

**Superconformal symmetry, the supercurrent, and non-BPS brane dynamics**

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The Noether currents associated with the nonlinearly realized super-Poincaré symmetries of the Green-Schwarz (Nambu-Goto-Akulov-Volkov) action for a non-Bogomol'nyi-Prasad-Sommerfield (non-BPS)  $p = 2$  brane embedded in a  $N = 1, D = 4$  target superspace are constructed. The  $R$  symmetry current, the supersymmetry currents, the energy-momentum tensor and the scalar central charge current are shown to be components of a world-volume supercurrent. The centrally extended superconformal transformations are realized on the Nambu-Goldstone boson and fermion fields of the non-BPS brane. The superconformal currents form supersymmetry multiplets with the world-volume conformal central charge current and special conformal current being the primary components of the supersymmetry multiplets containing all the currents. Correspondingly the superconformal symmetry breaking terms form supersymmetry multiplets the components of which are obtainable as supersymmetry transformations of the primary currents' symmetry breaking terms.

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**I. INTRODUCTION**

The formation of a membrane in target space spontaneously breaks its symmetries to the isometries of the world volume and its complement. The normal mode oscillations of the brane into the covolume are described by world-volume localized Nambu-Goldstone fields. In the case of a target superspace, Goldstino modes must accompany the broken space translational Nambu-Goldstone boson modes. The Green-Schwarz action [1] describes the dynamics of these world-volume fields. In Ref. [2] the Nambu-Goto-Akulov-Volkov action for a non-BPS  $p = 2$  brane embedded in  $N = 1, D = 4$  superspace was constructed via the nonlinear realization [3–5] of the spontaneously broken super-Poincaré symmetries of the target superspace. In Ref. [6] the Green-Schwarz action for this brane [7] was shown to be equivalent to the Nambu-Goto-Akulov-Volkov action by means of explicit nonlinear field redefinitions. The action described the motion of the brane in  $N = 1, D = 4$  superspace through the brane localized Nambu-Goldstone boson field  $\phi$  associated with motions in space directions transverse to the brane, hence in the direction of the broken space translation symmetry. It also involved brane localized  $D = 3$  Majorana Goldstino fields  $\theta_i$  and  $\lambda_i$ ,  $i = 1, 2$ , describing brane oscillations in Grassmann directions of superspace which are associated with the completely broken  $N = 1, D = 4$  supersymmetry (SUSY). The action, after application of the “inverse

Higgs mechanism” [8], is the  $N = 1, D = 4$  super-Poincaré invariant synthesis of the Akulov-Volkov [3] and Nambu-Goto [9] actions

$$\begin{aligned}\Gamma &= -\sigma \int d^3x \det \hat{e} \det N \\ &= -\sigma \int d^3x \det \hat{e} \sqrt{1 - \hat{\nabla}_a \phi \eta^{ab} \hat{\nabla}_b \phi} \\ &= -\sigma \int d^3x \det \hat{e} \sqrt{1 - [\hat{e}_a^{-1m} (\partial_m \phi + \bar{\theta} \overleftrightarrow{\partial}_m \lambda)]^2},\end{aligned}\quad (1.1)$$

where  $\sigma$  is the brane tension and, in the static gauge, the world-volume coordinates are  $x^m$ ,  $m = 0, 1, 2$ . The Akulov-Volkov dreibein is  $\hat{e}_m^a = \delta_m^a + i\bar{\theta} \gamma^a \partial_m \theta + i\bar{\lambda} \gamma^a \partial_m \lambda$ . The Nambu-Goto dreibein is given by  $N_a^b = \delta_a^b + [\hat{\nabla}_a \phi \hat{\nabla}^b \phi / (\hat{\nabla} \phi)^2] [\sqrt{1 - (\hat{\nabla} \phi)^2} - 1]$ , in which the Nambu-Goldstone boson covariant derivative,  $\hat{\nabla}_a \phi$ , is defined as

$$\begin{aligned}\hat{\nabla}_a \phi &= \hat{D}_a \phi + \bar{\theta} \hat{D}_a \lambda - \hat{D}_a \bar{\theta} \lambda \\ &= \hat{e}_a^{-1m} (\partial_m \phi + \bar{\theta} \overleftrightarrow{\partial}_m \lambda).\end{aligned}\quad (1.2)$$

The partial covariant derivative,  $\hat{D}_a$ , is defined by  $\hat{D}_a = \hat{e}_a^{-1m} \partial_m$ . The notation of Ref. [2] is followed throughout this paper.

The purpose of this paper is to determine the symmetry currents of this action, which are those associated with the nonlinearly realized  $N = 1, D = 4$  super-Poincaré symmetries. Further, in accordance with the equivalent  $N = 2, D = 3$  centrally extended SUSY algebra, it is shown that

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the  $R$  symmetry current, the supersymmetry currents, the energy-momentum tensor and the scalar central charge current are components of a supercurrent. That is a SUSY multiplet of currents. The primary current in this multiplet is the  $R$  current, the remaining derived currents can be obtained from it by SUSY transformations. Hence, since the  $R$  current is conserved, the conservation of the derived component SUSY currents, energy-momentum tensor and the central charge current is guaranteed. Besides the spontaneously broken super-Poincaré currents, the centrally extended superconformal currents are constructed. These also form supersymmetry multiplets. Since the scale symmetries are explicitly broken, the superconformal current nonconservation terms are similarly related by the SUSY multiplet structure of the currents. It is further shown that all superconformal as well as super-Poincaré currents are obtained as SUSY variations of the primary  $D = 3$  conformal central charge current and the primary  $D = 3$  special conformal current (both formerly comprise the  $D = 4$  special conformal symmetry current).

In Sec. II, Noether's theorem is stated and the Noether currents along with the variations of the Lagrangian (1.1) for the nonlinearly realized  $N = 1$ ,  $D = 4$  super-Poincaré symmetries are obtained. Since the unbroken symmetries are those of the  $D = 3$  Poincaré group, it is useful to express the  $D = 4$  charges in terms of their  $D = 3$  Lorentz group transformation properties. Appendix A summarizes the  $N = 1$ ,  $D = 4$  superconformal algebra expressed as the corresponding centrally extended  $N = 2$ ,  $D = 3$  superconformal algebra. In Appendix B the derivation of the nonlinear realization of this algebra on the world-volume Nambu-Goldstone boson field,  $\phi$ , and Goldstino fields,  $\theta$  and  $\lambda$ , is given for the case that the  $N = 1$ ,  $D = 4$  superconformal  $SU(2, 2 | 1)$  group is spontaneously broken to the  $D = 3$   $SO(3, 2)$  conformal group and  $U(1)$   $R$  symmetry (for fixed  $D = 4$  realizations see [10,11]). The underlying short distance models [12–14] that give rise to the non-BPS domain wall formation should also explicitly break the scale symmetries both radiatively and by the dimensionful brane tension parameter in the models. Hence physically no additional conformal Nambu-Goldstone modes occur. However, the superconformal algebra, as discussed in Appendix B, requires the conformal central charge symmetry (the spatial component normal to the brane of the  $D = 4$  special conformal transformations) and the conformal SUSY transformations to have a spontaneously broken component. Derivatives of the brane oscillation Nambu-Goldstone fields provide the required ground state expectation values for these broken symmetries and no additional fields are required in order to nonlinearly realize the above spontaneously broken symmetry transformations. In Sec. III, Noether's theorem is again utilized in order to construct the remainder of the superconformal Noether currents. Consistent with the charge algebra, these currents form SUSY multiplets of currents.

Since the superconformal currents are not conserved their explicit symmetry breaking terms also form SUSY multiplets. As in the case of all superconformal currents, all superconformal symmetry breaking terms are shown to be given by SUSY transformations of the  $D = 3$  conformal central charge-current nonconservation terms and the  $D = 3$  special conformal symmetry current nonconservation terms, the two independent primary component currents of the supermultiplets containing all currents. Finally, four tables are included at the end of Sec. III which summarize the symmetry charges and Noether currents associated with the Nambu-Goto-Akulov-Volkov action. Also the SUSY variations of the currents and their (non)conservation equations are recapitulated.

## II. SUSY AND THE SUPERCURRENT

Noether's theorem provides a relation between the divergence of a symmetry current and the associated variation of the Lagrangian for a given transformation of the fields. For a (Lie derivative) intrinsic symmetry transformation of the fields, denoted by  $\delta\phi$ ,  $\delta\theta$  and  $\delta\lambda$  and such that  $\delta(\partial_m\varphi^I) = \partial_m(\delta\varphi^I)$  where  $\varphi^I$  stands for any of the fields  $\varphi = (\phi, \theta, \lambda)$ , the variation of the Lagrangian yields

$$\delta\mathcal{L} = \partial_m j^m + w\Gamma. \quad (2.1)$$

The symmetry current  $j^m = \sum_I (\partial\mathcal{L}/\partial\partial_m\varphi^I)\delta\varphi^I$  and the local Ward-Takahashi functional differential operator for the symmetry transformation (often referred to as Euler-Lagrange terms when acting on the action) is  $w = \sum_I \delta\varphi^I(\delta/\delta\varphi^I)$  with  $\Gamma$  the action, in this case, Eq. (1.1). The intrinsic variation,  $\delta\varphi = \varphi'(x) - \varphi(x)$ , is related to the total variation  $\Delta\varphi = \varphi'(x') - \varphi(x)$ , by the subtraction of the first Taylor expansion term for the space-time coordinate variation; for the Lagrangian, this yields  $\delta\mathcal{L} = \Delta\mathcal{L} - \delta x^m \partial_m \mathcal{L}$ . Using the chain rule for the Taylor term, the final form of Noether's theorem is obtained

$$\partial_m J^m = [\Delta\mathcal{L} + (\partial_m \delta x^m)\mathcal{L}] - w\Gamma, \quad (2.2)$$

where the Noether current,  $J^m$ , is defined as

$$J^m \equiv j^m + \delta x^m \mathcal{L} = \sum_I \frac{\partial\mathcal{L}}{\partial\partial_m\varphi^I} \delta\varphi^I + \delta x^m \mathcal{L}. \quad (2.3)$$

