

Leptogenesis and $\mu - \tau$ symmetry

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If an exact $\mu \leftrightarrow \tau$ symmetry is the explanation of the maximal atmospheric neutrino mixing angle, it has interesting implications for the origin of matter via leptogenesis in models where small neutrino masses arise via the seesaw mechanism. For seesaw models with two right-handed neutrinos (N_μ, N_τ), lepton asymmetry vanishes in the exact $\mu \leftrightarrow \tau$ symmetric limit, even though there are nonvanishing Majorana phases in the neutrino mixing matrix. On the other hand, for three right-handed neutrino models, lepton asymmetry is nonzero and is given directly by the solar mass difference square. We also find an upper bound on the lightest neutrino mass.

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I. INTRODUCTION

One of the most puzzling aspects of neutrino mixings observed in various oscillation experiments is the near maximal value of the $\nu_\mu - \nu_\tau$ mixing angle (i.e., $\theta_{23} \approx \pi/4$). This was needed to explain the original atmospheric neutrino data and is now supported by data from the K2K experiment that uses accelerator neutrinos. The corresponding parameter in the quark sector is very small (about 4%) and is believed to be connected to the mass hierarchy among quarks. The large value of θ_{23} may therefore be telling us about some new symmetries of leptons that are not present in the quark sector and may provide a clue to understanding the nature of quark-lepton physics beyond the standard model.

To explore this further, the first step is to write down the neutrino mass matrix that leads to a near maximal θ_{23} and then try to see what physics leads to it. It is well known that the neutrino mixings are a combined effect of the structure of both the charged lepton and the neutrino mass matrices. If we write

$$\mathcal{L}_m = \nu_\alpha^T C^{-1} \mathcal{M}_{\nu, \alpha\beta} \nu + \bar{e}_{\alpha,L} M_{\alpha\beta}^e e_R + h.c., \quad (1)$$

diagonalizing the mass matrices by the transformations $U_\nu^T \mathcal{M}_\nu U_\nu = \mathcal{M}_{\text{diag}}^\nu$ and $U_\ell^\dagger M^e V = M_{\text{diag}}^e$, gives the lepton mixing matrix $U_{\text{PMNS}} = U_\ell^\dagger U_\nu$. It is conventional to parameterize U_{PMNS} in terms of three angles θ_{12} (the solar angle), θ_{23} (the atmospheric angle) and θ_{13} the reactor angle as well as three phases. Our goal is to understand the near maximal value of θ_{23} using a leptonic symmetry and study its implications.

A fundamental theory can of course determine the structure of both the charged lepton and the neutrino mass matrices and therefore will lead to predictions about lepton mixings. However, in the absence of such a theory, if one wants to adopt a model independent approach and look for symmetries that may explain the value of θ_{23} , it is useful to work in a basis where charged leptons are mass eigenstates and hope that any symmetries for leptons revealed in this basis are true or approximate symmetries of Nature.

In the basis where charged leptons are mass eigenstates, a symmetry that has proved useful in understanding maximal atmospheric neutrino mixing is $\mu \leftrightarrow \tau$ interchange symmetry [1]. The mass difference between the muon and the tau lepton of course breaks this symmetry. So we expect this symmetry to be an approximate one. It may however happen that the symmetry is truly exact at a very high scale; but at low mass scales, the effective theory only has the $\mu - \tau$ symmetry in the neutrino couplings but not in the charged lepton sector so that we have $m_\tau \gg m_\mu$ [2]. We will consider this class of theories in this note. For this case, a convenient parameterization of the neutrino mass matrix is (assuming the neutrinos to be Majorana fermions):

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m_A^2}}{2} \begin{pmatrix} c\epsilon^n & d\epsilon & d\epsilon \\ d\epsilon & 1 + \epsilon & -1 \\ d\epsilon & -1 & 1 + \epsilon \end{pmatrix}, \quad (2)$$

where $n \geq 1$. An immediate prediction of this mass matrix is that $\theta_{23} = \pi/4$ and $\theta_{13} = 0$; we also get $\epsilon \sim \sqrt{\Delta m_\odot^2 / \Delta m_A^2}$.

We can now use θ_{13} as a probe of how leptonic $\mu \leftrightarrow \tau$ symmetry is broken in Nature and through that one may hope for an understanding of the origin of the near maximal (maximal ?) θ_{23} , as has been emphasized in Ref. [3] (and also perhaps the $\mu - \tau$ mass difference). In particular, different ways of breaking $\mu \leftrightarrow \tau$ symmetry will lead to $\theta_{13} \sim \sqrt{\Delta m_\odot^2 / \Delta m_A^2}$ or $\theta_{13} \sim \Delta m_\odot^2 / \Delta m_A^2$. These predictions are clearly timely and interesting in view of many proposals to measure the parameter θ_{13} [4,5]¹.

