Analytical evolution of nucleon structure functions with power corrections at twist-4 and predictions for ultrahigh energy neutrino-nucleon cross section

R. Fiore,[1,*] L. L. Jenkovszky,[2,†] A. V. Kotikov,[3,‡] F. Paccanoni,[4,§] A. Papa,[1,||] and E. Predazzi[5,¶]

[1]*Dipartimento di Fisica, Università della Calabria, Istituto Nazionale di Fisica Nucleare,*
Gruppo collegato di Cosenza, I-87036 Arcavacata di Rende, Cosenza, Italy
[2]*Bogolyubov Institute for Theoretical Physics, Academy of Sciences of Ukraine, UA-03143 Kiev, Ukraine*
[3]*Bogolyubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, RU-141980 Dubna, Russia*
[4]*Dipartimento di Fisica, Università di Padova, Istituto Nazionale di Fisica Nucleare,*
Sezione di Padova, via F. Marzolo 8, I-35131 Padova, Italy
[5]*Dipartimento di Fisica Teorica, Università di Torino, Istituto Nazionale di Fisica Nucleare,*
Sezione di Torino, via P. Giuria 1, I-10125 Torino, Italy

In this paper we present an analytic result for the evolution in Q^2 of the structure functions for the neutrino-nucleon interaction, valid at twist-2 in the region of small values of the Bjorken x variable and for soft nonperturbative input. In the special case of flat initial conditions, we include in the calculation also the contribution of the twist-4 gluon recombination corrections, whose effect in the evolution is explicitly determined. Finally, we estimate the resulting charged-current neutrino-nucleon total cross section and discuss its behavior at ultrahigh energies.

I. INTRODUCTION

Saturation effects in the parton distributions for a nucleon, that could finally lead to unitarization of the cross sections, have been thoroughly discussed in the past. In the small Bjorken x region these effects can be accounted for by the introduction of nonlinear terms in the evolution equation for the gluon density [1]. The study of nonlinear evolution equations began 20 years ago and gave rise, in the following years, to different approaches to the small-x region [2,3]. More recently, generalizations of nonlinear evolution equations have been proposed with different physical motivations [4–6] and screening effects have been incorporated in the theoretical framework in different ways [7–9].

The approaches quoted above find interesting applications in the interaction of ultrahigh energy neutrinos with nucleons and nuclei [10–13]. While there is no general consensus on the importance of screening effects, one expects, at any rate, that the linear QCD evolution of parton distribution functions will be tamed in the very small-x region. Predictions for the cross section of the neutrino-nucleon interaction based on DGLAP [14,15] or BFKL [16] equations show a power increase with energy [17–19] that will finally violate the Froissart bound. We notice that the problem these papers address is rather intricate, since it requires a complete knowledge of nucleon structure functions in both variables x and Q^2. At ultrahigh energies the

integrals, giving the cross section in terms of structure functions, cover the whole permissible range of values for these variables and explore extreme regions of the (Q^2, x) phase space, where nonaccelerator data exist. Moreover, the mathematical complexity inherent in the solution of nonlinear evolution equations may conceal the physical essence of the problem.

Simplifications are well possible if we limit ourselves to the small-x region only, with a warning about the consequences on the value of the integrals giving the total cross section. For example, a powerlike behavior in x of the parton distributions is a simple solution of the DGLAP dynamics at next-to-leading order on the region $x < 0.1$ and for large Q^2 values, where the hard initial condition $x^{-\lambda} \gg$ constant can be applied for parton distribution functions [17]. Because of the asymptotic behavior of the DGLAP evolution [14], small-x data can also be interpreted in terms of the "double asymptotic scaling" [20–22] that provides an explicit solution to the problem. The straightforward application of these approximation schemes to the nonlinear evolution is however questionable. Moreover, it would be rather difficult to estimate the error induced on the cross section.

In a series of papers [23,24] a new set of evolution equations was suggested that includes parton recombination. These new equations are derived in the leading logarithmic (Q^2) approximation and differ from the traditional ones [1,2], that rely on the double logarithmic approximation (DLLA). DLLA means that only those terms in the splitting functions that generate large logarithms in x are important. In other words, in the DLLA a diagram consists of gluon ladders and any transition from gluon to quark is suppressed in this approximation. It becomes difficult however to reconcile small-x approximations with the

*Electronic address: fiore@cs.infn.it
†Electronic address: jenk@bitp.kiev.ua
‡Electronic address: kotikov@thsun1.jinr.ru
§Electronic address: paccanoni@pd.infn.it
||Electronic address: papa@cs.infn.it
¶Electronic address: predazzi@to.infn.it

twist-4 gluon recombination corrections of Ref. [24]. As emphasized in Ref. [25], the twist-4 coefficient function, driven by a two-particle gluon distribution, cannot be simplified by using DLLA or other small-x approximations.

There is, however, the possibility to replace, at small x, the convolution of two functions by a simple product. The method, introduced in Refs. [26–29], allows a correct treatment of the nonsingular part of parton distributions and has found numerous applications [29–31]. This approach, to be described at length in the following, has been applied also to the evaluation of the contributions from higher-twist operators of the Wilson operator product expansion [31]. The accuracy of this method, when applied to the modified DGLAP equations of Ref. [24], can be verified *a posteriori* with a suitable computer code.

In this paper we study the contributions of the twist-4 gluon recombination corrections to a previous twist-2 calculation [30]. In Ref. [30] we estimated the ultrahigh energy neutrino-nucleon cross section in the approach of Ref. [29]. The corrections introduced by the presence of nonlinear terms in the evolution equation, as given in Ref. [24], will change sensibly our previous estimate.

In the next section we will discuss the nonperturbative input and present an analytical form for the Q^2 evolution. Some restrictions on the value of the parameters of the input distributions, present in Ref. [30], are relaxed in this new formulation. However, gluon recombination corrections will be estimated in the particular case of starting flat initial conditions. In Sec. III results are presented for the structure function $F_2^{\nu N}(x, Q^2)$ and for the charged-current neutrino-nucleon total cross section. The asymptotic behavior of this cross section will be also discussed. In the Appendices the proof of the relevant analytical results is presented.

II. Q^2 EVOLUTION

As in our previous paper [30], we choose a soft non-perturbative input based on analyses of the nucleon structure functions [32–37]. If we denote by $f_q(x, Q^2)$ the sea quark distribution $xS(x, Q^2)$ and by $f_g(x, Q^2)$ the gluon distribution $xG(x, Q^2)$, that is if we put

$$f_q(x, Q^2) \equiv xS(x, Q^2), \qquad f_g(x, Q^2) \equiv xG(x, Q^2), \quad (1)$$

our soft nonperturbative input can be written in the form

$$f_a(x, Q_0^2) = \left[A_a + B_a \ln\left(\frac{1}{x}\right) \right] (1 - x)^{\nu(Q_0^2)} \qquad (a = q, g), \tag{2}$$

where A_a, B_a and $\nu(Q_0^2)$ are unknown parameters to be determined from data. Throughout this calculation at small x we will ignore the nonsinglet quark component and limit ourselves to the leading order (LO) of perturbation theory.

The factor $(1 - x)^{\nu(Q_0^2)}$ has been treated in detail in Ref. [30] and we neglect it in the following. At the end we will take it into account by multiplying the resulting structure function, $F_2^{\nu N}(x, Q^2) = f_q(x, Q^2)$ in the case of neutrino-nucleon DIS, by an effective large-x behavior $(1 - x)^\nu$, with constant ν. Furthermore, as in Ref. [30], we define

$$t = \ln\left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right] = \ln\left[\frac{\ln(Q^2/\Lambda_{\text{LO}}^2)}{\ln(Q_0^2/\Lambda_{\text{LO}}^2)} \right],$$

and the Ball-Forte scaling variables

$$\sigma = 2\sqrt{-\hat{d}_{gg} t \ln(1/x)}, \qquad \rho = \sqrt{\frac{-\hat{d}_{gg} t}{\ln(1/x)}} = \frac{\sigma}{2\ln(1/x)}, \tag{3}$$

where $\hat{d}_{gg} = -12/\beta_0$ and $\beta_0 = 11 - 2f/3$, with f the number of flavors, is the LO coefficient of the QCD β-function (in units of $-16\pi^2$). For brevity, we introduce the notation $\bar{d}_+(1) = 1 + 20f/(27\beta_0)$, $d_-(1) = 16f/(27\beta_0)$. At the LO, $\alpha_s(Q^2) = 4\pi/[\beta_0 \ln(Q^2/\Lambda_{\text{LO}}^2)]$.

A. DGLAP evolution

In Ref. [30] the results of Ref. [29] were used in order to obtain an approximate evolution, in LO perturbation theory, under the conditions $A_a \gg B_a$ (here and in the following the index a stands for q or g), so that no interference appears in the Q^2 evolution of the coefficients multiplying the different powers of the logarithm. Since here we relax these conditions, it is interesting to present the new expressions for f_q and f_g.

The method of solution adopted in Ref. [29] can be summarized as follows. Starting from the exact solution in the moment space, the anomalous dimensions and the coefficient functions are expanded in the neighborhood of $n = 1$. The singular part, when $n \rightarrow 1$, leads to Bessel functions but, in order to achieve the accuracy $O(\rho)$, also the regular part must be properly taken into account in the inverse Mellin transform.

