Transverse momentum distribution of Υ production in hadronic collisions

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We calculate the transverse momentum p_T distribution for production of the Υ states in hadronic reactions. For small $p_T(\leq M_Υ)$, we resum to all orders in the strong coupling α_s the process-independent large logarithmic contributions that arise from initial-state gluon showers. We demonstrate that the p_T distribution at low p_T is dominated by the region of small impact parameter b and that it may be computed reliably in perturbation theory. We express the cross section at large p_T by the $\mathcal{O}(\alpha_s^3)$ lowest-order nonvanishing perturbative contribution. Our results are consistent with data from the Fermilab Tevatron collider.

I. INTRODUCTION

The theoretical description of the transverse momentum p_T distribution of heavy quarkonium production in hadron collisions raises interesting challenges. Most calculations within the framework of perturbative quantum chromodynamics (QCD) consider the distribution at large p_T at collider energies and tend not to address the region of low p_T where the cross section is greatest and the bulk of the data lie [1–3]. A purely phenomenological fit to the low p_T data on Υ production [4] appears to require sizable nonperturbative parton-k_T smearing [5]. From a theoretical point of view, the region of low p_T is expected to be influenced strongly by initial-state gluon showering. A fixed-order perturbative treatment in QCD leads to singular terms in the region of small p_T of the type $1/p_T^2$, enhanced by large higher-order logarithmic contributions caused by initial-state gluon radiation. These contributions have the form $\alpha_s \log^2(M_Υ^2/p_T^2)$ for every power of the strong coupling α_s, and reliable predictions, especially in the regions of small and moderate p_T, require that the logarithmic contributions be summed to all orders in α_s.

The Collins-Soper-Sterman (CSS) impact parameter b-space resummation formalism [6] has been used successfully for the all-orders resummation of large initial-state logarithmic terms in several cases of physical interest [7–14]. In this paper, we argue and demonstrate that the resummation formalism should apply at the scale of the Υ mass in hadronic collisions at collider energies. We extend the formalism and use it to compute the p_T distribution of the Υ states [15]. We obtain good agreement with the data [4,16] on the p_T distribution of Υ production at Tevatron energies for all p_T.

Different from the production of the W, Z, and Higgs bosons, or of a virtual photon in the Drell-Yan process, the Υ is unlikely to be produced in pointlike fashion in a short-distance hard collision. Instead, a bottom quark $b\bar{b}$ pair is produced in the hard collision and then transmutes into a colorless Υ meson through soft radiation and coherent self-interaction. Therefore, there are issues to address before the CSS formalism can be applied to Υ production. These include the color structure of the lowest order production processes: $q\bar{q} \rightarrow b\bar{b}(Q)$ and $gg \rightarrow b\bar{b}(Q)$, and the relatively small value of the $b\bar{b}$ pair mass Q.

Most applications of the resummation formalism are to the production of systems that are color singlets whereas the $b\bar{b}$ system produced in $q\bar{q} \rightarrow b\bar{b}$ and $gg \rightarrow b\bar{b}$ need not be color neutral. Nevertheless, because the b quark mass is large, gluon radiation is suppressed from the final-state heavy quark lines and from virtual exchange lines that lead to the production of heavy quark pairs [10]. Correspondingly, the important logarithmic terms are associated with gluon radiation from the active initial-state partons, the same as those in massive-lepton-pair (Drell-Yan) and Higgs boson production. The process-independent leading logarithmic terms do not depend on the color of the heavy quark pair. Color dependence becomes relevant for the higher order terms, as explained in Sec. IV.

The overall center-of-mass energy $\sqrt{S}$ dependence of the CSS b-space distribution function is examined by Qiu and Zhang in Ref. [8]. They show that the location of the saddle point of this distribution can be well within the perturbative region of small b for Q as small as 6 GeV at the Tevatron collider energy. The resummed b-space distribution is peaked strongly in the perturbative region of small b, as we show in Sec. IV, and the p_T distribution of Υ production should be amenable to a resummation treatment. Despite the fact that the logarithmic term $\ln Q$ is not large, the large value of $\sqrt{S}$ opens a large region of phase space for gluon emission. Correspondingly, as is demonstrated in this paper, the shape of the p_T distribution for Υ production is determined by the resummable part of the gluon shower and is predictable quantitatively at low p_T.

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We begin in Sec. II with the basic assumption that the p_T distribution of Υ production is derived from the p_T distribution for the production of a pair $b\bar{b}$ of bottom quarks. We express the differential cross section in terms of a two-step factorization procedure. We present our fixed-order perturbative calculation applicable at large transverse momentum in Sec. III where we also describe models that specify the manner in which the $b\bar{b}$ pair transforms into the Υ. In Sec. IV, we specialize to the situation at small p_T and summarize the required parts of the all-orders resummation formalism. Section V is devoted to our numerical results and comparison with data. We provide of our conclusions and discuss potential improvements of our calculation in Sec. VI.

II. PRODUCTION DYNAMICS

We use a two-step factorization procedure to represent production of the Υ states, with particular attention to the prediction of transverse momentum distributions. We begin with the assumption that a pair of bottom quarks $b\bar{b}$ is produced in a hard-scattering short-distance process:

$$A(p_A) + B(p_B) \rightarrow b\bar{b}(Q)[\rightarrow \Upsilon(p) + \bar{X}] + X'. \quad (1)$$

Because the mass Q of the $b\bar{b}$ pair is large, the pair is produced at a distance scale $\sim 1/(2m_b) \sim 1/45$ fm. This scale is much smaller than the physical size of a Υ meson. The compact $b\bar{b}$ pair may represent the minimal Fock state of the Υ, but the overlap of this minimal Fock state with the full wave function of the Υ is perhaps small, as is suggested by the inadequacies of the color-singlet approach [17] in some situations [3], and other components of the wave function must be considered. Alternatively, one may realize that the compact $b\bar{b}$ system is unlikely to become an Υ meson at the production point. Instead, the pair must expand, and the b and $\bar{b}$ will interact with each other coherently until they transmute into a physical Υ meson.

Once produced in the hard-scattering, a $b\bar{b}$ pair of invariant mass $Q > 2M_B$ is more likely to become a pair of B mesons. Therefore, the virtuality of the intermediate $b\bar{b}$ pair should be limited if an Υ is to result. This limitation of the virtuality allows us to use perturbative factorization and to write the differential cross section in the usual way [18,19]. For $p_T \gg 2(M_B - m_b)$, we write

$$\frac{d\sigma_{AB\rightarrow\Upsilon X}}{dp_T^2 dy} = \sum_{a,b} \int dx_a \phi_{a/A}(x_a) dx_b \phi_{b/B}(x_b) \frac{d\hat{\sigma}_{ab\rightarrow\Upsilon X}}{dp_T^2 dy}.$$
$$(2)$$

In Eq. (2), p_T and y are the transverse momentum and rapidity of the final Υ. The functions $\phi_i(x)$ are parton distribution functions; x_a and x_b are fractional light-cone momenta carried by the incident partons; and Eq. (2) expresses initial-state collinear factorization. The spectator interactions between the beam remnants and the formation of the Υ meson are suppressed by one or more powers of $1/p_T^2$.

