# String theories with deformed energy-momentum relations, and a possible nontachyonic bosonic string

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We consider a prescription for introducing deformed dispersion relations in the bosonic string action. We find that in a subset of such theories it remains true that the embedding coordinates propagate linearly on the world sheet. While both the string modes and the center of mass propagate with deformed dispersion relations, the speed of light remains energy independent. We consider the canonical quantization of these strings and find that it is possible to choose theories so that ghost modes still decouple, as usual. We also find that there are examples where the tachyon is eliminated from the spectrum of the free bosonic string.

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## I. INTRODUCTION

Several experimental and theoretical developments point to the possibility that the usual relation between energy and momentum valid within the special theory of relativity,

$$E^2 = p^2 + m^2, \tag{1}$$

may be modified at Planck scales. For instance, the high-energy cosmic ray anomalies [1,2] may be solved if there are Planck scale departures from these relations [3–7]. It has also been shown that one may establish an observer independent border between the classical and quantum pictures of space-time [8–10], by means of specially designed deformed dispersion relations.

The usual argument linking these deformations to quantum gravity is simple [10]. The combination of gravity ($G$), the quantum ($\hbar$), and relativity ($c$) gives rise to the Planck length,

$$L_P = \sqrt{\frac{\hbar G}{c^3}}, \tag{2}$$

or its inverse, the Planck energy

$$E_P = \sqrt{\frac{\hbar c^5}{G}}. \tag{3}$$

These scales mark thresholds beyond which the old description of space-time breaks down and qualitatively new phenomena are expected to appear. However, the new theory has to agree with special relativity for experiments probing the nature of space-time at energy scales much smaller than $E_P$. The question, then, arises: *in whose reference frame are $L_P$ and $E_P$ the thresholds for new phenomena?* It is clear that the Lorentz-Fitzgerald contraction cannot apply all the way down to the Planck scale, if we are to avoid introducing preferred frame in quantum gravity. A possible way out is the introduction of nonlinear realizations of the Lorentz group, associated with deformed dispersion relations. This possibility, sometimes called "doubly special relativity," has been explored in a number of recent papers [7–10].

The most important reason to investigate this possibility is that it is an hypothesis that will be testable in experiments to be carried out over the next decade. What if a combination of low energy and astrophysical experiments found evidence of this kind of theory, such as energy dependent speed of light and/or modifications in relativistic energy-momentum conservation laws? What would be the implications for various candidate quantum theories of gravity? Given that the relevant experiments are planned, it is important if the different approaches to quantum gravity make predictions for their outcomes.

It has recently been shown that this possibility is realized in the case of quantum gravity coupled to point particles in $2 + 1$ dimensions [11]. This shows that there are physically sensible interacting quantum theories of gravity that are described in terms of deformed dispersion relations. As far as $3 + 1$ dimensions are concerned, there are as yet no definitive results, but there are preliminary indications [12,13] that such deformations may emerge naturally in loop quantum gravity [14].

In this paper, we thus take up the question of whether phenomena associated with deformed dispersion relations may be realized in a consistent string theory [15–17]. We consider realizations of deformed dispersion relations in which Lorentz symmetry is deformed rather than broken. For simplicity, we study here only the bosonic string.

In Sec. II we present the formalism for introducing deformed dispersion relations into the action of the bosonic string. We find that in most cases the vibrational modes do not linearize; however, there is a large class of examples in which they do. We study these in Sec. III and we are able then to first quantize the theory. Next, we examine the constraints imposed at the quantum level. In Sec. IV we study the Virasoro algebra and spectrum of the deformed strings.

In the following section (Sec. V) we look at three simple examples. The first employs a simple deformation of the energy-momentum relations, studied for particles in Ref. [10], in which there is a maximum invariant mass (or rest energy, to be more precise). We show that this remains the case in string theory, so that the rest energies of the string excitations are bounded from above.

In the next two examples, the deformed energy-momentum relations are chosen so that the tachyonic mode of the free bosonic string is eliminated. We find this an intriguing result, as the possibility of a consistent, tachyon free bosonic string would obviously be of great importance for string theory.