By analogy with Ref. [29], it is possible to obtain the small-x asymptotic results for parton distribution functions (PDFs) and the F_2 structure function at LO of perturbation theory by setting

$$f_a(x, Q^2) = f_a^+(x, Q^2) + f_a^-(x, Q^2), \tag{4}$$

where

$$f_a^-(x, Q^2) = \left[A_a^- + B_a^- \ln\left(\frac{1}{x}\right) \right] \cdot e^{-d_-(1)t} + O(x), \tag{5}$$

$$f_q^+(x, Q^2) = [A_q^+ \rho I_1(\sigma) + B_q^+ I_0(\sigma)] \cdot e^{-\bar{d}_+(1)t} \cdot [1 + O(\rho)], \tag{6}$$

$$f_g^+(x, Q^2) = \left[A_g^+ I_0(\sigma) + B_g^+ \frac{1}{\rho} I_1(\sigma) \right] \cdot e^{-\bar{d}_+(1)t} \cdot [1 + O(\rho)]. \tag{7}$$

Here, $I_n(z)$ are modified Bessel functions and the coefficients $A_a^{\pm}$ and $B_a^{\pm}$ are defined as

$$B_q^- = B_q, \quad B_g^- = -\frac{4}{9} B_q, \quad A_q^- = A_q - \frac{f}{9}\left(B_g + \frac{4}{9} B_q\right),$$

$$A_g^- = -\frac{4}{9} A_q - \frac{2}{27}\left[\left(1 - \frac{7f}{27}\right) B_q - \frac{2f}{3} B_g\right],$$

$$B_g^+ = B_g + \frac{4}{9} B_q, \quad B_q^+ = \frac{f}{9}\left(B_g + \frac{4}{9} B_q\right), \tag{8}$$

$$A_g^+ = A_g + \frac{4}{9} A_q + \frac{2}{27}\left[\left(1 - \frac{7f}{27}\right) B_q - \frac{2f}{3} B_g\right],$$

$$A_q^+ = \frac{f}{9}\left[A_g + \frac{4}{9} A_q - \frac{1}{6}\left(1 + \frac{7f}{27}\right) B_g - \frac{4f}{243} B_q\right].$$

B. Shadowing and antishadowing corrections

According to Ref. [24], sea quark and gluon distributions are modified by the introduction of gluon recombination as stated by the following modified DGLAP equations:

$$\frac{df_a^{full}(x, Q^2)}{d \ln Q^2} = \sum_{b=q,g} P_{ab}^{AP}(x) \otimes f_b^{full}(x, Q^2) + \frac{\alpha_s^2}{Q^2}$$

$$\times \left[K_1 \int_{x/2}^x \frac{dy}{y} F_{ag}\left(\frac{x}{y}\right) [f_g^{full}(y, Q^2)]^2 \right.$$

$$\left. - K_2 \int_x^{1/2} \frac{dy}{y} F_{ag}\left(\frac{x}{y}\right) [f_g^{full}(y, Q^2)]^2 \right]. \tag{9}$$

Here $P_{ab}^{AP}(x)$ are the Altarelli-Parisi kernels, $\otimes$ stands for the Mellin convolution, defined as

$$A(x) \otimes B(x) = \int_x^1 \frac{dy}{y} A\left(\frac{x}{y}\right) B(y),$$

and

$$F_{gg}(x) = \frac{27}{64}(2 - x)(99 - 136x + 132x^2 - 64x^3 + 16x^4), \tag{10}$$

$$F_{qg}(x) = \frac{x}{48}(2 - x)(36 - 60x + 49x^2 - 14x^3). \tag{11}$$

In Ref. [24] it was set $K_1 = K_2 = K$ and from a fit of the HERA data the resulting value for K in the evolution equations of Ref. [24] turned out to be very small, $K = 0.0014$. We will come back to this point in Sec. III A. The introduction of two parameters, K_1 and K_2, allows a clearer check of the importance of the antishadowing, with respect to the shadowing contribution. Moreover, they give the possibility to relate models introduced in Refs. [1,2,23,24] (see Sec. III A).

Since K_1 and K_2 are expected to be small numbers, the solution of Eq. (9) can be written as

$$f_a^{full}(x, Q^2) = f_a(x, Q^2) + T_a(x, Q^2), \tag{12}$$

where

$$\frac{df_a(x, Q^2)}{d \ln Q^2} = \sum_{b=q,g} P_{ab}^{AP}(x) \otimes f_b(x, Q^2) \tag{13}$$

and

$$\frac{dT_a(x, Q^2)}{d \ln Q^2} = \sum_{b=q,g} P_{ab}^{AP}(x) \otimes T_b(x, Q^2) + \alpha_s R_a(x, Q^2), \tag{14}$$

with

$$R_a(x, Q^2) = \frac{\alpha_s}{Q^2}\left[K_1 \int_{x/2}^x \frac{dy}{y} F_{ag}\left(\frac{x}{y}\right) [f_g(y, Q^2)]^2 \right.$$

$$\left. - K_2 \int_x^{1/2} \frac{dy}{y} F_{ag}\left(\frac{x}{y}\right) [f_g(y, Q^2)]^2 \right]. \tag{15}$$

Finally, the sea quark distribution is

$$xS(x, Q^2) = f_q^{full}(x, Q^2).$$

We note that in the integrands at the right-hand side of Eq. (15) f_g appears instead of f_g^{full}. This amounts to neglect terms which are of order K^2 or higher or, equivalently, terms of twist-6 or higher. This is consistent with our approach, where only twist-4 recombination effects were considered. Moreover, we anticipate here that we checked numerically *a posteriori* that the neglected terms in the right-hand side of Eq. (15) are indeed unimportant in our analysis.

C. Small-x solution of the complete equations

The solution of Eq. (13) with the boundary condition (2) has been found already and is expressed in Eqs. (4)–(7). This result has been obtained directly from the corresponding solution in the moment space (see Ref. [29]). In order to simplify the solution of the complete equations, we set in the following $B_q = B_g = 0$. Then our solution (5)–(7) assumes the simple form

$$f_a^-(x, Q^2) = A_a^- \cdot e^{-d_-(1)t} + O(x), \tag{16}$$

$$f_q^+(x, Q^2) = A_q^+ \rho I_1(\sigma) \cdot e^{-\bar{d}_+(1)t} \cdot [1 + O(\rho)], \tag{17}$$

$$f_g^+(x, Q^2) = A_g^+ I_0(\sigma) \cdot e^{-\bar{d}_+(1)t} \cdot [1 + O(\rho)]. \tag{18}$$

Equation (14) can be rewritten in the moment space, using the Mellin transform defined as

$$M(n) = \int_0^1 dx\, x^{n-2} M(x).$$

In the leading logarithmic approximation we obtain

$$\frac{dT_a(n, Q^2)}{dt} = - \sum_{b=q,g} d_{ab} T_b(n, Q^2) + r_a(n, Q^2), \quad (19)$$

where $d_{ab}(n) = \gamma_{ab}^{(0)}(n)/(2\beta_0)$ is the ratio between the anomalous dimension $\gamma_{ab}^{(0)}$ and twice β_0 and $r_a(n, Q^2) = 4\pi R_a(n, Q^2)/\beta_0$, with $R_a(n, Q^2)$ the Mellin moment of $R_a(x, Q^2)$. The solution of Eq. (19) can be written in the form

$$T_a(n, Q^2) = T_a^+(n, Q^2) + T_a^-(n, Q^2),$$

$$T_{\bar{a}}^\pm(n, Q^2) = - \int_t^\infty dw e^{d_\pm(w-t)} \sum_{b=q,g} \epsilon_{ab}^\pm r_b(n, M^2), \quad (20)$$

where

$$w = \ln\left(\frac{\ln(M^2/\Lambda_{\text{LO}}^2)}{\ln(Q_0^2/\Lambda_{\text{LO}}^2)}\right),$$

$d_\pm(n)$ are the eigenvalues of the d_{ab} matrix and $\epsilon_{ab}^\pm(n)$ are related to the components of the eigenvectors of the same matrix. Explicitly, we have

$$T_q^-(n, Q^2) = - \int_t^\infty dw e^{d_-(w-t)}[\eta r_q(n, M^2) + \tilde{\eta} r_g(n, M^2)],$$

$$T_q^+(n, Q^2) = - \int_t^\infty dw e^{d_+(w-t)}[(1-\eta)r_q(n, M^2) - \tilde{\eta} r_g(n, M^2)],$$

$$T_g^-(n, Q^2) = - \int_t^\infty dw e^{d_-(w-t)}[(1-\eta)r_g(n, M^2) + \epsilon r_q(n, M^2)],$$

$$T_g^+(n, Q^2) = - \int_t^\infty dw e^{d_+(w-t)}[\eta r_g(n, M^2) - \epsilon r_q(n, M^2)], \quad (21)$$

where $\epsilon_{ab}^\pm$ have been expressed in terms of η, $\tilde{\eta}$ and ϵ, which take the following values for $n \to 1$:

$$\eta = 1 - \frac{4f}{81}(n-1), \qquad \tilde{\eta} = -\frac{f}{9}(n-1),$$

$$\epsilon = -\frac{4}{9}. \quad (22)$$

The evaluation of the Mellin moments of $R_a(x, Q^2)$ requires particular care, since the integrals appearing in Eq. (15) are not exact Mellin convolutions. If we make the position

$$M(n|1/2) = \int_0^{1/2} dx x^{n-2} M(x),$$

we have, in addition to the usual Mellin convolution

$$\int_0^1 x^{n-2} dx \int_x^1 \frac{dy}{y} M_1\left(\frac{x}{y}\right) M_2(y) = M_1(n) M_2(n), \quad (23)$$

also the following one:

$$\int_0^1 x^{n-2} dx \int_{x/2}^{1/2} \frac{dy}{y} \tilde{M}_1\left(\frac{x}{2y}\right) M_2(y)$$

$$= 2^{n-1} \tilde{M}_1(n) M_2(n|1/2), \quad (24)$$

where the definition $\tilde{M}_1(x) \equiv M_1(2x)$ is used hereafter. Now, in order to calculate the Mellin moments of $R_a(x, Q^2)$ in Eq. (15), we should find a relation between the "Mellin transforms" $f_2(n)$ and $f_2(n|1/2)$, defined as

$$f_2(n) = \int_0^1 dx x^{n-2} f_g^2(x), \quad (25)$$

$$f_2(n|1/2) = \int_0^{1/2} dx x^{n-2} f_g^2(x). \quad (26)$$

This relation is

$$f_2(n|1/2) = f_2(n) + O[(n-1)^2], \quad (27)$$

the proof being given in Appendix A.