Substituting the Lagrangian derivatives associated with Eq. (1.1), the Noether current is obtained

$$J^m = -i[\delta\bar{\theta}\gamma^a\theta + \delta\bar{\lambda}\gamma^a\lambda] \left[ \hat{e}_a^{-1m} + \frac{\hat{\nabla}_a\phi\hat{\nabla}^b\phi\hat{e}_b^{-1m}}{(\det N)^2} \right] \mathcal{L} - \frac{\mathcal{L}\hat{\nabla}^a\phi\hat{e}_a^{-1m}}{(\det N)^2} [\delta\phi - \delta\bar{\theta}\lambda + \delta\bar{\lambda}\theta]. \quad (2.4)$$

The non-BPS  $p = 2$  brane action is world-volume space-time translationally invariant:  $\Delta^p(a)\mathcal{L} = 0$ . Recalling the space-time translations of the fields from Appendix B,  $\delta^p(a)x^m = a^m$  and  $\delta^p(a)\varphi = -a^m\partial_m\varphi$ , the Noether energy-momentum tensor is secured

$$T^m{}_a = -\mathcal{L}\hat{e}_a^{-1m} - \frac{\mathcal{L}}{(\det N)^2} \hat{\nabla}^b \phi \hat{e}_b^{-1m} \hat{\nabla}_a \phi. \quad (2.5)$$

Conservation of brane energy momentum yields  $\partial_m T^m{}_a = w_a^p(x)\Gamma$ , with the space-time translation Ward identity operator  $w_a^p(x) = -\sum_I \partial_a \varphi^I(x) [\delta/\delta\varphi^I(x)]$ . [Integration over the world volume leads to the global space-time translation Ward identity for the (tree level) one-particle irreducible generating functional,  $\Gamma$ , that is the effective action.] Likewise, the spontaneously broken  $D = 4$  space translation symmetry normal to the brane, now expressed as the  $D = 3$  spontaneously broken scalar central charge,  $Z$ , symmetry, is conserved. Its Noether current divergence is given simply by the  $\phi$  field equation, hence

$$Z^m = -\frac{\mathcal{L}}{(\det N)^2} \hat{\nabla}^a \phi \hat{e}_a^{-1m}. \quad (2.6)$$

Since only  $\phi$  transforms under  $Z$ ,  $\Delta^Z(z)\phi = z$ , Noether's theorem yields  $\partial_m Z^m = -w^Z(x)\Gamma$  with  $w^Z(x) = \frac{\delta}{\delta\phi(x)}$ .

All other currents, including those of the superconformal symmetries, can be expressed in terms of the  $D = 3$  energy-momentum tensor and the  $D = 3$  central charge symmetry current. The general form of the Noether current, Eq. (2.4), becomes

$$J^m = -T^m{}_a [\delta x^a - i(\Delta\bar{\theta}\gamma^a\theta + \Delta\bar{\lambda}\gamma^a\lambda)] + Z^m [\Delta\phi - \Delta\bar{\theta}\lambda + \bar{\theta}\Delta\lambda]. \quad (2.7)$$

Similarly, using these currents, the variation of the Lagrangian takes on a simplified form

$$\delta\mathcal{L} = -T^m{}_a \delta[\hat{e}_m^a] + Z^m \delta[\hat{e}_m^a \hat{\nabla}_a \phi]. \quad (2.8)$$

Applying this to the case of space-time translations, for instance, yields the simple derivative formula for the Lagrangian

$$\partial_n \mathcal{L} = -T^m{}_a \partial_n [\hat{e}_m^a] + Z^m \partial_n [\hat{e}_m^a \hat{\nabla}_a \phi]. \quad (2.9)$$

The brane Lagrangian is manifestly  $R$  invariant resulting in the  $R$  symmetry Noether current

$$R^m = -2T^m{}_a (\bar{\theta}\gamma^a\lambda) + iZ^m (\bar{\theta}\theta + \bar{\lambda}\lambda), \quad (2.10)$$

with the conservation equation  $\partial_m R^m = -w^R(x)\Gamma$ , where  $w^R(x) = -i[\lambda(x)\frac{\delta}{\delta\theta(x)} - \theta(x)\frac{\delta}{\delta\lambda(x)}]$ . Introducing  $q$ - and  $s$ -SUSY transformation parameters  $\xi$  and  $\eta$ , respectively, that are Grassmann  $D = 3$  Majorana spinors, the SUSY variations become

$$\begin{aligned} \delta^{q,s}(\xi, \eta)x^m &= -i(\bar{\xi}\gamma^m\theta + \bar{\eta}\gamma^m\lambda) \equiv -\Lambda^m(\xi, \eta), \\ \Delta^{q,s}(\xi, \eta)\theta_i &= \xi_i, \quad \Delta^{q,s}(\xi, \eta)\lambda_i = \eta_i, \\ \Delta^{q,s}(\xi, \eta)\phi &= -(\bar{\xi}\lambda - \bar{\eta}\theta). \end{aligned} \quad (2.11)$$

[Recall that  $\delta^{q,s}(\xi, \eta) = \bar{\xi}_i \delta_i^q + \bar{\eta}_i \delta_i^s$ , and so on.] As usual for SUSY the Lagrangian has a total derivative intrinsic variation,  $\delta^{q,s}(\xi, \eta)\mathcal{L} = \partial_m [\Lambda^m(\xi, \eta)\mathcal{L}]$ . The SUSY Noether currents are

$$\begin{aligned} Q^m(\xi) &= \bar{\xi}_i Q_i^m(x) = +2iT^m{}_a (\bar{\xi}\gamma^a\theta) - 2Z^m (\bar{\xi}\lambda), \\ S^m(\eta) &= \bar{\eta}_i S_i^m(x) = +2iT^m{}_a (\bar{\eta}\gamma^a\lambda) + 2Z^m (\bar{\eta}\theta). \end{aligned} \quad (2.12)$$

Noether's theorem yields the SUSY Ward identities

$$\partial_m [Q^m(\xi) + S^m(\eta)] = -w^{q,s}(x)\Gamma, \quad (2.13)$$

with the SUSY Ward identity operator given as  $w^{q,s}(x) = \sum_I \delta^{q,s}(\xi, \eta)\varphi^I(x) [\delta/\delta\varphi^I(x)]$ .

The remaining super-Poincaré transformations are the unbroken world-volume Lorentz transformations with angular momentum tensor  $M_{ab}^m = \epsilon_{abc} M^{mc}$  and the spontaneously broken  $D = 4$  Lorentz transformations with  $D = 3$  current  $N^m{}_a$ . The Lagrangian is invariant under  $D = 3$  Lorentz transformations yielding a conserved Noether current,  $\partial_m M_{ab}^m(x) = -w_{ab}^M(x)\Gamma$ , with

$$\begin{aligned} M_{ab}^m &= T^m{}_a x_b - T^m{}_b x_a + \frac{1}{2} T^m{}_c [\bar{\theta}\gamma_{ab}\gamma^c\theta + \bar{\lambda}\gamma_{ab}\gamma^c\lambda] \\ &\quad + iZ^m (\bar{\theta}\gamma_{ab}\lambda). \end{aligned} \quad (2.14)$$

Under spontaneously broken  $D = 4$  Lorentz transformations the Lagrangian transforms into a total divergence  $\delta^N(b)\mathcal{L} = \partial_m (b^m \phi \mathcal{L})$  resulting in a conserved Noether current  $\partial_m N^m{}_a(x) = -w_a^N(x)\Gamma$ , with

$$N^m{}_a = T^m{}_a \phi + iT^m{}_b (\bar{\theta}\gamma_a{}^b\lambda) - Z^m x_a. \quad (2.15)$$