## II. THE ACTION AND EQUATIONS OF MOTION

It is possible to write the bosonic string action in the form

$$S = \int d\sigma d\tau (\dot{x}^a p_a - N\mathcal{H} - M\mathcal{D}), \tag{4}$$

where $N$ and $M$ are Lagrange multipliers, $\mathcal{H}$ is the Hamiltonian constraint, and $\mathcal{D}$ is the constraint associated with spatial diffeomorphism invariance on the string. Together they generate diffeomorphisms on the world sheet. However, we now write the Hamiltonian constraint as

$$\mathcal{H} = \frac{f}{2T}\eta^{ab}p_a p_b + \frac{Tg}{2}\eta_{ab}x^{a'}x^{b'}, \tag{5}$$

where $T$ is the string tension and the prime represents a derivative with respect to $\sigma$. It can be checked that this action is reparametrization invariant. Here $f$ and $g$ are functions of the total energy

$$\mathbf{P}_0 = \int d\sigma p_0 \tag{6}$$

and are expected to encode all deviations from linear special relativity. Furthermore,

$$\mathcal{D} = \sqrt{fg}p^a x'_a. \tag{7}$$

With these modifications we can still write the algebra of constraints as

$$\begin{aligned} L_n &= \int d\sigma e^{in\sigma} 2(\mathcal{H} + \mathcal{D}) \\ &= \int d\sigma e^{in\sigma}\left(\sqrt{\frac{f}{T}}p_a + \sqrt{Tg}x'_a\right)^2. \end{aligned} \tag{8}$$

Notice that the theory we have just proposed is not a mere redefinition of the string tension. First, $f$ and $g$ multiply $T$ differently in the various terms of the constraints. Second, even if one considers each of these terms on its own, it looks as if the tension is being renormalized by a factor dependent on the total string energy. The latter is the zero component of a vector. Thus, such a "renormalized" tension would no longer be a scalar, invariant for all observers. One may expect a variety of new phenomena/pathologies to appear, an expectation we shall soon confirm.

The equations of motion can be found, e.g., from Hamilton's equation, and we note that the space and time coordinates in target space are now to be treated differently. For the spacial coordinates, we find

$$\dot{x}^i = \frac{\partial\mathcal{H}}{\partial p_i} = \frac{f}{T}p^i, \tag{9}$$

$$\dot{p}_i = -\frac{\partial\mathcal{H}}{\partial x^i} = Tg\partial_\sigma^2 x_i, \tag{10}$$

from which we may infer

$$\ddot{x}^i - fg\partial_\sigma^2 x^i = 0. \tag{11}$$

Hence, we find our first result: the $x^i$ coordinates satisfy the wave equation, but the speed of light on the string depends upon the total energy $\mathbf{P}_0$ stored in the string:

$$c_s = \sqrt{fg}. \tag{12}$$

We denote this speed as $c_s$ because it is really a speed of sound, i.e., a speed of propagation of vibrations along the string. It can be easily proved that $\mathbf{P}_0$ remains a constant of motion for general $f$ and $g$.

A similar set of equations may be found for the time target space coordinate,

$$\dot{x}^0 = \frac{\partial\mathcal{H}}{\partial p_i} = \frac{f}{T}p^0 + \frac{f'}{2T}p^2 + \frac{Tg'}{2}x'^2, \tag{13}$$

$$\dot{p}_0 = -\frac{\partial\mathcal{H}}{\partial x^0} = Tg\partial_\sigma^2 x_0. \tag{14}$$

As a result, we find that the $x^0$ coordinate in general satisfies a rather complicated nonlinear equation, coupled to the $x^i$ coordinates. The only exception occurs if $f = g$. Then the new terms are proportional to $\mathcal{H}$ and so vanish as a result of the Hamiltonian constraint. Hence, we learn that the deformations with $f = g$ play a special role in string theory, as they preserve the equation

$$\ddot{x}^a - fg\partial_\sigma^2 x^a = 0 \tag{15}$$

for $a = 0, i$.

It is very difficult to find solutions in the coupled case. The bosonic string has become nonlinear and the quanta travelling along the string interact with each other. This unpleasant property already plagues standard $p$-branes with $p > 1$.

