Role of the $a_1(1260)$ resonance in multipion decays of light vector mesons

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The contribution of the $a_1(1260)$ meson to the amplitudes of the decays $\rho(770) \to 4\pi$, $\omega(782) \to 5\pi$, and $\phi(1020) \to 5\pi$ is analyzed in the chiral model of pseudoscalar, vector, and axial vector mesons based on the generalized hidden local symmetry added with the anomalous terms. The analysis shows that inclusion of a_1 meson in the intermediate states results in enhancement of the branching ratios of the above decays by the factor ranging from 1.3 to 1.9 depending on the mass of a_1 meson ranging from 1.23 GeV to $m_{a_1} = m_\rho\sqrt{2} = 1.09$ GeV, the greater factor standing in case of lower mass of the a_1.

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I. INTRODUCTION

In the low-energy domain, quantum chromodynamics (QCD) manifests as an effective theory formulated in terms of colorless degrees of freedom [1]. They are introduced on the basis of chiral $G = \mathrm{U}(3)_L \times \mathrm{U}(3)_R$ symmetry of the QCD Lagrangian with approximately massless u, d, and s quarks. This symmetry is supposed to be spontaneously broken to $H = \mathrm{SU}(3)_{R+L}$. As is well known, the spontaneous symmetry breaking [2] is followed by the appearance of massless bosons [3], in the present case, nine pseudoscalar mesons $\pi^\pm$, π^0, $K^\pm$, K^0, $\bar{K}^0$, η, and η'. Their effective Lagrangian, including the interaction terms, is fixed by the symmetry breaking pattern $G \to H$, according to which the fields of Goldstone bosons are treated as the coordinates in the space G/H [4,5]. Adding the Wess-Zumino term [6] to the effective Lagrangian removes the spurious selection rule which forbids the processes with odd number of Goldstone bosons.

There are several models which incorporate the low-lying vector mesons $\rho(770)$, $\omega(782)$, $\phi(1020)$ etc. into the chiral theory, see Refs. [4,7–10]. As far as nonanomalous sector is concerned, there is the equivalence of such models, see [8,10,11]. However, the anomalous couplings are most conveniently incorporated into chiral theory in the framework of approach based on the hidden local symmetry (HLS) [9,12]. The above vector mesons are the gauge bosons of HLS. In particular, the convenience of HLS rests on the fact that ρ, ω, and ϕ mesons can be accounted for without violation of the low-energy theorems [9,11]. To avoid such a violation, other chiral models of vector and pseudoscalar mesons rely essentially on the subtraction to the gauged Wess-Zumino term [6]. The question of the validity of each specific model is acute because in the well-studied decays $\rho^0 \to \pi^+\pi^-$, $\omega \to \pi^+\pi^-\pi^0$ the final pions are not soft enough to use the decay amplitudes in the tree approximation. On the other hand, the multipion decays of vector mesons $\rho^0(770) \to \pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ [13–17], $\rho^\pm(770) \to \pi^\pm\pi^\pm\pi^\mp\pi^0$, $\pi^\pm\pi^0\pi^0\pi^0$ [17] and $\omega(782)$, $\phi(1020) \to \pi^+\pi^+\pi^-\pi^-\pi^0$, $\pi^+\pi^-\pi^0\pi^0\pi^0$ [18], where the final pions are truly soft to rely on the lowest order Born amplitudes, can be good candidates for testing the chiral models of vector and pseudoscalar mesons [17,18]. A brief accounts of the $\omega \to 5\pi$ and $\phi \to 5\pi$ results are given, respectively, in Refs. [19,20].

In Refs. [17,18] devoted to the evaluation of the branching ratios of the above multipion decays, we neglected the contribution of the axial vector $a_1(1260)$ meson. The present paper addresses the question to what extent the inclusion of this resonance affects the branching ratios of the decays listed above. As is known, chiral models admit the contribution of the axial vector mesons like $a_1(1260)$, see reviews [8,9]. We shall use the generalized hidden local symmetry model (GHLS) [21] because it accounts for the contributions of the vector and axial vector resonances in a most elegant way.

The material of the paper is organized as follows. In Sec. II, starting from the GHLS Lagrangian [21], the Lagrangian of π, ρ, ω, and a_1 mesons is obtained at the lowest number of derivatives necessary for the derivation of the $\rho \to 4\pi$, $\omega \to 5\pi$, and $a_1 \to 3\pi$ decay amplitudes. Sec. III is devoted to the derivation of the $a_1 \to 3\pi$ decay amplitude, with the emphasis on its behavior at the vanishing pion momenta. Using the derived expression, the $a_1 \to 3\pi$ decay width is evaluated assuming different masses of the a_1 meson. The contribution of the $a_1(1260)$ resonance to the $\rho \to 4\pi$ decay amplitude is found in Sec. IV. Its influence on the $\omega \to 5\pi$ and $\phi \to 5\pi$ decay amplitudes is discussed in the same section. The results of the evaluation of the branching ratios of the decays $\rho \to 4\pi$, $\omega \to 5\pi$, and $\phi \to 5\pi$, taking into account the contributions of the a_1 meson and the additional $\rho\rho\pi\pi$ vertex Eq. (2.11), are presented in Sec. V. Section VI is devoted to a brief discussion of the results obtained in the present paper.

II. CHIRAL INVARIANT LAGRANGIAN OF π, ρ, ω, AND a_1 MESONS WITH LOWEST NUMBER OF DERIVATIVES

The basis of the derivation is the Lagrangian of the generalized hidden local symmetry model [21] (GHLS)

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which, in the gauge $\xi_M = 1$, $\xi_L^\dagger = \xi_R = \xi$, looks as

$$\mathcal{L}^{(\mathrm{GHLS})} = a_0 f_\pi^2 \mathrm{Tr}\left(\frac{\partial_\mu \xi^\dagger \xi + \partial_\mu \xi \xi^\dagger}{2i} - g V_\mu\right)^2 + b_0 f_\pi^2 \mathrm{Tr}\left(\frac{\partial_\mu \xi^\dagger \xi - \partial_\mu \xi \xi^\dagger}{2i} + g A_\mu\right)^2 + c_0 f_\pi^2 g^2 \mathrm{Tr} A_\mu^2 + d_0 f_\pi^2 \mathrm{Tr}\left(\frac{\partial_\mu \xi^\dagger \xi - \partial_\mu \xi \xi^\dagger}{2i}\right)^2 - \frac{1}{2}\mathrm{Tr}(F_{\mu\nu}^{(V)2} + F_{\mu\nu}^{(A)2}) - i\alpha_4 g \mathrm{Tr}[A_\mu, A_\nu] F_{\mu\nu}^{(V)} + 2i\alpha_5 g \mathrm{Tr}\left(\left[\frac{\partial_\mu \xi^\dagger \xi - \partial \xi \xi^\dagger}{2ig}, A_\nu\right] + [A_\mu, A_\nu]\right) F_{\mu\nu}^{(V)}. \tag{2.1}$$

The notations, assuming the restriction to the sector of the nonstrange mesons, are

$$\begin{aligned} F_{\mu\nu}^{(V)} &= \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu] - i[A_\mu, A_\nu], \\ F_{\mu\nu}^{(A)} &= \partial_\mu A_\nu - \partial_\nu A_\mu - i[V_\mu, A_\nu] - i[A_\mu, V_\nu], \\ V_\mu &= \left(\frac{\boldsymbol{\tau}}{2}\cdot \boldsymbol{\rho}_\mu\right) + \frac{1}{2}\omega_\mu, \\ A_\mu &= \left(\frac{\boldsymbol{\tau}}{2}\cdot \boldsymbol{A}_\mu\right), \\ \xi &= \exp i \frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{2f_\pi}, \end{aligned} \tag{2.2}$$

where $\boldsymbol{\rho}_\mu$, ω_μ, $\boldsymbol{\pi}$ are the vector meson ρ, ω and pseudoscalar pion fields, respectively, $\boldsymbol{A}_\mu$ is the axial vector field (not a_1 meson!), $\boldsymbol{\tau}$ is the isospin Pauli matrices, $f_\pi = 92.4$ MeV is the pion decay constant, [,] stands for commutator. Hereafter the boldface characters, cross ($\times$), and dot ($\cdot$) stand for vectors, vector product, and scalar product, respectively, in the isotopic space. The constants a_0, b_0, c_0, d_0, $\alpha_{4,5}$ are specified below. The relevant terms of the Lagrangian describing a_1 meson and its couplings to the $\rho\pi$ and 3π systems can be obtained from Eq. (2.1) following the steps [21] outlined below. First, we exclude the mixing term