Then, the contributions to the Mellin transform of $R_a(x, Q^2)$ which are regular for $n \to 1$ assume the form (the upper index (r), here and in the following, stands for "regular contribution")

$$[\tilde{F}_{ag}^{(r)}(n) - F_{ag}^{(r)}(n)] f_2(n), \quad (28)$$

for the coefficient of K_1 and

$$F_{ag}^{(r)}(n) f_2(n), \quad (29)$$

for the coefficient of K_2, where [see also Eqs. (23) and (24)]

$$F_{ag}^{(r)}(n) = \int_0^1 dx x^{n-2} F_{ag}^{(r)}(x),$$

$$\tilde{F}_{ag}^{(r)}(n) = \int_0^1 dx x^{n-2} \tilde{F}_{ag}^{(r)}(x) = \int_0^1 dx x^{n-2} F_{ag}^{(r)}(2x), \quad (30)$$

since by definition $F_{ag}^{(r)}(x) \equiv \tilde{F}_{ag}^{(r)}(x/2)$.

Note that $F_{qg}^{(r)}(x) = F_{qg}(x)$, because the Mellin moments of $F_{qg}(x)$ have no singular part, and

$$F_{gg}^{(r)}(x) = \frac{27}{64}[-99x - 4x(2-x)(34 - 33x + 16x^2 - 4x^3)]. \quad (31)$$

By performing the Mellin integrals in Eqs. (30), we find

$$F_{qg}^{(r)}(n=1) = \frac{1813}{2880}, \qquad \tilde{F}_{qg}^{(r)}(n=1) = \frac{131}{180},$$

$$F_{gg}^{(r)}(n=1) = -\frac{31977}{320}, \qquad \tilde{F}_{gg}^{(r)}(n=1) = -\frac{23877}{160}. \quad (32)$$

In this way we obtain from Eqs. (15), (28), and (29)

$$R_q(n \to 1, M^2) = \left[\frac{283}{2880} K_1 - \frac{1813}{2880} K_2\right]$$

$$\times \frac{\alpha_s(M^2)}{M^2} f_2(n \to 1, M^2), \quad (33)$$

$$R_g(n \to 1, M^2) = 9\left[\left(\frac{297}{32}\ln 2 - \frac{1753}{320}\right)K_1\right.$$
$$\left. - \left(\frac{297}{32}\frac{1}{n-1} - \frac{3553}{320}\right)K_2\right]$$
$$\times \frac{\alpha_s(M^2)}{M^2} f_2(n \to 1, M^2) \qquad (34)$$

and, from Eqs. (21),

$$T_a^\pm(n \to 1, Q^2) = -\int_{Q^2}^\infty \frac{dM^2}{(M^2)^2} C_{ag}^\pm(n \to 1) f_2(n \to 1, M^2)$$
$$\times \frac{[\alpha_s(Q^2)]^{d_\pm(n \to 1)}}{[\alpha_s(M^2)]^{d_\pm(n \to 1) - 2}}. \qquad (35)$$

Here $C_{ag}^\pm(n \to 1)$ are the "coefficient functions" of the twist-4 corrections, because

$$R_a(n \to 1, Q^2) = \frac{\alpha_s(Q^2)}{Q^2} C_{ag}(n \to 1) f_2(n \to 1, Q^2),$$
$$(36)$$
$$C_{ag}(n \to 1) = C_{ag}^+(n \to 1) + C_{ag}^-(n \to 1),$$

i.e., they are the coefficients in front of the Mellin moments of the function $f_g^2(x)$. They can be written as

$$C_{ag}^\pm(n \to 1) = C_{ag}^{1,\pm} K_1 - C_{ag}^{2,\pm} K_2 \qquad (37)$$

and the definitions $C_{ag}^{i,\pm}(n \to 1) \equiv C_{ag}^{i,\pm}$ $(i = 1, 2)$ have been introduced for the sake of simplicity. The coefficients $C_{ag}^{i,\pm}$ are given in Appendix B.

The solution (35) in the moment space can be transformed to the x-space. Note that the product of moments in Eq. (35) leads to the convolution

$$M_1(n) \cdot M_2(n) \xrightarrow{M^{-1}} \int_x^1 \frac{dy}{y} M_1\left(\frac{x}{y}\right) M_2(y) \equiv M_1(x) \otimes M_2(x)$$
$$(38)$$

in the x-space. As in the case of the moment space, $T_a(x, Q^2)$ can be represented as the combination of the "+" and "−" components:

$$T_a(x, Q^2) = T_a^+(x, Q^2) + T_a^-(x, Q^2), \qquad (39)$$

which can be obtained from the corresponding components in Eq. (35) by an inverse Mellin transformation.

As for the "−" component, we note that the value $d_-(n)$ does not contain any singularity for $n \to 1$, hence (hereafter $v = w - t$)

$$e^{d_-(n)v} \approx e^{d_-(1)v} \xrightarrow{M^{-1}} e^{d_-(1)v} \delta(1 - x) \qquad (40)$$

[here $\delta(1 - x)$ is the Dirac δ-function], so that

$$C_{ag}^- e^{d_-(1)(w-t)} f_2(n \to 1, M^2) \xrightarrow{M^{-1}} C_{ag}^- e^{d_-(1)(w-t)} f_g^2(x, M^2). \qquad (41)$$

As for the "+" component, we have $d_+(n) = \hat{d}_+/(n - 1) + \overline{d}_+(n)$, with $\hat{d}_+ < 0$, hence

$$\frac{1}{n-1} e^{\hat{d}_+ v/(n-1)} = \sum_{k=0}^\infty \frac{1}{k!} \frac{(\hat{d}_+ v)^k}{(n-1)^{k+1}}$$
$$\xrightarrow{M^{-1}} \sum_{k=0}^\infty \frac{1}{(k!)^2} (\hat{d}_+ v)^k \left(\ln\frac{1}{x}\right)^k$$
$$= J_0(\tilde{\sigma}), \qquad (42)$$

$$e^{\hat{d}_+ v/(n-1)} = \sum_{k=0}^\infty \frac{1}{k!} \frac{(\hat{d}_+ v)^k}{(n-1)^k}$$
$$\xrightarrow{M^{-1}} \delta(1 - x) - \sum_{k=0}^\infty \frac{1}{k!(k-1)!} (\hat{d}_+ v)^k \left(\ln\frac{1}{x}\right)^{k-1}$$
$$= \delta(1 - x) - \tilde{\rho} J_1(\tilde{\sigma}), \qquad (43)$$

where

$$\hat{\rho} = \frac{\hat{\sigma}}{2\ln(1/x)}, \qquad \hat{\sigma} = \sigma \quad \text{with } t \to w, \qquad (44)$$

$$\tilde{\rho} = \frac{\tilde{\sigma}}{2\ln(1/x)}, \qquad \tilde{\sigma} = \sigma \quad \text{with } t \to (w - t) \qquad (45)$$

and $w = t$ when $Q^2 \to M^2$, i.e.,

$$w = \ln\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(M^2)}\right) = \ln\left(\frac{\ln(M^2/\Lambda_{LO}^2)}{\ln(Q_0^2/\Lambda_{LO}^2)}\right), \qquad (46)$$

$$w - t = \ln\left(\frac{\alpha_s(Q^2)}{\alpha_s(M^2)}\right) = \ln\left(\frac{\ln(M^2/\Lambda_{LO}^2)}{\ln(Q^2/\Lambda_{LO}^2)}\right). \qquad (47)$$

As a consequence

$$C_{qg}^+(n = 1) e^{\hat{d}_+(w-t)/(n-1)} e^{\overline{d}_+(1)(w-t)} f_2(n \to 1, M^2)$$
$$\xrightarrow{M^{-1}} C_{qg}^+(n = 1) e^{\overline{d}_+(1)(w-t)} \tilde{F}_1, \qquad (48)$$

$$\left[\hat{C}_{gg}^+ \frac{1}{n-1} + \overline{C}_{gg}^+(n = 1)\right] e^{\hat{d}_+(w-t)/(n-1)} e^{\overline{d}_+(1)(w-t)}$$
$$\times f_2(n \to 1, M^2)$$
$$\xrightarrow{M^{-1}} [\hat{C}_{gg}^+ \tilde{F}_2 + \overline{C}_{gg}^+(n = 1) \tilde{F}_1] e^{\overline{d}_+(1)(w-t)}, \qquad (49)$$

where

$$\tilde{F}_1 = \int_x^1 \left\{\frac{dy}{y} \delta(1 - y) - \tilde{\rho}(y) J_1[\tilde{\sigma}(y)]\right\} f_g^2(x/y, M^2)$$
$$\equiv [\delta(1 - x) - \tilde{\rho} J_1(\tilde{\sigma})] \otimes f_g^2(x, M^2), \qquad (50)$$

$$\tilde{F}_2 = \int_x^1 \frac{dy}{y} J_0[\tilde{\sigma}(y)] f_g^2(x/y, M^2) \equiv J_0(\tilde{\sigma}) \otimes f_g^2(x, M^2). \qquad (51)$$

Thus,

$$T_a^-(x, Q^2) = - \int_{Q^2}^{\infty} \frac{dM^2}{(M^2)^2} (K_1 \cdot C_{ag}^{1,-} - K_2 \cdot C_{ag}^{2,-})$$
$$\times [f_g(x, M^2)]^2 \frac{[\alpha_s(Q^2)]^{d_-(1)}}{[\alpha_s(M^2)]^{d_-(1)-2}}, \qquad (52)$$

$$T_q^+(x, Q^2) = - \int_{Q^2}^{\infty} \frac{dM^2}{(M^2)^2} (K_1 \cdot C_{qg}^{1,+} - K_2 \cdot C_{qg}^{2,+})$$
$$\times \tilde{F}_1 \frac{[\alpha_s(Q^2)]^{\overline{d}_+(1)}}{[\alpha_s(M^2)]^{\overline{d}_+(1)-2}}, \qquad (53)$$

$$T_g^+(x, Q^2) = - \int_{Q^2}^{\infty} \frac{dM^2}{(M^2)^2} (K_1 \cdot C_{gg}^{1,+} \tilde{F}_1 - K_2$$
$$\times [\hat{C}_{gg}^{2,+} \tilde{F}_2 + \overline{C}_{gg}^{2,+} \tilde{F}_1]) \frac{[\alpha_s(Q^2)]^{\overline{d}_+(1)}}{[\alpha_s(M^2)]^{\overline{d}_+(1)-2}}. \qquad (54)$$

The above formulas complete our calculation of the gluon recombination terms.