Since the momentum of heavy quark b ($\bar{b}$) in the pair's rest frame is much less than the mass of the pair, $Q - 2m_b < 2M_B - 2m_b \ll 2m_b$, the parton-level production cross section $d\hat{\sigma}_{ab\rightarrow\Upsilon X}/dp_T^2 dy$ in Eq. (2) might be factored further [19], as is sketched in Fig. 1. The incident partons labeled x_a and x_b interact inclusively to produce an off-shell $b\bar{b}$ system plus state X'. In turn, the $b\bar{b}$ system evolves into the Υ plus a system labeled $\bar{X}$; $X = X' + \bar{X}$. This second factored expression is

$$\frac{d\hat{\sigma}_{ab\rightarrow\Upsilon X}}{dp_T^2 dy} \approx \sum_{[b\bar{b}]} \int dQ^2 \left[\frac{d\hat{\sigma}_{ab\rightarrow[b\bar{b}](Q)X'}}{dQ^2 dp_T^2 dy} \right] \mathcal{F}_{[b\bar{b}]\rightarrow\Upsilon\bar{X}}(Q^2).$$
$$(3)$$

In writing Eq. (3), we approximate the transverse momentum and rapidity of the $b\bar{b}$ pair by p_T and y, respectively, because $Q^2 - 4m_b^2 \ll p_T^2$.

The function $d\hat{\sigma}_{ab\rightarrow[b\bar{b}](Q)X'}/dQ^2 dp_T^2 dy$ represents a partonic short-distance hard-part for inclusive production of a $b\bar{b}$ pair of invariant mass Q and quantum numbers $[b\bar{b}]$. This short-distance hard-part is calculable in perturbation theory with the parton momenta of all light partons off-mass-shell by at least $\min(4m_b^2, p_T^2)$. The function $\mathcal{F}_{[b\bar{b}]\rightarrow\Upsilon\bar{X}}(Q^2)$ represents a transition probability distribution for a $b\bar{b}$ pair of invariant mass Q and quantum numbers $[b\bar{b}]$ to transmute into an Υ meson. It includes all dynamical $b\bar{b}$ interactions of momentum scale less than $Q^2 - 4m_b^2$. Different assumptions and choices for the transition probability distribution $\mathcal{F}(Q^2)$ lead to different models of quarkonium production. We return to the topic of these models in Sec. III.

The basic assumptions of this section imply that the transverse momentum distributions of the Υ states at transverse momenta $p_T \sim M_\Upsilon$ will reflect the shape of the transverse momentum distribution for production of a $b\bar{b}$ pair whose mass $Q \sim M_\Upsilon$. In this paper, we focus on the region below $p_T \sim M_\Upsilon$. If $p_T^2 \gg Q^2$, all final-state loga-
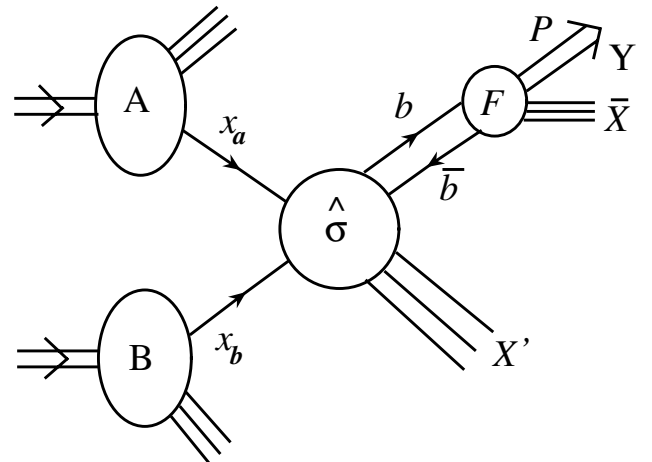

FIG. 1. Hadronic production of an Υ via an intermediate heavy quark pair b and $\bar{b}$.

rithmic terms of the form $[\alpha_s \log(p_T^2/Q^2)]^N$ can be resummed perturbatively to all orders in α_s [20].

III. FIXED-ORDER CALCULATION: $p_T \sim M_\Upsilon$

When the transverse momentum $p_T \sim \mathcal{O}(M_\Upsilon)$, the collinear factorized expression in Eqs. (2) and (3) remains reliable with the partonic short-distance hard-parts in Eq. (3) computed as a power series in α_s in QCD perturbation theory.

The transition probability distribution $\mathcal{F}(Q^2)$ is introduced in Sec. II. Different assumptions and choices for $\mathcal{F}(Q^2)$ correspond to different models of quarkonium production. In the color evaporation (or color-bleaching) model (CEM) [2,21], an assumption is made, based qualitatively on semilocal duality, that one may safely ignore the details of the formation of color-neutral bound states with specific quantum numbers J^{PC}. In particular, in the case of states such as the J/ψ and Υ that have $J^{PC} = 1^{--}$, soft-gluon effects are presumed to take care of whatever quantum numbers have to be arranged. Within our framework, this model is effectively represented by the statement that

$$\mathcal{F}_{[b\bar{b}] \to \Upsilon}(Q^2) = \begin{cases} C_\Upsilon & \text{if } 4m_b^2 \le Q^2 \le 4M_B^2 \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

The nonperturbative constant C_Υ sets the overall normalization of the cross section. Its value cannot be predicted. It changes with the specific state of the Υ meson. In the CEM model, the parton-level Υ cross section in Eq. (3) can be written as

$$\frac{d\hat{\sigma}_{ab \to \Upsilon X}^{\text{CEM}}}{dp_T^2 dy} \approx C_\Upsilon \int_{4m_b^2}^{4M_B^2} dQ^2 \left[\frac{d\hat{\sigma}_{ab \to b\bar{b}(Q)}}{dQ^2 dp_T^2 dy} \right], \quad (5)$$

where the $b\bar{b}$ final state includes a sum over all possible quantum states $[b\bar{b}]$ of the $b\bar{b}$ pair.

In the color-singlet model for quarkonium production [17], a projection operator is used to place the $b\bar{b}$ system in the spin state of the Υ, and explicit gluon radiation guarantees charge conjugation (C) and color conservation at the level of the hard-scattering amplitude. The distribution $\mathcal{F}_{[b\bar{b}] \to \Upsilon}(Q^2)$ is proportional to the square of the momentum-space wave function of the Υ, $|\tilde{\Psi}(q)|^2$, with the relative momentum of the $b\bar{b}$ pair $q^2 = Q^2 - 4m_b^2$.

Because the Υ wave function falls steeply, one can approximate $Q^2 \approx 4m_b^2$ in the $b\bar{b}$ partonic cross section. The integration $\int dQ^2 \mathcal{F}_{[b\bar{b}] \to \Upsilon}(Q^2)$ in Eq. (3) leads to the square of the Υ wave function at the origin $|\Psi(0)|^2$.

The nonrelativistic QCD model (NRQCD) [1,22] takes into consideration that the velocity of the heavy quark b ($\bar{b}$) in the rest frame of the $b\bar{b}$ pair is much less than the speed of light. The velocity expansion translates into statements that the distribution $\mathcal{F}(Q^2)$ is a steeply falling function of the relative heavy quark momentum, $q^2 \equiv Q^2 - 4m_b^2$, and that its moments satisfy the inequalities

$$\langle (q^2)^N \rangle \equiv \int dQ^2 (q^2)^N \mathcal{F}_{[b\bar{b}] \to \Upsilon}(Q^2) \ll (4m_b^2)^N, \quad (6)$$

for moments $N \ge 1$. Correspondingly, one can expand the partonic hard-part in Eq. (3) at $Q^2 = (2m_b)^2$ and obtain

$$\frac{d\hat{\sigma}_{ab \to \Upsilon X}^{\text{NRQCD}}}{dp_T^2 dy} \approx \sum_{[b\bar{b}]} \left[\frac{d\hat{\sigma}_{ab \to [b\bar{b}](Q)}}{dQ^2 dp_T^2 dy} (Q^2 = M_\Upsilon^2) \right]$$
$$\times \int dQ^2 \mathcal{F}_{[b\bar{b}] \to \Upsilon}(Q^2) + \mathcal{O}\left(\frac{\langle q^2 \rangle}{M_\Upsilon^2} \right), \quad (7)$$

with $m_b = M_\Upsilon/2$. The integral $\int dQ^2 \mathcal{F}_{[b\bar{b}] \to \Upsilon}(Q^2) \equiv \langle \hat{O}_{[b\bar{b}]}(0) \rangle$ corresponds to a local matrix element of the $b\bar{b}$ pair in the NRQCD model. In the NRQCD approach to heavy quarkonium production, the $b\bar{b}$ pair need not have the quantum numbers of the Υ. It is assumed that nonperturbative soft gluons take care of the spin and color of the Υ. The sum in Eq. (7) runs over all spin and color states of the $b\bar{b}$ system.