$$\Delta\mathcal{L}^{(a_1-\pi)} \propto \mathrm{tr} A_\mu \frac{\partial_\mu \xi^\dagger \xi - \partial_\mu \xi \xi^\dagger}{2i} \propto \boldsymbol{A}_\mu \left(\partial_\mu \boldsymbol{\pi} + \frac{1}{6f_\pi^2}[\boldsymbol{\pi}\times(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})] + \cdots\right) \tag{2.3}$$

by introducing the field of a_1 meson

$$a_\mu = \left(\frac{\boldsymbol{\tau}}{2}\cdot \boldsymbol{a}_\mu\right)$$

as follows:

$$A_\mu = a_\mu - \frac{b_0}{g(b_0 + c_0)} \frac{\partial_\mu \xi^\dagger \xi - \partial_\mu \xi \xi^\dagger}{2i}. \tag{2.4}$$

Note that in distinction with Ref. [21], where the mixing term $\boldsymbol{A}_\mu \partial_\mu \boldsymbol{\pi}$ with the lowest order derivative in pion field is rotated away, we do so with the entire nonlinear combination Eq. (2.3). We postpone the justification of our choice until discussing the 3π decay width of a_1 meson in Sec. III.

The above diagonalization introduces the unwanted momentum dependence of the $\rho\pi\pi$ vertex, which can be cancelled by the counter terms [21]. They are represented by the terms containing the parameters $\alpha_{4,5}$ in Eq. (2.1). Following Ref. [21], we retain only the terms with $-\alpha_4 = \alpha_5 = \alpha_6 \neq 0$, with the further fixing $-\alpha_4 = \alpha_5 = 1$ [21]. The second step is the renormalization $f_\pi \to Z^{-1/2} f_\pi$, $\boldsymbol{\pi} \to Z^{-1/2}\boldsymbol{\pi}$, $(a_0, b_0, c_0, d_0) = Z(a, b, c, d)$, where

$$\left(d_0 + \frac{b_0 c_0}{b_0 + c_0}\right) Z^{-1} = 1.$$

The last step is the choice [21] $a = b = c = 2$, $d = 0$ which results in the universality $g_{\rho\pi\pi} = g$ and vector dominance of the $\rho\pi\pi$ coupling, the Kawarabayashi-Suzuki-Riazzuddin-Fayyazuddin (KSRF) relation [22]

$$\frac{2g_{\rho\pi\pi}^2 f_\pi^2}{m_\rho^2} = 1, \tag{2.5}$$

and the Weinberg relation [23]

$$m_{a_1} = \sqrt{2} m_\rho = 1.09 \text{ GeV}, \tag{2.6}$$

see Eq. (2.8) and (2.11). The $\rho\pi\pi$ coupling constant resulting from Eq. (2.5) is $g_{\rho\pi\pi} = 5.9$. Finally, using the weak field expansion

$$\begin{aligned} \frac{\partial_\mu \xi^\dagger \xi - \partial_\mu \xi \xi^\dagger}{2i} &= -\frac{1}{2f_\pi}\boldsymbol{\tau}\cdot\left(\partial_\mu \boldsymbol{\pi} + \frac{1}{6f_\pi^2}[\boldsymbol{\pi}\times(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})] + \cdots\right), \\ \frac{\partial_\mu \xi^\dagger \xi + \partial_\mu \xi \xi^\dagger}{2i} &= -\frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot[\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}]\left(1 - \frac{\boldsymbol{\pi}^2}{12f_\pi^2}\cdots\right), \end{aligned} \tag{2.7}$$

one obtains the following Lagrangian of the ρ, ω, a_1, and π mesons at the order required for the evaluation of the a_1 meson contribution to the decay $\rho \to 4\pi$:

$$\mathcal{L}^{(\mathrm{GHLS})} \approx \mathcal{L}^{(\mathrm{HLS})} + \Delta\mathcal{L}^{(\mathrm{GHLS})},$$

where

$$\mathcal{L}^{\rm HLS} = -\frac{1}{4} - \frac{1}{4}\omega^2_{\mu\nu} + \frac{1}{2}ag^2f^2_\pi(\boldsymbol{\rho}^2_\mu + \omega^2_\mu) + \frac{1}{2}(\partial_\mu\boldsymbol{\pi})^2 - \frac{1}{2}m^2_\pi\boldsymbol{\pi}^2 + \frac{m^2_\pi}{24f^2_\pi}\boldsymbol{\pi}^4 + \frac{1}{2f^2_\pi}\left(\frac{a}{4} - \frac{1}{3}\right)[\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}]^2 + \frac{1}{2}ag\left(1 - \frac{\boldsymbol{\pi}^2}{12f^2_\pi}\right)(\boldsymbol{\rho}_\mu\cdot[\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}]) \qquad (2.8)$$

is the weak field limit of the Lagrangian of HLS including the terms $\propto m^2_\pi$ which explicitly violate the chiral symmetry,

$$\boldsymbol{\rho}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu + g[\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu], \quad \omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \qquad (2.9)$$

are the field strengths of the isovector ρ_μ and isoscalar ω_μ fields, g is the gauge coupling constant, $a = 2$ is HLS parameter. As is clear from Eq. (2.8),

$$g_{\rho\pi\pi} = \tfrac{1}{2}ag, \qquad m^2_\rho = ag^2f^2_\pi \qquad (2.10)$$

are the $\rho\pi\pi$ coupling constant and the ρ mass squared, respectively. Note that $m_\omega = m_\rho$ in HLS. The Lagrangian

$$\Delta\mathcal{L}^{\rm (GHLS)} = \mathcal{L}^{(a_1\rho\pi)} + \mathcal{L}^{(\rho\rho\pi\pi)} + \mathcal{L}^{(4\pi)}$$

is the contribution of that part of the GHLS Lagrangian Eq. (2.1) which contains the axial vector field A_μ, the terms originating from the diagonalization of $A - \pi$ mixing Eq. (2.3), and the counter terms. It consists of the terms responsible for the free a_1 field and its interaction with the $\rho\pi$ and 3π states,

$$\mathcal{L}^{(a_1\rho\pi)} = -\frac{1}{4}(\partial_\mu\boldsymbol{a}_\nu - \partial_\nu\boldsymbol{a}_\mu)^2 + \frac{1}{2}(b+c)g^2f^2_\pi\boldsymbol{a}^2_\mu - \frac{1}{f_\pi}(\partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu)\cdot[\boldsymbol{a}_\mu\times\partial_\nu\boldsymbol{\pi}] - \frac{1}{2f_\pi}(\partial_\mu\boldsymbol{a}_\nu - \partial_\nu\boldsymbol{a}_\mu)\cdot[\boldsymbol{\rho}_\mu\times\partial_\nu\boldsymbol{\pi}] - \frac{1}{8gf^3_\pi}[\boldsymbol{a}_\mu\times\partial_\nu\boldsymbol{\pi}]\cdot[\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi}] - \frac{1}{4gf^3_\pi}\partial_\mu\boldsymbol{a}_\nu\cdot[\boldsymbol{\pi}\times(\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi})], \qquad (2.11)$$

the term describing the $\rho\rho\pi\pi$ and the higher derivative pointlike $\rho\to 4\pi$ vertex vertices

$$\mathcal{L}^{(\rho\rho\pi\pi)} = -\frac{1}{16f^2_\pi}([\boldsymbol{\rho}_\mu\times\partial_\nu\boldsymbol{\pi}] - [\boldsymbol{\rho}_\nu\times\partial_\mu\boldsymbol{\pi}])^2 - \frac{1}{8gf^4_\pi}[\boldsymbol{\rho}_\mu\times\partial_\nu\boldsymbol{\pi}]\cdot[\boldsymbol{\pi}\times(\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi})], \qquad (2.12)$$

and the higher derivative 4π vertex:

$$\mathcal{L}^{(4\pi)} = \frac{1}{64g^2f^4_\pi}[\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi}]^2. \qquad (2.13)$$

Note that when deriving the above Lagrangians, we have not used the equation of motion of the fields π, ρ, and a_1. One should have in mind that in the decays of our interest the final pions are nonrelativistic, $p_\mu \approx (m_\pi, 0, 0, 0)$. The direct calculation shows that the ratio of the contribution from Eq. (2.13) to the lowest derivative $\pi\pi$ scattering amplitude is about $(m_\pi/4gf_\pi)^2 \approx 4\times 10^{-3}$, in agreement with the expectations of the chiral perturbation theory. Hence, we shall ignore this contribution in what follows. In the meantime, the higher derivative pointlike vertex $\rho\to 4\pi$ in Eq. (2.12) cannot be omitted, because it is essential for validity of the Adler condition for the contribution to the $\rho\to 4\pi$ decay amplitude originating from Eq. (2.12). See details in Sec. IV.