For models in  $N = 1$ ,  $D = 4$  superspace with linearly represented SUSY transformations, the  $R$  symmetry current, the SUSY currents and the energy-momentum tensor are components of a supercurrent superfield [15]. In addition all other superconformal currents are capable of being written in terms of space-time moments and SUSY covariant derivatives of this supercurrent. The explicit and anomalous breaking terms of the  $R$  and conformal symmetries also follow from space-time moments and SUSY covariant derivatives of the generalized trace of the supercurrent [16–18]. (For an analysis of the superconformal charges and the supercharge in supersymmetric quantum mechanics see Ref. [19].) In models with nonlinearly realized SUSY in  $D = 4$ , the  $R$  symmetry current, the SUSY currents and the energy-momentum tensor still form a SUSY multiplet [20] with the superconformal currents being given by space-time moments and field monomials times the components of this supercurrent (explicitly the  $R$  current and energy-momentum tensor) [11]. As shown below, in brane models these currents along with the central charge symmetry current form a SUSY multiplet. As well, all superconformal currents are shown to be world-volume coordinate moments and field monomials times the components of the supercurrent [explicitly the energy-momentum tensor and the central charge current as in Eq. (2.7)]. In general starting with a current  $J^m$ , its  $q$ - and  $s$ -SUSY transformations yield a relation of the form

$$\begin{aligned} \delta^{q,s}(\xi, \eta)J^m &= \partial_n [J^m \Lambda^n(\xi, \eta) - J^n \Lambda^m(\xi, \eta)] \\ &\quad + \Lambda^m(\xi, \eta) \partial_n J^n + J^m(\xi, \eta), \end{aligned} \quad (2.16)$$

where  $J^m(\xi, \eta)$  is another of the super-Poincaré or super-conformal currents or zero. If zero, then that  $J^m$  is the last (or highest weight) component in a supermultiplet. The first set of terms on the right-hand side are current improvement terms since they are algebraically divergenceless. They can be added to  $J^m(\xi, \eta)$  to define an improved and still conserved current

$$J_{\text{Imp}}^m(\xi, \eta) = J^m(\xi, \eta) + \partial_n [J^m \Lambda^n(\xi, \eta) - J^n \Lambda^m(\xi, \eta)], \quad (2.17)$$

and  $\partial_m J_{\text{Imp}}^m(\xi, \eta) = \partial_m J^m(\xi, \eta)$ . The improvement terms will be kept explicit in what follows. The set of currents which begins with some primary current  $J^m$  and continues upon SUSY variation to yield other currents until the last component current is reached comprise a supermultiplet of currents. Every superconformal current will form such a multiplet, even if the primary current is the sole member. Since differentiation commutes with intrinsic variation, the (non)conservation of the primary current implies the (possible non)conservation of the higher component currents. The second term on the right-hand side of Eq. (2.16) along with the relation between intrinsic and total variation results in the higher component conservation equation

$$\begin{aligned} \partial_m J^m(\xi, \eta) &= \Delta^{q,s}(\xi, \eta)(\partial_m J^m) \\ &+ [\partial_n \delta^{q,s}(\xi, \eta) x^n](\partial_m J^m). \end{aligned} \quad (2.18)$$

So the currents in any multiplet can be obtained from the primary current by SUSY variation. Likewise, the breaking of any symmetry in a multiplet can be obtained from the primary current symmetry breaking by SUSY variation.

The  $R^m$ ,  $Q_i^m$ ,  $S_i^m$ ,  $T_a^m$  and  $Z^m$  currents form the supercurrent multiplet with the  $R$  current as the primary component. Since  $R^m$  is conserved, so too are the other currents in this supercurrent as verified from their explicit construction. The SUSY variations are found to be (using  $\psi_i$  as a Grassmann,  $D = 3$  Majorana spinor parameter)

$$\begin{aligned} \delta^{q,s}(\xi, \eta) R^m &= \partial_n [R^m \Lambda^n(\xi, \eta) - R^n \Lambda^m(\xi, \eta)] \\ &+ \Lambda^m(\xi, \eta) \partial_n R^n - i Q^m(\eta) + i S^m(\xi), \\ \delta^{q,s}(\xi, \eta) Q^m(\psi) &= \partial_n [Q^m(\psi) \Lambda^n(\xi, \eta) - Q^n(\psi) \Lambda^m(\xi, \eta)] \\ &+ \Lambda^m(\xi, \eta) \partial_n Q^n(\psi) + 2i T_a^m (\bar{\psi} \gamma^a \xi) \\ &- 2Z^m (\bar{\psi} \eta), \\ \delta^{q,s}(\xi, \eta) S^m(\psi) &= \partial_n [S^m(\psi) \Lambda^n(\xi, \eta) - S^n(\psi) \Lambda^m(\xi, \eta)] \\ &+ \Lambda^m(\xi, \eta) \partial_n S^n(\psi) + 2i T_a^m (\bar{\psi} \gamma^a \eta) \\ &+ 2Z^m (\bar{\psi} \xi), \\ \delta^{q,s}(\xi, \eta) T_a^m &= \partial_n [T_a^m \Lambda^n(\xi, \eta) - T^n \Lambda^m(\xi, \eta)] \\ &+ \Lambda^m(\xi, \eta) \partial_n T_a^n, \\ \delta^{q,s}(\xi, \eta) Z^m &= \partial_n [Z^m \Lambda^n(\xi, \eta) - Z^n \Lambda^m(\xi, \eta)] \\ &+ \Lambda^m(\xi, \eta) \partial_n Z^n. \end{aligned} \quad (2.19)$$

From the super-Poincaré symmetry group perspective, the angular momentum tensor and the broken Lorentz transformation current are primary currents whose SUSY multiplets involve the SUSY currents, the energy-momentum tensor and the central charge current

$$\begin{aligned} \delta^{q,s}(\xi, \eta) M_{ab}^m &= \partial_n [M_{ab}^m \Lambda^n(\xi, \eta) - M_{ab}^n \Lambda^m(\xi, \eta)] \\ &+ \Lambda^m(\xi, \eta) \partial_n M_{ab}^n - i(\bar{\xi} \gamma_{ab} Q^m) \\ &+ i(\bar{\eta} \gamma_{ab} S^m), \\ \delta^{q,s}(\xi, \eta) N_a^m &= \partial_n [N_a^m \Lambda^n(\xi, \eta) - N_a^n \Lambda^m(\xi, \eta)] \\ &+ \Lambda^m(\xi, \eta) \partial_n N_a^n - i(\bar{\xi} \gamma_a S^m) \\ &+ i(\bar{\eta} \gamma_a Q^m). \end{aligned} \quad (2.20)$$

### III. SUPERCONFORMAL CURRENTS AND SUSY MULTIPLETS

Noether's theorem applies to the construction of the superconformal currents and their (non)conservation equations as well. Utilizing Eq. (2.8), the Lagrangian varies under scale transformations as  $\delta^D(\epsilon) \mathcal{L} = -\epsilon x^m \partial_m \mathcal{L} = 3\epsilon \mathcal{L} - \partial_m(\epsilon x^m \mathcal{L})$ . This results in a dilatation Noether current

$$D^m = -T_a^m x^a + Z^m \phi. \quad (3.1)$$

The scale symmetry is explicitly violated so that the divergence of the dilatation current obeys the broken Ward identity

$$\begin{aligned} \partial_m D^m &= -T_a^m \hat{\epsilon}_m^a + Z^m \hat{\epsilon}_m^a \hat{\nabla}_a \phi - w^D \Gamma \\ &= 3\sigma \frac{\partial}{\partial \sigma} \mathcal{L} - w^D \Gamma, \end{aligned} \quad (3.2)$$

where as usual the local Ward identity dilatation operator is  $w^D(x) = \sum_I \delta^D \varphi^I(x) (\delta / \delta \varphi^I)$ .

The  $D = 4$  special conformal transformations lead to the  $D = 3$  special conformal transformations and the conformal central charge symmetry. The Noether current for the latter is secured as

$$\begin{aligned} Y^m &= +2T_a^m [x^a \phi + i \epsilon^{abc} x_b (\bar{\theta} \gamma_c \lambda)] \\ &- Z^m [x^2 + \phi^2 - (\bar{\theta} \theta) (\bar{\lambda} \lambda)]. \end{aligned} \quad (3.3)$$

Noether's theorem yields the explicitly broken (but realized as spontaneously broken, as discussed in Appendix B)  $Y$  symmetry Ward identity

$$\partial_m Y^m = [\Delta^Y \mathcal{L} + (\partial_m \delta^Y x^m) \mathcal{L}] - w^Y \Gamma, \quad (3.4)$$

where  $w^Y = \sum_I \delta^Y \varphi^I (\delta / \delta \varphi^I)$ . The breaking terms are obtained from Eq. (2.8)

$$\begin{aligned} \delta^Y \mathcal{L} &= 2x^m \partial_m (\phi \mathcal{L}) + T_a^m [2x^a (\hat{\epsilon}_m^b \hat{\nabla}_b \phi) - 2i \epsilon_m^{ab} (\bar{\theta} \gamma_b \lambda) \\ &+ 2i(\bar{\theta} \theta - \bar{\lambda} \lambda) (\bar{\theta} \gamma^a \vec{\partial}_m \lambda)] - 2Z^m \hat{\epsilon}_m^a x_a. \end{aligned} \quad (3.5)$$

Applying this to the divergence equation, the nonconser-

vation of the  $Y$  current is found

$$\begin{aligned} [\Delta^Y \mathcal{L} + (\partial_m \delta^Y x^m) \mathcal{L}] &= -6\phi \mathcal{L} + T^m_a [2x^a (\hat{\epsilon}_m^b \hat{\nabla}_b \phi) \\ &\quad - 2i\epsilon_m^{ab} (\bar{\theta} \gamma_b \lambda) \\ &\quad + 2i(\bar{\theta}\theta - \bar{\lambda}\lambda) (\bar{\theta} \gamma^a \overleftrightarrow{\partial}_m \lambda)] \\ &\quad - 2Z^m \hat{\epsilon}_m^a x_a. \end{aligned} \quad (3.6)$$

The special conformal current  $K^m_a$  is determined to be

$$\begin{aligned} K^m_a &= -T^m_b [2x^b x_a - x^2 \delta^b_a + \phi^2 \delta^b_a + (\bar{\theta}\theta - \bar{\lambda}\lambda) \delta^b_a \\ &\quad - 2i\epsilon_a^{bc} \phi (\bar{\theta} \gamma_c \lambda) - \epsilon_a^{bc} x_c (\bar{\theta}\theta - \bar{\lambda}\lambda)] \\ &\quad + Z^m [2x_a \phi - 2i\epsilon_a^{bc} x_b (\bar{\theta} \gamma_c \lambda)]. \end{aligned} \quad (3.7)$$

