J_{CP} implies a large deviation for $1 - \sin^2 2\theta_{\rm atm}$. Figure 2 and 3 also show that the deviation of the atmospheric mixing angle from $\pi/4$ is negligibly small, which is the main difference between this model and Model II. In Fig. 4, we show the correlation between the leptonic Jarlskog invariant and the prediction for neutrinoless double beta decay matrix element, which ranges between 5×10^{-4} to 3×10^{-2} eV. Except for the region around $J_{CP} \simeq 0$, the value for $\langle m_{ee} \rangle$ increases as the value of J_{CP}. The total amount of lepton number asymmetry, $\epsilon_{\rm total} = \epsilon^{\Delta_L}$, is proprtional to $\Delta\epsilon'$, defined as

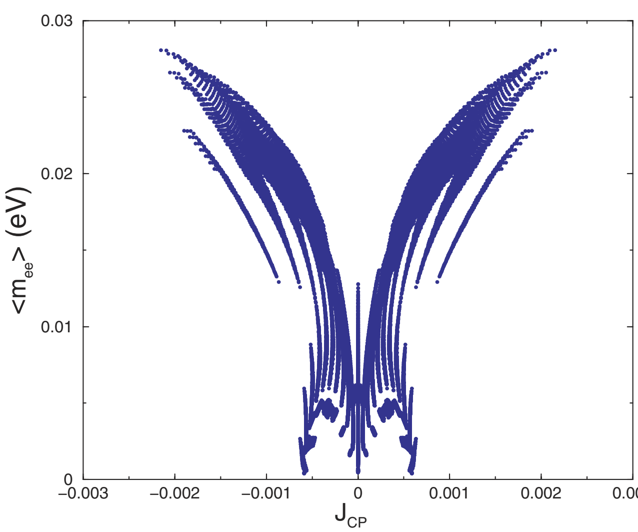

FIG. 4 (color online). Correlation between the matrix element of neutrinoless double beta decay, $\langle m_{ee} \rangle$, and the leptonic Jarlskog invariant in Model I.

$$\Delta\epsilon' = \frac{3}{16\pi} \frac{f_1^0}{v_L} \cdot \frac{{\rm Im}[\mathcal{M}_D (v_L f e^{i\alpha_L})^* \mathcal{M}_D^T]_{11}}{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}} = \frac{\epsilon^{\Delta_L}}{\beta}. \quad (36)$$

In deriving the above expression, we have used the property that the mass of the lightest RH neutrino, M_1, is proportional to (f_1^0/v_L), where f_1^0 is the smallest eigenvalue of the matrix f. So for fixed β, the ratio of $\Delta\epsilon'$ to ϵ^{Δ_L} is universal for all data points. Thus it suffices to consider $\Delta\epsilon'$ when extracting the correlation. In order for the lepton number asymmetry not to be washed out by the scattering processes, the out-of-equilibrium condition, characterized by the ratio of the decay rate of the lightest RH neutrino, Γ_1, to the Hubble constant at temperature equal to its mass, $H|_{T=M_1}$,

$$\gamma \equiv \frac{\Gamma_1}{H|_{T=M_1}} = \frac{M_{\rm Pl}}{(1.7)(32\pi)\sqrt{g_*} v^2} \cdot \frac{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}}{M_1} < 1, \quad (37)$$

with $\sqrt{g_*}$ being the number of relativistic degrees of freedom, must be satisfied. This condition can be rewritten as

$$\frac{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}}{M_1} < 0.01 \text{ eV}. \quad (38)$$

In our model, the quantity on the left hand side is given by

$$\frac{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}}{M_1} \simeq 2s \frac{(O_R)_{12}^2}{f_1^0} \left(\frac{m_c}{m_t}\right)^2 v_L, \quad (39)$$

which is highly suppressed by the factor $(m_c/m_t)^2$. In addition, as $f_1^0 \sim t$ is of order $\mathcal{O}(0.1)$, which reflects the fact that the hierarchy in the Majorana mass matrices is small, it does not off-set the suppression from (m_c/m_t). We have checked numerically and found that this quantity is of order $\mathcal{O}(10^{-7})$ eV for all points. Thus the condition given in Eq. (38) is satisfied and consequently there are no effects