For the purpose of calculating the inclusive p_T distributions of S-wave bound states at large enough p_T, both the CEM in Eq. (5) and the leading order NRQCD approach in Eq. (7) are expected to yield distributions similar in shape because of the relatively weak Q^2 dependence of the partonic hard-part in the limited range of Q^2. For example, p_T distributions of J/ψ and ψ' production at Tevatron energies are consistent with both CEM [2] and NRQCD [3] calculations for $p_T \gtrsim 5$ GeV. For production of $\Upsilon(nS)$ states, we choose a $\mathcal{F}(Q^2)$ that covers both the CEM and main properties of the leading order NRQCD treatment of heavy quarkonium production [23]

$$\mathcal{F}_{[b\bar{b}] \to \Upsilon(nS)}(Q^2) = \begin{cases} C_{\Upsilon(nS)}(1-z)^{\alpha_{\Upsilon(nS)}} & \text{if } M_{\Upsilon(nS)}^2 \le Q^2 \le 4M_B^2 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

with $z = (Q^2 - M_{\Upsilon(nS)}^2)/(4M_B^2 - M_{\Upsilon(nS)}^2)$. In Eq. (8), $C_{\Upsilon(nS)}$ and $\alpha_{\Upsilon(nS)}$ are parameters determined from data as discussed in Sec. V. With the choice of $\mathcal{F}(Q^2)$ in Eq. (8), we reproduce the CEM by setting $\alpha_{\Upsilon(nS)} = 0$ and replacing the lower limit $M_{\Upsilon(nS)}^2$ by $4m_b^2$. Other than the color degree of freedom, we could mimic the features of NRQCD by choosing a very large value for $\alpha_{\Upsilon(nS)}$.

With our choice of $\mathcal{F}(Q^2)$, the transverse momentum distribution of Υ production becomes

$$\frac{d\sigma_{AB \to \Upsilon(nS)X}}{dp_T^2 dy} = C_{\Upsilon(nS)} \int_{M_{\Upsilon(nS)}^2}^{4M_B^2} dQ^2 \left[\frac{d\sigma_{AB \to b\bar{b}(Q)X}}{dQ^2 dp_T^2 dy} \right]$$
$$\times \left(1 - \frac{Q^2 - M_{\Upsilon(nS)}^2}{4M_B^2 - M_{\Upsilon(nS)}^2} \right)^{\alpha_{\Upsilon(nS)}}. \quad (9)$$

The $b\bar{b}$ cross section is factored in terms of parton densities and the partonic cross section as

$$\frac{d\sigma_{AB\to b\bar{b}(Q)X}}{dQ^2dp_T^2dy} = \sum_{a,b} \int dx_a \phi_{a/A}(x_a)dx_b \phi_{b/B}(x_b)$$
$$\times \frac{d\hat{\sigma}_{ab\to b\bar{b}(Q)X}}{dQ^2dp_T^2dy}. \qquad (10)$$

The sum $\sum_{a,b}$ runs over gluon and light quark flavors up to and including charm. The partonic cross sections, $d\hat{\sigma}_{ab\to b\bar{b}(Q)X}/dQ^2dp_T^2dy$, are computed at $\mathcal{O}(\alpha_s^3)$ from all 2-parton to 3-parton Feynman diagrams for the subprocesses $q\bar{q} \to b\bar{b}g$, $qg \to b\bar{b}q$, and $gg \to b\bar{b}g$, with the squared amplitudes summed over the spins and colors of the $b\bar{b}$ pair [24].

IV. THE REGION OF SMALL TRANSVERSE MOMENTUM

When p_T (or Q_T of the $b\bar{b}$ pair) becomes small, the perturbatively calculated hard-part $d\hat{\sigma}_{ab\to[b\bar{b}](Q)X'}/dQ^2dp_T^2dy$ in Eq. (3) becomes singular

$$\frac{d\hat{\sigma}_{ab\to[b\bar{b}](Q)X'}}{dQ^2dp_T^2dy} \propto \frac{1}{p_T^2}. \qquad (11)$$

The $1/p_T^2$ singularity arises from the collinear region of initial-state parton splitting. Gluon radiation from the final-state heavy quark lines does not contribute a $1/p_T^2$ collinear singularity because the heavy quark mass regulates this singularity. However, this gluon radiation does lead to a $1/p_T^2$ infrared divergence which should be absorbed into the nonlocal transition probability distribution $\mathcal{F}(Q^2)$ [19]. When $p_T^2 \ll q^2 = Q^2 - 4m_b^2$, soft-gluon interactions between the spectator partons in the beam jets and the partons in $\mathcal{F}(Q^2)$ most likely break the factorization expressed in Eqs. (2) and (3). In this paper, our principal interest is to investigate how the large logarithmic terms from the initial-state gluon shower modify the $1/p_T^2$ distribution when $p_T^2 \ll M_\Upsilon^2$.

A. Resummation of Sudakov logarithms in b-space

Additional gluon radiation from the initial-state partons, recoiling against the $b\bar{b}$ pair as shown in Fig. 2, leads to (Sudakov) logarithmic contributions of the form $\alpha_s\log^2(Q^2/p_T^2)$ for each gluon radiation [25]. The effects of the large Sudakov logarithmic contributions, very important in the region of small p_T, can be resummed to all orders in α_s when $p_T \ll Q$ [26]. The resummation procedure tames the divergence seen in Eq. (11). Adopting the Collins, Soper, and Sterman (CSS) impact-parameter b-space (Fourier conjugate to p_T) approach [6], we write the resummed transverse momentum distribution for $b\bar{b}$ production as
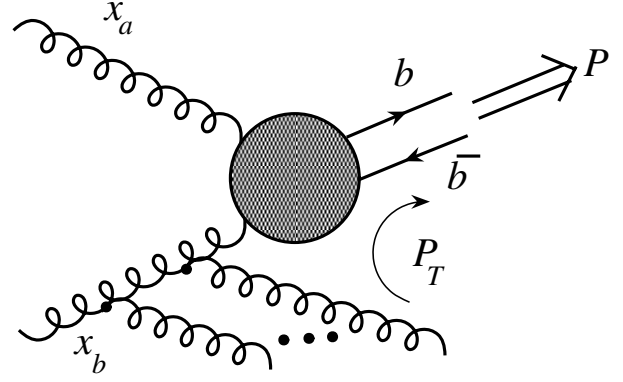

FIG. 2. Diagram that illustrates multiple gluon radiation from an initial-state parton.

$$\frac{d\sigma_{AB\to[b\bar{b}](Q)X'}^{\text{resum}}}{dQ^2dp_T^2dy} = \frac{1}{(2\pi)^2}$$
$$\times \int d^2b \, e^{i\vec{p}_T \cdot \vec{b}} \mathcal{W}_{AB\to[b\bar{b}](Q)}(b,Q,x_A,x_B)$$
$$= \int \frac{db}{2\pi} J_0(p_T b) b \, \mathcal{W}_{AB\to[b\bar{b}](Q)}(b,Q,x_A,x_B). \qquad (12)$$

The function $\mathcal{W}_{AB\to[b\bar{b}](Q)}(b,Q,x_A,x_B)$ resums to all orders in QCD perturbation theory the singular terms from initial-state gluon showers that otherwise behave as $\delta^2(p_T)$ and $(1/p_T^2)\log^m(Q^2/p_T^2)$ for all $m \geq 0$. In Eq. (12), the fractional partonic momenta are $x_A = \frac{Q}{\sqrt{S}}e^y$ and $x_B = \frac{Q}{\sqrt{S}}e^{-y}$, with $\sqrt{S}$ the overall center-of-mass collision energy, and y the rapidity of the $b\bar{b}$ pair; x_A and x_B are independent of the transverse momentum p_T of the pair. The entire dependence on p_T appears in the argument of the Bessel function J_0.