D. Evaluation of $\tilde{F}_1$ and $\tilde{F}_2$ at $O(\rho)$

The functions $\tilde{F}_1$ and $\tilde{F}_2$ can be evaluated approximately. Note that we can write

$$\tilde{F}_1 = \tilde{F}_1^{--} + 2\tilde{F}_1^{+-} + \tilde{F}_1^{++},$$
$$\tilde{F}_2 = \tilde{F}_2^{--} + 2\tilde{F}_2^{+-} + \tilde{F}_2^{++}, \qquad (55)$$

according to the decomposition [see Eq. (4)]

$$f_g^2(x, M^2) = [f_g^-(x, M^2)]^2 + 2f_g^+(x, M^2)f_g^-(x, M^2)$$
$$+ [f_g^+(x, M^2)]^2.$$

The details of the complete calculation, at order $O(\rho)$ and with $B_q = B_g = 0$, will be given in Appendix C. Here we present only the results.

For the "$--$" component we find

$$\tilde{F}_1^{--} = (A_g^-)^2 J_0(\tilde{\sigma}) \cdot e^{-2d_-(1)w} [1 + O(\rho)], \qquad (56)$$

$$\tilde{F}_2^{--} = (A_g^-)^2 \frac{1}{\tilde{\rho}} J_1(\tilde{\sigma}) \cdot e^{-2d_-(1)w} [1 + O(\rho)], \qquad (57)$$

while the result for the "$+-$" component is

$$\tilde{F}_1^{+-} \equiv -\tilde{\rho} J_1(\tilde{\sigma}) \otimes [f_g^+(x, M^2)f_g^-(x, M^2)]$$
$$= A_g^- A_g^+ I_0(\sigma) \cdot e^{-[d_-(1)+\overline{d}_+(1)]w} [1 + O(\rho)], \qquad (58)$$

$$\tilde{F}_2^{+-} \equiv J_0(\tilde{\sigma}) \otimes [f_g^+(x, M^2)f_g^-(x, M^2)]$$
$$= A_g^- A_g^+ \frac{1}{\rho} I_1(\sigma) \cdot e^{-[d_-(1)+\overline{d}_+(1)]w} [1 + O(\rho^2)]. \qquad (59)$$

The term corresponding to the component "$++$" requires a more involved treatment. As shown in Appendix C, one obtains

$$\tilde{F}_1^{++} \equiv [\delta(1-x) - \tilde{\rho} J_1(\tilde{\sigma})] \otimes [f_g^+(x, M^2)]^2$$
$$= (A_g^+)^2 I_0(zy^{1/2}) I_0(\bar{z}y^{1/2}) \cdot e^{-2\overline{d}_+(1)w} \cdot [1 + O(\rho)] \quad (60)$$

and

$$\tilde{F}_2^{++} \equiv J_0(\tilde{\sigma}) \otimes [f_g^+(x, M^2)]^2$$
$$= (A_g^+)^2 \frac{1}{2\sqrt{\alpha\beta}} y^{1/2} [z I_0(\bar{z}y^{1/2}) I_1(zy^{1/2})$$
$$- \bar{z} I_0(zy^{1/2}) I_1(\bar{z}y^{1/2})] \cdot e^{-2\overline{d}_+(1)w} \cdot [1 + O(\rho^2)], \qquad (61)$$

where $y = -\log(x)$, $z = \alpha^{1/2} + i|\beta^{1/2}|$, with $\alpha = -\hat{d}_+(3w + t)$ and $\beta = -\hat{d}_+(t - w)$, and $\bar{z}$ is the complex conjugate of z.

These approximations do not make the integrations in Eqs. (53) and (54) avoidable, but greatly simplify the analytic structure of the answer.

III. THE NEUTRINO-NUCLEON CROSS SECTION

A. The fitting procedure

We consider the parton distribution functions from the ZEUS Collaboration [38] in the region $2.5 \text{ GeV}^2 \leq Q^2 \leq 20 \text{ GeV}^2$, for values of x in the range $10^{-4} \leq x \leq 5 \times 10^{-3}$, where the ZEUS NLO fit favorably compares with existing HERA data. From the parton distributions it is possible to reconstruct the isoscalar structure function $2xF_1$ for neutrino-nucleon scattering

$$2xF_1^{\nu N}(x) \simeq 2xF_1^{\bar{\nu} N}(x)$$
$$\simeq xu(x) + x\bar{u}(x) + xd(x) + x\bar{d}(x)$$
$$+ 2xs(x) + 2xc(x) + \dots \qquad (62)$$

where ... stands for b and t quarks and we have assumed $s = \bar{s}$ and $c = \bar{c}$. At the LO $F_2(x) = 2xF_1(x)$ and the difference between the two structure functions decreases when Q^2 increases. Parton distribution functions are needed at any rate; the knowledge of the experimental $F_2^{\ell N}$ must in fact be supplemented by PDFs since

$$F_2^{\nu N} = \frac{18}{5} F_2^{\ell N} + \frac{6}{5}(xs - xc) + \dots$$

For the Q^2 values under consideration, there are no CCFR data [39] below $x = 0.0125$ and the comparison between our fit and F_2 measurements in the process $(\nu^\mu + \bar{\nu}^\mu) + \text{nucleon} \rightarrow (\mu^- + \mu^+) + X$ is impossible. A single high energy HERA data point [40] for the process $e^- p \rightarrow \nu_e X$, with $p_\perp > 25$ GeV, gives $\sigma = 55 \pm 15$ pb at $\sqrt{s} = 296$ GeV. This means for the neutrino-nucleon cross section a value of $(2.0 \pm 0.55) \times 10^{-34} \text{ cm}^2$ at $\sqrt{s} = 306.4$ GeV.

The possibility to test other saturation models, such as that of Ref. [1], relies on the presence of two different coefficients K_1 and K_2 for shadowing and antishadowing contributions. In the following, we set $K_1 = K_2 = K$, as in

Ref. [24], thus reducing the number of parameters in our approach and the related errors. Then, the free parameters become Λ_{LO}, f, Q_0, A_q, A_g, K. We choose to fix $f = 4$, as in Ref. [31], and $\Lambda_{LO} = 0.19$ GeV. The remaining parameters should be determined by a fit. There is one more parameter to be considered, namely, the "mean power" ν of the factor $(1 - x)$ present in $F_2^{\nu N}(x, Q^2)$. The term $(1 - x)^\nu$ will be considered, for the sake of simplicity, only at the end of the calculation and will not evolve in our model. This will surely affect the result, but, since we are interested in the total cross section at very high energy, and hence at $\langle x \rangle$ very small, the error should not influence too much the result.

The values of the parameters Q_0 and K can be estimated from other sources. In the paper by W. Zhu *et al.* [24] a fit to the HERA small-x data for F_2^{ep} has been performed starting from GRV-like [41] input distributions at $Q_0^2 = 0.34$ GeV^2. MD-DGLAP evolution equations [24] determine new parameters for the sea quark distributions at Q_0^2, with respect to GRV98LO [41], and give for the nonlinear coefficient K the value $K = 0.0014$.

On the other hand, the fit of Ref. [31] imposes a constraint on the parameter Q_0. This constraint originates from the conditions of applicability of the "generalized double asymptotic scaling," that gives a satisfactory description of the experimental data in a region $Q^2 > Q_{cut}^2$, where Q_{cut}^2 is a cutoff larger than Q_0^2. For small x and large Q^2 values it is reasonable to neglect the valence quarks and the disagreement at small Q^2 is a consequence of this approximation. The value of Q_0^2 in this approach, that we follow in our paper, turns out to be rather small and, for the LO fit, is approximately $Q_0^2 \sim 0.3 \div 0.4$ GeV^2. This value of Q_0^2 has been obtained by fitting HERA data with higher-twist contribution evaluated in the renormalon model [42].

At this point it can be useful to remember that the purpose of this calculation is to reproduce the neutrino-nucleon cross section at very high energy, where presumably valence quarks do not contribute. Looking at Ref. [19], in particular, at the Fig. 2 in this paper, one sees that the valence quark contribution to the total νN charged-current cross section is larger than the u and d sea quarks contribution up to the energy $E_\nu \sim 10^5$ GeV. Hence we cannot rely on the H1 Collaboration data point at $E_\nu \sim 5 \times 10^4$ GeV and we expect that our model can be trusted only at higher energies.

B. Results

We have reconstructed the structure function $F_2^{\nu N}(x, Q^2)$ from the ZEUS PDFs [38] and performed a fit to the asymptotic formula for $F_2^{\nu N}(x, Q^2) = f_q^{full}(x, Q^2)$, where $f_q^{full}(x, Q^2)$ has been defined in Eq. (12) and the ingredients for calculating it have been given in Sec. II. Basing ourselves on the arguments of Ref. [31], we impose an upper bound on Q_0^2, $Q_0^2 \leq 0.45$ GeV^2, and consider separately

two possible values for the mean power ν of the factor $(1 - x)$: $\nu = 4$ or $\nu = 5$. The lowest χ^2/d.o.f. for this constrained fit has been obtained having chosen $Q_0^2 = 0.45$ GeV^2 and $\nu = 4$, with the result (χ^2/d.o.f. = 0.553)

$$A_q = 1.002(39) \qquad A_g = 0.565(29)$$
$$K = 0.0130(49). \tag{63}$$

There are several reasons why the parameter K in (63) is much larger than the corresponding result in Ref. [24]. First of all, the change from the leptonic structure function of Ref. [24] to the neutrino one requires a factor near to $18/5$ that affects also the coefficient K. In addition, our fit shows that there is a strong correlation between the starting value of Q^2, i.e., Q_0^2, and the value of K, which turns out to increase with Q_0^2. These reasons suffice to say that, within errors, our result is compatible with that of Ref. [24].