In this paper, we discuss implications of exact $\mu \leftrightarrow \tau$ symmetry for the origin of matter via leptogenesis [6] and find several new results: (i) we find that if there are only two right-handed neutrinos (N_μ, N_τ) that via seesaw

¹The $\mu - \tau$ symmetry in supersymmetric seesaw models also leads to other phenomenological predictions such as the $B(\mu \rightarrow e + \gamma)/B(\mu \rightarrow e \nu \bar{\nu}) = B(\tau \rightarrow e + \gamma)/B(\tau \rightarrow e \nu \bar{\nu})$.

mechanism lead to neutrino masses, then primordial lepton asymmetry arising from right-handed neutrino decay vanishes in the $\mu - \tau$ symmetric limit even though in the low energy neutrino mass matrix may have Majorana phases; (ii) secondly, for the case of three right-handed neutrinos, the primordial lepton asymmetry is directly proportional to the solar mass difference square. These predictions are very different from the generic three neutrino case [7]. In both these case we assume that neutrino masses arise via the type I seesaw formula [8]. These results are independent of any detailed model.

II. PRIMORDIAL LEPTON ASYMMETRY WITH TWO RIGHT-HANDED NEUTRINOS

We start with the neutrino part of the superpotential:

$$W = e^{cT} \mathbf{Y}_\ell L H_d + N^{cT} \mathbf{Y}_\nu L H_u + \frac{1}{2} \mathbf{M}_R N^{cT} N^c \quad (3)$$

where we assume that $N^c \equiv (N_\mu^c, N_\tau^c)$. As noted earlier, we work in a basis where \mathbf{Y}_ℓ is diagonal. While naively, one may think that in such models $m_\mu = m_\tau$, there are models where one can split the muon and tau masses consistent with this symmetry in the neutrino sector [2].

The basic assumption of this work is that we have models where \mathbf{Y}_ν and \mathbf{M}_R obey $\mu \leftrightarrow \tau$ symmetry under which $(N_\mu \leftrightarrow N_\tau)$ and $L_\mu \leftrightarrow L_\tau$ whereas the $m_\mu \neq m_\tau$. The general structure of \mathbf{Y}_ν and \mathbf{M}_R are then given by:

$$\mathbf{M}_R = \begin{pmatrix} M_{22} & M_{23} \\ M_{23} & M_{22} \end{pmatrix} \quad (4)$$

$$\mathbf{Y}_\nu = \begin{pmatrix} h_{11} & h_{22} & h_{23} \\ h_{11} & h_{23} & h_{22} \end{pmatrix}.$$

The seesaw formula in our notation is

$$\mathcal{M}_\nu = -\mathbf{Y}_\nu^T \mathbf{M}_R^{-1} \mathbf{Y}_\nu v_{wk}^2 \quad (5)$$

and the formula for primordial lepton asymmetry in this case, caused by right-handed neutrino decay is[9]

$$\epsilon_1 = \frac{1}{4\pi} \sum_j \frac{Im[\tilde{Y}_\nu \tilde{Y}_\nu^\dagger]_{12}^2}{(\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{11}} F\left(\frac{M_1}{M_2}\right), \quad (6)$$

where \tilde{Y}_ν is defined in a basis where right-handed neutrinos are mass eigenstates and $F(x) \simeq -\frac{3}{2}x$ for small x which follows from our assumption that the right-handed neutrino masses are hierarchical. In order to use this formula, we must diagonalize the right-handed neutrino mass matrix and change the Y_ν to \tilde{Y}_ν . Since \mathbf{M}_R is a symmetric complex 2×2 matrix, it can be diagonalized by a transformation matrix

$$U(\pi/4) \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix},$$

i.e., $U(\pi/4) \mathbf{M}_R U^T(\pi/4) = \text{diag}(M_1, M_2)$ where $M_{1,2}$ are

complex numbers. In this basis we have $\tilde{\mathbf{Y}}_\nu = U(\pi/4) \mathbf{Y}_\nu$. We can therefore rewrite the formula for n_ℓ as

$$\epsilon_1 \propto \sum_j Im[U(\pi/4) \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger U^T(\pi/4)]_{12}^2 F\left(\frac{M_1}{M_2}\right). \quad (7)$$

Now note that $\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger$ has the form

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}$$

which can be diagonalized by the matrix $U(\pi/4)$. Therefore it follows that $n_\ell = 0$.