The expressions for the lowest-order subprocesses $gg \to b\bar{b}$ and $q\bar{q} \to b\bar{b}$ are independent of p_T. Therefore, the finite lowest-order partonic cross sections can be used as prefactors in the overall b-space distribution functions [25]. We write

$$\mathcal{W}_{AB\to[b\bar{b}](Q)}(b,Q,x_A,x_B) \equiv \sum_q W_{q\bar{q}}(b,Q,x_A,x_B)$$
$$\times \frac{d\hat{\sigma}_{q\bar{q}\to[b\bar{b}](Q)}^{(\text{LO})}}{dQ^2}$$
$$+ W_{gg}(b,Q,x_A,x_B)$$
$$\times \frac{d\hat{\sigma}_{gg\to[b\bar{b}](Q)}^{(\text{LO})}}{dQ^2}. \qquad (13)$$

The sum $\sum_q$ runs over all flavors of light quarks in the initial state. The lowest-order partonic cross sections in

Eq. (13), $d\hat{\sigma}^{(\text{LO})}_{ij\to[b\bar{b}]}(Q)/dQ^2$ with $ij = q\bar{q}, gg$, depend on the choice of the production model. For our choice of $\mathcal{F}_{b\bar{b}\to\Upsilon}(Q^2)$, they are

$$\frac{d\hat{\sigma}^{(\text{LO})}_{ij\to b\bar{b}}(Q)}{dQ^2} = \sum_{[b\bar{b}]}\frac{d\hat{\sigma}^{(\text{LO})}_{ij\to[b\bar{b}]}(Q)}{dQ^2} = \frac{x_a x_b}{Q^2}\hat{\sigma}^{(\text{LO})}_{ij}(Q^2) \quad (14)$$

with [21]

$$\hat{\sigma}^{(\text{LO})}_{q\bar{q}}(Q^2) = \frac{2}{9}\frac{4\pi\alpha_s^2}{3Q^2}\left[1 + \frac{1}{2}\gamma\right]\sqrt{1-\gamma};$$

$$\hat{\sigma}^{(\text{LO})}_{gg}(Q^2) = \frac{\pi\alpha_s^2}{3Q^2}\left[\left(1 + \gamma + \frac{1}{16}\gamma^2\right)\ln\left(\frac{1+\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}}\right)\right.$$
$$\left. - \left(\frac{7}{4} + \frac{31}{16}\gamma\right)\sqrt{1-\gamma}\right]; \quad (15)$$

and $\gamma = 4m_b^2/Q^2$.

When the impact parameter b lies in the region much less than 1 GeV^{-1} where perturbation theory applies, the distributions $W_{q\bar{q}}(b, Q, x_A, x_B)$ and $W_{gg}(b, Q, x_A, x_B)$ in Eq. (13) can be expressed as [6]

$$W^{\text{pert}}_{ij}(b, Q, x_A, x_B) = e^{-S_{ij}(b,Q)}f_{i/A}\left(x_A, \mu, \frac{c}{b}\right)$$
$$\times f_{j/B}\left(x_B, \mu, \frac{c}{b}\right)H_{ij}, \quad (16)$$

where $ij = q\bar{q}$ and gg, μ is the factorization scale, and $c = 2e^{-\gamma_E} = \mathcal{O}(1)$, with Euler's constant $\gamma_E \approx 0.577$. All large Sudakov logarithmic terms from $\log(c^2/b^2)$ to $\log(Q^2)$ are resummed to all orders in α_s in the exponential factors with

$$S_{ij}(b, Q) = \int_{c^2/b^2}^{Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right)\mathcal{A}_{ij}[\alpha_s(\bar{\mu})]\right.$$
$$\left. + \mathcal{B}_{ij}[\alpha_s(\bar{\mu})]\right]. \quad (17)$$

The functions $\mathcal{A}_{ij}$ and $\mathcal{B}_{ij}$ may be expanded in perturbative power series in α_s; $\mathcal{A}_{ij} = \sum_{n=1}\mathcal{A}^{(n)}_{ij}(\alpha_s/\pi)^n$, and $\mathcal{B}_{ij} = \sum_{n=1}\mathcal{B}^{(n)}_{ij}(\alpha_s/\pi)^n$. The first two coefficients in the power series for $\mathcal{A}_{ij}$ and the first term in the series for $\mathcal{B}_{ij}$ are *process-independent*. For $ij = q\bar{q}$ they are the same as those that appear in resummation of the transverse momentum distribution for massive-lepton-pair production (Drell-Yan production) [6–8]. For $ij = gg$, they are the same as the coefficients that are appropriate for resummation of the p_T distribution of Higgs boson production [11–14].

The modified parton distribution functions in Eq. (16) are expressed as [6]

$$f_{i/A}\left(x_A, \mu, \frac{c}{b}\right) = \sum_a\int_{x_A}^1\frac{d\xi}{\xi}\phi_{a/A}(\xi, \mu)C_{a\to i}\left(\frac{x_A}{\xi}, \mu, \frac{c}{b}\right). \quad (18)$$

In Eq. (18), $\phi_{a/A}$ are the usual parton distribution func-

tions. The functions $C_{a\to i} = \sum_{n=0}C^{(n)}_{a\to i}(\alpha_s/\pi)^n$ are b-space coefficient functions with the lowest-order terms normalized to $C^{(0)}_{a\to i}(z, \mu, c/b) = \delta_{ai}\delta(1 - z)$. All higher-order coefficient functions are computed perturbatively from the Fourier transform of the singular terms in p_T-space from initial-state gluon showers with $Q^2 = c^2/b^2$.

In the CSS formalism, $H_{ij} = 1$ in Eq. (16). All coefficient functions, $C^{(n)}_{a\to i}$ with $n \geq 1$, are *process-dependent* representing the nonlogarithmic short-distance partonic contributions to σ^{resum}. Alternatively, one may be able to reorganize Eq. (16) such that all *process-dependent* short-distance contributions are moved into the *process-dependent* hard-part H_{ij}, leaving the coefficient functions $C_{a\to i}$ and the modified parton distributions *process-independent* [12]. Expressions for H^n_{ij} with $n \geq 1$ depend on the "resummation scheme," the *choices* made when the *process-dependent* finite pieces are moved from the higher-order terms in the $\mathcal{A}$'s, $\mathcal{B}$'s, and C's to the H_{ij} functions [12].

For production of a colorless object, such as the W, Z, and Higgs bosons or a virtual photon in the Drell-Yan process, all resummed $(1/p_T^2)\log^m(Q^2/p_T^2)$ singular terms arise from initial-state gluon showers. For the Υ, which is not produced directly in the hard collision, additional singular $1/p_T^2$ terms can originate from soft-gluon radiation from the $b\bar{b}$ pair. These additional $1/p_T^2$ terms should not be included in the resummation of Sudakov logarithms from initial-state gluon showers. The calculation of the $n \geq 1$ corrections to the coefficient functions $C^{(n)}_{a\to i}$ and hard-part $H^{(n)}_{ij}$ (or $C^{(n)}_{a\to i}$ in the CSS formalism) should involve a systematic removal of these additional $1/p_T^2$ singular terms.