In order to build further intuition about the meaning of the case $f = g$, in the appendix we consider the analogue construction for point particles.

## III. SOLUTIONS AND CANONICAL QUANTIZATION

The particular case where the string remains linear is, therefore, the only one where we shall be able to perform a full study. In this case, we may introduce light-cone variables:

$$\sigma^{\pm} = \sqrt{fg}\tau \pm \sigma. \tag{16}$$

It is important that in this definition the speed of sound multiplies the coordinate $\tau$, rather than divides the coordinate $\sigma$. In terms of $\sigma^{\pm}$, the wave Eq. (15) becomes

$$\partial_+\partial_- x^a = 0. \tag{17}$$

Its most general solution is

$$x^a = \mathbf{x}^a + \frac{\mathbf{P}^a_{\rm CM}}{\pi T}\tau + v(\sigma^+) + w(\sigma^-), \tag{18}$$

where $\mathbf{x}^a$ and $\mathbf{P}^{\rm CM}_a$ are center of mass integration constants. If we consider an open string with $\sigma \in [0, \pi]$, we can therefore write

$$x^a = \mathbf{x}^a + \frac{\mathbf{P}^a_{\rm CM}}{\pi T}\tau + \frac{i}{\sqrt{\pi T}}\sum_{n\neq 0}\frac{\alpha^a_n}{n}e^{-in\sqrt{fg}\tau}\cos(n\sigma). \tag{19}$$

Note that we must have $\alpha^a_{-n} = \alpha^{a\dagger}_n$, for $x^a$ to be real. The associated string momentum is

$$p_a = \frac{T}{f}\dot{x}_a = \frac{\mathbf{P}^a_{\rm CM}}{\pi f} + \sqrt{\frac{gT}{\pi f}}\sum_{n\neq 0}\alpha^a_n e^{-in\sqrt{fg}\tau}\cos(n\sigma). \tag{20}$$

We now proceed to canonically quantize this string, starting from the equal-time commutation relations:

$$[x^a(\sigma,\tau), p_b(\sigma',\tau)] = i\delta(\sigma-\sigma')\delta^a_b. \tag{21}$$

Using

$$\delta(\sigma-\sigma') = \frac{1}{\pi}\left[1 + 2\sum_{n=1}^{\infty}\cos(n\sigma)\cos(n\sigma')\right], \tag{22}$$

we thus arrive at

$$[\mathbf{x}^a, \mathbf{P}^{\rm CM}_b] = if\delta^a_b, \tag{23}$$

$$[\alpha^a_n, \alpha^b_{-n}] = \sqrt{\frac{f}{g}}n\eta^{ab}. \tag{24}$$

The first relation suggests an energy dependent Planck's constant, a phenomenon we have found before in nonlinear realizations of Lorentz invariance [7]. Given that we have assumed $f = g$, the second relation leads to trivial definitions of creation and annihilation operators:

$$\alpha^a_n = \sqrt{n}a^a_n, \tag{25}$$

$$\alpha^s_{-n} = \sqrt{n}a^{a\dagger}_n, \tag{26}$$

with

$$[a^a_n, a^{b\dagger}_m] = \delta_{mn}\eta^{ab}. \tag{27}$$

Should $f \neq g$, the whole quantization procedure should be different. One may argue that the extra nonlinear terms may be seen as an interaction and the system described by means of an $S$ matrix. If this is true, one should bear in mind that $f/g$ will still appear in the definition of creation and annihilation operators and thus in the asymptotic states.