The terms of the effective Lagrangian necessary for the calculation of the $\omega\to 5\pi$ decay amplitude are obtained from the weak field limit of the terms [9,11] induced by the anomalous term of Wess and Zumino [6]. The corresponding expression looks as

$$\mathcal{L}^{\rm an} = \frac{n_cg}{32\pi^2f^3_\pi}(c_1 - c_2 - c_3)\varepsilon_{\mu\nu\lambda\sigma}\omega_\mu(\partial_\nu\boldsymbol{\pi}\cdot[\partial_\lambda\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi}]) + \frac{n_cg}{128\pi^2f^5_\pi}\left[-c_1 + \frac{5}{3}\left(c_2 + c_3\right)\right]\varepsilon_{\mu\nu\lambda\sigma}\omega_\mu(\partial_\nu\boldsymbol{\pi}\cdot[\partial_\lambda\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi}])\boldsymbol{\pi}^2 - \frac{n_cg^2c_3}{8\pi^2f_\pi}\varepsilon_{\mu\nu\lambda\sigma}\partial_\mu\omega_\nu\left\{(\boldsymbol{\rho}_\lambda\cdot\partial_\sigma\boldsymbol{\pi}) + \frac{1}{6f^2_\pi}[(\boldsymbol{\rho}_\lambda\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\partial_\sigma\boldsymbol{\pi}) - \boldsymbol{\pi}^2(\boldsymbol{\rho}_\lambda\cdot\partial_\sigma\boldsymbol{\pi})]\right\} - \frac{n_Cg^2}{8\pi^2f_\pi}(c_1 + c_2 - c_3)\varepsilon_{\mu\nu\lambda\sigma}\omega_\mu\left\{\frac{1}{4f^2_\pi}(\partial_\nu\boldsymbol{\pi}\cdot\boldsymbol{\rho}_\lambda)(\boldsymbol{\pi}\cdot\partial_\sigma\boldsymbol{\pi}) - \frac{g}{4}([\boldsymbol{\rho}_\nu\times\boldsymbol{\rho}_\lambda]\cdot\partial_\sigma\boldsymbol{\pi})\right\} \qquad (2.14)$$

[18], where $n_c = 3$ is the number of colors, $c_{1,2,3}$ are arbitrary constants multiplying three independent structures in the solution [9,11] of the Wess-Zumino anomaly equation [6]. The normalization of $c_{1,2,3}$ is in accord with Ref. [11]. As is evident from Eq. (2.14), the $\omega\rho\pi$ coupling constant is

$$g_{\omega\rho\pi} = -\frac{n_c g^2 c_3}{8\pi^2 f_\pi}. \qquad (2.15)$$

Assuming in what follows the relation

$$c_1 - c_2 - c_3 = 0, \qquad (2.16)$$

i.e., the absence of the pointlike $\omega \to \pi^+\pi^-\pi^0$ amplitude [24], and using the $\omega \to \pi^+\pi^-\pi^0$ partial width to extract $g_{\omega\rho\pi}$, the $\rho \to \pi^+\pi^-$ partial width and Eq. (2.10) to extract $g = g_{\rho\pi\pi} \approx 6$ (assuming $a = 2$), one finds $c_3 \approx 1$. Hereafter we use the particle parameters (masses, full and partial widths, etc.) taken from Ref. [25]. The decay $\phi \to 5\pi$ is described by the effective Lagrangian similar to Eq. (2.14), see Ref. [18]. The evaluation of the branching ratios of the decays $\omega, \phi \to 5\pi$ with the neglect of the a_1 meson and counter term contributions is performed in Ref. [18].

III. THE WIDTH OF a_1 RESONANCE IN GHLS

Let us find the width of the decay $a_1 \to 3\pi$ in GHLS. This task is necessary, because the original Ref. [21] contains only the discussion of the $a_1 \to \rho\pi$ decay width which, as it will be clear, overestimates the true $a_1 \to 3\pi$ decay width. When so doing, the pointlike $a_1 \to 3\pi$ vertex is essential. The amplitude of, say, the decay $a_1^0 \to \pi^+\pi^-\pi^0$ can be found from the Lagrangian Eq. (2.11):

$$iM(a^0_{1Q} \to \pi^+_{q_1}\pi^-_{q_2}\pi^0_{q_3}) = \epsilon_\mu J_\mu(a^0_{1Q} \to \pi^+_{q_1}\pi^-_{q_2}\pi^0_{q_3}), \qquad (3.1)$$

where ϵ stands for the four-vector of polarization of the a_1 meson, particles are labeled by their four-momenta, and

$$J_\mu(a^0_{1Q} \to \pi^+_{q_1}\pi^-_{q_2}\pi^0_{q_3}) = \frac{ag}{4f_\pi}(1 + P_{12})(1 - P_{13})\left\{\frac{1}{D_\rho(q_1+q_3)}[2(1-P_{13})q_{1\mu}(q_2q_3) + (1-P_{12})q_{1\mu}(Qq_2)] - \frac{1}{2m_\rho^2}q_{1\mu}[2(Qq_3) + (q_2q_3)]\right\}. \qquad (3.2)$$

Hereafter P_{ij} is the operator that interchanges the four-momenta q_i and q_j,

$$D_\rho(k) = m_\rho^2 - k^2 - i\sqrt{k^2}\Gamma_\rho(\sqrt{k^2}) \qquad (3.3)$$

is the inverse propagator of the ρ meson whose energy dependent width above the $\pi^+\pi^-$ threshold and below the $K^*\bar{K}$ one includes the $\pi^+\pi^-$, $K\bar{K}$, and $\omega\pi$ decay modes:

$$\Gamma_\rho(\sqrt{k^2}) = \frac{g^2_{\rho\pi\pi}}{6\pi k^2}q^3_{\pi\pi}(k^2) + \frac{g^2_{\rho K\bar{K}}}{3\pi k^2}q^3_{K\bar{K}}(k^2)\theta(\sqrt{k^2} - 2m_K) + \frac{g^2_{\rho\omega\pi}}{12\pi}q^3_{\omega\pi}(k^2)\theta(\sqrt{k^2} - m_\omega - m_\pi). \qquad (3.4)$$

Here θ is the usual step function, while

$$q_{ab}(k^2) = \frac{1}{2\sqrt{k^2}}\sqrt{[k^2 - (m_a + m_b)^2][k^2 - (m_a - m_b)^2]} \qquad (3.5)$$

is the momentum of the final state particle in the rest frame system of the decaying particle. In case of energies $E \sim m_\phi$ discussed in the present paper, only the $\pi^+\pi^-$ decay mode is essential. In the quark model, the coupling constants are related in the following way: $g^2_{\rho K\bar{K}} = \frac{1}{2}g^2_{\phi K\bar{K}}$, $g_{\rho\omega\pi} = g_{\omega\rho\pi}$, where $g_{\phi K\bar{K}}$ is calculated from the $\phi \to K\bar{K}$ decay width.