The long-distance nature of soft-gluon radiation means that the additional singular terms from final-state radiation should be included in the nonperturbative transformation of the $b\bar{b}$ pair to the Υ. Therefore, the removal of the additional $1/p_T^2$ terms depends on the models of Υ production. In terms of the two-step factorization procedure discussed in Sec. II, the $1/p_T^2$ terms should be absorbed into the transition probability distributions $\mathcal{F}_{[b\bar{b}]\to\Upsilon X}$ defined in terms of matrix elements of nonlocal operators. In the NRQCD model of heavy quarkonium production, these $1/p_T^2$ singularities are a consequence of the kinematic end point of the quarkonium transverse momentum spectrum. Although the kinematic effect of soft-gluon emission from the heavy quark pair is usually a higher-order effect in the nonrelativistic expansion, the high order nonperturbative contributions are enhanced in the region of the kinematic end point as $p_T \to 0$, leading to a breakdown of the NRQCD expansion and the introduction of "shape functions" [27,28].

Faced with the model dependence of the α_s corrections and the complication of separating singular terms of differ-

ent origins, we resum only *process-independent* logarithmic terms from initial-state gluon showers in this paper. That is, we keep only $\mathcal{A}_{q,g}^{(1)}$, $\mathcal{A}_{q,g}^{(2)}$, and $\mathcal{B}_{q,g}^{(1)}$ in the Sudakov exponential functions $S_{q,g}(b, Q)$ in Eq. (17), the lowest-order coefficient function $C_{a\to i}^{(0)}$, and the lowest-order short-distance hard-parts $H_{ij}^{(0)} = 1$ in Eq. (16). We choose the factorization scale $\mu = c/b$ for the resummed b-space distribution in Eq. (16).[1] We defer to a future study the calculation of *process- and model-dependent* higher-order corrections $C_{a\to i}^{(1)}$ and $H_{ij}^{(1)}$ as well as $\mathcal{A}^{(3)}$ and $\mathcal{B}^{(2)}$. As a result of these restrictions, we anticipate that our calculation will somewhat underestimate the magnitude of the differential cross section in the region of small p_T, and we return to this point in next section.

B. Predictive power

The predictive power of the Fourier transformed formalism in Eq. (12) depends critically on the shape of the b-space distribution function $b\mathcal{W}(b, Q, x_A, x_B)$ [8]. Indeed, the resummed calculation of the transverse mo-

$$W_{ij}(b, Q, x_A, x_B) = \begin{cases} W_{ij}^{\text{pert}}(b, Q, x_A, x_B) & b \le b_{\max} \\ W_{ij}^{\text{pert}}(b_{\max}, Q, x_A, x_B)F_{ij}^{NP}(b, Q; b_{\max}) & b > b_{\max} \end{cases} \quad (19)$$

for $ij = q\bar{q}$ and gg. The perturbative distribution $W_{ij}^{\text{pert}}(b, Q, x_A, x_B)$ is given in Eq. (16). The nonperturbative function in the large b region is

$$F_{ij}^{NP} = \exp\left\{-\ln\left(\frac{Q^2 b_{\max}^2}{c^2}\right)\{g_1[(b^2)^\alpha - (b_{\max}^2)^\alpha] + g_2(b^2 - b_{\max}^2)\} - \bar{g}_2(b^2 - b_{\max}^2)\right\}. \quad (20)$$

The $(b^2)^\alpha$ term with $\alpha < 1/2$ represents a direct extrapolation of the resummed function $W_{ij}^{\text{pert}}(b, Q, x_A, x_B)$. The parameters, g_1 and α are fixed from W_{ij}^{pert} if we require that the first and second derivatives of $W_{ij}(b, Q, x_A, x_B)$ be continuous at $b = b_{\max}$. The x_A and x_B dependences of the nonperturbative function F_{ij}^{NP} are included in the parameters g_1 and α.

The two terms proportional to b^2 correspond to power corrections in the evolution equation. The g_2 term represents a power correction from soft-gluon showers. The $\bar{g}_2$ term is associated with the finite intrinsic transverse momentum of the incident partons. Since g_1 and α are fixed by the continuity of the $W_{ij}(b, Q, x_A, x_B)$ at $b = b_{\max}$, the (in)sensitivity of the calculated p_T distribution to the numerical values of $b_{\max}$, g_2, and $\bar{g}_2$ is a good quantitative measure of the predictive power of the resummmation formalism [8].

---

[1]Since there is only one momentum scale, $1/b$, involved in the functions $W_{ij}(b, c/b, x_A, x_B)$, it is natural to choose $\mu = c/b$ in the modified parton densities $f(x, \mu, c/b)$ in order to remove the logarithmic terms in the coefficient functions, C. However, this choice is not required. For more discussion, see Ref. [13].

mentum distribution at low p_T can be reliable *only if* the Fourier transformation in Eq. (12) is dominated by the region of small b, where perturbation theory applies, and is not sensitive to the extrapolation to the region of large b. This condition is achieved if the distribution $b\mathcal{W}(b, Q, x_A, x_B)$ has a prominent saddle point for $b_{\text{sp}} \ll 1 \text{ GeV}^{-1}$.

The location of the saddle point in the b-space distribution depends not only on the value of Q but also strongly on the collision energy $\sqrt{S}$ (or, equivalently, on the values of the parton momentum fractions x_a and x_b that control the cross section) [8]. At $\sqrt{S} = 1.8$ TeV, the saddle point can be within the perturbative region ($b_{\text{sp}} < 0.5 \text{ GeV}^{-1}$) for Q as low as 6 GeV [8].

An extrapolation into the region of large b is needed in order for us to perform the Fourier transformation to the p_T distribution in Eq. (12). We choose the Qiu-Zhang prescription which has the desirable property that it separates cleanly the perturbative prediction at small b from nonperturbative contributions in the large b region.

In Fig. 3, we show the b-space distributions that result from Eq. (19) at rapidity $y = 0$. The functions are integrated over the mass range $2m_b < Q < 2M_B$. In evaluating the perturbative distribution, we keep only the *process-independent* terms in the Sudakov exponential functions $S_{q,g}(b, Q)$ in Eq. (17) and use the CTEQ6M parton densities [29]. For the extrapolation to the large b region, we choose $b_{\max} = 0.5 \text{ GeV}^{-1}$ and $g_2 = \bar{g}_2 = 0$. The magnitude of $bW_{gg}(b, Q, x_A, x_B)$ is scaled by a factor of 20, as is indicated in the figure. The gg contribution far exceeds the $q\bar{q}$ contribution to Υ production at Tevatron energies. For both channels, the saddle points are clearly defined and have numerical values well within the perturbative region.
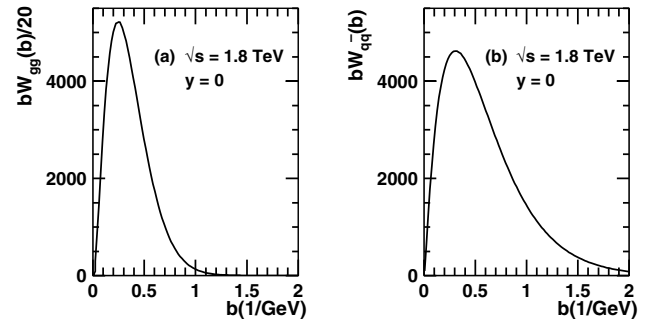

FIG. 3. The b-space distributions for Υ production: (a) gg channel, and (b) *the sum of all $q\bar{q}$ channels*. Note that the magnitude of the gg distribution has been scaled by a factor of 20. The functions are evaluated at rapidity $y = 0$ and are integrated over the mass range $2m_b < Q < 2M_B$.