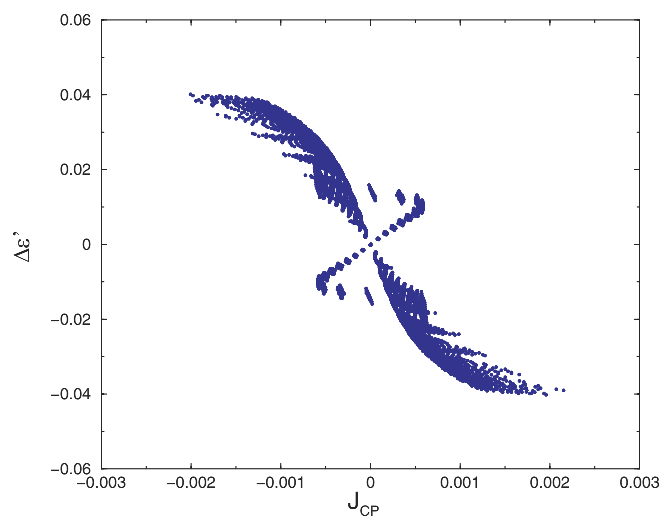

FIG. 5 (color online). Correlation between the amount of leptogenesis and the leptonic Jarlskog invariant in Model I.

due to dilution. Figure 5 shows the correlation between the leptonic Jarlskog invariant and the amount of leptogenesis, characterized by $\Delta\epsilon'$. As both $\Delta\epsilon'$ and J_{CP} are proportional to $\sin\alpha_L$, the plot has a reflection symmetry in the second and the forth quadrants. This is not surprising as both $\Delta\epsilon'$ and J_{CP} are proportional to $\sin\alpha_L$. And similar to Fig. 4, a large value for J_{CP} implies a large value for $\Delta\epsilon'$, in the region when $|J_{CP}| > 0.0005$. To generate the observed amount of the baryon asymmetry of the Universe (BAU), $n_b/s_e \sim 10^{-10}$, where n_b and s_e are, respectively, the baryon number and entropy, requires the lepton number asymmetry $\epsilon_{\rm total}$ to be of the order of 10^{-8}. As the parameter $s = q^2/2\beta$ is of order $\mathcal{O}(10^2 \sim 10^3)$, if q assumes a natural value ~ 1, β is roughly $\mathcal{O}(10^{-2} \sim 10^{-3})$. This leads to a total lepton number asymmetry $\epsilon_{\rm total} = \beta\Delta\epsilon'$ of order $\mathcal{O}(10^{-4} \sim 10^{-5})$, sufficient to generate the observed BAU. We note that an amount of lepton number asymmetry $\epsilon_{\rm total} \sim 10^{-8}$ corresponds to a leptonic Jarlskog invariant $|J_{CP}| \sim 10^{-5}$. A value of β in the range of $(10^{-2} \sim 10^{-3})$ corresponds to a $SU(2)_R$ breaking scale of $v_R \sim (10^{12} - 10^{13})$ GeV. And, due to the fact that the hierarchy in the Majorana mass matrices is small, the mass of the lighest RH neutrino M_1 is lighter than v_R only by about 1 order of magnitude. The scale of v_R in our model is consistent with the bounds given in Ref. [20].

B. Model II

Assuming the type-II see-saw term, $f_{ij}v_L$, has the following form,

$$f_{ij} = \begin{pmatrix} A & B & -B \\ B & D + \frac{A}{2} & D - \frac{A}{2} \\ -B & D - \frac{A}{2} & D + \frac{A}{2} \end{pmatrix}, \qquad (40)$$

where

$$A = \frac{1}{2}(f_1^0 + f_2^0), \quad B = \frac{1}{2\sqrt{2}}(f_2^0 - f_1^0), \quad D = \frac{1}{2} f_3^0, \qquad (41)$$

with $f_{1,2,3}^0$ being the eigenvalues of the matrix f. This matrix gives rise to exactly bi-maximal mixing pattern, and a vanishing value for $|U_{e3}|$. Adding the conventional type-I see-saw term,

$$M_\nu^I = -\frac{v_L}{\beta v^2}(M_D^T f^{-1} M_D) \equiv -s v_L \frac{(P^T f^{-1} P)}{q^2}, \qquad (42)$$

where $s = q^2/\beta$ in this case, gives rise to a deviation from maximal solar mixing, and a nonvanishing value for $|U_{e3}|$. The (33) element of M_D dominates the Dirac neutrino mass matrix. Hence if we set $m_u = m_c = 0$ in the matrix P_{ij}, the conventional type-I see-saw mass term contributes only to the (33) element in the total effective neutrino mass matrix; this contribution called x is given by $x = \frac{s}{4}(\frac{1}{f_1^0} + \frac{1}{f_2^0} + \frac{2}{f_3^0})$. In this approximation, the Jarlskog invariant, J_{CP}, can be obtained analytically; it is given by,