## IV. THE QUANTUM CONSTRAINTS AND THE SPECTRUM

We now examine the constraints and their enforcement at quantum level, starting with the Hamiltonian constraint. Using Eqs. (19) and (20), the Hamiltonian (5) may be written in terms of the amplitudes $\alpha^a_n$. The quantum Hamiltonian, however, should contain only normal ordered $\alpha^a_n$, and so an ordering constant $a$ has to be added at quantum level. The result is

$$H = \int d\sigma\mathcal{H} = \frac{\mathbf{P}^2_{\rm CM}}{2\pi fT} + \frac{g}{2}\left(\sum_{n\neq 0}|\alpha^a_n|^2 - a\right). \tag{28}$$

By setting $H = 0$, we thus arrive at the string center of mass dispersion relations:

$$\frac{\eta_{ab}\mathbf{P}^a_{\rm CM}\mathbf{P}^b_{\rm CM}}{fg} = -M^2 = 2\pi T\left(a - \sum_{n\neq 0}|\alpha^a_n|^2\right). \tag{29}$$

Given (24), it is convenient to define

$$\beta^a_n = \left(\frac{g}{f}\right)^{1/4}\alpha^a_n \tag{30}$$

so that their algebra is energy independent. It is also convenient to define

$$\beta^a_0 = \frac{1}{f^{3/4}g^{1/4}}\mathbf{P}^a_{\rm CM}. \tag{31}$$

Their algebra is

$$[\mathbf{x}^a, \beta^b_0] = i\left(\frac{f}{g}\right)^{1/4}\delta^a_b, \tag{32}$$

$$[\beta^a_n, \beta^b_m] = n\eta^{ab}\delta_{n+m}. \tag{33}$$

We then define, as usual,

$$\tilde{M}^2 = \sum_{m>0}\beta_{-m}\beta_m. \tag{34}$$

In terms of these, the modified generators of the Virasoro algebra are

$$L_0 = \frac{1}{2\pi Tf}[\mathbf{P}^2_{\rm CM} + \sqrt{fg}\tilde{M}^2], \tag{35}$$

$$L_l = \sqrt{fg}\sum_{m>0}\beta_{l-m}\beta_m = \sqrt{fg}\tilde{L}_l, \tag{36}$$

where $\tilde{L}_l$ are, for $l \neq 0$, the conventional Viraroso generators. The algebra is

$$[L_m, L_n] = \sqrt{fg}(m-n)L_{m-n} + fg\frac{Dm(m^2-1)}{12}\delta_{m+n}. \tag{37}$$

We see that the anomaly appears to be energy dependent,

$$c = fg\frac{Dm(m^2-m)}{12}. \tag{38}$$

Of course, as long as $fg > 0$ for all $\mathbf{P}_0$ we can use the rescaled generators $\tilde{L}_n = (fg)^{-1/2}L_n$, which have the conventional Virasoro algebra, with an energy independent anomaly. However, as we will see shortly, there are interesting cases in which $fg$ vanishes for finite $\mathbf{P}_0$. In these cases, we should be careful about which version of the Virasoro algebra is defined on the whole space of physical states.

Now we study the spectrum of the theory. It is most convenient to define $k$ as

$$\mathbf{P}^a_{\mathrm{CM}}|0, k\rangle = k^a|0, k\rangle, \tag{39}$$

as $\mathbf{P}^a_{\mathrm{CM}}$ is in fact by (19) and (20) the quantity that defines the velocity of the center of mass of the string.

The ground state is defined as usual by $L_n|0, k\rangle = 0$. The ground state energy is given by

$$\mathbf{P}^2_{\mathrm{CM}} = k^2 = 2\pi T af\sqrt{fg}, \tag{40}$$

where $a$ is the usual energy independent constant resulting from the normal ordering of $\tilde{M}$.

We see from these relations that the spectrum of the deformed string will be the same as the spectrum for the ordinary bosonic string, so long as we express momentum in terms of the nonlinear variable

$$\tilde{p}^a = f^{-3/4}g^{-1/4}\mathbf{P}^a_{\mathrm{CM}}. \tag{41}$$

If $f\sqrt{fg} > 0$ and is nonvanishing and nonsingular, all the standard results on the string spectrum will go through regarding the elimination of ghosts, the existence of a tachyon, $a = 1$, etc. For example, we see directly from (40) that so long as $f\sqrt{fg} > 0$ the ground state remains tachyonic. However, if we choose to violate this condition, we can eliminate the tachyonic ground state, as we will describe below. With a suitable choice of functions, this can be done without reintroducing ghosts into the theory.