An interesting feature of this model is that one can determine the neutrino masses and mixings explicitly in terms of the parameters of the model. We find a hierarchical mass pattern, i.e., $m_1 \ll m_2 \ll m_3$ with the lightest neutrino being massless, i.e.,

$$m_1 = 0; \quad m_2 = \frac{2}{M_+} (h_+^2 + 2h_{11}^2); \quad (8)$$

$$m_3 = \frac{2}{M_-} (h_-^2),$$

where M_\pm are the masses of the two right-handed neutrinos with $M_- \ll M_+$ and $h_\pm = (h_{22} \pm h_{23})$.

Even though there is no lepton asymmetry in the model, there are Majorana CP phases in the light neutrino mixing which we denote by $K = (e^{i\alpha}, e^{-i\alpha}, 1)$. It is easy to see the origin of the phases: by appropriate choice of the phases of the fields one can show that M_R has only one phase and Y_ν also has only one phase. After using the seesaw formula, one gets the light neutrino mass matrix which therefore has only one phase after redefinition of the light neutrino fields.

A. $\mu - \tau$ SYMMETRY BREAKING WITH TWO RIGHT-HANDED NEUTRINOS

From the above discussion, it is natural to expect the model to have nonzero lepton asymmetry once $\mu - \tau$ symmetry is broken, as well as also a nonvanishing θ_{13} . One may then expect that $\epsilon_1 \propto \theta_{13}$. The details however depend on how the symmetry is broken. As an example we note that when the symmetry is broken only by the masses of the RH neutrinos, i.e., a RH neutrino mass matrix of the form $M_R = \text{diag}(M_1, M_2)$ and no off diagonal terms, since $Y_\nu Y_\nu^\dagger$ is a real matrix, $\epsilon_1 \propto Im[Y_\nu Y_\nu^\dagger]_{12} = 0$ despite the $\mu - \tau$ symmetry breaking. It is easy to check that $\theta_{13} \simeq c(\theta_A - \frac{\pi}{4}) \propto (M_1 - M_2) \neq 0$.

One may however break $\mu \leftrightarrow \tau$ symmetry in the Dirac mass terms for the neutrinos, i.e., in Y_ν . This can be done in many ways, e.g., by choosing

$$\mathbf{Y}_\nu = \begin{pmatrix} h_{11} & h_{22} & h_{23} \\ h_{12} & h_{23} & h_{22} \end{pmatrix}$$

or

$$\mathbf{Y}_\nu = \begin{pmatrix} h_{11} & h_{22} & h_{23} \\ h_{11} & h_{23} & h_{33} \end{pmatrix},$$

etc. In all these cases, one gets $\epsilon_1 \neq 0$ and also $\theta_{13} \neq 0$ and $\theta_A \neq \pi/4$. One lesson one can draw from this observation is that, if leptogenesis is the true mechanism for the origin of matter, then the limit on θ_{13} going down by an order of magnitude could teach us about the nature of right-handed neutrino spectrum. For instance, a very small θ_{13} (i.e., $\theta_{13} \leq \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$) would indicate a nearly exact $\mu - \tau$ symmetry and therefore sufficient leptogenesis would then require the existence of three right-handed neutrinos or some complicated way of breaking $\mu - \tau$ symmetry.

III. THE CASE OF THREE RIGHT-HANDED NEUTRINOS

In this case, the right-handed neutrino mass matrix M_R and the Dirac Yukawa coupling Y_ν can be written, respectively, as:

$$\mathbf{M}_R = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & M_{22} & M_{23} \\ M_{12} & M_{23} & M_{22} \end{pmatrix} \quad (9)$$

$$\mathbf{Y}_\nu = \begin{pmatrix} h_{11} & h_{12} & h_{12} \\ h_{21} & h_{22} & h_{23} \\ h_{21} & h_{23} & h_{22} \end{pmatrix},$$

where M_{ij} and h_{ij} are all complex.² An important property of these two matrices is that they can be cast into a block diagonal form by the transformation matrix

$$U_{23}(\pi/4) \equiv \begin{pmatrix} 1 & 0 \\ 0 & U(\pi/4) \end{pmatrix}$$

and then be subsequently diagonalized by the most general 2×2 unitary matrix as follows:

$$V^T(2 \times 2)U_{23}^T(\pi/4)M_R U_{23}(\pi/4)V(2 \times 2) = M_R^d, \quad (10)$$

where

$$V(2 \times 2) = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix},$$

where V is the most general 2×2 unitary matrix given by $V = e^{i\alpha}P(\beta)R(\theta)P(\gamma)$ with $P(\beta) = \text{diag}(e^{i\beta}, e^{-i\beta})$;

$$R(\theta) = \begin{pmatrix} c & s \\ -s & c \end{pmatrix};$$

(c, s being cosine and sine of θ respectively). We will denote $V(2 \times 2)$ simply by $V_{L,R}$ depending on whether it acts on left-handed or the right-handed neutrinos.