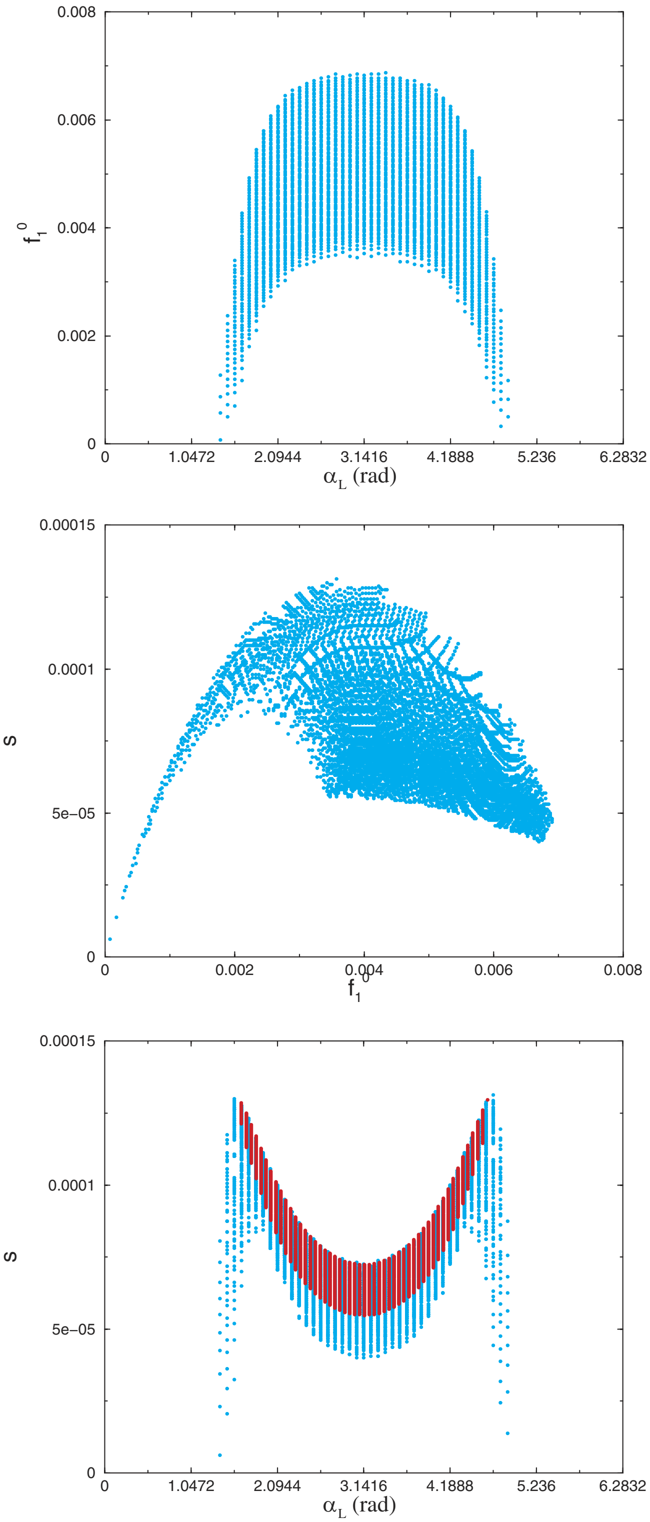

FIG. 6 (color online). Full allowed parameter space on the (f_1^0, α_L, s) plane in Model II. The shaded area in cyan (light shade) is the full allowed region, while the area in red (dark shade) corresponds to $f_1^0 = 0.00424$.

$$J_{CP} = -\frac{1}{16}\frac{(\Delta m_{21}^{0})^2(m_1^0 - m_2^0)(m_2^0 - m_3^0)(m_3^0 - m_1^0)}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2} x \sin(\alpha_L), \tag{43}$$

and the total amount of lepton number asymmetry is given by,

$$\epsilon^{\Delta_L} = -\frac{3\sqrt{2}}{16\pi}\frac{M_1}{v^2}\left(D + \frac{A}{2}\right)\sin\alpha_L. \tag{44}$$

So both quantities to the leading order are proportional to $\sin\alpha_L$.