For the dominant gg channel, the location of the saddle point is $b_{\rm sp} \sim 0.25$ GeV^{-1}. For perspective, we remark that this value is smaller than that for the saddle point of W and Z boson production at Tevatron energies [8]. This feature arises because the gluon anomalous dimension at small x is much larger than that of the quarks, compensating for the fact that the mass Q here is much less than the mass of W and Z bosons. This analysis leads us to expect that the QCD resummed p_T distribution of Υ production in Eq. (12) can be predicted reliably in the region of small and intermediate p_T because it is dominated by perturbative contributions in the region of small b.

V. NUMERICAL RESULTS

In this section we present the results of our numerical computation, including a comparison with data. We compute the Υ transverse momentum distribution from Eq. (9). For the region of large transverse momentum, $p_T \sim \mathcal{O}(M_\Upsilon)$, the $b\bar{b}$ cross section is given in Eq. (10). For the region of small p_T, we use the all-orders resummed $b\bar{b}$ cross section in Eq. (12) with the b-space distribution $\mathcal{W}_{AB \to b\bar{b}}$ specified in Eqs. (13) and (19). We set $m_b = 4.5$ GeV, and we use a two-loop expression for α_s, in keeping with our use of the CTEQ6M parton densities [29].

To distinguish the production of $\Upsilon(nS)$ states with different n, we choose different powers $\alpha_{\Upsilon(nS)}$ and normalization constants $C_{\Upsilon(nS)}$, in addition to the differences in mass threshold on the limits of the dQ^2 integration in Eq. (9). The values of $\alpha_{\Upsilon(nS)}$ and $C_{\Upsilon(nS)}$ are correlated. A larger value of $\alpha_{\Upsilon(nS)}$ leads to a larger value of $C_{\Upsilon(nS)}$.

A. Matching of results at small and large p_T

In a complete calculation, one would expect a seamless joining of the results applicable at small and at large p_T. In the CSS resummation formalism for production of a color-singlet heavy boson, this matching is accomplished through the introduction of an "asymptotic" term, $\sigma^{\rm asym}$, and

$$d\sigma = d\sigma^{\rm resum} + (d\sigma^{\rm pert} - d\sigma^{\rm asym}). \quad (21)$$

The term $\sigma^{\rm asym}$ is constructed to cancel the singular behavior of $\sigma^{\rm pert}$ as $p_T \to 0$ and to cancel $\sigma^{\rm resum}$ when $p_T \sim Q$. It is obtained from the fixed-order terms in the expansion of $\sigma^{\rm resum}$ in a power series in α_s.

The procedure just described is not immediately applicable in our case. Because the $b\bar{b}$ system is not necessarily in a color-neutral state, $\sigma^{\rm pert}$ includes radiation from the heavy quark system as well as from the incoming partons. This final-state radiation is not included in either $\sigma^{\rm resum}$ or $\sigma^{\rm asym}$. Soft-gluon radiation from the heavy quark system leads to an infrared divergent $1/p_T^2$ singularity [10] which should be absorbed into $\mathcal{F}(Q^2)$ [19]. To avoid extrapolation of $\sigma^{\rm pert}$ into region of low p_T, we adopt the following matching procedure:

$$\frac{d\sigma_{AB \to \Upsilon(nS)X}}{dp_T^2 dy} = \begin{cases} \frac{d\sigma^{\rm resum}_{AB \to \Upsilon(nS)X}}{dp_T^2 dy} & p_T < p_{T_M} \\ \frac{d\sigma^{\rm pert}_{AB \to \Upsilon(nS)X}}{dp_T^2 dy} & p_T \geq p_{T_M}. \end{cases} \quad (22)$$

Matching is done at a value p_{T_M} chosen as the location of intersection of the resummed and perturbative components of the p_T distribution. From other work on resummed p_T spectra [7–13], we expect $p_{T_M} \sim M_\Upsilon/2$.

To ensure a smooth parameter-free matching of $\sigma^{\rm resum}$ to the perturbative p_T distribution $\sigma^{\rm pert}$ computed at $\mathcal{O}(\alpha_s^3)$, we would need to calculate the *process-dependent* $\mathcal{O}(\alpha_s)$ corrections $C^{(1)}_{a \to i}$ and $H^{(1)}_{ij}$ (or $C^{(1)}_{a \to i}$ in the CSS formalism) for $\sigma^{\rm resum}$. If these $\mathcal{O}(\alpha_s)$ corrections were included, $\sigma^{\rm resum}$ would also be of order $\mathcal{O}(\alpha_s^3)$ at the matching point where the logarithms are not important. Based on prior experience [13], we expect that these effects will increase the predicted normalization of $d\sigma^{\rm resum}/dp_T^2 dy$, and change the shape of the p_T distribution somewhat, increasing (decreasing) the spectrum at small (large) p_T. For the reasons stated in last section, we do not calculate the order α_s corrections to $\sigma^{\rm resum}$ in this paper. To account for the size of the order α_s corrections, we introduce a *resummation enhancement factor* K_r such that

$$\left[C^{(0)}_{a \to i} + C^{(1)}_{a \to i} \frac{\alpha_s}{\pi} \right] \otimes \left[C^{(0)}_{b \to j} + C^{(1)}_{b \to j} \frac{\alpha_s}{\pi} \right] \otimes \left[H^{(0)}_{ij} + H^{(1)}_{ij} \frac{\alpha_s}{\pi} \right]$$
$$\equiv K_r C^{(0)}_{a \to i} \otimes C^{(0)}_{a \to i} \otimes H^{(0)}_{ij}. \quad (23)$$

We assume that K_r is a constant. The factor K_r should not be confused with a "K-factor" for the overall p_T distribution. It is invoked because we do not calculate the order α_s corrections to $\sigma^{\rm resum}$, and our $\sigma^{\rm resum} \sim \mathcal{O}(\alpha_s^2)$ when $p_T \sim \mathcal{O}(M_\Upsilon)$.

Displayed in Fig. 4 are curves that show the differential cross section for production of the $\Upsilon(1S)$ as a function of p_T. The curves in the region of large p_T illustrate the dependence of the fixed-order $\mathcal{O}(\alpha_s^3)$ perturbative cross section on the common renormalization/factorization scale μ. We vary μ over the range $0.5 < \mu/\mu_0 < 2$ where $\mu_0 = \sqrt{Q^2 + p_T^2}$. This variation demonstrates the inevitable theoretical uncertainty of a fixed-order calculation. It could be reduced if a formidable $\mathcal{O}(\alpha_s^4)$ calculation is done in perturbation theory. We fix $\mu = 0.5\sqrt{Q^2 + p_T^2}$ for the remainder of our discussion. The $1/p_T^2$ divergence mentioned in Eq. (11) is evident in the fixed-order curves. Shown for purposes of perspective is a dot-dashed line that represents the resummed prediction, applicable at small p_T, obtained with $b_{\rm max} = 0.5$ GeV^{-1} and $g_2 = \bar{g}_2 = 0$. The set of curves illustrates the range of possibilities for the value of the matching point p_{T_M}. To obtain these results, we set $C_\Upsilon = 0.044$, $\alpha_\Upsilon = 0$, and $K_r = 1.22$, for reasons that are explained in the next subsection.
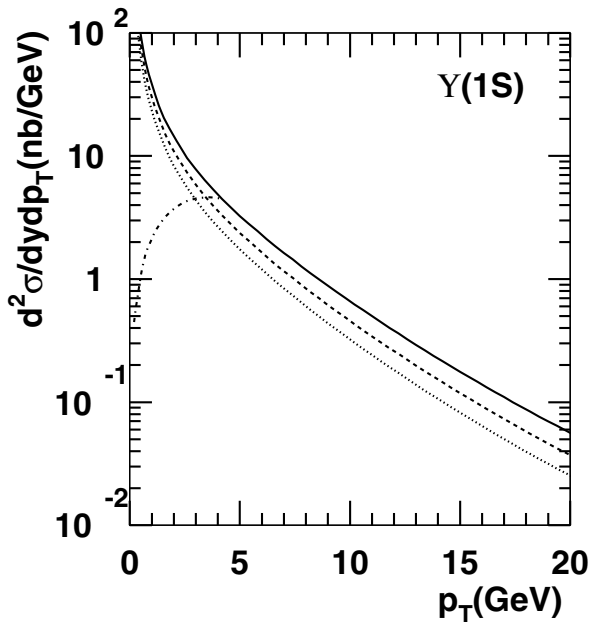
FIG. 4. Inclusive transverse momentum distribution of the $\Upsilon(1S)$ in $p\bar{p}$ interactions at $\sqrt{S} = 1.8$ TeV. The solid, dashed, and dotted lines in the region of large p_T are obtained from fixed-order $\mathcal{O}(\alpha_s^3)$ perturbative QCD for three different values of the scale μ; solid for $\mu = 0.5\mu_0$, dashed for $\mu = \mu_0$, and dotted for $\mu = 2\mu_0$; with $\mu_0 = \sqrt{Q^2 + p_T^2}$. The dot-dashed line in the region of small p_T is the resummed prediction.