## V. EXAMPLES

### A. A simple example

Let us finally look at several examples that illustrate how much freedom is allowed by string theory, with regard to deforming the energy-momentum relations. We have found that, to keep the embedding degrees of freedom linear, we must take $f = g$. We have also found that the character of the spectrum is unchanged if $f^2$ is positive, nonvanishing, and nonsingular.

One consequence is that a varying speed of light is ruled out [18–21]. By computing $c = dE/dp$, for $M = 0$, one finds that massless modes always move at the speed of light. On the other hand, it is easy to accommodate the dispersion relation discussed in Ref. [10], by choosing

$$f = g = 1 - L_P\mathbf{P}^{\mathrm{CM}}_0. \tag{42}$$

Such a dispersion relation leads to a modified mass-energy relation [10]. Setting $l^2 = 1/(2\pi T)$ (the string "length"), the usual string spectrum mass spectrum [inferred from the right-hand side of (29)] is

$$l^2M^2_n = N - 1, \tag{43}$$

where $N = \sum_{n\neq 0}|\alpha^a_n|^2$. However, the rest energy spectrum is now

$$E_n = \frac{M_n}{1 + M_nL_P}. \tag{44}$$

If $l \gg L_P$, the lowest string states are uncorrected, but as the string energy approaches the Planck energy, states accumulate just below $E_P = L_P^{-1}$ and can never exceed this energy. This is precisely the property sought in Ref. [10], ensuring an invariant border between classical and quantum gravity.

### B. Eliminating the tachyon

It is also possible to choose deformations for which the ground state with negative $M^2$ is NOT a tachyon, a phenomenon already discussed in the context of neutrino flavor states [22]. As we are about to see, this can be done by choosing deformations such that $f^2$ is not positive over its entire range. Note that there is nothing wrong with a negative $f^2$, because, even though $f$ is then imaginary, it always leads to real Lorentz transformations (see [10] for how to construct them).

We can, for example, choose

$$f^2(\mathbf{P}^{\mathrm{CM}}_0) = 1 - (L_P\mathbf{P}^{\mathrm{CM}}_0)^2. \tag{45}$$

This fails to be positive only for high energies $\mathbf{P}^0_{\mathrm{CM}} > L_P^{-1}$. We get the rest mass-energy relation:

$$(\mathbf{P}^0_{\mathrm{CM}})^2 = \frac{M^2}{1 + (L_PM)^2}. \tag{46}$$

Using (43) leads to the energy spectrum. In this case, if we choose the string scale $l < L_P$ the tachyon state ($N = 0$) is no longer a tachyon. From (40), we have that at rest,

$$(\mathbf{P}^0_{\mathrm{CM}})^2 = \frac{2\pi T}{\frac{L_P^2}{l^2} - 1}. \tag{47}$$

Thus, whenever $L_P > l$ we have $(\mathbf{P}^0_{\mathrm{CM}})^2 > 0$. Indeed, the

$N = 0$ state is the only state with $\mathbf{P}^0_{\rm CM} > L_P^{-1}$; states with $N \gg 1$ accumulate just under $L_P^{-1}$. The lowest energy state is the first "excited" state $N = 1$ for which $M^2 = 0$. As a result, the rest of the spectrum is as in the ordinary bosonic string, expressed in terms of $\tilde{p}^2 = f^{-2}\mathbf{P}^2_{\rm CM}$.

Another example is given by

$$f^2(\mathbf{P}_0^{\rm CM}) = \frac{(\lambda \mathbf{P}_0^{\rm CM})^2}{2}\left[\pm\sqrt{1 + \left(\frac{2}{\lambda \mathbf{P}_0^{\rm CM}}\right)^2} - 1\right], \qquad (48)$$

where $\lambda$ is a length scale. The negative branch is chosen whenever $(\mathbf{P}_0^{\rm CM})^2 < 0$. With this choice, we can use $E^2/f^2 = M^2$ to find the rest mass-energy relation:

$$E_n^2 = \frac{M_n^2}{1 + \frac{1}{(\lambda M_n)^2}}. \qquad (49)$$

If $l > \lambda$ (a condition satisfied if $\lambda = L_P \ll l$), then the $N = 0$ state has $M^2 = -1/l^2$, but its rest energy squared is positive. The ground state is again the massless state $N = 1$ (for which both $M$ and the rest $E$ are zero). The $N = 0$ state has negative $M^2$ but behaves like a normal massive particle.