As we assume the hierarchical mass pattern for the neutrinos, the two heavier eigenvalues of the matrix f can be approximated as $f_3^0 v_L \sim \sqrt{\Delta m_{\rm atm}^2}$ and $f_2^0 v_L \sim \sqrt{\Delta m_\odot^2}$. Hence we choose $f_2^0 = 0.01$ and $f_3^0 = 0.049$ in our analysis with $v_L \sim 1$ eV, and treat f_1^0, along with s and α_L, as a free parameter. For the Dirac neutrino mass matrix M_D, we set $m_u/m_t = (0.22)^8$ and $m_c/m_t = (0.22)^4$. Following the steps described in Model I, we search the full allowed region for (f_1, s, α_L), which is shown in Fig. 6, and the corresponding $SU(2)_L$ triplet VEV $v_L = r \times (1 \text{ eV})$ with $r = (0.847 - 1.21)$. The correlation between $|U_{e3}|$, J_{CP} and $1 - \sin^2 2\theta_{\rm atm}$, which, contrary to Model I, can be as large as 0.03, are shown in Fig. 7 and Fig. 8, respectively. The predicted range of $|U_{e3}|$ is $\mathcal{O}(0.001 - 0.01)$ and that of J_{CP} is $(0 - 0.03)$. Unlike in Model I, a small value for $|J_{CP}|$ implies a large deviation for $1 - \sin^2 2\theta_{\rm atm}$. In Fig. 9, we show the correlation between the leptonic Jarlskog invariant and the prediction for neutrinoless double beta decay matrix element, the range of which is $(3 - 10) \times 10^{-3}$ eV. Except for the region around $J_{CP} \simeq 0$, a large value of $|J_{CP}|$ implies a large value for $\langle m_{ee}\rangle$. The correlation between the leptonic Jarlskog invariant and the amount of leptogenesis, which is proportional to $\Delta\epsilon'$, is shown in Fig. 10, which shows a reflection symmetry in the second and the forth quadrants. A large value for $|J_{CP}|$ implies a large value for $\Delta\epsilon'$, which reaches a plateau when $|J_{CP}| > 0.0005$. A natural way to have s of order $\mathcal{O}(10^{-4})$ is to have $\beta \sim 1$ and $q \sim 10^{-2}$. With $\beta \sim 1$, the total amount of lepton number asymmetry ϵ can be as large as 10^{-5}. This is sufficient for generating the obaserved BAU. The quantity given in Eq. (39) is of order $\mathcal{O}(10^{-7})$ in this case, and thus the out-of-equilibrium is

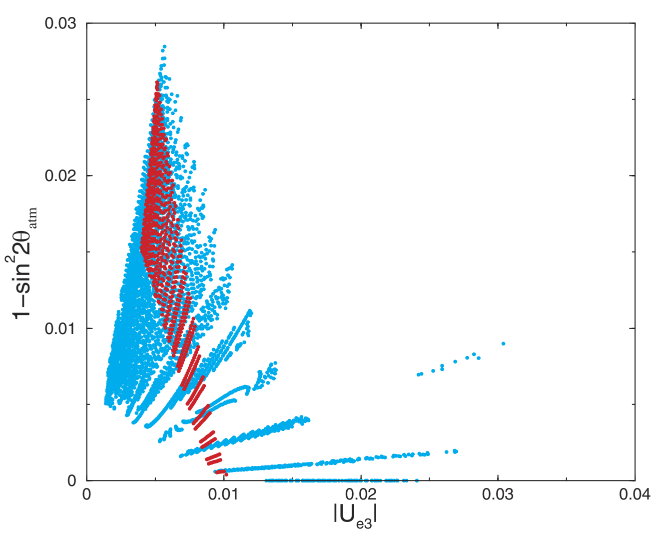

FIG. 7 (color online). Correlation between the matrix element of the leptonic mixing matrix, $|U_{e3}|$, and the deviation of the atmospheric mixing angle $\sin^2 2\theta_{\rm atm}$ from 1 in Model II. The shaded area in cyan (light shade) is the full allowed region, while the area in red (dark shade) corresponds to $f_1^0 = 0.00424$.