The special conformal current is explicitly not conserved, obeying the divergence equation

$$\partial_m K^m_a = [\Delta^k_a \mathcal{L} + (\partial_m \delta^k_a x^m) \mathcal{L}] - w_a^k \Gamma, \quad (3.8)$$

where the Ward identity term  $w_a^k(x) = \sum_l \delta_a^k \varphi^l (\delta / \delta \varphi^l)$ . The special conformal variation, with transformation parameter  $\epsilon^a$ , of the Lagrangian is determined directly using the field variations found in Appendix B. This yields the total variation of the Lagrangian,  $\Delta^k(\epsilon) \mathcal{L} = \delta^k(\epsilon) \mathcal{L} + [\delta^k(\epsilon) x^m] \partial_m \mathcal{L}$ , and from this the current breaking terms are found

$$\begin{aligned} \{\Delta^k(\epsilon) \mathcal{L} + [\partial_m \delta^k(\epsilon) x^m] \mathcal{L}\} &= 6\epsilon^m x_m \mathcal{L} - T^m_a \{2\epsilon^a \phi (\hat{\epsilon}_m^b \hat{\nabla}_b \phi) + 2(\epsilon_b x^a - \epsilon^a x_b) \hat{\epsilon}_m^b - \epsilon^a \partial_m [(\bar{\theta}\theta) (\bar{\lambda}\lambda)] \\ &\quad + (\bar{\theta}\theta + \bar{\lambda}\lambda) \epsilon^a_{bc} \epsilon^c (\hat{\epsilon}_m^b - 2\delta^b_m) + 4(\bar{\theta} \gamma^a \lambda) \partial_m (\bar{\theta} \not{\epsilon} \lambda) - 2i\epsilon^{abc} \epsilon_c (\bar{\theta} \gamma_b \lambda) (\hat{\epsilon}_m^d \hat{\nabla}_d \phi)\} \\ &\quad + Z^m [2\epsilon_a \phi \hat{\epsilon}_m^a - 2i\epsilon_{mab} \epsilon^b (\bar{\theta} \gamma^a \lambda) - 2i(\bar{\theta}\theta - \bar{\lambda}\lambda) (\bar{\theta} \not{\epsilon} \overleftrightarrow{\partial}_m \lambda)]. \end{aligned} \quad (3.9)$$

Finally the  $u_i$ - and  $v_i$ -conformal SUSY spinor currents are obtained using Eq. (2.7) (with the explicitly broken conformal SUSY realized as a spontaneously broken symmetry, as discussed in Appendix B)

$$\begin{aligned} U_i^m &= 2T^m_a [x_b (\gamma^b \gamma^a \theta)_i - i\phi (\gamma^a \lambda)_i + i(\bar{\lambda}\lambda) (\gamma^a \theta)_i] \\ &\quad + 2Z^m [i(\not{x}\lambda)_i - \phi \theta_i + (\bar{\theta}\theta) \lambda_i], \\ V_i^m &= -2T^m_a [x_b (\gamma^b \gamma^a \lambda)_i + i\phi (\gamma^a \theta)_i + i(\bar{\theta}\theta) (\gamma^a \lambda)_i] \\ &\quad + 2Z^m [i(\not{x}\theta)_i + \phi \lambda_i + (\bar{\lambda}\lambda) \theta_i]. \end{aligned} \quad (3.10)$$

The nonconservation equations have the form

$$\begin{aligned} \partial_m U_i^m &= [\Delta_i^u \mathcal{L} + (\partial_m \delta_i^u x^m) \mathcal{L}] - w_i^u \Gamma, \\ \partial_m V_i^m &= [\Delta_i^v \mathcal{L} + (\partial_m \delta_i^v x^m) \mathcal{L}] - w_i^v \Gamma, \end{aligned} \quad (3.11)$$

where the Ward identity operators are  $w_i^{u,v}(x) = \sum_l \delta_i^{u,v} \varphi^l (\delta / \delta \varphi^l)$ . The superconformal symmetry breaking terms can be determined directly, however, as shown below, the  $U_i^m$  and  $V_i^m$  currents are components of a conformal current SUSY multiplet and so the breaking can be found from the SUSY variation of the primary current's breaking terms.

As in the case of the super-Poincaré symmetry currents, the superconformal currents belong to  $q$ - and  $s$ -SUSY multiplets consistent with the superconformal algebra. The SUSY variations of the currents are just parts of the superconformal charge-current algebra. In the superconformal case, the conformal central charge current,  $Y^m$ , and the special conformal current,  $K^m_a$ , act as independent primary currents from which the remainder of the superconformal and even the super-Poincaré currents can be obtained. The  $q$ - and  $s$ -SUSY variations of the  $Y^m$  and  $K^m_a$  currents are

$$\begin{aligned} \delta^{q,s}(\xi, \eta) Y^m &= \partial_n [Y^m \Lambda^n(\xi, \eta) - Y^n \Lambda^m(\xi, \eta)] \\ &\quad + \Lambda^m(\xi, \eta) \partial_n Y^n + (\bar{\eta} U^m) + (\bar{\xi} V^m), \\ \delta^{q,s}(\xi, \eta) K^m_a &= \partial_n [K^m_a \Lambda^n(\xi, \eta) - K^n_a \Lambda^m(\xi, \eta)] \\ &\quad + \Lambda^m(\xi, \eta) \partial_n K^n_a + i(\bar{\xi} \gamma_a U^m) \\ &\quad - i(\bar{\eta} \gamma_a V^m). \end{aligned} \quad (3.12)$$

The SUSY variations of the conformal SUSY currents lead to the dilatation current and the super-Poincaré Lorentz and  $R$  currents, introducing the  $D = 3$  Majorana spinor parameter  $\psi_i$ ,

$$\begin{aligned} \delta^{q,s}(\xi, \eta) U^m(\psi) &= \partial_n [U^m(\psi) \Lambda^n(\xi, \eta) - U^n(\psi) \Lambda^m(\xi, \eta)] \\ &\quad + \Lambda^m(\xi, \eta) \partial_n U^n(\psi) + 2i(\bar{\eta} \gamma^a \psi) N^m_a \\ &\quad - 3i(\bar{\eta} \psi) R^m + 2i(\bar{\xi} \gamma^a \psi) M^m_a \\ &\quad - 2(\bar{\xi} \psi) D^m, \\ \delta^{q,s}(\xi, \eta) V^m(\psi) &= \partial_n [V^m(\psi) \Lambda^n(\xi, \eta) - V^n(\psi) \Lambda^m(\xi, \eta)] \\ &\quad + \Lambda^m(\xi, \eta) \partial_n V^n(\psi) + 2i(\bar{\xi} \gamma^a \psi) N^m_a \\ &\quad - 3i(\bar{\xi} \psi) R^m - 2i(\bar{\eta} \gamma^a \psi) M^m_a \\ &\quad + 2(\bar{\eta} \psi) D^m. \end{aligned} \quad (3.13)$$

Completing the multiplet, the SUSY variation of the dilatation current yields the  $q$ - and  $s$ -SUSY currents

$$\begin{aligned} \delta^{q,s}(\xi, \eta) D^m &= \partial_n [D^m \Lambda^n(\xi, \eta) - D^n \Lambda^m(\xi, \eta)] \\ &\quad + \Lambda^m(\xi, \eta) \partial_n D^n + \frac{1}{2}(\bar{\xi} Q^m) + \frac{1}{2}(\bar{\eta} S^m). \end{aligned} \quad (3.14)$$

From the conformal central charge symmetry breaking terms, Eq. (3.6), or the special conformal symmetry break-

TABLE I. Symmetries, charges and currents.

$D = 4$ symmetry and charge	$D = 3$ symmetry and charge	World-volume Noether current
Target space translations $P_\mu$	World-volume translations $p_a = P_{\mu=a}$ Central charge $Z = P_{\mu=3}$	$T^m_a$ $Z^m$
Lorentz transformations $M_{\mu\nu}$	World-volume Lorentz transformations $M_{ab} = M_{\mu=ab}$ Broken Lorentz automorphisms $N_a = -M_{\mu=av=3}$	$M^m_{ab}$ $N^m_a$
$N = 1, D = 4$ SUSY	$N = 2, D = 3$ SUSY	$Q^m_i$
$Q_\alpha, \bar{Q}_{\dot{\alpha}}$	$\begin{pmatrix} q_i \\ s_i \end{pmatrix} = \frac{1}{2} e^{i(\pi/4)} \begin{pmatrix} \sigma & i\sigma\sigma_z \\ -i\sigma & -\sigma\sigma_z \end{pmatrix} \begin{pmatrix} Q_\alpha \\ \bar{Q}_{\dot{\alpha}} \end{pmatrix}$	$S^m_i$
$R$ -transformation $R$	$R$ -transformation $R = R$	$R^m$
Dilatation $D$	Dilatation $D = D$	$D^m$
Special conformal $K_\mu$	Special conformal $k_a = K_{\mu=a}$ Conformal central charge $Y = K_{\mu=3}$	$K^m_a$ $Y^m$
$N = 1, D = 4$ superconformal	$N = 2, D = 3$ superconformal	$U^m_i$
$S_\alpha, \bar{S}_{\dot{\alpha}}$	$\begin{pmatrix} u_i \\ v_i \end{pmatrix} = \frac{1}{2} e^{i(\pi/4)} \begin{pmatrix} \sigma & i\sigma\sigma_z \\ -i\sigma & -\sigma\sigma_z \end{pmatrix} \begin{pmatrix} S_\alpha \\ \bar{S}_{\dot{\alpha}} \end{pmatrix}$	$V^m_i$

ing terms, Eq. (3.9), the remainder of the superconformal symmetry breaking terms can be obtained by means of SUSY variations of the primary current's breaking terms as per Eq. (2.18).