One can convince oneself that the expression (3.1) vanishes at the vanishing four-momentum of any final pion. This property called the Adler condition, expresses the chiral invariance of the underlying theory. [To be more precise, the check based on the Adler condition hereafter is applied in the narrow ρ width approximation. Indeed, it should be recalled that the finite width effects are attributed to the loop corrections which are beyond the tree approximation adopted in the present paper. Numerically, at energies of our concern the invariant mass of a pion pair is $m < 0.6$ GeV, so that $m\Gamma_\rho(m)/(m_\rho^2 - m^2) < 0.26$, and the effects of the ρ width in the diagrams with the non resonant ρ meson are small.] The ρ pole contribution without the pointlike $a_1 \to \pi^+\pi^-\pi^0$ vertex does not possess this property. Remarkably, the Adler condition for the $a_1 \to 3\pi$ decay amplitude Eq. (3.1) is valid even in the case of the off-mass-shell a_1 meson. This is very useful because one can safely add the a_1 contribution to the amplitudes which satisfy the Adler condition, without spoiling this property. At this point, one can justify the choice of the diagonalization of the $A - \pi$ Lagrangian used in Sec. II. Indeed, when the $A - \pi$ mixing is excluded in the first order in the π field, it is equivalent to the adding the term

$$i\Delta M(a^0_{1Q} \to \pi^+_{q_1}\pi^-_{q_2}\pi^0_{q_3}) = \frac{g}{3f_\pi}(\epsilon, q_1 + q_2 - 2q_3) + \frac{1}{4gf_\pi^3}[(\epsilon q_3)(Q, q_1 + q_2) - (\epsilon, q_1 + q_2)(Qq_3)] \qquad (3.6)$$

to the right hand side of Eq. (3.1). Hereafter (a, b) stands for the Lorentz scalar product in cases when the four-vectors a or b are sums of other four-vectors. As is evident from Eq. (3.6), $i\Delta M$ does not vanish at $q_1 = 0$ but rather reduces to the expression

$$iM(a^0_{1Q} \to \pi^+_{q_1=0}\pi^-_{q_2}\pi^0_{q_3}) = \frac{g}{f_\pi}(\epsilon q_2)\left(1 - \frac{Q^2}{m^2_{a_1}}\right),$$

where $m^2_{a_1} = 4g^2 f^2_\pi$, which vanishes only on the mass shell of the a_1 meson. This would result in the breaking of the Adler condition for the $\omega, \phi \to 5\pi$ decay amplitudes upon taking the a_1 resonance into account. In turn, it would demand adding further counter terms, besides those proposed in Ref. [21], to make the amplitude chirally invariant. The above amplitudes with the neglect of the a_1 meson were shown to obey the Adler condition [18].

The energy dependence of the $a^0_1 \to \pi^+\pi^-\pi^0$ decay width can be found from the expression

$$\Gamma_{a^0_1\to\pi^+\pi^-\pi^0}(m) = \frac{1}{3\times 2^8 \times \pi^3 \times m^3}\int_{4m^2_\pi}^{(m-m_\pi)^2} ds_1 \times \int_{u_{1-}}^{u_{1+}} du_1 |M(a^0 \to \pi^+_{q_1}\pi^-_{q_2}\pi^0_{q_3})|^2, \tag{3.7}$$

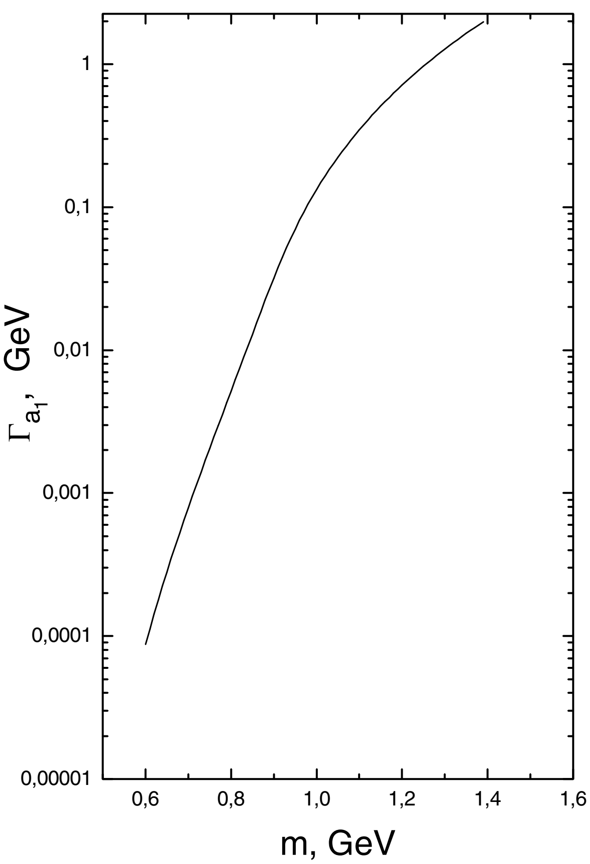

FIG. 1. The energy dependence of the $a^0_1 \to \pi^+\pi^-\pi^0$ decay width calculated in the generalized hidden local symmetry model.

where $|M(a^0 \to \pi^+_{q_1}\pi^-_{q_2}\pi^0_{q_3})|^2$ is the modulus squared of the amplitude Eq. (3.1) summed over three polarization states of the a_1. It should be expressed through the invariant Kumar variables [26] $m^2 = (q_1 + q_2 + q_3)^2$, $s_1 = (q_2 + q_3)^2$, $u_1 = (q_1 + q_3)^2$. The limits of integration over u_1 are

$$u_{1\pm} = \frac{1}{2}(m^2 + 3m^2_\pi - s_1) \pm \frac{1}{2s_1} \times \sqrt{\lambda(s_1, m^2_\pi, m^2_\pi)\lambda(m^2, s_1, m^2_\pi)},$$

where

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz. \tag{3.8}$$

The results of the evaluation are shown in Fig. 1. The above width rises rapidly with increasing m. In particular, one obtains $\Gamma_{a_1} \equiv \Gamma_{a^0_1\to\pi^+\pi^-\pi^0}(m) = 320$, 860, 1024 MeV at, respectively, $m = 1090$, 1230, 1260 MeV. For comparison, the $a_1 \to \rho\pi$ decay width in the narrow ρ width approximation is 420, 1100, 1240 MeV, respectively. Since $\sqrt{s} \leq m_\phi = 1020$ MeV is of our main concern, the upper kinematical bound of the invariant mass of the three pion system is 740 MeV. In the mass range $m \leq 740$ MeV the a_1 width is rather small, $\Gamma_{a_1} < 1.7$ MeV, and can be safely neglected.

IV. THE a_1 AND COUNTER TERM CONTRIBUTIONS TO THE $\rho \to 4\pi$ DECAY AMPLITUDE

The $\rho \to 4\pi$ decay amplitudes obtained in Refs. [17,18] from the HLS Lagrangian Eq. (2.8), upon neglecting the a_1 meson contribution, obey the Adler condition. In the GHLS approach, the additional terms originate, first, from the Lagrangian Eq. (2.11) and are represented by the diagram Fig. 2(a), where, for each specific decay $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$, $\rho^\pm \to \pi^\pm\pi^\pm\pi^\mp\pi^0$, $\pi^\pm\pi^0\pi^0\pi^0$, one should take the sum of the diagrams with all possible permutations of the final pion momenta. Second, there are the terms which do not contain a_1 meson explicitly but result from the exclusion of the axial vector-pseudoscalar mixing term Eq. (2.3). They are represented by the diagrams Fig. 2(b) and 2(c) and correspond to the first and second term in the right hand side of Eq. (2.12). Again, one should include the sum of the diagrams with all possible permutations of the final pion momenta. The a_1 contribution to the $\rho \to 4\pi$ decay amplitude can be obtained in the following way. When so doing, we present the details for the $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$ decay mode only, since other modes can be treated similarly. One has