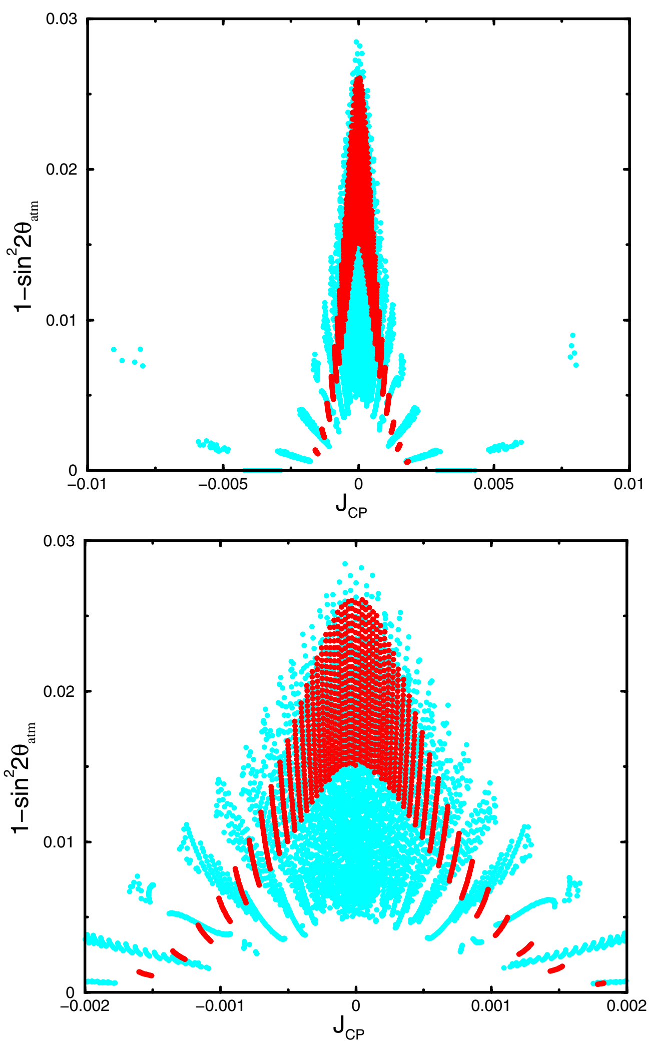

FIG. 8 (color online). Correlation between the leptonic Jarlskog invariant and the deviation of the atmospheric mixing angle $\sin^2 2\theta_{\rm atm}$ from 1 in Model II. The shaded area in cyan (light shade) is the full allowed region, while the area in red (dark shade) corresponds to $f_1^0 = 0.00424$.