The Noether currents associated with the nonlinearly realized super-Poincaré symmetries of the Nambu-Goto-Akulov-Volkov action, Eq. (1.1), for a non-BPS  $p = 2$  brane embedded in a  $N = 1, D = 4$  target superspace are summarized in Table I. The target space symmetry transformation generators are given with their  $D = 3$  Lorentz group decomposition and the related Noether currents are

listed. In Table II the forms of these Noether currents are specified for the effective action given in Eq. (1.1). The SUSY variations of the Noether currents are listed in Table III. The  $R$  symmetry current is the primary current for the supercurrent multiplet of centrally extended supertranslation Noether currents. The  $R$  symmetry current, the  $q$ -SUSY and  $s$ -SUSY currents, the energy-momentum tensor and the scalar central charge current are the components of this supercurrent. Because the  $R$  current is conserved, the conservation of the derived component SUSY currents, energy-momentum tensor and central charge cur-

TABLE II. Noether currents.

Noether current	Form of Noether current
$J^m$	$J^m = -T^m_a[\delta x^a - i(\Delta\bar{\theta}\gamma^a\theta + \Delta\bar{\lambda}\gamma^a\lambda)] + Z^m[\Delta\phi - \Delta\bar{\theta}\lambda + \bar{\theta}\Delta\lambda]$
$T^m_a$	$T^m_a = -\mathcal{L}\hat{e}_a^{-1m} - [\mathcal{L}/(\det N)^2]\hat{\nabla}^b\phi\hat{e}_b^{-1m}\hat{\nabla}_a\phi$
$Z^m$	$Z^m = -[\mathcal{L}/(\det N)^2]\hat{\nabla}^a\phi\hat{e}_a^{-1m}$
$M^m_{ab}$	$M^m_{ab} = T^m_ax_b - T^m_bx_a + \frac{1}{2}T^m_c[\bar{\theta}\gamma_{ab}\gamma^c\theta + \bar{\lambda}\gamma_{ab}\gamma^c\lambda] + iZ^m(\bar{\theta}\gamma_{ab}\lambda)$
$N^m_a$	$N^m_a = T^m_a\phi + iT^m_b(\bar{\theta}\gamma_a{}^b\lambda) - Z^m x_a$
$Q^m_i$	$Q^m(\xi) = \bar{\xi}_i Q^m_i(x) = +2iT^m_a(\bar{\xi}\gamma^a\theta) - 2Z^m(\bar{\xi}\lambda)$
$S^m_i$	$S^m(\eta) = \bar{\eta}_i S^m_i(x) = +2iT^m_a(\bar{\eta}\gamma^a\lambda) + 2Z^m(\bar{\eta}\theta)$
$R^m$	$R^m = -2T^m_a(\bar{\theta}\gamma^a\lambda) + iZ^m(\bar{\theta}\theta + \bar{\lambda}\lambda)$
$D^m$	$D^m = -T^m_ax^a + Z^m\phi$
$K^m_a$	$K^m_a = -T^m_b[2x^bx_a - x^2\delta^b_a + \phi^2\delta^b_a + (\bar{\theta}\theta - \bar{\lambda}\lambda)\delta^b_a - 2i\epsilon_a{}^{bc}\phi(\bar{\theta}\gamma_c\lambda) - \epsilon_a{}^{bc}x_c(\bar{\theta}\theta - \bar{\lambda}\lambda)] + Z^m[2x_a\phi - 2i\epsilon_a{}^{bc}x_b(\bar{\theta}\gamma_c\lambda)]$
$Y^m$	$Y^m = +2T^m_a[x^a\phi + i\epsilon^{abc}x_b(\bar{\theta}\gamma_c\lambda)] - Z^m[x^2 + \phi^2 - (\bar{\theta}\theta)(\bar{\lambda}\lambda)]$
$U^m_i$	$U^m_i = 2T^m_a[x_b(\gamma^b\gamma^a\theta)_i - i\phi(\gamma^a\theta)_i + i(\bar{\lambda}\lambda)(\gamma^a\theta)_i] + 2Z^m[i(\not{x}\lambda)_i - \phi\theta_i + (\bar{\theta}\theta)\lambda_i]$
$V^m_i$	$V^m_i = -2T^m_a[x_b(\gamma^b\gamma^a\lambda)_i + i\phi(\gamma^a\theta)_i + i(\bar{\theta}\theta)(\gamma^a\lambda)_i] + 2Z^m[i(\not{x}\theta)_i + \phi\lambda_i + (\bar{\lambda}\lambda)\theta_i]$

TABLE III. SUSY multiplets of Noether currents.

Noether current	SUSY transformation of Noether current
$J^m$	$\delta^{q,s}(\xi, \eta)J^m = \partial_n[J^m \Lambda^n(\xi, \eta) - J^n \Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n J^n + J^m(\xi, \eta)$
$T^m_a$	$\delta^{q,s}(\xi, \eta)T^m_a = \partial_n[T^m_a \Lambda^n(\xi, \eta) - T^n_a \Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n T^n_a$
$Z^m$	$\delta^{q,s}(\xi, \eta)Z^m = \partial_n[Z^m \Lambda^n(\xi, \eta) - Z^n \Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n Z^n$
$M^m_{ab}$	$\delta^{q,s}(\xi, \eta)M^m_{ab} = \partial_n[M^m_{ab} \Lambda^n(\xi, \eta) - M^n_{ab} \Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n M^m_{ab} - i(\bar{\xi}\gamma_{ab}Q^m) + i(\bar{\eta}\gamma_{ab}S^m)$
$N^m_a$	$\delta^{q,s}(\xi, \eta)N^m_a = \partial_n[N^m_a \Lambda^n(\xi, \eta) - N^n_a \Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n N^m_a - i(\bar{\xi}\gamma_a S^m) + i(\bar{\eta}\gamma_a Q^m)$
$Q^m_i$	$\delta^{q,s}(\xi, \eta)Q^m(\psi) = \partial_n[Q^m(\psi)\Lambda^n(\xi, \eta) - Q^n(\psi)\Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n Q^m(\psi) + 2iT^m_a(\bar{\psi}\gamma^a\xi) - 2Z^m(\bar{\psi}\eta)$
$S^m_i$	$\delta^{q,s}(\xi, \eta)S^m(\psi) = \partial_n[S^m(\psi)\Lambda^n(\xi, \eta) - S^n(\psi)\Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n S^m(\psi) + 2iT^m_a(\bar{\psi}\gamma^a\eta) + 2Z^m(\bar{\psi}\xi)$
$R^m$	$\delta^{q,s}(\xi, \eta)R^m = \partial_n[R^m \Lambda^n(\xi, \eta) - R^n \Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n R^n - iQ^m(\eta) + iS^m(\xi)$
$D^m$	$\delta^{q,s}(\xi, \eta)D^m = \partial_n[D^m \Lambda^n(\xi, \eta) - D^n \Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n D^n + \frac{1}{2}(\bar{\xi}Q^m) + \frac{1}{2}(\bar{\eta}S^m)$
$K^m_a$	$\delta^{q,s}(\xi, \eta)K^m_a = \partial_n[K^m_a \Lambda^n(\xi, \eta) - K^n_a \Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n K^m_a + i(\bar{\xi}\gamma^a U^m) - i(\bar{\eta}\gamma_a V^m)$
$Y^m$	$\delta^{q,s}(\xi, \eta)Y^m = \partial_n[Y^m \Lambda^n(\xi, \eta) - Y^n \Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n Y^n + (\bar{\eta}U^m) + (\bar{\xi}V^m)$
$U^m_i$	$\delta^{q,s}(\xi, \eta)U^m(\psi) = \partial_n[U^m(\psi)\Lambda^n(\xi, \eta) - U^n(\psi)\Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n U^m(\psi) + 2i(\bar{\eta}\gamma^a\psi)N^m_a - 3i(\bar{\eta}\psi)R^m + 2i(\bar{\xi}\gamma^a\psi)M^m_a - 2(\bar{\xi}\psi)D^m$
$V^m_i$	$\delta^{q,s}(\xi, \eta)V^m(\psi) = \partial_n[V^m(\psi)\Lambda^n(\xi, \eta) - V^n(\psi)\Lambda^m(\xi, \eta)] + \Lambda^m(\xi, \eta)\partial_n V^m(\psi) + 2i(\bar{\xi}\gamma^a\psi)N^m_a - 3i(\bar{\xi}\psi)R^m - 2i(\bar{\eta}\gamma^a\psi)M^m_a + 2(\bar{\eta}\psi)D^m$

rent is guaranteed. This, as well as the (non)conservation equations of the remaining super-Poincaré and centrally extended superconformal currents are listed in Table IV. The superconformal currents also form supersymmetry multiplets. Since the scale symmetries are explicitly broken, the superconformal current nonconservation terms are similarly related by the SUSY multiplet structure of the currents. It was further shown that all superconformal, as

well as super-Poincaré, currents are obtained as SUSY variations of the primary  $D = 3$  conformal central charge current and the primary  $D = 3$  special conformal current.

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TABLE IV. Current (non)conservation.