²After this paper was posted, it was brought to our attention that leptogenesis for a $\mu - \tau$ symmetric model with the specific restriction that $Y_\nu = \text{diag}(a, b, b)$ was considered in Ref. [10]. Our consideration is more general.

We now change to the basis where the right-handed neutrino mass matrix is diagonal [Eq. (10)]. The Dirac Yukawa coupling in this basis has the form

$$\tilde{Y}_\nu = V^T(2 \times 2)U_{23}^T(\pi/4)Y_\nu \quad (11)$$

Because of the special form of Y_ν dictated by $\mu \leftrightarrow \tau$ symmetry, it is easy to see that

$$\tilde{Y}_\nu = V^T(2 \times 2)Y'_\nu U_{23}^T(\pi/4) \quad (12)$$

where Y'_ν is in block diagonal form. An important point to realize at this stage is that the 3×3 matrix problem has reduced to a 2×2 problem. So all the matrices from now on will be 2×2 and the third neutrino (the heaviest of the light neutrinos) has completely ‘‘decoupled’’ from the considerations below of both seesaw formula for neutrino masses as well as lepton asymmetry. This is a direct consequence of $\mu - \tau$ symmetry and of course considerably simplifies the discussions.

Restricting to the 2×2 case, we can use the seesaw formula to write down the left-handed neutrino mass matrix as follows in units of $-v_{wk}^2$:

$$\mathcal{M}_\nu = -\tilde{Y}_\nu^T M_R^{d,-1} \tilde{Y}_\nu \quad (13)$$

Next, we go to a basis where \mathcal{M}_ν (the upper 2×2 block of it) is diagonalized by a matrix V_L , i.e., $V_L^T \mathcal{M}_\nu V_L = \mathcal{M}_\nu^d$. In this basis, the Dirac Yukawa coupling \tilde{Y}_ν becomes $V_L^T \tilde{Y}_\nu^T \equiv Y_\nu'^T$. Let us write $Y_\nu'^T$, which is a 2×2 matrix as

$$Y_\nu'^T = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}. \quad (14)$$

The Z_{ij} obey the constraints: $Z_{12} = -Z_{21} \frac{Z_{22} M_1}{Z_{11} M_2}$ and the neutrino masses are given by

$$m_1 = \frac{Z_{11}^2}{M_1} \rho e^{i\eta}, \quad m_2 = \frac{Z_{22}^2}{M_2} \rho e^{i\eta}, \quad (15)$$

where $\rho e^{i\eta} = (1 + \frac{M_1 Z_{21}^2}{M_2 Z_{11}^2})$.

Let us now calculate the out of equilibrium for the decay of the lightest right-handed neutrino, which we assume to be the lighter of the two mass eigenstates of the 2×2 right-handed neutrino mass matrix considered above. It is given by:

$$\Gamma_1 = \frac{1}{8\pi} (Y_\nu' Y_\nu'^{\dagger})_{11} M_1 = \frac{M_1 (|Z_{11}|^2 + |Z_{12}|^2)}{8\pi} \leq 14 \frac{M_1^2}{M_{Pl}}, \quad (16)$$

where M_{Pl} appears in the right-hand side from the Hubble expansion formula $H^2 \simeq \sqrt{g_*} T^2 / M_{Pl}$ in a radiation dominated Universe. Using Eq. (15), which gives $(|Z_{11}|^2 + |Z_{12}|^2) \simeq \frac{M_1}{v_{wk}^2} [|m_1| + |\rho e^{i\eta} - 1| |m_2|]$, we can rewrite this inequality as a constraint on the following combination of the masses of the two lightest neutrino eigenstates:

$$\frac{[|m_1| + |\rho e^{i\eta} - 1||m_2|]}{\rho} \leq 10^{-3} \text{ eV}. \quad (17)$$

For hierarchical right-handed neutrino mass spectrum (i.e., $M_2 \gg M_1$), $\rho \sim 1$ and we get

$$|m_1| + 2|m_2||\sin\eta/2| \leq 10^{-3} \text{ eV}. \quad (18)$$

This puts a limit on the two lightest neutrino masses. For instance, it implies that the lightest neutrino mass $m_1 \leq 10^{-3}$ eV. The solar neutrino oscillation would require $\sin\eta/2 \sim 0.07$ so that m_2 will match the central value required by data.