In order to test our solution, we have calculated $F_2^{\nu N}(x, Q^2)$ from the ZEUS PDFs [38] at $Q^2 = 50$ GeV^2 and compared with our theoretical result. The percentage error at $x = 10^{-4}$ is nearly 2% and is of the same order of magnitude in the whole range of x: $10^{-4} \leq x \leq 5 \times 10^{-3}$ (see Fig. 1).

Another test of this model regards the slope $dF_2^{\nu N}(x, Q^2)/d \ln Q^2$. As noticed in Ref. [25], higher-twist effects are more easily revealed in the slope than in $F_2(x, Q^2)$. Since in Ref. [25] it was stated that one cannot rely on the DLLA for the calculation of the twist-4 contributions, a critical examination of our approximation becomes important. It is easy to write in our approach (see Secs. II B and II C)
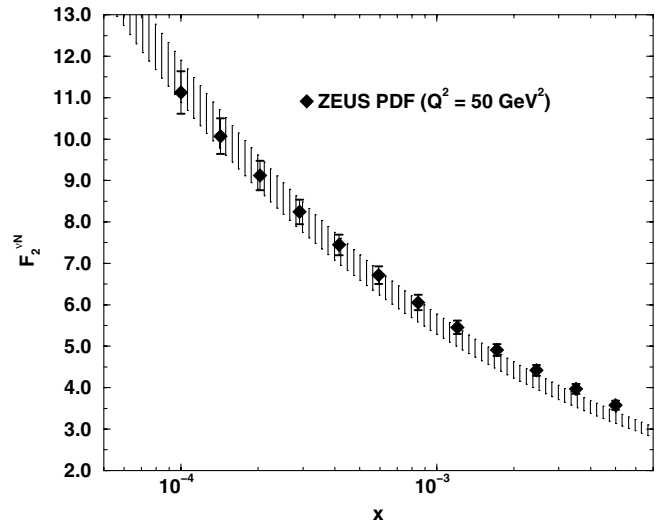

FIG. 1. Comparison at $Q^2 = 50$ GeV^2 between $F_2^{\nu N}(x, Q^2)$ as obtained from the ZEUS PDFs [38] and from our theoretical calculation (the vertical bars represent the uncertainties coming from the error in the fitted parameters). The values of the parameters of Eqs. (63) entering our results were obtained by a fit performed in the region 2.5 GeV$^2 \leq Q^2 \leq 20$ GeV^2, $10^{-4} \leq x \leq 5 \times 10^{-3}$.

$$\frac{dF_2^{\nu N}(x, Q^2)}{d\ln Q^2} = \frac{df_q^{full}(x, Q^2)}{d\ln Q^2} \sim \frac{df_q(x, Q^2)}{d\ln Q^2}$$
$$+ \frac{\alpha_s^2}{Q^2} K\left[-\frac{17}{32} f_g^2(x, Q^2) \right]. \qquad (64)$$

In Fig. 2 we show the behavior of the slope as a function of x for $Q^2 = 10$ GeV^2 in the range $10^{-10} \leq x \leq 10^{-3}$ according to our theoretical results, with the parameters given in Eqs. (63). A similar behavior is observed for other values of Q^2 in the range 2.5 GeV$^2 \leq Q^2 \leq 20$ GeV^2. From this Figure it is possible to see the higher-twist effects and to have the idea of the uncertainty resulting from the errors on the fitted parameters.

Finally, we consider the total cross section. With the usual notation, we can write for charged-current neutrino interactions

$$\left(\frac{d\sigma}{dxdy}\right)^\nu = \frac{G_F^2 M E_\nu}{\pi} \left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 \left\{ \frac{1 + (1-y)^2}{2} F_2^{\nu N}(x, Q^2) \right.$$
$$\left. + \left(1 - \frac{y}{2}\right) y x F_3^{\nu N}(x, Q^2) \right\}. \qquad (65)$$

For antineutrino charged-current processes one must change the sign in front of $F_3^{\nu N}(x, Q^2)$. In Eq. (65), G_F is the Fermi constant, M is the nucleon mass and the variable y is related to the Bjorken x through the relation

$$y = \frac{Q^2}{x(s - M^2)} \simeq \frac{Q^2}{xs}. \qquad (66)$$

The laboratory neutrino energy $E_\nu = (s - M^2)/(2M)$ is approximately $s/(2M)$ in the energy region of interest. The F_2 contribution to the total cross section can be written in the form
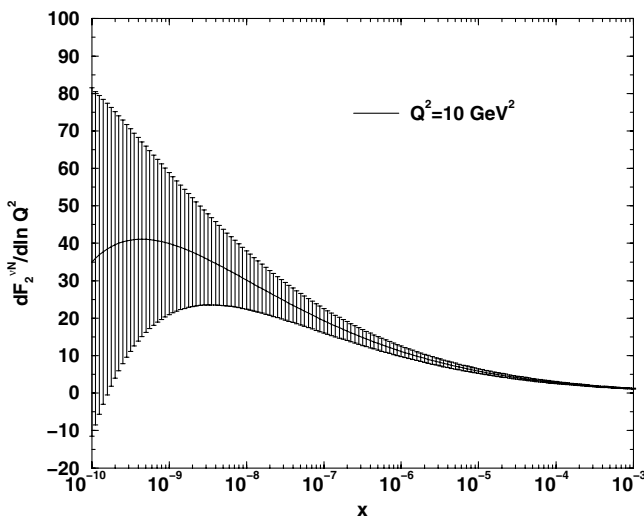
FIG. 2. Slope $dF_2^{\nu N}(x, Q^2)/d\ln Q^2$ at $Q^2 = 10$ GeV^2 according to our results. The uncertainties on the data come from the errors in the values of the fitted parameters of Eq. (63).
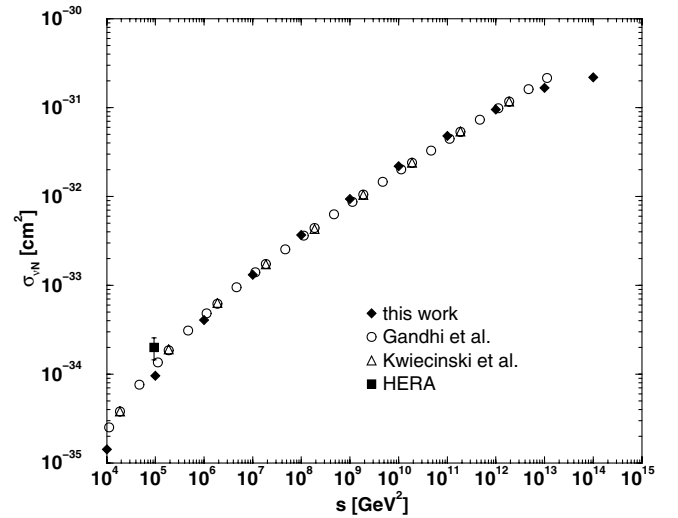
FIG. 3. Cross section $\bar{\sigma}^{\nu N}$ as a function of s according to our results (filled diamonds). Data do not include the uncertainties coming from the errors on the values of the fitted parameters of Eq. (63) (see, instead, Fig. 4). For comparison, data from Ref. [18] (open circles) and from Ref. [19] (open triangles) are also shown. The isolated point at $\sqrt{s} = 306.42$ GeV (filled square) represents the HERA measurement.

$$\bar{\sigma}^{\nu N} \equiv \frac{\sigma^{\nu N} + \sigma^{\bar{\nu} N}}{2}$$
$$= \frac{G_F^2}{2\pi} \int_{Q_0^2}^s dQ^2 \left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 \int_{Q^2/s}^1 \frac{dx}{x}$$
$$\times \frac{1 + [1 - Q^2/(xs)]^2}{2} F_2^{\nu N}(x, Q^2) \qquad (67)$$

and the formulas obtained for the $F_2^{\nu N}(x, Q^2)$ allow the evaluation of $\bar{\sigma}^{\nu N}$ that, for large s, is a good approximation for the total νN charged-current cross section.

In Fig. 3 we show the behavior of $\bar{\sigma}^{\nu N}$ as a function of s, in the range 10^4 GeV$^2 \leq s \leq 10^{14}$ GeV^2. The relative error is 4.4% at $s = 10^4$ GeV^2 and 14.0% at $s = 10^{12}$ GeV^2. For comparison we draw on the same plot the results obtained in Ref. [18] and in Ref. [19] as well as the HERA measurement at $\sqrt{s} = 306.42$ GeV. At this value of s our estimate of the cross section is much lower than the H1 data point. However, this is not surprising in view of the fact that we neglected valence quarks, which still can play a residual role at these relatively small values of s. For larger values of s, our results nicely compare with those of Refs. [18,19], showing good agreement till $s \sim 10^{13}$ GeV^2. For even larger values of s the effect of higher-twist starts to become visible.

These findings support the conclusion that our approach, based on an asymptotic calculation, succeeded in singling out the relevant part of the cross section.