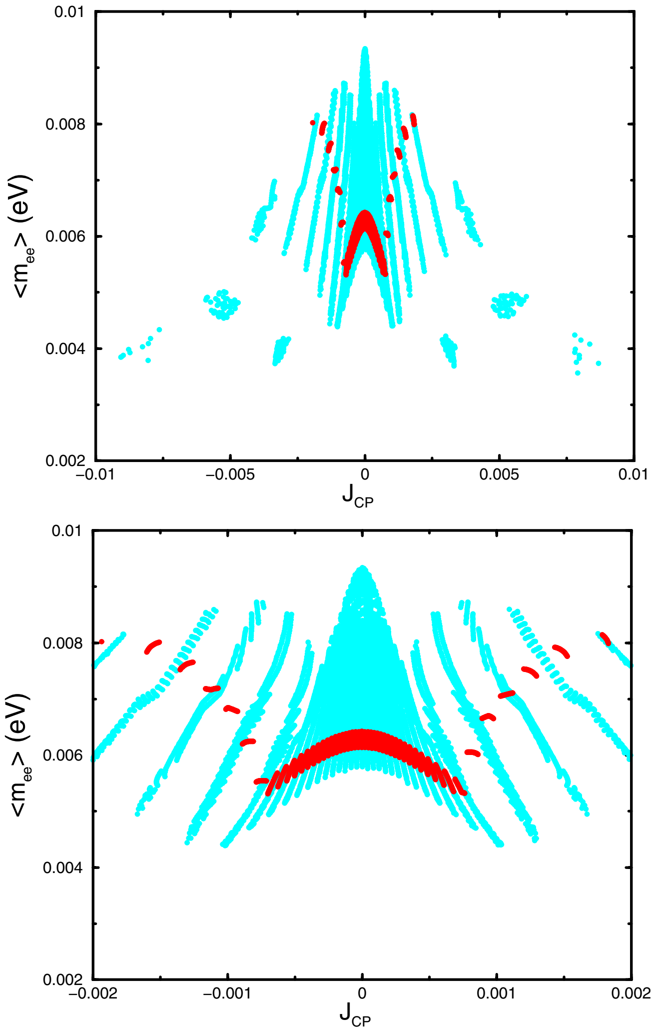

FIG. 9 (color online). Correlation between the matrix element of neutrinoless double beta decay, $\langle m_{ee}\rangle$, and the leptonic Jarlskog invariant in Model II. The shaded area in cyan (light shade) is the full allowed region, while the area in red (dark shade) corresponds to $f_1^0 = 0.00424$.

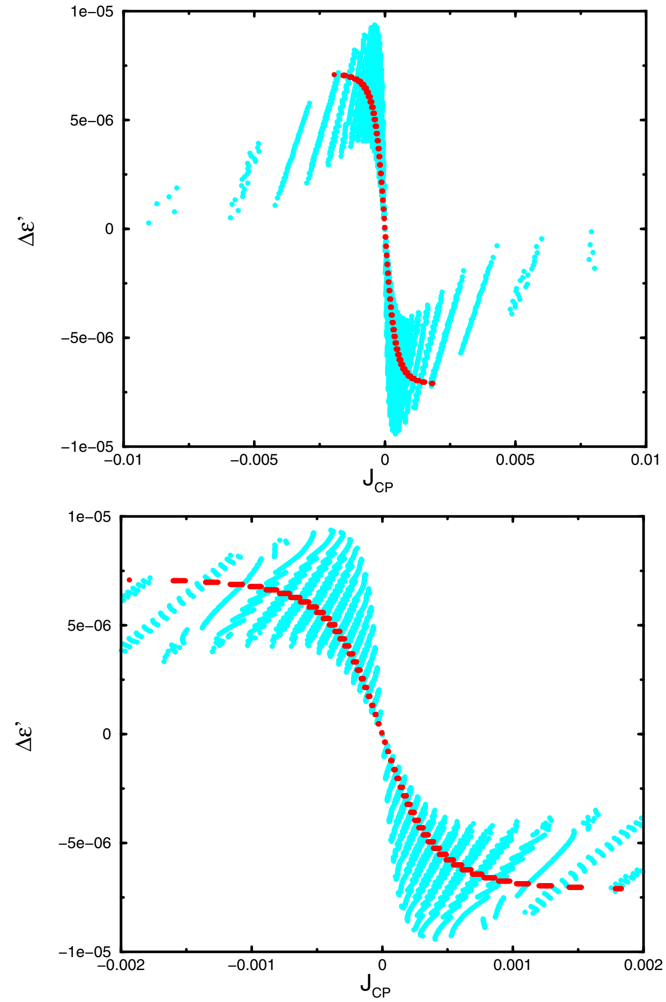

FIG. 10 (color online). Correlation between the amount of leptogenesis and the leptonic Jarlskog invariant in Model II. The shaded area in cyan (light shade) is the full allowed region, while the area in red (dark shade) corresponds to $f_1^0 = 0.00424$.

satisfied. The v_R in this case is 10^{15} GeV, and the mass of the lighest RH neutrino is given by $M_1 \sim 10^{12}$ GeV.

IV. CONCLUSION

In this paper, we have shown that there exist correlations among the *CP* violation in leptogenesis, neutrino oscillation and neutrinoless double beta decay in the minimal left-right symmetric model with spontaneous *CP* violation in which there are only two intrinsic *CP* violating phases to account for all *CP* violation in both the quark and lepton sectors. We construct two realistic models and exhibit such correlations explicitly. Even though these two specific models have very different predictions for the neutrino oscillation parameters, both models exhibit the feature that a large leptonic Jarlskog invariant implies a large value for leptogenesis. When $|J_{CP}| = 0$, leptogenesis vanishes. This statement is true for any model having nonvanishing $|U_{e3}|$. Except for $|J_{CP}| \simeq 0$, in both models, a large value for $|J_{CP}|$ also implies a large value for the matrix element of the neutrinoless double beta decay. The connection between the *CP* violation in the leptonic sector and that in the quark sector is rather weak due to the large hierarchy in the bi-doublet VEV required by a realistic quark sector.

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