We now proceed to calculate the primordial lepton asymmetry ϵ_1 in this model. It turns out that ϵ_1 is directly proportional to the solar mass difference square as we show below. We start with the expression for ϵ_1 ,

$$\begin{aligned} \epsilon_1 &\simeq \frac{3}{8\pi} \frac{Im(Y'_\nu Y'^{\dagger}_{\nu'})^2_{12}}{(Y'_\nu Y'^{\dagger}_{\nu'})_{11}} \frac{M_1}{M_2} \\ &\equiv \frac{3}{8\pi} \frac{M_1}{M_2} \frac{Im(Z_{11}Z_{12}^* + Z_{21}Z_{22}^*)^2}{|Z_{11}|^2 + |Z_{12}|^2}. \end{aligned} \quad (19)$$

Using the constraints on Z_{ij} discussed in Eq. (15) and the relation just prior to it, we get, $Im(Z_{11}Z_{12}^* + Z_{21}Z_{22}^*)^2 = |Z_{11}|^4 Im(\frac{Z_{12}^2}{Z_{11}^2}) + |Z_{22}|^4 \frac{M_1^2}{M_2^2} Im(\frac{Z_{12}^*}{Z_{11}^*})$. Plugging this expression into Eq. (19), we can express the primordial lepton asymmetry ϵ_1 in terms of neutrino masses $m_{1,2}$ and the parameters ρ and η as follows:

$$\begin{aligned} \epsilon_1 &= \frac{3}{8\pi} \frac{M_1}{v_{wk}^2} \frac{\Delta m_\odot^2 \sin\eta}{|m_1| + |(\rho e^{i\eta} - 1)m_2|} \\ &\simeq 10^{-7} \left(\frac{M_1}{10^{10} \text{ GeV}} \right) \left(\frac{\Delta m_\odot^2}{8 \times 10^{-5} \text{ eV}^2} \right) \\ &\quad \times \frac{10^{-3} \text{ eV}}{[|m_1| + |\rho e^{i\eta} - 1||m_2|]} (\sin\eta/0.14). \end{aligned} \quad (20)$$

We see that the origin of matter in this model is predicted primarily in terms of the solar mass difference square and the unknown phase η whose value is already determined

by Eq. (18). Thus given a value for the lightest right-handed neutrino mass, the model predicts the value of primordial lepton asymmetry ϵ_1 . In Eq. (20), we have assumed $M_1 \simeq 10^{10}$ GeV. Note that our result is based on only three assumptions: (i) type I seesaw formula for neutrino masses and (ii) the existence of $\mu \leftrightarrow \tau$ symmetry and (iii) hierarchy among right-handed neutrinos. It may appear that lepton asymmetry goes to infinity as the neutrino masses go to zero but in this limit, the lightest right-handed neutrino in our model does not decay and the numerator of the ϵ_1 expression in Eq. (17) also goes to zero and no lepton asymmetry appears.

To the best of our knowledge, the connection between ϵ_1 and solar neutrino mass difference that we have found is not present in generic seesaw models without $\mu \leftrightarrow \tau$ symmetry. It is also interesting that origin of matter is tied not only to the existence of solar neutrino oscillations but also it is the large mixing angle solution to the solar neutrino problem that reproduces the correct order of magnitude for the lepton asymmetry which after taking into the dilution factor [11] and sphaleron effects, can give rise to the magnitude for the observed baryon to photon ratio. The value of 10^{10} GeV for the mass of the lightest right-handed neutrino is chosen to show that the model, when embedded into an extension of minimal supersymmetric standard model, can avoid the reheat temperature constraint coming from gravitino production.

In conclusion, we have discussed the consequences of the hypothesis that the large atmospheric neutrino mixing angle arises from an intrinsic $\mu - \tau$ symmetry for leptons for origin of matter via leptogenesis. We point out that if there are two right-handed neutrinos obeying $\mu - \tau$ interchange symmetry, then lepton asymmetry vanishes whereas for three right-handed neutrinos, it is given directly the solar mass difference square provided one assumes type I seesaw formula for neutrino masses. We also obtain an upper limit on the lightest neutrino mass of a milli-eV under these assumptions.

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