Noether current	Current (non)conservation
$J^m$	$\partial_m J^m = [\Delta \mathcal{L} + (\partial_m \delta x^m) \mathcal{L}] - w \Gamma$
$T^m_a$	$\partial_m T^m_a = w^p_a(x) \Gamma$
$Z^m$	$\partial_m Z^m = -w^Z(x) \Gamma$
$M^m_{ab}$	$\partial_m M^m_{ab}(x) = -w^M_{ab}(x) \Gamma$
$N^m_a$	$\partial_m N^m_a(x) = -w^N_a(x) \Gamma$
$Q^m_i$	$\partial_m Q^m_i = -w^Q_i(x) \Gamma$
$S^m_i$	$\partial_m S^m_i = -w^S_i(x) \Gamma$
$R^m$	$\partial_m R^m = -w^R(x) \Gamma$
$D^m$	$\partial_m D^m = -T^m_a \hat{e}^a_m + Z^m \hat{e}^a_m \hat{\nabla}_a \phi - w^D \Gamma = 3\sigma \frac{\partial}{\partial \sigma} \mathcal{L} - w^D \Gamma$
$K^m_a$	$\partial_m K^m_a = 6\epsilon^m x_m \mathcal{L} - T^m_a [2\epsilon^a \phi [\hat{e}^b_m \hat{\nabla}_b \phi] + 2(\epsilon_b x^a - \epsilon^a x_b) \hat{e}^b_m - \epsilon^a \partial_m ((\bar{\theta}\theta)(\bar{\lambda}\lambda)) + (\bar{\theta}\theta + \bar{\lambda}\lambda) \epsilon^a_{bc} \epsilon^c (\hat{e}^b_m - 2\delta^b_m) + 4(\bar{\theta}\gamma^a \lambda) \partial_m (\bar{\theta} \not{\epsilon} \lambda) - 2i\epsilon^{abc} \epsilon_c (\bar{\theta} \gamma_b \lambda) (\hat{e}^d_m \hat{\nabla}_d \phi)] + Z^m [2\epsilon_a \phi \hat{e}^a_m - 2i\epsilon_{mab} \epsilon^b (\bar{\theta} \gamma^a \lambda) - 2i(\bar{\theta}\theta - \bar{\lambda}\lambda) (\bar{\theta} \not{\epsilon} \partial_m \lambda)] - w^k_a \Gamma$
$Y^m$	$\partial_m Y^m = -6\phi \mathcal{L} + T^m_a [2x^a (\hat{e}^b_m \hat{\nabla}_b \phi) - 2i\epsilon^{mab} (\bar{\theta} \gamma_b \lambda) + 2i(\bar{\theta}\theta - \bar{\lambda}\lambda) (\bar{\theta} \gamma^a \partial_m \lambda)] - 2Z^m \hat{e}^a_m x_a - w^Y \Gamma$
$U^m_i$	$\partial_m U^m_i = [\Delta^u_i \mathcal{L} + (\partial_m \delta^u_i x^m) \mathcal{L}] - w^u_i \Gamma$ (see text)
$V^m_i$	$\partial_m V^m_i = [\Delta^v_i \mathcal{L} + (\partial_m \delta^v_i x^m) \mathcal{L}] - w^v_i \Gamma$ (see text)

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### APPENDIX A: CENTRALLY EXTENDED SUPERCONFORMAL ALGEBRA

The  $N = 1, D = 4$  space-time translation generator  $P^\mu$ ,  $\mu = 0, 1, 2, 3$ , consists of a  $D = 3$  Lorentz group vector,  $p^m = P^m$ , with  $m = 0, 1, 2$ , and a  $D = 3$  scalar central charge,  $Z \equiv P_3$ . Likewise, the Lorentz transformation charges  $M^{\mu\nu}$  consist of two  $D = 3$  vector representation charges:  $M^{mn} = \epsilon^{mnr} M_r$ , and  $N^m \equiv M^{m3}$ . The  $R$  charge is a singlet from both points of view. Finally the  $D = 4$  SUSY  $(\frac{1}{2}, 0)$  spinor  $Q_\alpha$  and the  $(0, \frac{1}{2})$  spinor  $\bar{Q}^{\dot{\alpha}}$  consist of two  $D = 3$  two-component Majorana spinors:  $q_i$  and  $s_i$ , with  $i = 1, 2$ . These are the charges that comprise the (centrally extended)  $N = 2, D = 3$  SUSY algebra. The spinor charges are given as linear combinations of  $Q_\alpha$  and  $\bar{Q}^{\dot{\alpha}}$  according to

$$\begin{pmatrix} q_i \\ s_i \end{pmatrix} = \frac{1}{2} e^{i(\pi/4)} \begin{pmatrix} \sigma & i\sigma\sigma_z \\ -i\sigma & -\sigma\sigma_z \end{pmatrix} \begin{pmatrix} Q_\alpha \\ \bar{Q}^{\dot{\alpha}} \end{pmatrix}, \quad (\text{A1})$$

where  $(\sigma_x, \sigma_y, \sigma_z)$  are the Pauli matrices and

$$\sigma \equiv \begin{pmatrix} 1 & -1 \\ -i & -i \end{pmatrix}. \quad (\text{A2})$$

The  $N = 1, D = 4$  super-Poincaré algebra can be written in terms of the  $D = 3$  Lorentz representation charges of the centrally extended  $N = 2, D = 3$  SUSY algebra as

$$\begin{aligned} [p^m, p^n] &= 0 & [M^m, M^n] &= -i\epsilon^{mnr} M_r \\ [p^m, Z] &= 0 & [M^m, N^n] &= -i\epsilon^{mnr} N_r \\ & & [N^m, N^n] &= +i\epsilon^{mnr} M_r \\ [M^m, p^n] &= -i\epsilon^{mnr} p_r & [N^m, p^n] &= +i\eta^{mn} Z \\ [M^m, Z] &= 0 & [N^m, Z] &= +ip^m \\ [M^{mn}, q_i] &= -\frac{1}{2}\gamma_{ij}^{mn} q_j & [N^m, q_i] &= +\frac{1}{2}\gamma_{ij}^m s_j \\ [M^{mn}, s_i] &= -\frac{1}{2}\gamma_{ij}^{mn} s_j & [N^m, s_i] &= -\frac{1}{2}\gamma_{ij}^m q_j \\ [R, q_i] &= +is_i & \{q_i, q_j\} &= +2(\gamma^m C)_{ij} p_m \\ [R, s_i] &= -iq_i & \{s_i, s_j\} &= +2(\gamma^m C)_{ij} p_m \\ & & \{q_i, s_j\} &= -2iC_{ij} Z. \end{aligned} \quad (\text{A3})$$

Note, the notation used in this paper is that of Ref. [2], in particular, the charge conjugation matrix and the  $2 + 1$  ( $D = 3$ ) dimensional gamma matrices in the appropriate associated representation are given there.

The  $N = 1, D = 4$  superconformal  $[SU(2, 2 | 1)]$  algebra [21,22] includes the additional charges: the dilatation charge  $D$ , a scalar from both points of view, the special conformal charge  $K^\mu$  which consists of a  $D = 3$  Lorentz group vector,  $k^m = K^m$ , with  $m = 0, 1, 2$ , and a  $D = 3$

scalar conformal central charge,  $Y \equiv K_3$ . The  $D = 4$  conformal SUSY  $(\frac{1}{2}, 0)$  spinor  $S_\alpha$  and the  $(0, \frac{1}{2})$  spinor  $\bar{S}^{\dot{\alpha}}$  consist of two  $D = 3$  two-component Majorana spinors:  $u_i$  and  $v_i$ , with  $i = 1, 2$ . These are the additional charges that comprise the centrally extended  $N = 2, D = 3$  superconformal algebra. The spinor charges are given as linear combinations of  $S_\alpha$  and  $\bar{S}^{\dot{\alpha}}$  according to

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} = \frac{1}{2} e^{i(\pi/4)} \begin{pmatrix} \sigma & i\sigma\sigma_z \\ -i\sigma & -\sigma\sigma_z \end{pmatrix} \begin{pmatrix} S_\alpha \\ \bar{S}^{\dot{\alpha}} \end{pmatrix}. \quad (\text{A4})$$

The remaining nonzero (anti)commutators for the  $N = 2, D = 3$  centrally extended superconformal algebra are

$$\begin{aligned} [M^m, k^n] &= -i\epsilon^{mnr} k_r & [N^m, k^n] &= +i\eta^{mn} Y \\ [M^m, Y] &= 0 & [N^m, Y] &= +ik^m \\ [M^{mn}, u_i] &= -\frac{1}{2}\gamma_{ij}^{mn} u_j & [N^m, u_i] &= +\frac{1}{2}\gamma_{ij}^m v_j \\ [M^{mn}, v_i] &= -\frac{1}{2}\gamma_{ij}^{mn} v_j & [N^m, v_i] &= -\frac{1}{2}\gamma_{ij}^m u_j \\ [R, u_i] &= -iv_i & \{u_i, u_j\} &= +2(\gamma^m C)_{ij} k_m \\ [R, v_i] &= +iv_i & \{v_i, v_j\} &= +2(\gamma^m C)_{ij} k_m \\ & & \{u_i, v_j\} &= -2iC_{ij} Y. \end{aligned}$$

$$\begin{aligned} \{q_i, u_j\} &= +2(\gamma^m C)_{ij} M_m + 2iC_{ij} D \\ \{q_i, v_j\} &= +2(\gamma^m C)_{ij} N_m + 3iC_{ij} R \\ \{s_i, u_j\} &= +2(\gamma^m C)_{ij} N_m + 3iC_{ij} R \\ \{s_i, v_j\} &= -2(\gamma^m C)_{ij} M_m - 2iC_{ij} D \end{aligned}$$

$$\begin{aligned} [k^m, q_i] &= \gamma_{ij}^m u_j & [p^m, u_i] &= \gamma_{ij}^m q_j \\ [k^m, s_i] &= -\gamma_{ij}^m v_j & [p^m, v_i] &= -\gamma_{ij}^m s_j \\ [Y, q_i] &= -iv_i & [Z, u_i] &= -is_i \\ [Y, s_i] &= -iv_i & [Z, v_i] &= -iq_i \\ [D, q_i] &= -\frac{i}{2} q_i & [D, u_i] &= +\frac{i}{2} u_i \\ [D, s_i] &= -\frac{i}{2} s_i & [D, v_i] &= +\frac{i}{2} v_i \end{aligned}$$

$$\begin{aligned} [D, p^m] &= -ip^m & [D, k^m] &= +ik^m \\ [D, Z] &= -iZ & [D, Y] &= +iY \end{aligned}$$

$$\begin{aligned} [p^m, k^n] &= 2i(\eta^{mn} D - \epsilon^{mnr} M_r) & [Z, Y] &= -2iD \\ [p^m, Y] &= +2iN^m & [k^m, Z] &= +2iN^m. \end{aligned} \quad (\text{A5})$$