In order to make the higher-twist effects more visible, we determined also $\bar{\sigma}^{\nu N}$ without considering the recombination terms in the evolution equations, i.e., with $K = 0$ from the beginning. In this case the fitted parameters turn
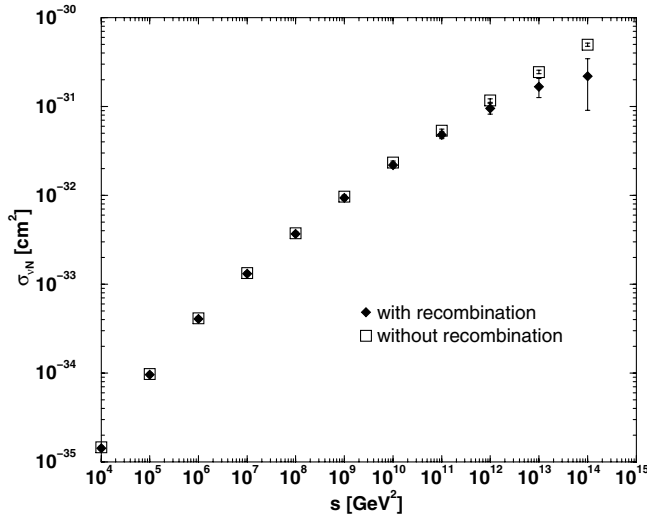
FIG. 4. Cross section $\bar{\sigma}^{\nu N}$ as a function of s according to our results (filled diamonds, same as Fig. 3, but with the inclusion of error bars) and without the contribution from the recombination (open squares).

to be $A_q = 1.040(36)$ and $A_g = 0.548(28)$ ($\chi^2/\text{d.o.f.} = 0.665$). In Fig. 4 we compare our results for $\bar{\sigma}^{\nu N}$ with and without the inclusion of the recombination effect. The higher-twist effects become visible for $s = 10^{13}$ GeV^2.

IV. CONCLUSIONS

Twist-4 corrections to the structure function F_2 have been estimated at small x in leading order QCD following the method developed in Ref. [29]. This estimate leads to an analytical parametrization for the gluon recombination effects and completes, in this respect, the program outlined in our previous paper [30], where an approximate QCD evolution at twist-2 was presented.

Our approach to the saturation phenomenon follows the scheme proposed in Ref. [23]. The nonlinear evolution equations we study are the MD-DGLAP equations [24], where momentum conservation gives rise to an antishadowing term that influences appreciably the screening effects. A compact and analytical solution of these equations at small x is possible only if some conditions are satisfied. The most relevant of these conditions regards the input parton distributions that must be flat. The proof of Eq. (27), which allows a simple treatment of the gluon recombination terms, requires this assumption. Moreover, the kernels in the MD-DGLAP equations are given in leading order of perturbation theory and this approximation reflects on our approach. In spite of these assumptions and approximations, the results are satisfactory. Examples of the Q^2 evolution and of the behavior of the slope $dF_2/d\ln Q^2$, shown in Figs. 1 and 2, make us confident on the accuracy of the method.

The most interesting application of our simplified formulas is, in our opinion, the study of the interaction of ultrahigh energy cosmic neutrinos with nucleons. This process can probe an energy region far beyond the largest energy reached by existing accelerators. The simplified and reasonable expressions for the structure function $F_2^{\nu N}$, disposable in this paper, renders the evaluation of the neutrino-nucleon cross section and the estimate of the twist-4 contributions simpler and more transparent. Our result for the cross section, shown in Fig. 3, is in perfect agreement with the calculations of other models and, only above $s = 10^{13}$ GeV^2 ($E_\nu \simeq 5.3 \times 10^{12}$ GeV), the effect of twist-4 gluon recombination becomes visible. The Froissart limit will be eventually satisfied, but in a region where the complete solution for the gluon distribution becomes necessary.

It is quite possible, at such high energies, to simplify further the integrals leading to the νN cross section. Work on this problem is in progress.

We stress, finally, that there is a lower limit on the energy range where our formulas are valid, since we neglected the contribution from valence quarks—see the discussion at the end of Sec. III B. As shown in Fig. 2 of Ref. [19], the contribution of valence quarks becomes $1/10$ of the total cross section for $s \simeq 10^6$ GeV^2. Therefore, our approach cannot be valid for values of s much smaller than this value. Our Fig. 3 indeed shows that our results for the total cross section are in agreement with those of Refs. [18,19] for values of $s \geq 10^6$ GeV^2. For smaller values of s our results turn out to be expectedly smaller (by a factor of about 2.5 at $s = 10^4$ GeV^2).

APPENDIX A: PROOF OF EQ. (27)

We consider separately the three terms coming from the decomposition $f_g^2 = (f_g^-)^2 + 2f_g^- f_g^+ + (f_g^+)^2$, which will be named in the following "$++$," "$+-$" and "$--$" components, respectively. Moreover, we will use the notation $\xrightarrow{M}$ to denote the Mellin transform and $\xrightarrow{M^{-1}}$ to denote the inverse Mellin transform.

(a) For the "$--$" component (hereafter A is arbitrary) we get

$$\int_x^{1/A} \frac{dy}{y} [f_g^-(y)]^2 \sim \int_x^{1/A} \frac{dy}{y}$$
$$= \ln\frac{1}{x} - \ln A \sim \left(\ln\frac{1}{x} - \ln A\right)$$
$$\times [f_g^-(x)]^2 \xrightarrow{M} \left(\frac{1}{n-1} - \ln A\right) f_2^{--}(n), \quad (A1)$$

where we have used the definition

$$f_2^{ij}(n) = \int_0^1 dx x^{n-2} f_g^i(x) f_g^j(x) \quad (i,j = \pm). \quad (A2)$$

The same argument gives

$$\int_{x/2}^{1/A} \frac{dy}{y} [f_g^-(y)]^2 \sim \int_{x/2}^{1/A} \frac{dy}{y}$$

$$= \ln\frac{2}{x} - \ln A \sim \left(\ln\frac{2}{x} - \ln A\right) [f_g^-(x)]^2$$

$$\xrightarrow{M} \left(\frac{1}{n-1} + \ln 2 - \ln A\right) f_2^{--}(n). \quad (A3)$$

Then, from Eqs. (A1) and (A3) we obtain

$$\int_{x/2}^x \frac{dy}{y} [f_g^-(y)]^2 = \ln 2 [f_g^-(x)]^2 \xrightarrow{M} \ln 2 f_2^{--}(n). \quad (A4)$$

(b) For the "+−" component (hereafter $z = \ln(1/y)$ and $\Delta = |\hat{d}_{gg}|t$) we get

$$\int_x^{1/A} \frac{dy}{y} f_g^+(y) f_g^-(y) \sim \int_x^{1/A} \frac{dy}{y} I_0[\sigma(y)]$$

$$= \int_{\ln A}^{\ln(1/x)} dz \sum_{k=0}^\infty \frac{\Delta^k z^k}{(k!)^2}$$

$$= \sum_{k=0}^\infty \frac{\Delta^k}{k!(k+1)!} \left[\left(\ln\frac{1}{x}\right)^{k+1} - (\ln A)^{k+1}\right]$$

$$= \frac{1}{\rho} I_1(\sigma) - \frac{1}{\rho_A} I_1(\sigma_A)$$

$$= \frac{1}{\rho} I_1(\sigma) \cdot [1 + O(\rho^2)]$$

$$\xrightarrow{M} \left[\frac{1}{n-1} + O(n-1)\right] f_2^{+-}(n), \quad (A5)$$

where

$$\rho_A = \rho|_{x\to 1/A}, \qquad \sigma_A = \sigma|_{x\to 1/A}. \quad (A6)$$

Similarly, we obtain

$$\int_{x/2}^{1/A} \frac{dy}{y} f_g^+(y) f_g^-(y) \sim \frac{1}{\rho_2} I_1(\sigma_2) - \frac{1}{\rho_A} I_1(\sigma_A), \quad (A7)$$

where

$$\rho_2 = \rho|_{x\to x/2}, \qquad \sigma_2 = \sigma|_{x\to x/2}, \quad (A8)$$

Note that

$$\frac{1}{\rho_2} I_1(\sigma_2) = \sum_{k=0}^\infty \frac{\Delta^k}{k!(k+1)!} \left[\ln\frac{2}{x}\right]^{k+1}$$

$$= \sum_{k=0}^\infty \frac{\Delta^k}{k!(k+1)!} \left\{\left(\ln\frac{1}{x}\right)^{k+1}\right.$$

$$\left. + (k+1)\ln 2\left(\ln\frac{1}{x}\right)^k + O\left[\left(\ln\frac{1}{x}\right)^{k-1}\right]\right\}$$

$$= \frac{1}{\rho} I_1(\sigma) + \ln 2 I_0(\sigma) \cdot [1 + O(\rho)]. \quad (A9)$$

Then we arrive at

$$\int_{x/2}^{1/A} \frac{dy}{y} f_g^+(y) f_g^-(y) \sim \frac{1}{\rho} I_1(\sigma) + \ln 2 I_0(\sigma)$$

$$\cdot [1 + O(\rho)]$$

$$\xrightarrow{M} \left[\frac{1}{n-1} + \ln 2 + O(n-1)\right]$$

$$\times f_2^{+-}(n). \quad (A10)$$

Finally, from Eqs. (A5) and (A10), we obtain

$$\int_{x/2}^x \frac{dy}{y} f_g^+(y) f_g^-(y) = \ln 2 f_g^+(x) f_g^-(x) \cdot [1 + O(\rho)]$$

$$\xrightarrow{M} [\ln 2 + O(n-1)] f_2^{+-}(n). \quad (A11)$$

(c) For the "++" component we get

$$\int_x^{1/A} \frac{dy}{y} [f_g^+(y)]^2 \sim \int_x^{1/A} \frac{dy}{y} I_0^2[\sigma(y)]$$

$$= \int_{\ln A}^{\ln(1/x)} dz I_0^2[\sigma(z)]$$

$$= \frac{1}{\rho} I_1(\sigma) I_0(\sigma) \cdot [1 + O(\rho^2)]$$

$$- \int_{\ln A}^{\ln(1/x)} dz I_1^2[\sigma(z)], \quad (A12)$$

where we have used the integration by parts and the results from the previous case (b).
Since, for $\sigma \to \infty$, $I_1(\sigma) = I_0(\sigma) \cdot [1 + O(\rho)]$, we have

$$\int_x^{1/A} \frac{dy}{y} [f_g^+(y)]^2 \sim \frac{1}{2\rho} I_1(\sigma) I_0(\sigma)$$

$$\cdot [1 + O(\rho^2)]$$

$$\xrightarrow{M} \left[\frac{1}{n-1} + O(n-1)\right] f_2^{++}(n). \quad (A13)$$

In the same way, we obtain

$$\int_{x/2}^{1/A} \frac{dy}{y} [f_g^+(y)]^2 \sim \frac{1}{2\rho_2} I_1(\sigma_2) I_0(\sigma_2) \cdot [1 + O(\rho^2)]$$