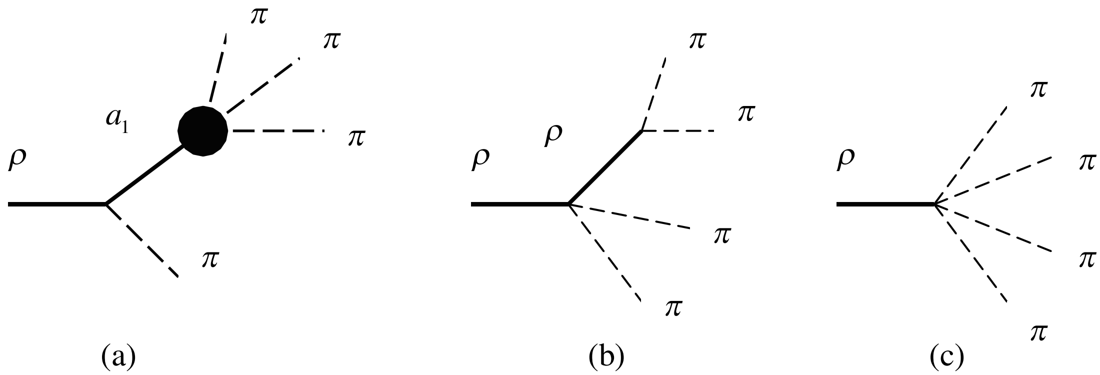

FIG. 2. Diagrams corresponding to the contribution of the intermediate a_1 meson (a), $\rho\rho\pi\pi$ (b) and pointlike $\rho \to 4\pi$ (c) vertices, respectively. The shaded circle in (a) denotes the $a_1 \to 3\pi$ decay amplitude similar to Eq. (3.1).

$$i\Delta M^{(a_1\rho\pi)}(\rho_q^0 \to \pi_{q_1}^+\pi_{q_2}^+\pi_{q_3}^-\pi_{q_4}^-) = (1 + P_{34})J_\mu(\rho_q^0 \to a_1^+\pi_{q_4}^-)\frac{i\left[\eta_{\mu\nu} - \frac{(q-q_4)_\mu(q-q_4)_\nu}{m_{a_1}^2}\right]}{D_{a_1}(q - q_4)}J_\nu(a_1^+ \to \pi_{q_1}^+\pi_{q_2}^+\pi_{q_3}^-) + (1 + P_{12})J_\mu(\rho_q^0 \to a_1^+\pi_{q_2}^-)\frac{i\left[\eta_{\mu\nu} - \frac{(q-q_2)_\mu(q-q_2)_\nu}{m_{a_1}^2}\right]}{D_{a_1}(q - q_2)}J_\nu(a_1^- \to \pi_{q_1}^+\pi_{q_3}^-\pi_{q_4}^-), \quad (4.1)$$

where the inverse propagator of the a_1 meson is

$$D_{a_1}(k) = m_{a_1}^2 - k^2. \quad (4.2)$$

The decay current is

$$J_\mu(\rho_q^0 \to a_1^+\pi_{q_4}^-) = \frac{1}{2f_\pi}[(\epsilon q_4)(2q - q_4)_\mu - (3qq_4 - q_4^2)\epsilon_\mu], \quad (4.3)$$

ϵ is the polarization four-vector of the initial ρ meson, and $J_\nu(a_1^+ \to \pi_{q_1}^+\pi_{q_2}^+\pi_{q_3}^-)$ is obtained from Eq. (3.2) by inverting the overall sign. The expression for $J_\nu(\rho_q^0 \to a_1^-\pi_{q_2}^+)$ is obtained from Eq. (4.3) by inverting the sign while the expression for $J_\nu(a_1^- \to \pi_{q_1}^+\pi_{q_3}^-\pi_{q_4}^-)$ is obtained from $J_\nu(a_1^+ \to \pi_{q_1}^+\pi_{q_2}^+\pi_{q_3}^-)$ upon the charge conjugation followed by the replacements $q_1 \longleftrightarrow q_3$, $q_2 \longleftrightarrow q_4$ of the final pion momenta. One can directly show that the amplitude Eq. (4.1) obeys the Adler condition at the vanishing of any pion momentum.

Next let us give the expressions for the contribution to the $\rho \to 4\pi$ decay amplitudes generated by the terms Eq. (2.12). They are

$$i\Delta M^{(\rho\rho\pi\pi)}(\rho_q^0 \to \pi_{q_1}^+\pi_{q_2}^+\pi_{q_3}^-\pi_{q_4}^-) = -i\frac{ag}{8f_\pi^2}(1 + P_{12})(1 + P_{34})(1 + P_{24})(1 - P_{13})\left[\frac{(1 - P_{12})(\epsilon q_1)(q_2q_4)}{D_\rho(q_1 + q_3)} - \frac{(\epsilon q_1)(q_2q_3)}{m_\rho^2}\right],$$
$$i\Delta M^{(\rho\rho\pi\pi)}(\rho_q^0 \to \pi_{q_1}^+\pi_{q_2}^-\pi_{q_3}^0\pi_{q_4}^0) = -i\frac{ag}{8f_\pi^2}(1 + P_{34})(1 - P_{12})\left[(1 - P_{13})\frac{(1 - P_{14})(\epsilon q_1)(q_2q_4)}{D_\rho(q_1 + q_3)} - \frac{(\epsilon q_1)(q_2q_3)}{m_\rho^2}\right],$$
$$i\Delta M^{(\rho\rho\pi\pi)}(\rho_q^- \to \pi_{q_1}^+\pi_{q_2}^-\pi_{q_3}^-\pi_{q_4}^0) = -i\frac{ag}{8f_\pi^2}(1 + P_{23})\left[(1 - P_{24})\frac{(1 - P_{12})(\epsilon q_1)(q_2q_3)}{D_\rho(q_2 + q_4)} + (1 - P_{12})\frac{(1 - P_{13})(\epsilon q_1)(q_3q_4)}{D_\rho(q_1 + q_2)} + (1 - P_{14})\frac{(1 - P_{12})(\epsilon q_1)(q_2q_3)}{D_\rho(q_1 + q_4)} + (1 - P_{24})(1 + P_{13})\frac{(\epsilon q_2)(q_1q_4)}{m_\rho^2}\right],$$
$$i\Delta M^{(\rho\rho\pi\pi)}(\rho_q^- \to \pi_{q_1}^-\pi_{q_2}^0\pi_{q_3}^0\pi_{q_4}^0) = i\frac{ag}{8f_\pi^2}(1 + P_{23} + P_{24})(1 - P_{12})(1 + P_{34})\left[\frac{(1 - P_{13})(\epsilon q_1)(q_3q_4)}{D_\rho(q_1 + q_2)} - \frac{(\epsilon q_1)(q_2q_3)}{m_\rho^2}\right]. \quad (4.4)$$

The total amplitude for the decay $\rho \to 4\pi$ is obtained upon adding the pure HLS contribution $M^{(\mathrm{HLS})}$ from Refs. [17,18] and the above mentioned Eq. (4.1) (and similar expressions) together with Eq. (4.4):

$$M_{\rho\to 4\pi} = M^{(\mathrm{HLS})}_{\rho\to 4\pi} + \Delta M^{(a_1\rho\pi)}_{\rho\to 4\pi} + \Delta M^{(\rho\rho\pi\pi)}_{\rho\to 4\pi} \equiv \epsilon_\mu J_\mu(\rho \to \pi_{q_1}\pi_{q_2}\pi_{q_3}\pi_{q_4}). \quad (4.5)$$

The expressions for $J_\mu(\rho \to \pi_{q_1}\pi_{q_2}\pi_{q_3}\pi_{q_4})$ are excessively lengthy, even with the use of the permutation operators P_{ij}, so we do not give them here.