## APPENDIX B: SUPERCONFORMAL TRANSFORMATIONS

The presence of the non-BPS brane in  $N = 1$ ,  $D = 4$  superspace spontaneously breaks the super-Poincaré group to the  $D = 3$  Poincaré symmetry and  $R$  symmetry groups. Commensurate with this is the appearance of the broken target space translation symmetry Nambu-Goldstone boson  $\phi$  and the broken SUSY Goldstino  $D = 3$  Majorana spinor fields  $\theta_i$  and  $\lambda_i$ ,  $i = 1, 2$ . Recalling the coset method of construction for the realization of this spontaneously broken super-Poincaré group on these fields from Ref. [2], the world-volume coordinate variation and the linearly represented total variations of the fields were found to be

$$\begin{aligned} x'^m &= x^m + a^m - i(\bar{\xi}\gamma^m\theta + \bar{\eta}\gamma^m\lambda) - \phi b^m + \epsilon^{mnr}\alpha_n x_r, \\ \phi' &= \phi + z + (\xi\gamma^0\lambda - \theta\gamma^0\eta) - b^m x_m, \\ \theta'_i &= \theta_i + \xi_i - \frac{i}{2}b_m(\gamma^m\lambda)_i - i\rho\lambda_i - \frac{i}{2}\alpha_m(\gamma^m\theta)_i, \\ \lambda'_i &= \lambda_i + \eta_i + \frac{i}{2}b_m(\gamma^m\theta)_i + i\rho\theta_i - \frac{i}{2}\alpha_m(\gamma^m\lambda)_i. \end{aligned} \quad (\text{B1})$$

Thus, applying the relation between total and intrinsic variations of a field,  $\delta\varphi = \Delta\varphi - \delta x^m\partial_m\varphi$ , the nonlinear realization of the  $N = 2$ ,  $D = 3$  super-Poincaré algebra, given in Eq. (A3), on the  $\phi$ ,  $\theta$  and  $\lambda$  fields is

$$\begin{aligned} \delta^P(a)\phi &= -a^m\partial_m\phi & \delta^Z(z)\phi &= z \\ \delta^P(a)\theta &= -a^m\partial_m\theta & \delta^Z(z)\theta &= 0 \\ \delta^P(a)\lambda &= -a^m\partial_m\lambda & \delta^Z(z)\lambda &= 0 \\ \\ \delta^q(\xi)\phi &= -\bar{\xi}\lambda + i\bar{\xi}\gamma^m\theta\partial_m\phi & \delta^s(\eta)\phi &= +\bar{\eta}\theta + i\bar{\eta}\gamma^m\lambda\partial_m\phi \\ \delta^q(\xi)\theta_i &= \xi_i + i\bar{\xi}\gamma^m\theta\partial_m\theta_i & \delta^s(\eta)\theta_i &= +i\bar{\eta}\gamma^m\lambda\partial_m\theta_i \\ \delta^q(\xi)\lambda_i &= +i\bar{\xi}\gamma^m\theta\partial_m\lambda_i & \delta^s(\eta)\lambda_i &= \eta_i + i\bar{\eta}\gamma^m\lambda\partial_m\lambda_i \\ \\ \delta^M(\alpha)\phi &= -\epsilon^{mnr}\alpha_n x_r\partial_m\phi & \delta^N(b)\phi &= -b^m x_m + \phi b^m\partial_m\phi \\ \delta^M(\alpha)\theta_i &= -\frac{i}{2}\alpha_m(\gamma^m\theta)_i - \epsilon^{mnr}\alpha_n x_r\partial_m\theta_i & \delta^N(b)\theta_i &= +\frac{i}{2}b_m(\gamma^m\lambda)_i + \phi b^m\partial_m\theta_i \\ \delta^M(\alpha)\lambda_i &= -\frac{i}{2}\alpha_m(\gamma^m\lambda)_i - \epsilon^{mnr}\alpha_n x_r\partial_m\lambda_i & \delta^N(b)\lambda_i &= -\frac{i}{2}b_m(\gamma^m\theta)_i + \phi b^m\partial_m\lambda_i \\ \\ \delta^R(\rho)\phi &= 0 \\ \delta^R(\rho)\theta_i &= -i\rho\lambda_i \\ \delta^R(\rho)\lambda_i &= +i\rho\theta_i. \end{aligned} \quad (\text{B2})$$

The brane tension explicitly breaks the superconformal symmetries at low energies (not to mention the hard superconformal symmetry breaking by radiative corrections at all scales). Hence, no new Nambu-Goldstone fields are expected to arise from the spontaneously broken component of these hard broken superconformal symmetries. Yet, in addition to the explicit hard breaking, the  $Y$ ,  $u_i$  and  $v_i$  conformal symmetries have a spontaneously broken component as required by the superconformal algebra. Consider the ground state,  $|0\rangle$ , expectation value of the Jacobi identity involving  $p^m$ ,  $Y$  and  $\partial^n\phi$  along with the fact that the brane is  $D = 3$  translation invariant,  $p^m|0\rangle = 0$ , and the superconformal algebra contains the commutator  $[p^m, Y] = 2iN^m$  while the  $D = 3$  translations are represented by world-volume derivatives,  $[p^m, \partial^n\phi] = i\partial^m\partial^n\phi$ , to obtain

$$\langle 0|[Y, \partial^m\partial^n\phi]|0\rangle = -2\langle 0|[N^m, \partial^n\phi]|0\rangle. \quad (\text{B3})$$

On the other hand, the Jacobi identity involving  $p^n$ ,  $N^m$  and  $\phi$  along with the algebraic relation  $[N^m, p^n] = i\eta^{mn}Z$  implies that

$$\langle 0|[N^m, i\partial_n\phi]|0\rangle = -i\delta_n^m\langle 0|[Z, \phi]|0\rangle = \delta_n^m \neq 0, \quad (\text{B4})$$

the right-hand side being nonzero since  $\phi$  is the  $Z$  symmetry Nambu-Goldstone boson which transforms into a constant. Hence, the  $N^m$  symmetry has a spontaneously broken component with  $\partial^m\phi$  acting as the corresponding Nambu-Goldstone boson. Further, from Eq. (B3), it is found that the  $Y$  symmetry is spontaneously broken with  $\partial^2\phi$  acting as the corresponding Nambu-Goldstone boson. Likewise, the conformal SUSY symmetries are spontaneously broken with derivatives of  $\theta$  and  $\lambda$  acting as their Goldstino modes. Even though the conformal central charge symmetry  $Y$ , the broken Lorentz symmetry generator  $N^m$  as well as the conformal SUSY charges  $u$  and  $v$  all have a spontaneously broken symmetry component, no new Nambu-Goldstone modes are required to realize these spontaneously broken symmetry transformations.

Thus the remaining superconformal transformations are to be realized on the  $\phi$ ,  $\theta$  and  $\lambda$  Nambu-Goldstone fields. These transformations can be most easily found by using the coset construction with a considerable simplification.

Consider the coset element  $\Omega \in G/H$  with  $G = \text{SU}(2, 2 | 1)$ , the superconformal group, and  $H$  generated by the set of unbroken charges  $\{M^{mn}, R, D, k^m\}$ .  $\Omega$  can be written as the product  $\Omega = \Omega_o \hat{\Omega}$  where

$$\Omega_o = e^{ix^m P_m} e^{i(\phi Z + \bar{\theta} q + \bar{\lambda} s)} = e^{ix^\mu P_\mu} e^{i(\theta Q + \bar{\theta} \bar{Q})} \quad (\text{B5})$$

and

$$\hat{\Omega} = e^{iv^m N_m} e^{i(fY + \bar{\chi}u + \bar{\psi}v)}. \quad (\text{B6})$$

$g \in G$  acts on  $\Omega$  to yield  $g\Omega = \Omega'_o \hat{h} \hat{\Omega} = \Omega'_o \hat{\Omega}' h$  with  $h \in H$  while  $\hat{h}$  is generated by  $\{M^{mn}, N^m, R, D, k^m, Y, u, v\}$ . The simplification in finding the superconformal transformations of  $\phi$ ,  $\theta$  and  $\lambda$  comes from the observation

$$\hat{h} \hat{\Omega} = \hat{\Omega}' h; \quad (\text{B7})$$

that is  $\hat{h}$  acting on  $\hat{\Omega}$  does not feedback to a change in  $\Omega'_o$ . Hence, as far as the superconformal transformations of  $\phi$ ,  $\theta$  and  $\lambda$  are concerned, only the action of  $g\Omega_o$  needs to be considered. With  $g$  a superconformal transformation generated by  $\{D, K^\mu, S^\alpha, \bar{S}_{\dot{\alpha}}\}$  the transformation has the form

$$g\Omega_o = e^{ix'^\mu P_\mu} e^{i(\theta' Q + \bar{\theta}' \bar{Q})} \hat{h}. \quad (\text{B8})$$