B. Comparison with data

In order to make contact with data we must determine values for K_r in Eq. (23) and p_{T_M} in Eq. (22), and for the two nonperturbative parameters C_Υ and α_Υ in the transition probability distribution $\mathcal{F}(Q^2)$. The structure of Eq. (22) indicates that the data at *high* p_T determine C_Υ and α_Υ, and the data at *low* p_T fix the enhancement factor K_r. Dependence on the parameter α_Υ turns out to be very weak, as might be expected from the limited range in Q^2 over which $\mathcal{F}(Q^2)$ is probed. This weak dependence confirms that the production models we consider predict very similar inclusive p_T distributions. We choose to set $\alpha_\Upsilon = 0$ for all three Υ states. Second, common values of K_r and p_{T_M} work adequately for all three Υ states, as might be expected since the differences are small among the three Upsilon masses.

In our approach to the data, C_Υ represents not just the normalization in the transition probability distribution $\mathcal{F}(Q^2)$, but the product of this normalization times the unknown K-factor from order $\mathcal{O}(\alpha_s^4)$ perturbative contributions at large p_T. It should not be surprising that nonperturbative free parameters enter the comparison with data at large p_T. The reliance on nonperturbative parameters to set the normalization is true of all models other than the color-singlet model [3].

We compare our calculation with data published by the Collider Detector at Fermilab (CDF) collaboration

[4,16] obtained in run I of the Tevatron collider at $\sqrt{S} = 1.8$ TeV. In the second of the two publications, it is noted that the measured rates are lower than those reported in the first paper. To account for the difference in our fits to the data, we include an overall multiplicative normalization factor C_n, whose value we determine from our χ^2 fitting routine. This factor is used only for the 1995 data [4].

Following our initial qualitative exploration of the data, we are left with the three parameters C_Υ, the common values of K_r and p_{T_M}, and the data adjustment factor C_n. We use a χ^2 minimization procedure to determine these quantities. We find best fit values $K_r = 1.22 \pm 0.02$, and $p_{T_M} \sim 4.27$ GeV. The value of K_r is comparable to typical K-factors found in next-to-leading order calculations, but, as remarked above, the origin here is different. The matching point is fixed essentially by the location where the resummed and perturbative cross section intersect. Its value, $p_{T_M} \sim M_\Upsilon/2$, is similar to results found in other work on resummed p_T spectra [7–13]. We find that the values of C_Υ: 0.044 ± 0.001, 0.040 ± 0.006, and 0.041 ± 0.003 for $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$, respectively, are approximately independent of M_Υ, meaning that the differences in rates for the three S-wave Υ states are accounted for by the different threshold values of the integrals in Eq. (9).

In our determination of C_Υ, K_r and p_{T_M}, we keep only *process-independent* terms in the Sudakov exponential functions $S_{ij}(b, Q)$ in Eq. (17), and the parameters of the nonperturbative function F^{NP} are fixed at $b_{\max} = 0.5$ GeV^{-1}, and $g_2 = \bar{g}_2 = 0$. Because of the dominance of the perturbative small-b region under the curves of bW_{ij} in Fig. 3, any reasonable values of g_2 and $\bar{g}_2$ lead to transverse momentum distributions that do not differ more than 1% from those calculated with $g_2 = \bar{g}_2 = 0$ [8,13]. Without adjusting the normalization, we find a few percent change in the resummed distributions over the entire low p_T region when we vary $b_{\max}$ from 0.3 to 0.7 GeV^{-1}.

The principal predictive power of our calculation is the shape of the p_T-distribution for the full p_T region. In Fig. 5, we present our calculation of the transverse momentum distribution for hadronic production of $\Upsilon(nS)$, $n = 1 - 3$, as obtained from our Eq. (22), and multiplied by the leptonic branching fractions B. We use the values of B from Ref. [30]. The solid lines are for $b_{\max} = 0.5$ GeV^{-1} while the dashed and dotted lines are for $b_{\max} = 0.3$ and 0.7 GeV^{-1}, respectively. Also shown in Fig. 5 are data from the CDF collaboration [4,16]. We determine a data normalization adjustment of $C_n = 0.88 \pm 0.05$ and use this value to multiply only the 1995 cross sections shown in the figure. The shapes of the p_T distributions are consistent with experimental results.

The essential similarity of the production differential cross sections for the three $\Upsilon(nS)$ states is illustrated in
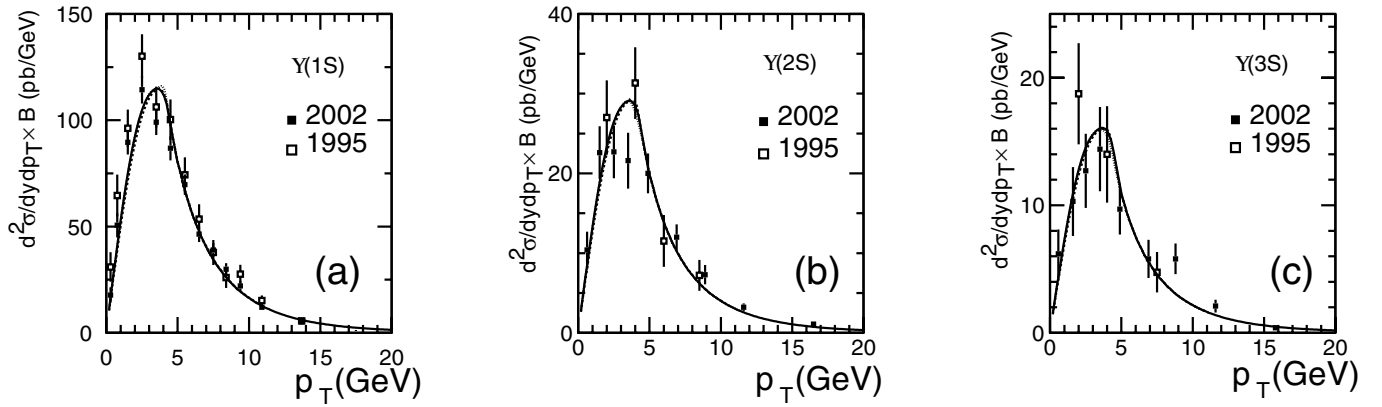
FIG. 5.   Calculated differential cross sections times leptonic branching fractions B, evaluated at $y = 0$, as functions of transverse momentum for hadronic production of (a) ϒ(1S), (b) ϒ(2S), and (c) ϒ(3S), along with CDF data [4,16] at $\sqrt{S} = 1.8$ TeV. The dashed, solid, and dotted lines show the result of our full calculation for $b_{\text{max}} = 0.3$, 0.5, and 0.7 GeV^{-1}, respectively. The 1995 CDF cross sections are multiplied by $C_n = 0.88$.