$$= \frac{1}{2\rho} I_1(\sigma) I_0(\sigma) + \ln 2 I_0^2(\sigma) \cdot [1 + O(\rho)]$$

$$\xrightarrow{M} \left[\frac{1}{n-1} + \ln 2 + O(n-1)\right] f_2^{++}(n) \quad (A14)$$

and, from Eqs. (A13) and (A14),

$$\int_{x/2}^x \frac{dy}{y} [f_g^+(y)]^2 = \ln 2 (f_g^+)^2 \cdot [1 + O(\rho)]$$

$$\xrightarrow{M} [\ln 2 + O(n-1)] f_2^{++}(n). \quad (A15)$$

Finally we get

$$\int_x^{1/A} \frac{dy}{y} f_g^2(y) = \int_x^1 \frac{dy}{y} f_g^2(y) \cdot [1 + O(\rho^2)]$$
$$- \ln A [f_g^-(x)]^2$$
$$= \int_x^1 \frac{dy}{y} f_g^2(y) \cdot [1 + O(\rho^2)]$$
$$\xrightarrow{M} \left[\frac{1}{n-1} + O(n-1) \right] f_2(n), \quad \text{(A16)}$$

because [see Eqs. (16) and (18)]

$$\frac{[f_g^-(x)]^2}{f_g^2(x)} \sim I_0^{-2}(\sigma) \ll O(\rho^2). \tag{A17}$$

Thus, the result of (A16) does not depend on the specific value of A. Analogously we find

$$\int_{x/2}^{1/A} \frac{dy}{y} f_g^2(y) = \int_x^1 \frac{dy}{y} f_g^2(y) + \ln 2 f_g^2(x) \cdot [1 + O(\rho)]$$
$$\xrightarrow{M} \left[\frac{1}{n-1} + \ln 2 + O(n-1) \right] f_2(n). \tag{A18}$$

By comparing Eq. (A18) for $A = 2$ and Eq. (24), we get

$$2^{n-1} \frac{1}{n-1} f_2(n|1/2)$$
$$= \left[\frac{1}{n-1} + \ln 2 + O(n-1) \right] f_2(n) \tag{A19}$$

and conclude that

$$f_2(n|1/2) = f_2(n) + O[(n-1)^2]. \tag{A20}$$

This completes the proof of Eq. (27). Equation (A20) is very important and helps us to sum the regular parts of $F_{ag}(x)$. We should remember, however, that this result holds only for x-independent input distributions, that is for $B_q = B_g = 0$ in Eq. (2).

Comparing Eqs. (A4), (A11), and (A15), we get finally

$$\int_{x/2}^x \frac{dy}{y} f_g^2(y) = \ln 2 f_g^2(x) \cdot [1 + O(\rho)]$$
$$\xrightarrow{M} \ln 2 f_2(n) + O(n-1). \tag{A21}$$

APPENDIX B: THE COEFFICIENTS $C_{ag}^{i,\pm}$

Here we give the explicit values of the coefficients $C_{ag}^{i,\pm}(n = 1) \equiv C_{ag}^{i,\pm}$ appearing in Eqs. (35) and (37):

$$C_{qg}^{1,-} = \frac{283}{2880}, \qquad C_{qg}^{2,-} = \frac{1813}{2880} - \frac{297}{32} f, \qquad C_{qg}^{1,+} = 0,$$
$$C_{qg}^{2,+} = \frac{297}{32} f, \qquad C_{gg}^{1,-} = -\frac{283}{6480}, \qquad C_{gg}^{2,-} = \frac{33}{8} f - \frac{1813}{6480},$$
$$C_{gg}^{1,+} = \frac{27}{32} \left(99 \ln 2 - \frac{255361}{4374} \right). \tag{B1}$$

For the coefficient $C_{gg}^{2,+}$ one must consider separately the singular and regular part,

$$C_{gg}^{2,+} = \hat{C}_{gg}^{2,+} \frac{1}{n-1} + \overline{C}_{gg}^{2,+}(n = 1),$$

where

$$\hat{C}_{gg}^{2,+} = \frac{2673}{32}, \qquad \overline{C}_{gg}^{2,+}(n = 1) = -\frac{33}{8} f - \frac{516577}{5184}. \tag{B2}$$

APPENDIX C: PROOF OF THE SIMPLIFIED FORM FOR THE FUNCTIONS $\tilde{F}_1$ AND $\tilde{F}_2$

The validity of the simplified form for the functions $\tilde{F}_1$ and $\tilde{F}_2$, given in Eqs. (56)–(61), is based on the following estimates. As in Appendix A, we consider separately the three components of the functions $\tilde{F}_1$ and $\tilde{F}_2$ given in Eq. (55).

(a) Consider first the "$--$" component. The function $[f_g^-(x, M^2)]^2$ is x-independent since we assumed $B_q = B_g = 0$: $[f_g^-(x, M^2)]^2 = [f_g^-(M^2)]^2$, so

$$f_2^{--}(n, M^2) = \frac{1}{n-1} [f_g^-(M^2)]^2. \tag{C1}$$

Then [see Eqs. (42) and (43)] we have

$$[\delta(1 - x) - \tilde{\rho} J_1(\tilde{\sigma})] \otimes [f_g^-(x, M^2)]^2$$
$$\xrightarrow{M} e^{\hat{d}_+ v/(n-1)} \frac{1}{n-1} [f_g^-(M^2)]^2$$
$$\xrightarrow{M^{-1}} J_0(\tilde{\sigma}) [f_g^-(x, M^2)]^2, \tag{C2}$$

$$J_0(\tilde{\sigma}) \otimes [f_g^-(x, M^2)]^2 \xrightarrow{M} e^{\hat{d}_+ v/(n-1)} \frac{1}{(n-1)^2}$$
$$\times [f_g^-(M^2)]^2$$
$$\xrightarrow{M^{-1}} \frac{1}{\tilde{\rho}} J_1(\tilde{\sigma}) [f_g^-(x, M^2)]^2. \tag{C3}$$

Thus,

$$\tilde{F}_1^{--} = (A_g^-)^2 J_0(\tilde{\sigma}) \cdot e^{-2d_-(1)w} [1 + O(\rho)], \tag{C4}$$

$$\tilde{F}_2^{--} = (A_g^-)^2 \frac{1}{\tilde{\rho}} J_1(\tilde{\sigma}) \cdot e^{-2d_-(1)w} [1 + O(\rho)]. \tag{C5}$$

(b) Consider now the "$+-$" component. The function $f_g^+(x, M^2)$ can be represented as

$$f_g^+(x, M^2) = A_g^+ I_0(\hat{\sigma}) e^{-\overline{d}_+(1)w}$$
$$\xrightarrow{M} \frac{1}{n-1} A_g^+ e^{-\hat{d}_+ w/(n-1)} e^{-\overline{d}_+(1)w}. \tag{C6}$$

Note that

$$[\delta(1 - x) - \tilde{\rho} J_1(\tilde{\sigma})] \otimes I_0(\hat{\sigma}) \xrightarrow{M} e^{\hat{d}_+ v/(n-1)}$$
$$\times \frac{1}{n-1} e^{-\hat{d}_+ w/(n-1)}$$
$$= \frac{1}{n-1} e^{-\hat{d}_+ t/(n-1)}$$
$$\xrightarrow{M^{-1}} I_0(\sigma). \tag{C7}$$

In the same way we obtain

$$J_0(\tilde{\sigma}) \otimes I_0(\hat{\sigma}) \xrightarrow{M} \frac{1}{n-1} e^{\hat{d}_+ v/(n-1)} \frac{1}{n-1} e^{-\hat{d}_+ w/(n-1)}$$

$$= \frac{1}{(n-1)^2} e^{-\hat{d}_+ t/(n-1)} \xrightarrow{M^{-1}} \frac{1}{\rho} I_1(\sigma). \quad (C8)$$

Thus, we get

$$\tilde{F}_1^{+-} \equiv -\tilde{\rho} J_1(\tilde{\sigma}) \otimes [f_g^+(x, M^2) f_g^-(x, M^2)]$$

$$= A_g^- A_g^+ I_0(\sigma) \cdot e^{-[d_-(1)+\overline{d}_+(1)]w} [1 + O(\rho)], \quad (C9)$$

$$\tilde{F}_2^{+-} \equiv J_0(\tilde{\sigma}) \otimes [f_g^+(x, M^2) f_g^-(x, M^2)]$$

$$= A_g^- A_g^+ \frac{1}{\rho} I_1(\sigma) \cdot e^{-[d_-(1)+\overline{d}_+(1)]w} [1 + O(\rho^2)]. \quad (C10)$$

(c) Finally, let us consider the "$++$" component. The problem of finding the small-x behavior of $J_0(\tilde{\sigma}) \otimes I_0^2(\hat{\sigma})$ can be solved as follows. From the tables of the inverse Mellin transforms in Ref. [43], we obtain

$$\int_0^1 I_\nu[(\alpha^{1/2} + \beta^{1/2})y^{1/2}] I_\nu[(\alpha^{1/2}$$

$$- \beta^{1/2})y^{1/2}] x^{\gamma-1} dx$$

$$= \frac{1}{\gamma} \exp\left(\frac{\alpha+\beta}{2\gamma}\right) I_\nu\left(\frac{\alpha-\beta}{2\gamma}\right), \quad (C11)$$

where $y = -\log x$ and $\mathrm{Re}\,\nu > -1$, $\mathrm{Re}\,\gamma > 0$.
Equation (C11) gives, for $\beta = \nu = 0$, $\alpha = -4\hat{d}_{gg}w$ and $\gamma = n-1$, the Mellin transform of $I_0^2(\hat{\sigma})$:

$$I_0^2(\hat{\sigma}) \xrightarrow{M} \frac{1}{n-1} e^{-2\hat{d}_+ w/(n-1)} I_0\left(\frac{-2\hat{d}_+ w}{n-1}\right). \quad (C12)$$