This is just the usual linear representation of a superconformal motion in  $N = 1, D = 4$  superspace. For the transformations

$$g = e^{i(\epsilon D + \epsilon^\mu K_\mu + \eta S + \bar{\eta} \bar{S})} \quad (\text{B9})$$

the superspace motion is given by [16,17]

$$\begin{aligned} x'^\mu &= (1 + \epsilon)x^\mu + 2\epsilon^\rho x_\rho x^\mu - x^2 \epsilon^\mu + \theta^2 \bar{\theta}^2 \epsilon^\mu - ix^\mu \eta \theta \\ &\quad - x_\rho (\theta \sigma^{\mu\rho} \eta) - \theta^2 (\eta \sigma^\mu \bar{\theta}) + ix^\mu \bar{\eta} \bar{\theta} + x_\rho (\bar{\eta} \bar{\sigma}^{\rho\mu} \bar{\theta}) \\ &\quad + \bar{\theta}^2 (\bar{\eta} \bar{\sigma}^\mu \theta), \\ \theta'^\beta &= (1 + \frac{1}{2}\epsilon)\theta^\beta + \epsilon^\rho x_\rho \theta^\beta - i\epsilon^\mu x^\rho (\theta \sigma_{\mu\rho})^\beta \\ &\quad + 2i(\theta \not{\epsilon} \bar{\theta})\theta^\beta - 2i\theta^2 \eta^\beta - (\not{\epsilon} \bar{\eta})^\beta + 2i(\bar{\eta} \bar{\theta})\theta^\beta, \\ \bar{\theta}'^{\dot{\beta}} &= (1 + \frac{1}{2}\epsilon)\bar{\theta}^{\dot{\beta}} + \epsilon^\rho x_\rho \bar{\theta}^{\dot{\beta}} - i\epsilon^\mu x^\rho (\bar{\theta} \bar{\sigma}_{\mu\rho})^{\dot{\beta}} \\ &\quad - 2i(\theta \not{\epsilon} \bar{\theta})\bar{\theta}^{\dot{\beta}} + 2i\bar{\theta}^2 \bar{\eta}^{\dot{\beta}} - (\eta \not{\epsilon})^{\dot{\beta}} - 2i(\eta \theta)\bar{\theta}^{\dot{\beta}}. \end{aligned} \quad (\text{B10})$$

Recalling that  $x^3 = \phi$ ,  $P_3 = Z$ ,  $K_3 = Y$ , etc., while the  $N = 1, D = 4$  superspace Grassmann coordinates  $\theta^\alpha$  and  $\bar{\theta}_{\dot{\alpha}}$  are related to the  $N = 2, D = 3$  Goldstinos  $\theta_i$  and  $\lambda_i$  according to

$$\begin{pmatrix} \theta^\alpha \\ \bar{\theta}_{\dot{\alpha}} \end{pmatrix} = \frac{1}{2} e^{i(\pi/4)} \begin{bmatrix} \sigma_z \sigma^T (\theta - i\lambda) \\ i\sigma^T (\theta + i\lambda) \end{bmatrix}, \quad (\text{B11})$$

and the conformal SUSY charges obey the inverse relation to Eq. (A4)

$$\begin{pmatrix} S_\alpha \\ \bar{S}^{\dot{\alpha}} \end{pmatrix} = \frac{1}{2} e^{i(\pi/4)} \begin{bmatrix} -i\sigma^T \sigma_z (u + iv) \\ -i\sigma^T \sigma_x (u - iv) \end{bmatrix}, \quad (\text{B12})$$

the various  $N = 2, D = 3$  superconformal variations of the

Nambu-Goldstone fields are determined. Isolating the dilatations with parameter  $\epsilon$ , the world-volume coordinate transformation and total variation of the fields are found

$$\begin{aligned} \delta^D(\epsilon)x^m &= \epsilon x^m, & \Delta^D(\epsilon)\phi &= \epsilon\phi, \\ \Delta^D(\epsilon)\theta_i &= \frac{1}{2}\epsilon\theta_i, & \Delta^D(\epsilon)\lambda_i &= \frac{1}{2}\epsilon\lambda_i. \end{aligned} \quad (\text{B13})$$

In general the (Lie) intrinsic variation of a field  $\varphi$  is related to the total variation of the field according to  $\delta\varphi = \Delta\varphi - \delta x^m \partial_m \varphi$ . Hence the intrinsic dilatation transformation has the linear representation

$$\begin{aligned} \delta^D(\epsilon)\phi &= \epsilon(1 - x^m \partial_m)\phi, \\ \delta^D(\epsilon)\theta_i &= \epsilon(\frac{1}{2} - x^m \partial_m)\theta_i, \\ \delta^D(\epsilon)\lambda_i &= \epsilon(\frac{1}{2} - x^m \partial_m)\lambda_i. \end{aligned} \quad (\text{B14})$$

The special conformal transformations with generators  $k^m$  and conformal central charge transformations with charge  $Y$  are obtained from Eq. (B10) by means of the identification  $\epsilon^\mu = (\epsilon^m, y)$  and  $\epsilon^\mu K_\mu = \epsilon^m k_m + yY$

$$\begin{aligned} \delta^Y(y)x^m &= -2y\phi x^m, \\ \Delta^Y(y)\phi &= -y[x^2 + \phi^2 + (\bar{\theta}\theta)(\bar{\lambda}\lambda)], \\ \Delta^Y(y)\theta &= y[i(\not{x}\lambda) - \phi\theta - (\bar{\theta}\theta)\lambda], \\ \Delta^Y(y)\lambda &= y[-i(\not{x}\theta) - \phi\lambda + (\bar{\lambda}\lambda)\theta], \\ \delta^k(\epsilon)x^m &= 2\epsilon^n x_n x^m - x^2 \epsilon^m + \phi^2 \epsilon^m - (\bar{\theta}\theta)(\bar{\lambda}\lambda)\epsilon^m, \\ \Delta^k(\epsilon)\phi &= 2\epsilon^n x_n \phi, \\ \Delta^k(\epsilon)\theta &= \epsilon^n x_n \theta - 2i(\bar{\theta}\not{\epsilon}\lambda)\lambda - i\phi(\not{\epsilon}\lambda) \\ &\quad - i\epsilon^{mnr} \epsilon_m x_n (\gamma_r \theta), \\ \Delta^k(\epsilon)\lambda &= \epsilon^n x_n \lambda + 2i(\bar{\theta}\not{\epsilon}\lambda)\theta + i\phi(\not{\epsilon}\theta) \\ &\quad - i\epsilon^{mnr} \epsilon_m x_n (\gamma_r \lambda). \end{aligned} \quad (\text{B15})$$

Finally the conformal SUSY transformations with charges  $u_i$  and  $v_i$  are determined to be, using explicit indices,

$$\begin{aligned} \delta_i^u x^m &= -x^m \theta_i + i(\bar{\lambda}\lambda)(\gamma^m \theta)_i + i\phi(\gamma^m \lambda)_i \\ &\quad + i\epsilon^{mnr} x_n (\gamma_r \theta)_i, \\ \Delta_i^u \phi &= +i(\not{x}\lambda)_i - \phi\theta_i - (\bar{\theta}\theta)\lambda_i, \\ \Delta_i^u \theta_j &= +i(\not{x}C)_{ij} + \frac{1}{2}(\bar{\theta}\theta)C_{ij} - \frac{3}{2}(\bar{\lambda}\lambda)C_{ij}, \\ \Delta_i^u \lambda_j &= +\phi C_{ij} + 2(\bar{\theta}\lambda)C_{ij} - [\theta_i \lambda_j - \lambda_i \theta_j], \\ \delta_i^v x^m &= +x^m \lambda_i - i(\bar{\theta}\theta)(\gamma^m \lambda)_i + i\phi(\gamma^m \theta)_i \\ &\quad - i\epsilon^{mnr} x_n (\gamma_r \lambda)_i, \\ \Delta_i^v \phi &= +i(\not{x}\theta)_i + \phi\lambda_i - (\bar{\lambda}\lambda)\theta_i, \\ \Delta_i^v \theta_j &= +\phi C_{ij} - 2(\bar{\theta}\lambda)C_{ij} - [\theta_i \lambda_j - \lambda_i \theta_j], \\ \Delta_i^v \lambda_j &= -i(\not{x}C)_{ij} + \frac{3}{2}(\bar{\theta}\theta)C_{ij} - \frac{1}{2}(\bar{\lambda}\lambda)C_{ij}. \end{aligned} \quad (\text{B16})$$

As usual, the intrinsic variations can be found according to  $\delta\varphi = \Delta\varphi - \delta x^m \partial_m \varphi$  for each field  $\varphi$ . Note that  $Y$ ,  $k^m$ ,  $u_i$  and  $v_i$  variations are all nonlinearly realized, however the special conformal symmetry generated by  $k^m$  is not

spontaneously broken while the  $Y$ ,  $u_i$  and  $v_i$  generated symmetries are spontaneously broken due to the field independent but world-volume coordinate dependent terms in their transformation equations as discussed above.

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