Fig. 6. Shown are the differential cross sections divided by their respective integrals over the range $0 < p_T < 20$ GeV. The integrated values are computed from the theoretical cross sections and used to scale the experimental as well as the theoretical results. The three theory curves are practically indistinguishable. The transverse momentum distribution is described well over the full range of p_T. Since the curves in Fig. 6 are normalized by the integrated cross sections, dependence on the normalization parameters C_Υ cancels in the ratio. The shape for $p_T < M_\Upsilon/2$ is predicted quantitatively. It reflects the resummation of the gluon

FIG. 6.   Normalized transverse momentum distributions for ϒ production: ϒ(1S) (solid), ϒ(2S) (dashed), and ϒ(3S) (dotted), along with the 2002 CDF data [16].

shower and is independent of parameter choices. The good agreement with data over the full range in p_T is based on the choice of only two adjustable constants, the resummation enhancement factor K_r and the matching point p_{T_M}.

VI. SUMMARY AND CONCLUSIONS

In this paper we calculate the transverse momentum p_T distribution for production of the ϒ states in hadronic reactions, applicable over the full range of values of p_T. Our starting assumption is that the p_T distribution of ϒ production may be derived from the p_T distribution for the production of a pair $b\bar{b}$ of bottom quarks. We express the differential cross section in terms of a two-step factorization procedure. We justify the validity of an all-orders soft-gluon resummation approach to compute the p_T distribution in the region of small and intermediate p_T where $p_T < M_\Upsilon$. Resummation is necessary to deal with the perturbative $1/p_T^2$ singularity and the large logarithmic enhancements that arise from initial-state gluon showers. We demonstrate that the p_T distribution at low p_T is dominated by the region of small impact-parameter b and that it may be computed reliably in perturbation theory. We express the cross section at large p_T by the $\mathcal{O}(\alpha_s^3)$ lowest-order nonvanishing perturbative contribution. Our results are in good agreement with data from $p\bar{p}$ interactions at the Fermilab Tevatron collider at center-of-mass energy $\sqrt{S} = 1.8$ TeV, and they confirm that the resummable part of the initial-state gluon showers provides the correct shape of the p_T distribution in the region of small p_T.

An improvement of our calculation in the region $p_T < M_\Upsilon$ would require inclusion of the order α_s process-dependent corrections associated with the coefficient functions $C^{(1)}$ in the CSS formalism [or equivalently, $C^{(1)}$ and $H_{ij}^{(1)}$ in Eq. (16)]. Based on prior experience [13], we expect that these effects will increase the predicted normalization of $d\sigma^{\text{resum}}/dp_T^2 dy$, and change the shape of
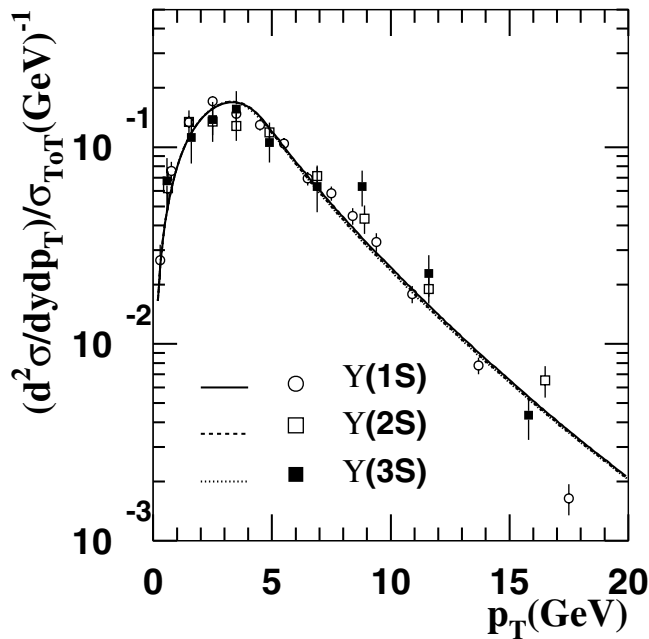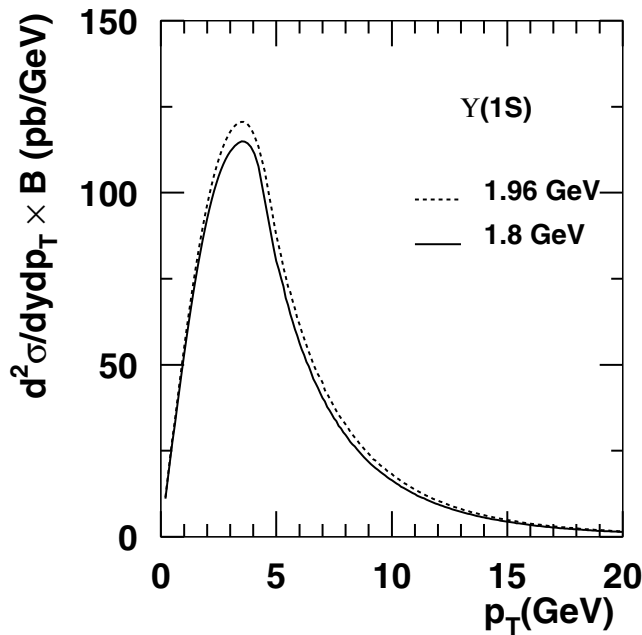
FIG. 7. Differential cross sections times leptonic branching fractions B, evaluated at $y = 0$, as functions of transverse momentum for hadronic production of the $\Upsilon(1S)$ at $\sqrt{S} = 1.8$ TeV (solid line) and 1.96 TeV (dashed line).

value of the fractional momentum x carried by the gluons is determined by the ratio $M_\Upsilon/\sqrt{S}$. The growth of the gluon density as x decreases leads to two expected changes in our predictions for larger $\sqrt{S}$. First, and perhaps obvious, the magnitude of the cross section near the peak in, e.g., Fig. 5 will increase. Second, and more subtle, is the prediction that the peak location should shift to a greater value of p_T as $\sqrt{S}$ grows. We use the same parameters as those at $\sqrt{S} = 1.8$ TeV, Fig. 5, to compute $\Upsilon(1S)$ production at the Tevatron in run-II at $\sqrt{S} = 1.96$ TeV; our results are shown in Fig. 7. The change of $\sqrt{S}$ from 1.8 to 1.96 TeV does not produce a marked difference in the spectrum, but we expect the shift of the peak in p_T to be about 1 GeV at the LHC energy of $\sqrt{S} = 14$ TeV.

Our focus on Υ production may motivate questions about the analogous production of the J/ψ states. The mass of the J/ψ is relatively small, meaning that inverse power contributions of the form $1/Q$ are potentially as significant as the logarithmic terms $\log(Q)$ that we resum. In addition, the saddle point in the b-space distribution moves into, or close to, the region in which perturbation theory can no longer be claimed to dominate the p_T distribution.

the p_T distribution somewhat, increasing (decreasing) the spectrum at small (large) p_T. A complete calculation of the order α_s corrections $C^{(1)}_{a \to i}$ and $H^{(1)}_{ij}$ in Eq. (16) would provide a better test of QCD predictions. In the region of large p_T, an improved prediction of the normalization and shape of the differential cross section would require a formidable $\mathcal{O}(\alpha_s^4)$ calculation of $b\bar{b}$ production.

Inclusive production of the Υ states in the central region of rapidity at Tevatron energies and above is controlled by partonic subprocesses initiated by gluons. The typical

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