The detailed analysis of the $\omega \to 5\pi$ and $\phi \to 5\pi$ decay amplitudes is given elsewhere [18]. As was shown there, the $\rho \to 4\pi$ transition amplitude enters into the dominant diagrams in Fig. 3(a) corresponding to the process $\omega, \phi \to \rho\pi \to 5\pi$, in the following way:

$$M_{\omega_q,\phi_q\to 5\pi} = \frac{g_{\rho\pi\pi} g_{\omega,\phi\to\rho\pi}}{f_\pi^2} \varepsilon_{\mu\nu\lambda\sigma} q_\mu \times \epsilon_\nu \left[\frac{q_{5\lambda} J_\sigma(\rho \to \pi_{q_1}\pi_{q_2}\pi_{q_3}\pi_{q_4})}{D_\rho(q - q_5)} + \cdots \right], \quad (4.6)$$

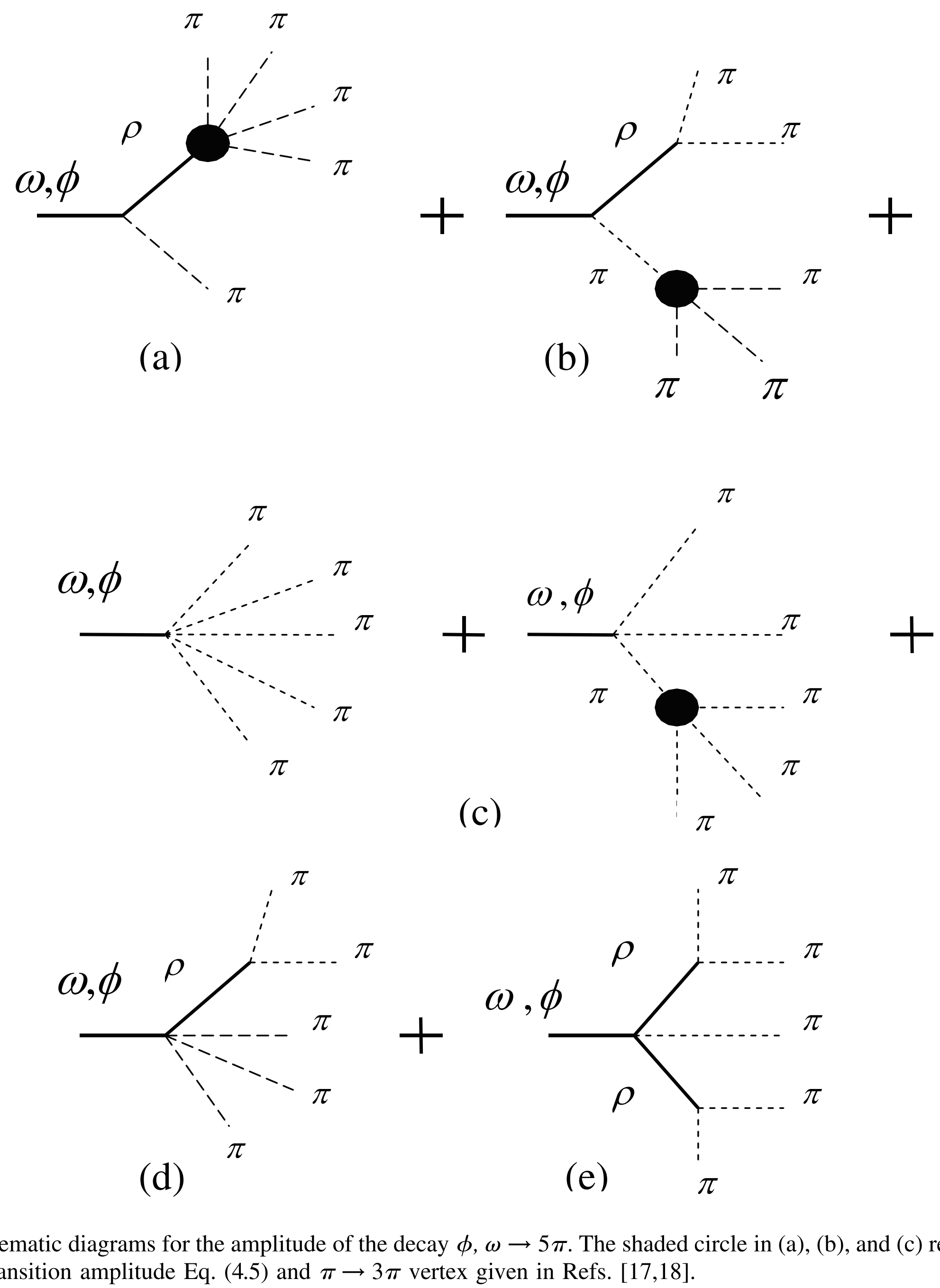

FIG. 3. The schematic diagrams for the amplitude of the decay $\phi, \omega \to 5\pi$. The shaded circle in (a), (b), and (c) refers, respectively, to the $\rho \to 4\pi$ transition amplitude Eq. (4.5) and $\pi \to 3\pi$ vertex given in Refs. [17,18].

where particles are labeled by their four-momentum, ϵ_ν is the polarization four-vector of the decaying ω, ϕ, and $\cdots$ means the terms obtained from the written one by the permutation of the pion momenta plus the contributions from the remaining diagrams in Fig. 3(b)–3(e). Taking into account the a_1 resonance in the generalized hidden local symmetry approach reduces to the use of the total $\rho \to 4\pi$ decay current in that part of the $\omega, \phi \to 5\pi$ decay amplitude which corresponds to the process $\omega, \phi \to 5\pi$ with the resonant intermediate ρ meson, see the diagram Fig. 3(a). The latter term means hereafter that the upper kinematical bound on the invariant mass of the four-pion system in the final state of the decay $\omega, \phi \to 5\pi$ can be greater than the ρ mass. As can be seen from Eq. (4.6), the amplitude obeys the Adler condition. Indeed, the contribution without the a_1 resonance was shown to obey this condition [18], while the $a_1\rho\pi$ and $\rho\rho\pi\pi$ terms discussed earlier in this paper satisfy this property separately. We do not give here the explicit expressions for the full amplitudes because they are very cumbersome.

V. BRANCHING RATIOS OF THE DECAYS $\rho \to 4\pi$, $\omega \to 5\pi$, AND $\phi \to 5\pi$ EVALUATED WITH THE a_1 CONTRIBUTION

Using Eq. (4.5), the $\rho \to 4\pi$ decay width is evaluated according to the expression

$$\Gamma_{\rho\to 4\pi}(s) = \frac{1}{3\times\pi^6\times s^{3/2}\times 2^{12}\times N_s}\int_{s_{1-}}^{s_{1+}} ds_1 \int_{s_{2-}}^{s_{2+}} ds_2 \int_{u_{1-}}^{u_{1+}} \frac{du_1}{\lambda^{1/2}(s, s_2, s_2')}\int_{u_{2-}}^{u_{2+}} du_2 \int_{-1}^{1}\frac{d\zeta_2}{(1-\zeta_2^2)^{1/2}} \times |M_{\rho\to 4\pi}[s, s_1, s_2, u_1, u_2, t_2(\zeta_2)]|^2, \tag{5.1}$$

where the modulus squared of the matrix element summed over the polarization states of the initial ρ meson, $|M_{\rho\to 4\pi}[s, s_1, s_2, u_1, u_2, t_2(\zeta_2)]|^2$, is expressed through the Mandelstam-like invariant variables $s = q^2$, $s_1 = (q - q_1)^2$, $s_2 = (q_3 + q_4)^2$, $u_1 = (q - q_2)^2$, $u_2 = (q - q_3)^2$, $t_2 = (q_1 + q_4)^2$, $s_2' = (q_1 + q_2)^2$. See Ref. [26], where the expressions for the limits of integration and $t_2 \equiv t_2(\zeta_2)$ are given. The Bose symmetry factor is $N_s = 2$ for the decay modes $\rho^0 \to \pi^+\pi^-\pi^0\pi^0$, $\rho^\pm \to \pi^\pm\pi^\pm\pi^\mp\pi^0$, $N_s = 4$ for the mode $\rho^0 \to \pi^+\pi^-\pi^+\pi^-$, and $N_s = 6$ for the decay mode $\rho^\pm \to \pi^\pm\pi^0\pi^0\pi^0$. Notice that the isotopic mass difference of the charged and neutral pion is taken into account both in the phase space volume and the decay matrix element. The results of calculation are given in Table I, where the $\rho \to 4\pi$ decay widths are presented in the cases of $m_{a_1} = m_\rho\sqrt{2} = 1.09$ GeV (the Weinberg relation), $m_{a_1} = 1.23$ GeV (the PDG value [25]), and in case when the a_1 and counter term contributions are neglected. One can see that the lower mass of the a_1 meson results in a greater decay rate. This enhancement is due to the low-energy tail of the a_1 Breit-Wigner factor.