The same result can be obtained in an alternative way by performing the Mellin transform of the series

$$I_0^2(\hat{\sigma}) = \frac{1}{\sqrt{\pi}} \sum_0^\infty \frac{(\hat{\sigma})^{2m} \Gamma(m+1/2)}{\Gamma^2(m+1)}.$$

From this first step we get

$$J_0(\tilde{\sigma}) \otimes I_0^2(\hat{\sigma}) \xrightarrow{M} \frac{1}{(n-1)^2} e^{-\hat{d}_+(w+t)/(n-1)} I_0\left(\frac{-2\hat{d}_+ w}{n-1}\right) \quad (C13)$$

and the inverse Mellin transform of the right hand side of (C13) will be the final answer for $\tilde{F}_2^{++}$ [apart

from the overall factor $(A_g^+)^2$].
We consider now Eq. (C11) for $\nu = 0$. By taking the sum of the derivative of Eq. (C11) with respect to α with its derivative with respect to β, we obtain the important formula

$$\frac{1}{2\sqrt{\alpha\beta}} \int_0^1 dx\, x^{\gamma-1} y^{1/2} \{(-\alpha^{1/2} + \beta^{1/2})$$

$$\times I_0[(\alpha^{1/2} + \beta^{1/2})y^{1/2}]$$

$$\times I_1[(\alpha^{1/2} - \beta^{1/2})y^{1/2}] + (\alpha^{1/2} + \beta^{1/2})$$

$$\times I_0[(\alpha^{1/2} - \beta^{1/2})y^{1/2}]$$

$$\times I_1[(\alpha^{1/2} + \beta^{1/2})y^{1/2}]\}$$

$$= \frac{1}{\gamma^2} e^{(\alpha+\beta)/(2\gamma)} I_0\left(\frac{\alpha-\beta}{2\gamma}\right), \quad (C14)$$

where, as before, $\gamma \equiv n-1$ and $y = -\log x$. With the substitutions

$$\alpha \to -\hat{d}_+(3w+t), \quad \beta \to -\hat{d}_+(t-w), \quad (C15)$$

Equation (C14) provides the final answer, since the coefficient of $x^{\gamma-1}$ in the integral at the left hand side gives the required inverse Mellin transform.
A similar procedure can be applied to determine $\tilde{\rho} J_1(\tilde{\sigma}) \otimes I_0^2(\hat{\sigma})$, which is needed to calculate $\tilde{F}_1^{++}$, starting from the formula (C11), with the same values of the parameters α and β as in (C15) and $y = -\log x$. Notice that $\beta < 0$ and hence $\beta^{1/2}$ is pure imaginary.
After introducing, for the sake of simplicity, the notation $z = \alpha^{1/2} + i|\beta^{1/2}|$, we get

$$\tilde{F}_1^{++} \equiv [\delta(1-x) - \tilde{\rho} J_1(\tilde{\sigma})] \otimes [f_g^+(x, M^2)]^2$$

$$= (A_g^+)^2 I_0(zy^{1/2}) I_0(\bar{z}y^{1/2}) \cdot e^{-2\overline{d}_+(1)w} \cdot [1 + O(\rho)], \quad (C16)$$

while, with the same definition of the variables α, β and y, we get

$$\tilde{F}_2^{++} \equiv J_0(\tilde{\sigma}) \otimes [f_g^+(x, M^2)]^2$$

$$= (A_g^+)^2 \frac{1}{2\sqrt{\alpha\beta}} y^{1/2} [z I_0(\bar{z}y^{1/2}) I_1(zy^{1/2})$$

$$- \bar{z} I_0(zy^{1/2}) I_1(\bar{z}y^{1/2})] \cdot e^{-2\overline{d}_+(1)w}$$

$$\cdot [1 + O(\rho^2)]. \quad (C17)$$

This completes the calculation of the simplified form of the functions $\tilde{F}_1$ and $\tilde{F}_2$.

[1] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rep. **100**, 1 (1983).

[2] A. H. Mueller and J. Qiu, Nucl. Phys. B **268**, 427 (1986); A. H. Mueller, Nucl. Phys. **B335**, 115 (1990).

[3] J. Bartels, J. Blümlein, and G. Schuler, Z. Phys. C **50**, 91 (1991).

[4] E. Laenen and E. M. Levin, Nucl. Phys. **B451**, 207 (1995); A. L. Ayala, M. B. Gay Ducati, and E. M. Levin, Nucl. Phys. **B511**, 355 (1998).

[5] J. Jalilian-Marian, A. Kovner, and H. Weigert, Phys. Rev. D **59**, 014015 (1999); J. Jalilian-Marian *et al.*, Phys. Rev. D **59**, 034007 (1999).

[6] M. H. Reno *et al.*, hep-ph/0110235.

[7] J. K. Golec-Biernat and M. Wüsthoff, Phys. Rev. D **59**, 014017 (1999); **60**, 114023 (1999).

[8] I. Balitski, Nucl. Phys. **B463**, 99 (1996).

[9] Yu. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999).

[10] R. Basu, D. Choudhury, and S. Majhi, J. High Energy Phys. 10 (2002) 012.

[11] J. Jalilian-Marian, Phys. Rev. D **68**, 054005 (2003).

[12] K. Kutak and J. Kwieciński, Eur. Phys. J. C **29**, 521 (2003).

[13] M. V. T. Machado, Phys. Rev. D **70**, 053008 (2004).

[14] V. N. Gribov and L. N. Lipatov, Yad. Fiz. **15**, 781 (1972); [Sov. J. Nucl. Phys. **15**, 438 (1972)].

[15] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977); L. N. Lipatov, Yad. Fiz. **20**, 181 (1974); [Sov. J. Nucl. Phys. **20**, 94 (1974)]; Yu. L. Dokshitzer, Zh. Eksp. Teor. Fiz. **73**, 1216 (1977)[Sov. Phys. JETP **46**, 641 (1977)].

[16] V. S. Fadin, E. A. Kuraev, and L. N. Lipatov, Phys. Lett. B **60**, 50 (1975); E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Zh. Eksp. Teor. Fiz. **71**, 840 (1976); Sov. Phys. JETP **44**, 443 (1976); Zh. Eksp. Teor. Fiz. **72**, 377 (1977); Sov. Phys. JETP **45**, 199 (1977); Ya. Ya. Balitsky and L. N. Lipatov, Yad. Fiz. **28**, 1597 (1978); Sov. J. Nucl. Phys. **28**, 822 (1978).

[17] D. W. McKay and J. P. Ralston, Phys. Lett. B **167**, 103 (1986); G. M. Frichter, D. W. McKay, and J. P. Ralston, Phys. Rev. Lett. **74**, 1508 (1995).

[18] R. Gandhi *et al.*, Phys. Rev. D **58**, 093009 (1998); Astropart. Phys. **5**, 81 (1996).

[19] J. Kwieciński, A. D. Martin, and A. M. Staśto, Phys. Rev. D **59**, 093002 (1999).

[20] A. D. Rújula *et al.*, Phys. Rev. D **10**, 1649 (1974).

[21] R. D. Ball and S. Forte, Phys. Lett. B **336**, 77 (1994).

[22] L. Mankiewicz, A. Saalfeld, and T. Weigl, Phys. Lett. B **393**, 175 (1997).

[23] W. Zhu, Nucl. Phys. **B551**, 245 (1999); W. Zhu and J. Ruan, Nucl. Phys. **B559**, 378 (1999).

[24] W. Zhu *et al.*, Phys. Rev. D **68**, 094015 (2003).

[25] J. Blümlein *et al.*, Phys. Lett. B **504**, 235 (2001).

[26] C. Lopez and F. J. Ynduráin, Nucl. Phys. **B171**, 231 (1980); **B183**, 157 (1981).

[27] F. Martin, Phys. Rev. D **19**, 1382 (1979).

[28] A. V. Kotikov, Yad. Fiz. **57**, 142 (1994); [Phys. At. Nucl. **57**, 133 (1994)]; Phys. Rev. D **49**, 5746 (1994).

[29] A. V. Kotikov and G. Parente, Nucl. Phys. **B549**, 242 (1999).

[30] R. Fiore *et al.*, Phys. Rev. D **68**, 093010 (2003).

[31] A. Yu. Illarionov, A. V. Kotikov, and G. Parente, hep-ph/0402173.

[32] L. Jenkovszky, F. Paccanoni, and E. Predazzi, Nucl. Phys. B, Proc. Suppl. **25**, 80 (1992).

[33] P. Desgrolard *et al.*, Phys. Lett. B **309**, 191 (1993).

[34] M. Bertini *et al.*, Rivista Nuovo Cimento **19**, 1 (1996).

[35] P. Desgrolard *et al.*, Phys. Lett. B **459**, 265 (1999).

[36] L. Csernai *et al.*, Eur. Phys. J. C **24**, 205 (2002).

[37] L. Jenkovszky, A. Lengyel, and F. Paccanoni, Nuovo Cimento A **111**, 551 (1998).

[38] ZEUS Collaboration, S. Chekanov *et al.*, Phys. Rev. D **67**, 012007 (2003); see also http://durpdg.dur.ac.uk/HEPDATA/PDF that includes PDF error calculations from ZEUS 2002 PDFs.

[39] CCFR/NuTeV Collaboration, U. K. Yang *et al.*, Phys. Rev. Lett. **87**, 251802 (2001); U. K. Yang, Ph.D. thesis, University of Rochester, 2001.

[40] H1 Collaboration, T. Ahmed *et al.*, Phys. Lett. B **324**, 241 (1994).

[41] M. Glück, E. Reya, and A. Vogt, Eur. Phys. J. C **5**, 461 (1998).

[42] E. Stein *et al.*, Nucl. Phys. **B536**, 318 (1998).

[43] Bateman Manuscript Project, *Tables of Integral Transforms* (McGraw-Hill, New York, 1954), Vol. I, p. 358.