## VI. CONCLUSIONS

In this paper, we introduced deformed dispersion relations into string theory, considering the bosonic string and using perturbative canonical quantization methods. The results found here are only a first step, and more work is required to see if a consistent theory can be constructed along these lines.

The basic results we found may be summarized as follows.

(i) The deformations of the string action affect both the vibrational and center of mass modes. In most cases, we find that the vibrational modes no longer decouple and that only a very specific class of deformations preserve mode decoupling, those with $f = g$. This condition implies that the speed of light remains energy independent. Thus, it appears that the observation of an energy dependent speed of light would be difficult to fit into a consistent string theory.

(ii) So long as $fg > 0$ on the whole space of states, we can define conventional Virasoro generators, and the spectrum is as usual but with a transformed center of mass energy and momentum. But there are interesting cases in which $fg$ vanishes at finite $\mathbf{P}_0$, in which case care must be used to define the Virasoro generators for all states. The result can be an energy dependent central charge. It may still be possible to choose the ghost action so as to cancel the energy dependence of the central charge; this remains an interesting question for future work.

(iii) For a large class of theories, those for which $f = g$ and $f^2 > 0$ and nonsingular for its whole range, the usual conclusions concerning the elimination of ghosts and the presence of a tachyon hold.

(iv) By choosing $f^2 < 0$ for energies near the string scale, the tachyonic mode of the bosonic string can be eliminated.

(v) The choice

$$f^2 = 1 - (L_P \mathbf{P}_0^{\rm CM})^2 \qquad (50)$$

with $l_P < l$ is promising. It appears to produce a bosonic string without a tachyon, and it also appears to be ghost free.

Two final remarks are in order.

We stress that the prescription proposed here for introducing modified dispersion relations into string theories is by no means unique. It could be that other methods exist which bypass some of the potential problems and predictions of our approach.

Finally, string theory might shed new light into the multiparticle sector of doubly special relativity (and the so-called "soccer ball problem"; see, e.g., [7]). Within string theory, solutions to this problem must take as elementary objects strings, not point particles. Given that in this paper we do not consider multistring configurations (e.g., no string collisions are considered), we do not need to address the issue of how to add energies for the center of mass of different strings. Note that adding energies for the string internal degrees of freedom is a distinct matter. By construction, our treatment of the internal degrees of freedom does not break the world-sheet diffeomorphism invariance.

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## APPENDIX: PARTICLE ANALOGUE

In order to understand the significance of the $f = g$ case, we now consider a simple model of a relativistic massive particle in $D$-dimensional Minkowski space-time moving in a static potential. The action is given by

$$S = \int dt p_a \dot{x}^a - N\mathcal{H}, \qquad (A1)$$

where the Hamiltonian constraint is

$$\mathcal{H} = \frac{f(E)}{2}\eta^{ab}p_a p_b + g(E)[m^2 + V(x^i)], \qquad (A2)$$

where $f$ and $g$ are, as before, functions of $E = p_0$ and the potential is a function only of the spatial coordinates $x^i$, so that $E$ is still a conserved quantity. We find that the equations of motion are

$$\ddot{x}^i = -\frac{fgN}{m}\frac{\partial V}{\partial x^i}, \tag{A3}$$

$$\ddot{x}^0 = -\frac{N^2}{m}p_i\frac{\partial V}{\partial x^i}[fg' - f'g]. \tag{A4}$$

So we see that there is an anomalous acceleration of $x^0$ unless we eliminate the last factor by choosing $f = g$. In this case, we note that if we do not make such a choice we will not be able to fix the gauge in which $x^0 = Ct$, which appears to contradict reparametrization invariance. The resolution of this apparent paradox is that, when there is a potential and these modified energy-momentum relations, we cannot assume that a relativistic particle does not reverse direction in time, because there will be trajectories that pass through $\dot{x}^0 = 0$. So the failure of the $x^0$ equations of motion to linearize appears necessary if these trajectories are to be included as part of the theory.


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