The partial width of the decay $\omega, \phi \to \pi_{q_1}\pi_{q_2}\pi_{q_3}\pi_{q_4}\pi_{q_5}$, where the pions are labeled by their four-momenta, is evaluated according to the expression

$$\Gamma_{\omega,\phi\to 5\pi}(s) = \frac{\pi^2\sqrt{s}}{8\times 3\times N_{\rm sym}\times(2\pi)^{11}}\int_{s_{1-}}^{s_{1+}} ds_1 \int_{s_{2-}}^{s_{2+}} ds_2 \int_{s_{3-}}^{s_{3+}} ds_3 \int_{u_{1-}}^{u_{1+}} du_1 \int_{u_{2-}}^{u_{2+}} \frac{du_2}{[\lambda(s, s_2, s_2')\lambda(s, m_3^2, u_2)]^{1/2}} \times \int_{u_{3-}}^{u_{3+}} \frac{du_3}{[\lambda(s, s_3, s_3')\lambda(s, m_4^2, u_3)]^{1/2}} \int_{t_{2-}}^{t_{2+}} \frac{dt_2}{[\lambda(s, t_1, t_1')(1-\xi_2^2)(1-\eta_2^2)(1-\zeta_2^2)]^{1/2}} \times \int_{t_{3-}}^{t_{3+}} \frac{dt_3 |M_{\omega,\phi\to 5\pi}(s, s_1, s_2, s_3, u_1, u_2, u_3, t_2, t_3)|^2}{[\lambda(s, t_2, t_2')(1-\xi_3^2)(1-\eta_3^2)(1-\zeta_3^2)]^{1/2}}, \tag{5.2}$$

where $s = (\sum_{a=1}^5 q_a)^2$; the Bose symmetry factor is $N_{\rm sym} = 4$, six in case of the final state $2\pi^+2\pi^-\pi^0$, $\pi^+\pi^-3\pi^0$, respectively. The basic integration variables due to Kumar [26] are

TABLE I. The width of the decay $\rho \to 4\pi$ [keV] evaluated in the model of generalized hidden local symmetry [21], at different masses of the a_1 resonance. The uncertainty of the quoted central values set to about 10% is due to the difference in the value of gauge coupling constant $g = g_{\rho\pi\pi}$ found from the $\rho \to \pi^+\pi^-$ decay width or from KSRF relation Eq. (2.5).

m_{a_1} [GeV]	$\Gamma_{\rho^0\to 2\pi^+2\pi^-}(m_\rho^2)$	$\Gamma_{\rho^0\to \pi^+\pi^-2\pi^0}(m_\rho^2)$	$\Gamma_{\rho^\pm\to 2\pi^\pm\pi^\mp\pi^0}(m_\rho^2)$	$\Gamma_{\rho^\pm\to \pi^\pm 3\pi^0}(m_\rho^2)$
1.09	1.84	0.81	1.53	1.17
1.23	1.59	0.75	1.38	1.00
no a_1	0.94	0.59	0.99	0.59

$$s_1 = (q - q_1)^2, \qquad s_2 = (q - q_1 - q_2)^2,$$
$$s_3 = (q - q_1 - q_2 - q_3)^2, \qquad u_1 = (q - q_2)^2,$$
$$u_2 = (q - q_3)^2, \qquad u_3 = (q - q_4)^2,$$
$$t_2 = (q - q_2 - q_3)^2, \qquad t_3 = (q - q_2 - q_3 - q_4)^2, \tag{5.3}$$

$t_1 \equiv u_1$, $t'_1 \equiv m_2^2$. The variables $s'_2 = (q_1 + q_2)^2$, $s'_3 = (q_1 + q_2 + q_3)^2$, $t'_2 = (q_2 + q_3)^2$, $\xi_{2,3}$, $\eta_{2,3}$, and $\zeta_{2,3}$ can be expressed through the ones Eq. (5.3), see Ref. [26]. The limits of integration in Eq. (5.2) are also given there. $|M_{\omega,\phi\to 5\pi}(s, s_1, s_2, s_3, u_1, u_2, u_3, t_2, t_3)|^2$ is the modulus squared of the $\omega, \phi \to 5\pi$ decay amplitude summed over polarization states of the decaying particle. It should be expressed through the same variables. The necessary expressions of the scalar products $(q_a q_b)$, $a, b = 1, \cdots 5$ can be found in Ref. [18]. The latter reference is devoted to the evaluation of the branching ratios of the decays $\omega, \phi \to \pi^+\pi^-3\pi^0$ and $\omega, \phi \to 2\pi^+2\pi^-\pi^0$ in the HLS scheme using the Lagrangian Eq. (2.14) in the case of the $\omega(782)$ and analogous Lagrangian in the case of $\phi(1020)$. Taking the a_1 resonance into account in GHLS model reduces to using the total $\rho \to 4\pi$ decay current obtained in the previous section, in that part of the $\omega, \phi \to 5\pi$ decay amplitude which corresponds to the process $\omega, \phi \to \rho\pi \to 5\pi$ with the resonant intermediate ρ meson. See Eq. (4.6).

As was pointed out in Ref. [18], the Lagrangian Eq. (2.14) induced by the anomalous term of Wess and Zumino (and analogous expression in the case of ϕ) can be used for the evaluation of the $\omega, \phi \to 5\pi$ decay rates only under the definite assumptions about arbitrary parameters $c_{1,2,3}$ (and analogous parameters in the case of ϕ). The choice

$$c_1 = c_3, \qquad c_2 = 0, \qquad a = 2. \tag{5.4}$$

made in Ref. [18] is used here, too. With this choice, the $\omega \to 5\pi$ decay rate is determined by the coupling constant Eq. (2.15) only. The variation of $c_{1,2,3}$ within rather wide margins around the values given by Eq. (5.4) imply no significant changes in the branching ratio. As for the $\phi \to 5\pi$ decay, its branching ratio is determined within the accuracy 20% by the effective coupling constant $g_{\phi\rho\pi}$ extracted from the $\phi \to \pi^+\pi^-\pi^0$ decay width. The results are insensitive to the choice of free parameters analogous to Eq. (5.4). See Ref. [18] for the detailed study of this question. The results of the evaluation are presented in Table II. Notice the difference in the central value of $B_{\phi\to 2\pi^+2\pi^-\pi^0} = 5.0 \times 10^{-7}$ in the lower line of this table with the figure $(6.9 \pm 1.4) \times 10^{-7}$ given in Ref. [18]. This is due to the typesetting error in the program code for the nonleading contribution represented by the anomaly induced terms corresponding to the process $\rho \to \omega\pi \to 4\pi$. Such terms refer to higher derivatives in the effective Lagrangian. The tail of this error disappears upon the energy decrease. Indeed, the value of $B_{\omega\to 2\pi^+2\pi^-\pi^0}$ here and in Ref. [18] differs by the factor 1.06. The error is fixed when preparing the present paper.

It is interesting to plot the mass spectrum of the four-pion subsystem in the final five-pion state. This can be fulfilled straightforwardly for the distribution over the Kumar variable $\sqrt{s_1}$, see Eq. (5.3). The corresponding curves for the decays $\phi \to 2\pi^+2\pi^-\pi^0$ and $\phi \to \pi^+\pi^-3\pi^0$ are shown in Fig. 4 and 5 in the cases when, first, no a_1 resonance is present and, second, the a_1 resonance is included with the above chosen masses $m_{a_1} = 1.09$ and 1.23 GeV. The spectra look different. Specifically, both spectra has the peak due to the ρ pole. In the meantime, the mass spectrum of the subsystem $\pi^+\pi^-\pi^-\pi^0$ possesses the second peak, while the $\pi^-\pi^0\pi^0\pi^0$ one does not. This is due to the presence of the strong energy dependent anomaly induced contribution $\rho^- \to \omega\pi^- \to \pi^+\pi^-\pi^-\pi^0$ in the decay $\phi \to \rho^-\pi^+ \to \pi^+\pi^+\pi^-\pi^-\pi^0$ followed by the necessary phase space kinematical cutoff. There is no anomaly induced enhancement in the decay $\phi \to \rho^-\pi^+ \to \pi^+\pi^-\pi^0\pi^0\pi^0$. The distributions over invariant mass of the remaining four-pion subsystems, $\sqrt{u_{1,2,3}}$ and $\sqrt{s'_5}$, where $s'_5 = (q - q_5)^2$ [26], in principle, can be obtained upon inserting the corresponding δ function into Eq. (5.2). In practice, however, this demands the complex rearrangements of the sequential integration bounds in Eq. (5.2) [26]. We do not make this task here. Instead, we restrict ourselves by drawing the qualitative conclusions that the mass spectra of the subsystems $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\pi^+\pi^-$ should look similar to ones shown in Fig. 4 and 5, respectively, because in the former, like in the plotted $\pi^+\pi^-\pi^-\pi^0$ one, there is also

TABLE II. The branching ratios of the decays $\omega(782) \to 5\pi$ and $\phi(1020) \to 5\pi$ evaluated in the model of generalized hidden local symmetry [21] added with the anomaly induced terms [9], at different masses of the a_1 resonance. The uncertainty of the central values due to the parameter dependence of the anomaly induced terms is set to $\pm 20\%$, see Ref. [18].

m_{a_1} [GeV]	$B_{\omega\to\pi^+\pi^-3\pi^0}(m_\omega^2)$	$B_{\omega\to 2\pi^+2\pi^-\pi^0}(m_\omega^2)$	$B_{\phi\to\pi^+\pi^-3\pi^0}(m_\phi^2)$	$B_{\phi\to 2\pi^+2\pi^-\pi^0}(m_\phi^2)$
1.09	4.2×10^{-9}	3.8×10^{-9}	4.4×10^{-7}	8.8×10^{-7}
1.23	4.1×10^{-9}	3.7×10^{-9}	3.9×10^{-7}	7.7×10^{-7}
no a_1	3.6×10^{-9}	3.3×10^{-9}	2.5×10^{-7}	5.0×10^{-7}

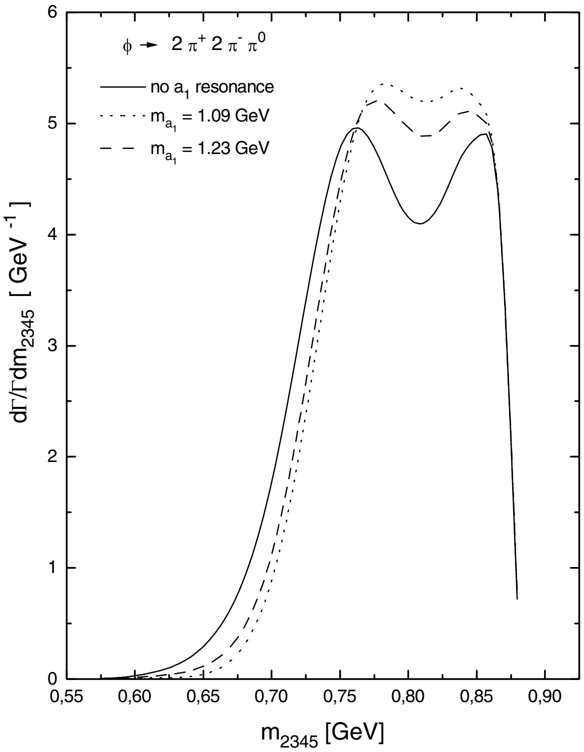

FIG. 4. The mass spectrum of the system $\pi^+_{q_2}\pi^-_{q_3}\pi^-_{q_4}\pi^0_{q_5}$ in the decay $\phi \to \pi^+_{q_1}\pi^+_{q_2}\pi^-_{q_3}\pi^-_{q_4}\pi^0_{q_5}$ normalized to the respective 5π decay width, and calculated at $\sqrt{s} = m_\phi$. The invariant mass squared is $m^2_{2345} = (q_2 + q_3 + q_4 + q_5)^2 \equiv s_1$.

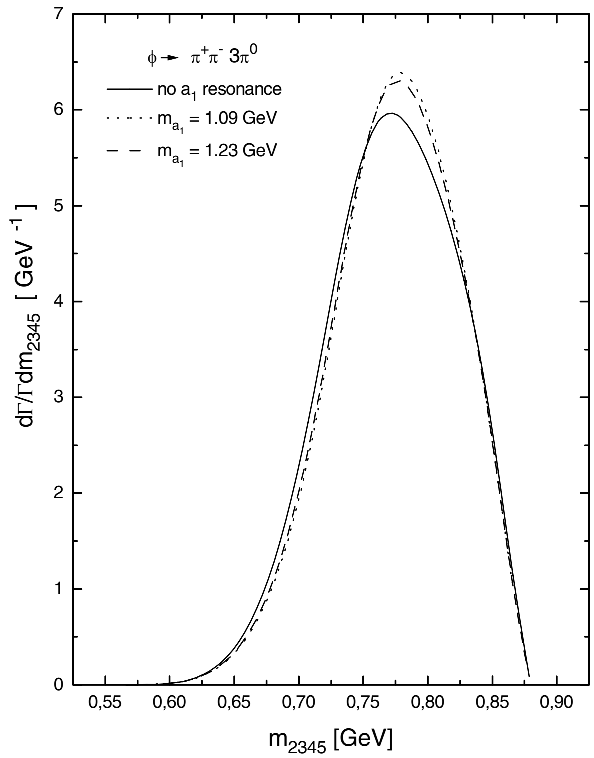

FIG. 5. The mass spectrum of the system $\pi^-_{q_2}\pi^0_{q_3}\pi^0_{q_4}\pi^0_{q_5}$ in the decay $\phi \to \pi^+_{q_1}\pi^-_{q_2}\pi^0_{q_3}\pi^0_{q_4}\pi^0_{q_5}$ normalized to the respective 5π decay width, and calculated at $\sqrt{s} = m_\phi$. The invariant mass squared is $m^2_{2345} = (q_2 + q_3 + q_4 + q_5)^2 \equiv s_1$.

the anomaly induced contribution $\rho^0 \to \omega\pi^0 \to \pi^+\pi^-\pi^0\pi^0$ while in the latter one there is no such contribution, see Ref. [17] for more detail. To summarize the discussion of the mass spectrum of the four-pion subsystem, we point out that, as is evident from Fig. 4 and 5, the greater part of the total number of events of the decay $\phi \to 5\pi$ should originate from the $\pm\frac{1}{2}\Gamma_\rho$ vicinity of the ρ peak in the process $\phi \to \rho\pi \to 5\pi$.

VI. DISCUSSION

Let us compare the part of our results concerning the widths of the decays $\rho^0 \to 2\pi^+2\pi^-$ and $\rho^0 \to \pi^+\pi^-2\pi^0$ with those of Ref. [16]. One can see that our calculation in cases when the a_1 resonance is taken into account, gives the partial widths which exceed those obtained in Ref. [16] by a factor ranging from 1.5 to 1.8, depending on the mass of the a_1 meson. In the meantime, our calculation gives the coinciding results in the model without a_1 meson. When making such a comparison, note, first, that here we take into account the mass difference of the charged and neutral pions both in matrix elements and the phase space volume, while the authors of Ref. [16] set all pion masses equal to the mass of $\pi^\pm$. Second, we fix $g_{\rho\pi\pi}$ from the $\rho \to \pi^+\pi^-$ width while in Ref. [16] it is fixed by Eq. (2.5). The mentioned difference between the results, in all appearance, could be attributed to the way of taking into account the contribution of the a_1 resonance. Indeed, as is discussed in Sec. III, there are different ways of taking into account the additional terms arising due to the diagonalization of the axial-pseudoscalar mixing. This could result in the terms similar to Eq. (3.6), which, in principle, could affect the specific value of the $\rho \to 4\pi$ width. Unfortunately, the authors of Ref. [16] did not give the necessary details to make the comparison and reveal the reason of the discussed discrepancy.

The KLOE collaboration at DAΦNE ϕ factory has collected the total number of events at $\sqrt{s} = m_\phi$ equivalent to the luminosity integral $\int \mathcal{L}dt \approx 500$ pb^{-1} [27]. Using the Table II, one can estimate the expected number of events $N_{\phi\to5\pi}$ of the decay $\phi \to 5\pi$ which already could be present in the whole KLOE statistics. One obtains $N_{\phi\to5\pi} \approx 1340$, 2070, 2360, respectively, in the HLS model without a_1 meson, in the GHLS model which incorporates the a_1 meson with the mass $m_{a_1} =$ 1.23 GeV, 1.09 GeV.

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